Absolute Stability Criteria of Singularly Perturbed Lur'e Systems with Time-Varying Delays

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***Abstract*—This paper is concerned with the absolute stability of a class of singularly perturbed Lur'e systems with an interval time-varying delay and sector-bounded nonlinearity. By constructing a Lyapunov-Krasovskii functional (LKF), the absolute stability criteria are proposed in terms of linear matrix inequalities (LMIs). One numerical example is presented to show the effectiveness of the result.**

***Index Terms*—singularly perturbed Lur'e systems, absolute stability, time-varying delay, sector-bounded nonlinearity.**

I. INTRODUCTION

In many engineering systems, state variables of two different time scales exist in the meantime, which leads to the curse of dimensionality, computational complexity and stiffness[1]. One common method to deal with the two-time-scale characteristic is modeling these systems as singularly perturbed forms, which are called singularly perturbed systems (SPSs) [2], [3]. The original singularly perturbed systems can be analyzed and controlled based on the decomposition of the original system into slow and fast subsystems, such that the computational complexities for the analysis and design of singularly perturbed systems are reduced. The wind energy conversion systems are modeled as singularly perturbed systems and robust control algorithms are proposed in [4]. Singular perturbation method is applied to analyze the fully-constrained parallel cable robots with elastic cables in [5] and a composite controller consisting of two main components is developed. Singular perturbation formulation of quad rotor unmanned aerial vehicles is developed in [6] and a dual-loop control system scheme has been proposed in the presence of wind disturbances. During the past several decades singularly perturbed systems have been a popular research topic due to its comprehensive applications in electrical and electronic systems, economic models, flexible robot systems, unmanned aerial vehicles, etc. [7], [8].

On another research front line, many nonlinear physical systems can be represented as Lur'e systems which consist of a feedback connection of a linear dynamical system and a nonlinearity satisfying the sector condition [9], [10], [11]. The absolute stability of Lur'e systems has been widely studied. Reference [12] deals with a robust stability problem for uncertain Lur'e systems with time-varying delays and sector-bounded nonlinearities. An improved delay-dependent robust stability criterion is proposed via a modified Lyapunov-Krasovskii functional (LKF) approach. And, a state feedback sampled-data control for continuous-time Lur'e systems is studied preserving global asymptotic stability and minimizing a guaranteed quadratic cost in [13]. In the last few years, synchronization of Lur'e systems has drawn much attention. Finite-time H_∞ synchronization of semi-Markov jump Lur'e systems [14], chaos synchronization of fractional-order Lur'e systems [15], cluster synchronization of heterogeneous Lure networks [16] and synchronization conditions for chaotic fractional-order Lur'e systems [17] are all studied in succession. In addition, the control problems of Markovian Lur'e systems have obtained progress to a certain extent [14], [18], [19], [20], [21], and more related work need to be done.

If the Lur'e systems have two time scale property, then these systems can be modeled as Lur'e Singularly Perturbed Systems (Lur'e SPSs), such as an inverted pendulum controlled by a DC motor via a gear [9]. In [8] a general class of Lur'e SPSs whose nonlinear terms depend on both the fast and slow dynamics are studied, and absolute stability criteria for the slow and fast subsystems are proposed. Reference [9] investigates the absolute stability problem for Lur'e SPSs with multiple nonlinearities, and a stability criterion expressed in terms of LMIs is derived. Saksena in [22] considers the absolute stability of single-input-single-output Lur'e SPSs whose nonlinear terms only depend on the slow dynamics. Saksena [22] proves that the original system is absolutely stable for all sufficiently small singular perturbation parameter if the reduced-order slow subsystem is absolutely stable. Wang [23] investigates the integral sliding mode control problem for Lur'e SPSs with sector-constrained nonlinearities. In [24] the absolute stability and feedback control problems of Lur'e singularly perturbed uncertain systems are investigated.

As known to all, time-delays are commonly encountered in the control loops and are often attributed as a source of poor performance and instability of systems [25], [26], [27]. Therefore, this paper is concerned with the absolute stability of Lur'e SPSs with time delay. This research topic has drawn some attention, but is not studied extensively yet. In [28] absolutely stable and passive performance problem for Lur'e SPSs with time-delay based on state feedback

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control is considered. Liu [29] considers Lur'e SPSs with time-delay, and designs a robust observer-based feedback control law such that the resulting closed-loop system is absolutely stable.

In the existing references [28], [29], the time delays are assumed to be constants. However, in practice, the time delay probably changes with time or some varying elements. Motivated by the above consideration, we assume that the time delay of the Lur'e SPSs is a function of time in this paper, and the absolute stability criteria are derived by constructing a novel time-varying delay dependent Lyapunov-Krasovskii functional. One numerical example is carried out to verify the effectiveness of the results obtained. Compared to the existing results, the main contributions of this paper are outlined as below:

The time-varying delay is put into consideration, for the first time, of the absolute stability problem of Lur'e SPSs, and this gives rise to a more complicated analysis process to obtain the absolute stability conditions in LMIs form. We have proposed theoretical conditions to guarantee the absolute stability based on Lyapunov stability theory and intricate mathematical derivation.

II. Problem Statement

Consider the following singularly perturbed Lur'e systems with time-varying delays and sector bounded nonlinearity as below:

$$\begin{cases} \dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) + B_{11}x_1(t-h(t)) \\ \qquad +B_{12}x_2(t-h(t)) + C_1 f(\delta(t)) \\ \epsilon\dot{x}_2(t) = A_{21}x_1(t) + A_{22}x_2(t) + B_{21}x_1(t-h(t)) \\ \qquad +B_{22}x_2(t-h(t)) + C_2 f(\delta(t)) \\ \delta(t) = H^T x(t) = [h_1 \quad \ldots \quad h_l]^T x(t), \ \forall t \geq 0 \end{cases} \tag{1}$$

where $x_1(t) \in \mathbb{R}^n$ and $x_2(t) \in \mathbb{R}^m$ are the slow and fast variables respectively, $x(t) = [x_1^T(t) \ x_2^T(t)]^T \in \mathbb{R}^{n+m}$, $\delta(t) \in \mathbb{R}^l$ is the output, $0 < \epsilon \ll 1$ is the singular perturbation parameter. The time-varying delay $h(t)$ is a continuous-time function which is assumed to satisfy the following conditions

$$0 \leq h(t) \leq h, \quad \dot{h}(t) \leq h_d \tag{2}$$

and $x_1(s) = \varphi_1(s)$, $\dot{x}_1(s) = \dot{\varphi}_1(s)$, $x_2(s) = \varphi_2(s)$, $\dot{x}_2(s) = \dot{\varphi}_2(s)$, $s \in [-h, 0]$, where $\varphi_1(s) \in \mathbb{R}^n$ and $\varphi_2(s) \in \mathbb{R}^m$ are continuous initial functions specified on $[-h, 0]$. $f(\delta(t))$ is the nonlinear function in the feedback path, which is given by $f(\delta(t)) = [f_1(\delta_1(t)) \ \ldots \ f_l(\delta_l(t))]^T$, wherein, each term $f_i(\delta_i(t)), (i = 1, \ \ldots, \ l)$ satisfies the finite sector condition:

$$\begin{aligned} f_i(\delta_i(t)) \in K_{[0,k_i]} = \{&f_i(\delta_i(t)) | f_i(0) = 0, \\ &0 < \delta_i(t) f_i(\delta_i(t)) \leq k_i \delta_i(t)^2, \delta_i(t) \neq 0\} \end{aligned} \tag{3}$$

or the the infinite sector condition:

$$\begin{aligned} f_i(\delta_i(t)) \in K_{[0,k_i]} = \{&f_i(\delta_i(t)) | f_i(0) = 0, \\ &\delta_i(t) f_i(\delta_i(t)) \geq 0, \delta_i(t) \neq 0\} \end{aligned} \tag{4}$$

This paper investigates the delay-dependent stability of the system (1) satisfying the time-varying delay conditions (2) and sector conditions (3). To achieve this purpose, the system (1) is rearranged into the following form:

$$E_\epsilon \dot{x}(t) = Ax(t) + Bx(t-h(t)) + Cf(\delta(t)) \tag{5}$$

where $E_\epsilon = \begin{bmatrix} I_n & 0 \\ 0 & \epsilon I_m \end{bmatrix}$, $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$, $C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$.

And following lemmas are required in deriving the stability criteria:

Lemma 2.1: [30] For any symmetric positive definite matrix $W \in \mathbb{R}^{n \times n}$, a scalar $\gamma > 0$, and vector function $\dot{x}$: $[-\gamma, 0] \to \mathbb{R}^n$ such that the integration $\int_{t-\gamma}^{t} \dot{x}^T(s) W \dot{x}(s) ds$ is well defined, then the following inequality holds:

$$-\gamma \int_{t-\gamma}^{t} \dot{x}^T(s) W \dot{x}(s) ds \leq \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}^T \begin{bmatrix} -W & W \\ * & -W \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}$$

Lemma 2.2: [31] Suppose $r_1 \leq r(t) \leq r_2$, where $r(.) : \mathbb{R}_+ \to \mathbb{R}_+$. Then, for any $R = R^T > 0$, following integral inequality holds:

$$\begin{aligned} -\int_{t-r_2}^{t-r_1} \dot{x}^T(s) R \dot{x}(s) ds \leq \delta^T(t)[&(r_2 - r(t)) T R^{-1} T^T \\ &+(r(t) - r_1) Y R^{-1} Y^T + [Y \ \ -Y + T \ \ -T] \\ &+[Y \ \ -Y + T \ \ -T]^T]\delta(t) \end{aligned} \tag{6}$$

where $\delta(t) = [x^T(t - r_1) \ x^T(t - r(t)) \ x^T(t - r_2)]^T$, $T = [T_1^T \ T_2^T \ T_3^T]^T$, and $Y = [Y_1^T \ Y_2^T \ Y_3^T]^T$ are free matrices of appropriate dimension.

Lemma 2.3: [32] Suppose $r_1 \leq r(t) \leq r_2$, where $r(.) : \mathbb{R}_+ \to \mathbb{R}_+$. Then for any constant matrices Ξ_1, Ξ_2, and Ξ with proper dimensions, the following matrix inequality

$$\Xi + (\gamma(t) - \gamma_1)\Xi_1 + (\gamma_2 - \gamma(t))\Xi_2 < 0$$

holds, if and only if

$$\begin{aligned} \Xi + (\gamma_2 - \gamma_1)\Xi_1 < 0 \\ \Xi + (\gamma_2 - \gamma_1)\Xi_2 < 0 \end{aligned}$$

Schur complement: Given constant symmetric matrices Σ_1, Σ_2 and Σ_3 where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, the $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$, if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, or \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0$$

III. Main Result

Now the main result of the work, i.e., absolute stability criteria for the system (1) satisfying the time-varying delay conditions (2) and sector conditions (3) are presented in the form of a theorem as below.

Theorem 3.1: The system (1) satisfying the time-varying delay conditions (2) and sector conditions (3) is absolutely stable, if there exist real symmetric positive definite matrices P, Q, Z_1 and Z_2; matrices Q_{11}, Q_{12} and Q_{22} such that $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0$; positive semidefinite matrices $R = diag(r_1, r_2, \ldots, r_m)$ and $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_l)$; slack matrices T_i, Y_i, M_i, N_i, $i = 1, 2, 3$ of appropriate dimensions such that the following LMIs are satisfied:

$$\begin{bmatrix} \Pi + \Pi' + \Pi_1 + \Pi_1^T & \frac{h}{2} T_a \\ * & -\frac{h}{2} Z_1 \end{bmatrix} < 0 \quad (7)$$

$$\begin{bmatrix} \Pi + \Pi' + \Pi_1 + \Pi_1^T & \frac{h}{2} Y_a \\ * & -\frac{h}{2} Z_1 \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} \Pi + \Pi'' + \Pi_2 + \Pi_2^T & \frac{h}{2} M_a \\ * & -\frac{h}{2} Z_2 \end{bmatrix} < 0 \quad (9)$$

$$\begin{bmatrix} \Pi + \Pi'' + \Pi_2 + \Pi_2^T & \frac{h}{2} N_a \\ * & -\frac{h}{2} Z_2 \end{bmatrix} < 0 \quad (10)$$

where

$$\Pi = \begin{bmatrix} PA + A^T P + Q + L_1 & Q_{12} & PB + L_2 & 0 & PC + L_3 \\ * & Q_{22} - Q_{11} & 0 & -Q_{12} & 0 \\ * & * & -(1-h_d)Q & 0 & L_4 \\ * & * & * & -Q_{22} & 0 \\ * & * & * & * & L_5 + L_5^T + L_6 \end{bmatrix}$$

$$\begin{aligned} L_1 &= A^T E_\epsilon^{-1} [\frac{h}{2}(Z_1 + Z_2)] E_\epsilon^{-1} A, \\ L_2 &= A^T E_\epsilon^{-1} [\frac{h}{2}(Z_1 + Z_2)] E_\epsilon^{-1} B, \\ L_3 &= A^T E_\epsilon^{-1} H \Lambda^T + A^T E_\epsilon^{-1} [\frac{h}{2}(Z_1 + Z_2)] E_\epsilon^{-1} C \\ &\quad + HKR, \\ L_4 &= B^T E_\epsilon^{-1} H \Lambda^T + B^T E_\epsilon^{-1} [\frac{h}{2}(Z_1 + Z_2)] E_\epsilon^{-1} C, \\ L_5 &= \Lambda H^T E_\epsilon^{-1} C, \\ L_6 &= C^T E_\epsilon^{-1} [\frac{h}{2}(Z_1 + Z_2)] E_\epsilon^{-1} C - 2R, \\ \Pi' &= (e_2 - e_4)^T (-\frac{2}{h} Z_2)(e_2 - e_4), \\ \Pi_1 &= [Y_a \quad -Y_a + T_a \quad -T_a \quad 0 \quad 0], \\ Y_a &= [Y_1^T \quad Y_2^T \quad Y_3^T \quad 0 \quad 0]^T, \\ T_a &= [T_1^T \quad T_2^T \quad T_3^T \quad 0 \quad 0]^T, \\ \Pi'' &= (e_1 - e_2)^T (-\frac{2}{h} Z_1)(e_1 - e_2), \\ \Pi_2 &= [0 \quad -N_a + M_a \quad N_a \quad -M_a \quad 0], \\ M_a &= [0 \quad M_1^T \quad M_2^T \quad M_3^T \quad 0]^T, \\ N_a &= [0 \quad N_1^T \quad N_2^T \quad N_3^T \quad 0]^T \end{aligned}$$

e_i, $i = 1, \ldots, 5$ are block entry matrices of appropriate dimension, i.e., $e_1 = [I \ 0 \ 0 \ 0 \ 0]$, $e_2 = [0 \ I \ 0 \ 0 \ 0]$, and so on, with $e_5 = [0 \ 0 \ 0 \ 0 \ I]$.

Proof: In order to simplify the symbols, denote $x_t \triangleq x(t)$ in the proof. Construct a Lyapunov-Krasovskii functional as below

$$V(x_t) = \sum_{i=1}^{4} V_i(x_t) \quad (11)$$

where

$$\begin{aligned} V_1(x_t) &= x^T(t) E_\epsilon P x(t), \\ V_2(x_t) &= 2 \sum_{i=1}^{l} \lambda_i \int_0^{\delta_i(t)} f_i(\delta) d\delta, \\ V_3(x_t) &= \int_{t-h(t)}^{t} x^T(s) Q x(s) ds \\ &+ \int_{t-\frac{h}{2}}^{t} \begin{bmatrix} x(s) \\ x(s - \frac{h}{2}) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s - \frac{h}{2}) \end{bmatrix} ds \\ V_4(x_t) &= \int_{-\frac{h}{2}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z_1 \dot{x}(s) ds d\theta \\ &+ \int_{-h}^{-\frac{h}{2}} \int_{t+\theta}^{t} \dot{x}^T(s) Z_2 \dot{x}(s) ds d\theta \end{aligned}$$

Take the derivative of $V(x_t)$ with the respect to t along the trajectory of system (1) , we obtain

$$\begin{aligned} \dot{V}_1(x_t) &= 2x^T(t) P E_\epsilon \dot{x}(t) \\ &= 2x^T(t) P \Big\{ Ax(t) + Bx(t - h(t)) + Cf(\delta(t)) \Big\} \\ \dot{V}_2(x_t) &= 2 \sum_{i=1}^{l} \lambda_i \dot{\delta}_i(t) f_i(\delta_i(t)) \\ &= 2 \sum_{i=1}^{l} \lambda_i f_i(\delta_i(t)) h_i^T \dot{x}(t) \\ &= 2 f^T(\delta(t)) \Lambda H^T \dot{x}(t) \\ &= 2 f^T(\delta(t)) \Lambda H^T E_\epsilon^{-1} \Big\{ Ax(t) + Bx(t - h(t)) \\ &\quad + Cf(\delta(t)) \Big\} \\ \dot{V}_3(x_t) &\le x^T(t) Q x(t) - (1 - h_d) x^T(t - h(t)) Q \\ &\quad \times x(t - h(t)) + \eta^T(t) \Phi \eta(t) \\ \dot{V}_4(x_t) &= \dot{x}^T(t) \Big[\frac{h}{2}(Z_1 + Z_2) \Big] \dot{x}(t) \\ &\quad - \int_{t-\frac{h}{2}}^{t} \dot{x}^T(s) Z_1 \dot{x}(s) ds \\ &\quad - \int_{t-h}^{t-\frac{h}{2}} \dot{x}^T(s) Z_2 \dot{x}(s) ds \end{aligned}$$

where $\eta(t) = [x^T(t) \ x^T(t - \frac{h}{2}) \ x^T(t - h)]^T$,

$$\Phi = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ * & Q_{22} - Q_{11} & -Q_{12} \\ * & * & -Q_{22} \end{bmatrix} \quad (12)$$

Now for systems satisfying the sector condition (3), for any $r_i \ge 0$, $i = 1, 2, \ldots, l$, it yields

$$r_i f_i(\delta_i(t))(k_i h_i^T x(t) - f_i(\delta_i(t))) \ge 0, \ i = 1, 2, \ldots, l \quad (13)$$

which is equivalent to

$$x^T(t) HKR f(\delta(t)) - f^T(\delta(t)) R f(\delta(t)) \ge 0 \quad (14)$$

Using the positive quantity (14), $\dot{V}(x_t)$ is bounded as follows

$$\begin{aligned} \dot{V}(x_t) &\le \sum_{i=1}^{4} \dot{V}_i(x_t) \\ &\quad + 2\big(x^T(t) HKR f(\delta(t)) - f^T(\delta(t)) R f(\delta(t)) \big) \end{aligned}$$

Define an augmented state vector $\xi(t) = [x^T(t)\ x^T(t-\frac{h}{2})\ x^T(t-h(t))\ x^T(t-h)\ f^T(\delta(t))]^T$, then the above inequality can be reformed into

$$\dot{V}(x_t) \le \xi^T(t)\Pi\xi(t) - \int_{t-\frac{h}{2}}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-h}^{t-\frac{h}{2}} \dot{x}^T(s)Z_2\dot{x}(s)ds \quad (15)$$

Next, according to Lemma 1 and Lemma 2, we will deal with the integral terms $-\int_{t-\frac{h}{2}}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds$ and $-\int_{t-h}^{t-\frac{h}{2}} \dot{x}^T(s)Z_2\dot{x}(s)ds$ differently in the delay intervals $h(t) \in [0, \frac{h}{2}]$ and $h(t) \in [\frac{h}{2}, h]$.

When $0 \le h(t) \le \frac{h}{2}$, we have

$$\begin{aligned} & -\int_{t-\frac{h}{2}}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds \\ \le\ & \zeta_1^T(t)\Big[\big(\frac{h}{2}-h(t)\big)TZ_1^{-1}T^T \\ & +h(t)YZ_1^{-1}Y^T + [Y\ \ -Y+T\ \ -T] \\ & +[Y\ -Y+T\ -T]^T\Big]\zeta_1(t) \end{aligned} \quad (16)$$

$$\begin{aligned} & -\int_{t-h}^{t-\frac{h}{2}} \dot{x}^T(s)Z_2\dot{x}(s)ds \\ \le\ & \begin{bmatrix} x(t-\frac{h}{2}) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -\frac{2}{h}Z_2 & \frac{2}{h}Z_2 \\ * & -\frac{2}{h}Z_2 \end{bmatrix} \\ & \times \begin{bmatrix} x(t-\frac{h}{2}) \\ x(t-h) \end{bmatrix} \end{aligned} \quad (17)$$

Similarly, when $\frac{h}{2} \le h(t) \le h$, we have

$$\begin{aligned} & -\int_{t-\frac{h}{2}}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds \\ \le\ & \begin{bmatrix} x(t) \\ x(t-\frac{h}{2}) \end{bmatrix}^T \begin{bmatrix} -\frac{2}{h}Z_1 & \frac{2}{h}Z_1 \\ * & -\frac{2}{h}Z_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\frac{h}{2}) \end{bmatrix} \end{aligned} \quad (18)$$

$$\begin{aligned} & -\int_{t-h}^{t-\frac{h}{2}} \dot{x}^T(s)Z_2\dot{x}(s)ds \\ \le\ & \zeta_2^T(t)\Big[(h-h(t))MZ_2^{-1}M^T \\ & +\big(h(t)-\frac{h}{2}\big)NZ_2^{-1}N^T \\ & +[-N+M\ \ N\ \ -M] \\ & +[-N+M\ \ N\ \ -M]^T\Big]\zeta_2(t) \end{aligned} \quad (19)$$

where

$$\begin{aligned} \zeta_1(t) &= \big[x^T(t)\ x^T(t-\frac{h}{2})\ x^T(t-h(t))\big]^T, \\ \zeta_2(t) &= \big[x^T(t-\frac{h}{2})\ x^T(t-h(t))\ x^T(t-h)\big]^T, \\ Y &= \big[Y_1^T\ Y_2^T\ Y_3^T\big]^T, \quad T = \big[T_1^T\ T_2^T\ T_3^T\big]^T, \\ M &= \big[M_1^T\ M_2^T\ M_3^T\big]^T, \quad N = \big[N_1^T\ N_2^T\ N_3^T\big]^T. \end{aligned}$$

For $0 \le h(t) \le \frac{h}{2}$, substituting inequalities (16) and (17) into (15) leads to

$$\begin{aligned} \dot{V}(x_t) \le\ & \xi^T(t)\Big[\Pi + \Pi' + \Pi_1 + \Pi_1^T \\ & +\big(\frac{h}{2}-h(t)\big)T_aZ_1^{-1}T_a^T + h(t)Y_aZ_1^{-1}Y_a^T\Big]\xi(t) \end{aligned}$$

Based on Lemma 3 and Schur complement, by combining Eqs. (7) and (8), we arrive at (20)

$$\Big[\Pi + \Pi' + \Pi_1 + \Pi_1^T + \big(\frac{h}{2}-h(t)\big)T_aZ_1^{-1}T_a^T + h(t)Y_aZ_1^{-1}Y_a^T\Big] < 0 \quad (20)$$

It can be easily seen that the inequality (20) can guarantee that $\dot{V}(x_t) < 0$ for $0 \le h(t) \le \frac{h}{2}$.

In the similar way, for $\frac{h}{2} \le h(t) \le h$, substituting inequality (18) and inequality (19) into (15) leads to

$$\begin{aligned} \dot{V}(x_t) \le\ & \xi^T(t)\Big[\Pi + \Pi'' + \Pi_2 + \Pi_2^T \\ & +\big(h-h(t)\big)M_aZ_2^{-1}M_a^T \\ & +\big(h(t)-\frac{h}{2}\big)N_aZ_2^{-1}N_a^T\Big]\xi(t) \end{aligned} \quad (21)$$

And inequalities (9) and (10) are sufficient conditions for inequality (22) which can guarantee that $\dot{V}(x_t) < 0$ for $\frac{h}{2} \le h(t) \le h$.

$$\Big[\Pi + \Pi'' + \Pi_2 + \Pi_2^T + \big(h-h(t)\big)M_aZ_2^{-1}M_a^T + \big(h(t)-\frac{h}{2}\big)N_aZ_2^{-1}N_a^T\Big] < 0 \quad (22)$$

This completes the proof. ■

Corollary 3.1: If the delay items in the system (1) are nonexistent, i.e., the $h = 0$, then the absolute stability conditions in the LMI forms are simplified as below:

$$\begin{bmatrix} PA + A^TP & PC + A^TE_\epsilon^{-1}H\Lambda \\ * & \Lambda H^TE_\epsilon^{-1}C \end{bmatrix} < 0 \quad (23)$$

Proof: Define the Lyapunov-Krasovskii functional $V(t) = V_1(t) + V_2(t)$, which are the same as $V_1(t)$, $V_2(t)$ in the theorem 1, and the corollary result can be obtained easily. ■

If the system satisfies the infinite sector condition (4), for any $r_i \ge 0$, $i = 1, 2, \ldots, l$, it yields

$$r_if_i(\delta_i(t))h_i^Tx(t) \ge 0, \ i = 1, 2, \ldots, l \quad (24)$$

which is equivalent to

$$x^T(t)HRf(\delta(t)) \ge 0 \quad (25)$$

Using this condition in the stability analysis, we can obtain the following corollary:

Corollary 3.2: The system (1) satisfying the time-varying delay conditions (2) and sector conditions (4) is absolutely stable, if there exist real symmetric positive definite matrices P, Q, Z_1 and Z_2; matrices Q_{11}, Q_{12} and Q_{22} such that $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0$; positive semidefinite matrices $R = diag(r_1, r_2, \ldots, r_m)$ and $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_l)$; slack matrices T_i, Y_i, M_i, N_i, $i = 1, 2, 3$ of appropriate dimensions such that the following LMIs (7-10) hold with L_3 replaced with $\bar{L}_3$ and L_6 replaced with $\bar{L}_6$, where

$$\begin{aligned} \bar{L}_3 &= A^TE_\epsilon^{-1}H\Lambda^T + A^TE_\epsilon^{-1}[\frac{h}{2}(Z_1+Z_2)]E_\epsilon^{-1}C + H, \\ \bar{L}_6 &= C^TE_\epsilon^{-1}[\frac{h}{2}(Z_1+Z_2)]E_\epsilon^{-1}C. \end{aligned}$$

The proof is omitted here since it is similar to the proof of Theorem 1.

IV. Numerical Example

In this section, one numerical example is adopted to verify the results obtained.

Example 4.1: In the light of system (1), the following Lur'e SPSs is considered:

$$\begin{cases} \dot{x}_1(t) = -2.0x_1(t) + 0.5x_2(t) + x_1(t-h(t)) \\ \qquad + 0.4x_2(t-h(t)) + C_1 f(\delta) \\ \epsilon \dot{x}_2(t) = -x_2(t) + 0.4x_1(t-h(t)) - x_2(t-h(t)) \\ \qquad + C_2 f(\delta) \end{cases} \tag{26}$$

where $\epsilon = 0.01$, $C_1 = [0.5\ 0]$, $C_1 = [0\ 0.5]$, $\delta = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

In order to test the theorem 1, we convert system (26) into the form of (5), where

$$E_\epsilon = \begin{bmatrix} 1 & 0 \\ 0 & \epsilon \end{bmatrix}, A = \begin{bmatrix} 2 & 0.5 \\ 0 & -1 \end{bmatrix},$$
$$B = \begin{bmatrix} -2.9 & 0.4 \\ 0.4 & -1.8 \end{bmatrix}, C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix},$$
$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$f(\delta) = \begin{bmatrix} 0.5\sin(x_1(t)) + 0.5x_1(t) \\ 0.5\sin(x_2(t)) + 0.5x_2(t) \end{bmatrix},$$

and it can be seen that $f(\delta(t))$ satisfies the finite sector conditions (3) with $k_1 = k_2 = 1$. In the first place, it is assumed that $h = h_d = 1$, and by using the LMI Toolbox of MATLAB, we obtain the following solution to the LMIs of Theorem 1:

$$\begin{aligned}
P &= \begin{bmatrix} 8.7594 & -0.5659 \\ -0.5659 & 13.5072 \end{bmatrix}, \\
Q &= \begin{bmatrix} 11.2720 & -0.2564 \\ -0.2564 & 3.7845 \end{bmatrix}, \\
Z_1 &= \begin{bmatrix} 5.4890 & 0.0013 \\ 0.0013 & 0.0011 \end{bmatrix}, \\
Z_2 &= \begin{bmatrix} 5.2023 & 0.0040 \\ 0.0040 & 0.0017 \end{bmatrix}, \\
Q_{11} &= \begin{bmatrix} 15.7417 & -0.0300 \\ -0.0300 & 19.7131 \end{bmatrix}, \\
Q_{12} &= \begin{bmatrix} -5.7868 & -0.2174 \\ -0.2174 & 0.3017 \end{bmatrix}, \\
Q_{22} &= \begin{bmatrix} 16.0719 & 0.0607 \\ 0.0607 & 10.3597 \end{bmatrix}, \\
R &= \begin{bmatrix} 10.4148 & 0 \\ 0 & 12.3882 \end{bmatrix}, \\
\Lambda &= \begin{bmatrix} 3.6829 & 0 \\ 0 & 0.0561 \end{bmatrix}, \\
T_1 &= \begin{bmatrix} -0.4978 & 0.0010 \\ 0.0010 & 0.0086 \end{bmatrix}, \\
T_2 &= \begin{bmatrix} -0.9590 & 0.0134 \\ 0.0134 & -0.0080 \end{bmatrix}, \\
T_3 &= \begin{bmatrix} -0.3462 & 0.0024 \\ 0.0024 & 0.0168 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
Y_1 &= \begin{bmatrix} -5.2013 & -0.6411 \\ -0.6411 & -4.0359 \end{bmatrix}, \\
Y_2 &= \begin{bmatrix} 0.9549 & -0.2829 \\ -0.2829 & -4.8027 \end{bmatrix}, \\
Y_3 &= \begin{bmatrix} 1.0500 & -1.3153 \\ -1.3153 & 1.4609 \end{bmatrix}, \\
M_1 &= \begin{bmatrix} 5.7868 & 0.2174 \\ 0.2174 & -0.3017 \end{bmatrix}, \\
M_2 &= \begin{bmatrix} -10.9980 & 5.0965 \\ 5.0965 & -23.0400 \end{bmatrix}, \\
M_3 &= \begin{bmatrix} -1.5790 & -0.0303 \\ -0.0303 & 1.2771 \end{bmatrix}, \\
N_1 &= \begin{bmatrix} 7.8280 & 0.3851 \\ 0.3851 & 1.0137 \end{bmatrix}, \\
N_2 &= \begin{bmatrix} -19.2575 & 5.2568 \\ 5.2568 & -24.6811 \end{bmatrix}, \\
N_3 &= \begin{bmatrix} -1.5790 & -0.0303 \\ -0.0303 & 1.2771 \end{bmatrix}.
\end{aligned}$$

Figure 1 shows the states responses for the system (26) which demonstrate the asymptotic stability of this system. And the stabilized results are coincident with computation outcomes from the LMI conditions which illustrate that the theorem 1 is effective to verify the absolute stability for the system (1).

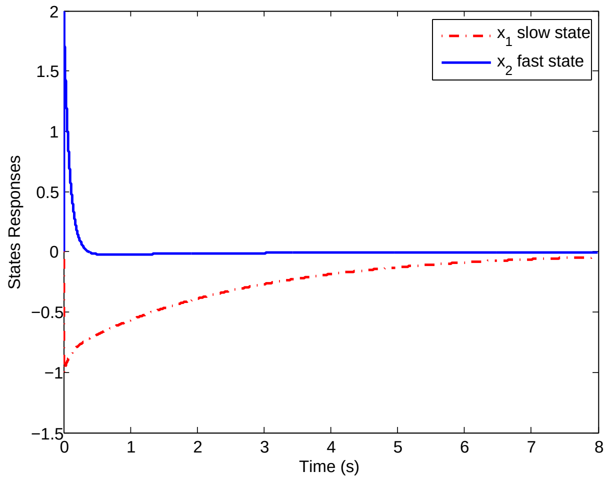

Fig. 1. The states responses of system

V. Conclusions

In this paper, the absolute stability of a class of singularly perturbed Lur'e systems with an interval time-varying delays and sector-bounded nonlinearity is considered. A Lyapunov-Krasovskii functional (LKF) is constructed to derive the absolute stability criteria in terms of LMIs. In the end, the effectiveness of the theoretical result is verified using the simulation of one numerical example.

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