Real-time Adaptive Level Control of a Multivariable Waste Water Treatment Plant

Jackson B. Renteria-Mena, Eduardo Giraldo

Abstract—**In this work, a real-time multivariable adaptive liquid level control is applied over a multivariable waste-water treatment plant, where multiple inputs and multiple level outputs are considered to control the water level at each tank. The system is identified in real-time by using an adaptive least-squares approach. The multivariable system is also controlled using a pole placement multivariable polynomial approach. It is worth noting that the controller is designed based on the identified model. To this end, the identified multivariable model parameters are used in the controller design in real-time. The system is evaluated in simulation and by using a real-time prototype by considering an equivalent electric circuit. A comparison to a decoupled PID controller is performed where the pole placement adaptive multivariable controller outperforms the classical PID controller. As a result, a new multivariable adaptive pole placement controller is obtained as an alternative technique for level control in waste treatment systems. In addition, it is worth mentioning that the proposed approach can be generalized to large order treatment plants.**

Index Terms—**Real-time, multivariable control, identification, waste-water treatment plant.**

I. Introduction

THE design and development of real-scale multivariable prototypes is a valuable method to evaluate the performance of controllers. Therefore, the multivariable controllers can be evaluated over a real environment by using an embedded controller. The prototype design allows an additional comparison with the simulated results and evaluates other implementation issues, such as noise corrupted measurements [1]. In [2] polynomial deterministic and stochastic adaptive controllers are applied over multivariable systems. In [3], an adaptive approach applied over simulated systems is also considered for the identification and control of multivariable systems in state-space.

In [1] a ball and beam plate prototype is designed and developed to evaluate the performance of several multivariable controllers. In [4] a multivariable unmanned aerial vehicle of two degrees of freedom is also designed and developed to evaluate intelligent controllers. In [5], a simulation using operational amplifiers of a third-order state-space model is implemented to evaluate the real-time environment and implementation issues.

In [6] a regulation strategy for the water level of clean water tank is designed, where optimization of the operation of parallel water-intake pump group and the minimization of electricity consumption is performed according to the required flow of water supply. In [7], an adaptive control strategy is applied over a dual tank system in order to evaluate decoupling strategies. In [8] a supervisory level control strategy is proposed for a waste-water treatment facility, and in [9] a tuning PID strategy is proposed for a waste-water treatment plant. In addition, several algorithms have been proposed for liquid level control of multivariable coupled tanks. For example, in [10] a double-tank liquid level control is performed by using generic algorithms, and in [11] a predictive controller over a three-tank coupled system is proposed. However, the methods mentioned above require detailed system modeling and must be adjusted for each case.

This work applies a real-time multivariable adaptive level control over a waste-water treatment plant. The system is identified in real-time by using an adaptive least-squares approach. The multivariable system is also controlled using an adaptive pole placement multivariable polynomial controller. To this end, the identified multivariable model parameters are used in the controller design in real-time. The system is evaluated in simulation and by using a real-time prototype by considering an equivalent electric circuit. A proposed adaptive pole placement approach is compared by considering a decoupled adaptive PID controller. This paper is organized as follows: in section II is presented the mathematical model of a waste-water treatment plant for level control and also is presented the adaptive identification and pole placement control method. In section III is given the evaluation of the controllers over a real model by using an electric circuit equivalent prototype for the identification stage and over a simulated multivariable model for the adaptive control stage. Finally, in section IV the conclusions and final remarks are presented.

II. Waste treatment plant for level control

A. Mathematical model

In Fig. 1 is depicted the five order interconnected tank facility for waste-water treatment, where the variable to be controlled is the liquid level of each tank by using flow inputs.

The system of Fig. 1 can be described in discrete-time as a polynomial difference equation, as follows:

$$\begin{aligned} y[k] = &-A_1 y[k-1] - A_2 y[k-2] - A_3 y[k-3] \\ &- A_4 y[k-4] - A_5 y[k-5] \\ &+ B_1 u[k-1] + B_2 u[k-2] + B_3 u[k-3] \\ &+ B_4 u[k-4] + B_5 u[k-5] \end{aligned} \quad (1)$$

being $y[k] \in \mathbb{R}^{5\times 1}$ the level outputs at each tank at sample k, and being $u[k] \in \mathbb{R}^{5\times 1}$ the flow inputs at each tank, and $A_i \in \mathbb{R}^{5\times 5}$ and $B_i \in \mathbb{R}^{5\times 5}$, the polynomial matrices with $i = 1, 2, 3, 4, 5$.

Manuscript received November 30, 2021; revised April 5, 2022.

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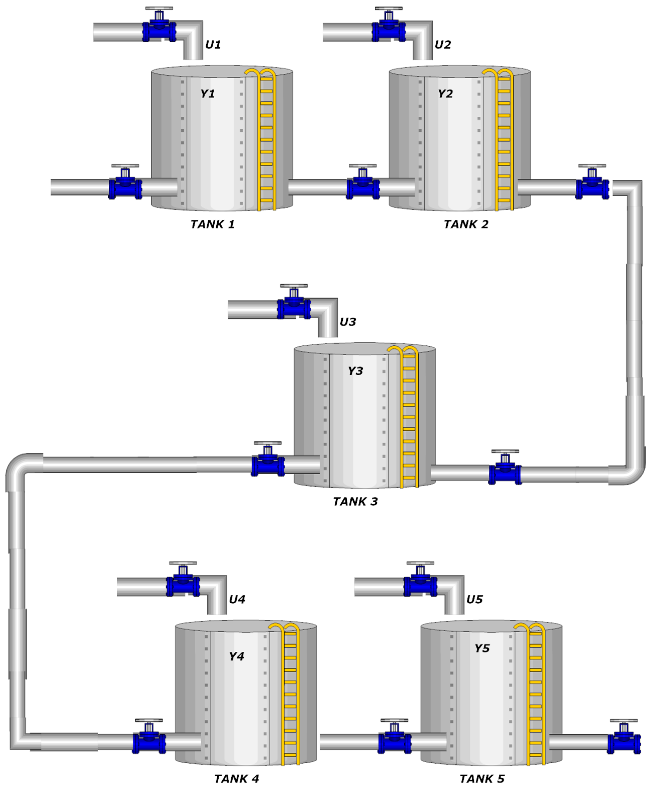

Fig. 1. Waste-water treatment plant with five interconnected tanks

The model can be identified by using an adaptive least-squares multivariable method as described in [2], where the (1) can be rewritten as follows:

$$y[k] = \theta^T \phi[k-1] \tag{2}$$

being θ^T the transpose of $\theta \in \mathbb{R}^{25\times 5}$, as follows:

$$\boldsymbol{\theta}^T = \begin{bmatrix} -A_1 & \cdots & -A_5 & B_1 & \cdots & B_5 \end{bmatrix} \tag{3}$$

and being $\phi[k-1] \in \mathbb{R}^{25\times 5}$ that holds the past inputs and outputs as follows:

$$\phi[k-1] = \begin{bmatrix} y[k-1] \\ \vdots \\ y[k-5] \\ u[k-1] \\ \vdots \\ u[k-5] \end{bmatrix} \tag{4}$$

As described in [2] the multivariable least squares algorithm can be defined as:

$$\begin{aligned} e[k] &= \mathbf{y}[k]^T - \phi[k-1]^T \hat{\theta}[k-1] \\ M[k] &= \frac{P[k-1]}{1+\phi[k-1]^T P[k-1]\phi[k-1]} \\ \hat{\theta}[k] &= \hat{\theta}[k-1] + M[k]\phi[k-1]e[k] \end{aligned} \tag{5}$$

and

$$P[k] = P[k-1] - \frac{P[k-1]\phi[k-1]\phi[k-1]^T P[k-1]}{1+\phi[k-1]^T P[k-1]\phi[k-1]} \tag{6}$$

with initial estimate $\hat{\theta}[0]$ given and $P[0]$ a positive diagonal matrix.

A simplified projection algorithm can also be used, where the estimation equation is defined by

$$\begin{aligned} e[k] &= \mathbf{y}[k]^T - \phi[k-1]^T \hat{\theta}[k-1] \\ M[k] &= \frac{1}{\phi[k-1]^T \phi[k-1]} \\ \hat{\theta}[k] &= \hat{\theta}[k-1] + M[k]\phi[k-1]e[k] \end{aligned} \tag{7}$$

B. Adaptive pole placement control

According to [2], a pole placement (PP) multivariable polynomial controller can be defined as follows:

$$E(z) = K_g R(z) - Y(z) \tag{8}$$

$$U(z) = C(z)E(z) \tag{9}$$

being the controller $C(z)$ defined as

$$C(z) = P^{-1}(z)L(z) \tag{10}$$

and K_g computed as

$$K_g = (B(1)L(1))^{-1}(A(1)P(1) + B(1)L(1)) \tag{11}$$

where the closed-loop characteristic equation is defined by

$$P_{CL}(z) = A(z)P(z) + B(z)L(z) \tag{12}$$

and therefore compared to a desired closed-loop polynomial as follows:

$$P_d(z) = P_{LC}(z) \tag{13}$$

III. Results

In order to evaluate the proposed adaptive PP approach, a comparison analysis with an adaptive decoupled PID controller is performed. The closed-loop design is developed by considering dead-beat behavior. The proposed approach is evaluated over an electric circuit by considering their dynamic equivalence to a waste treatment plant's liquid level control in tanks. In Fig. 2 a detailed schematic circuit of the implemented five order circuit prototype is depicted. The electric circuit is controlled by using an Arduino DUE.

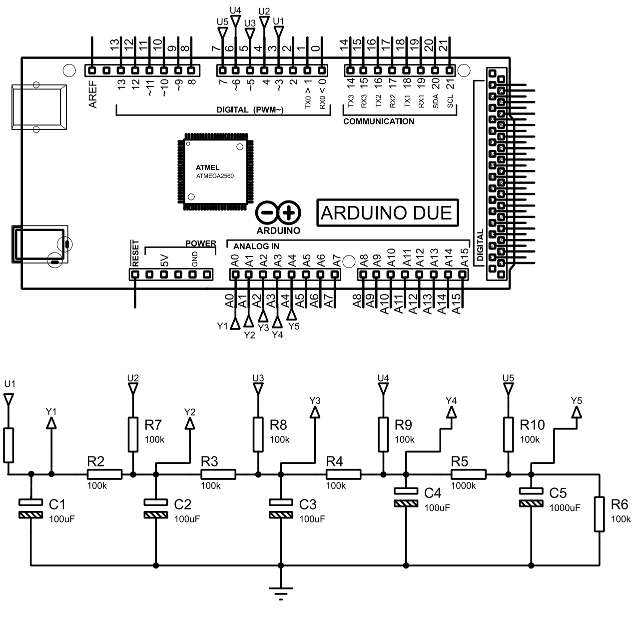

Fig. 2. Equivalent circuit of a waste-water treatment facility with five interconnected tanks

It is worth noting that the Arduino DUE is a microcontroller with twelve 8-bit PWM outputs, twelve 12-bit analog inputs, 512Kb flash memory, and an 84MHz clock. These features make the Arduino DUE a capable microcontroller for real-time adaptive multivariable control.

In addition, the estimation capability of the proposed approach is evaluated by a comparison analysis between the least-squares and the projection algorithms in an open loop.

In Fig. 3 is presented the estimation result by using the projection algorithm (y_1 P) and the least-squares algorithm (y_1 LS) for the level of tank 1. It can be seen that the output data are adequately approximated.

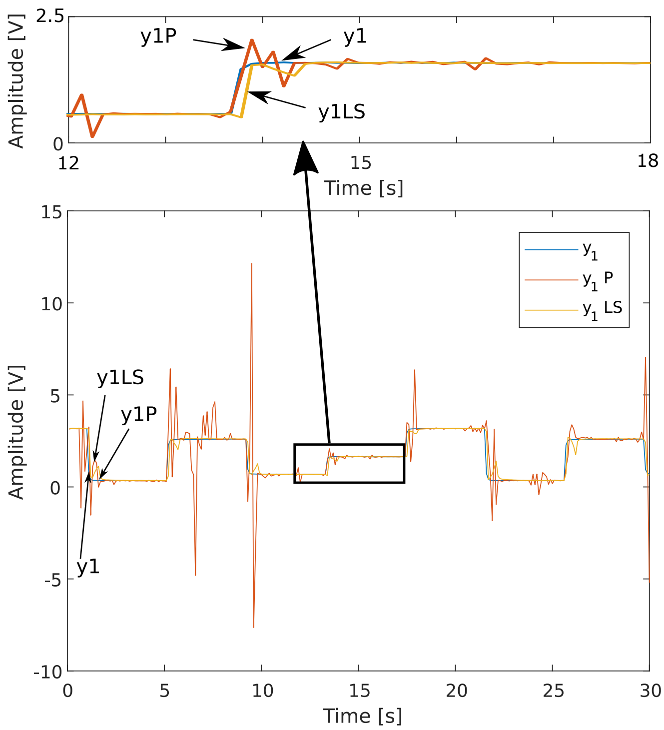

Fig. 3. Estimated output $y_1[k]$ and $\hat{y}_1[k]$ by using projection algorithm (y_1 P) and least-squares algorithm (y_1 LS)

A detailed analysis of the estimation of Fig. 3 is presented in Fig. 4.

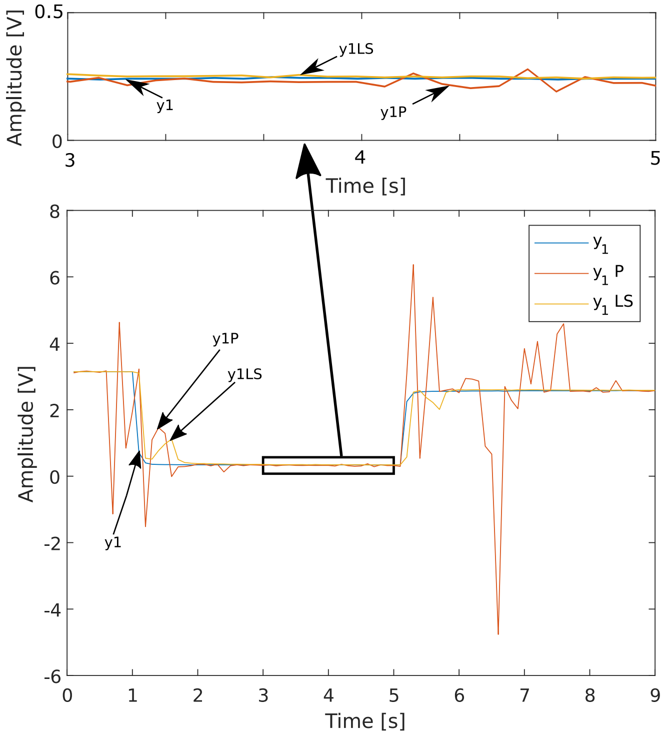

Fig. 4. Detailed analysis of estimated output $y_1[k]$ and $\hat{y}_1[k]$ by using projection algorithm (y_1 P) and least squares algorithm (y_1 LS)

In Fig. 5 is presented the estimation result by using projection algorithm (y_2 P) and least-squares algorithm (y_2 LS) for the level of the tank 2. It can be seen that the output data are adequately approximated.

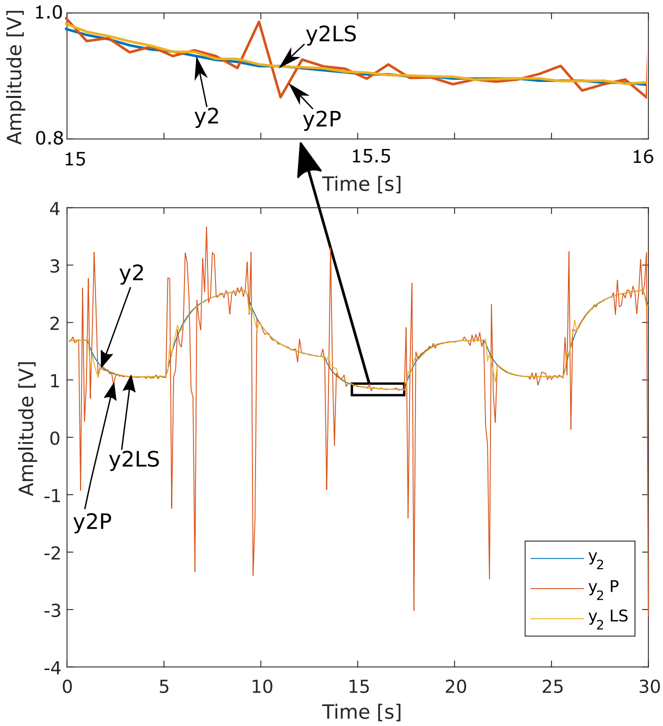

Fig. 5. Estimated output $y_2[k]$ and $\hat{y}_2[k]$ by using projection algorithm (y_2 P) and least squares algorithm (y_2 LS)

A detailed analysis of the estimation of Fig. 5 is presented in Fig. 6.

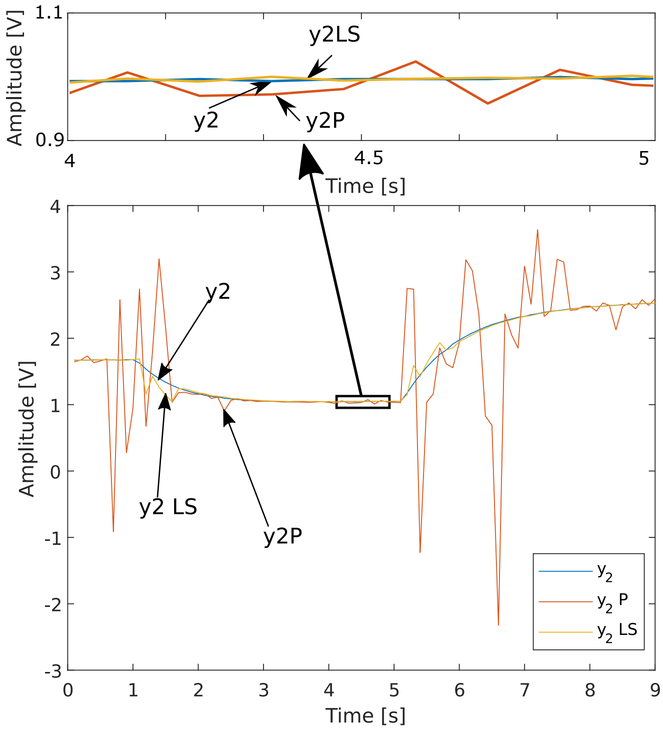

Fig. 6. Detailed analysis of estimated output $y_2[k]$ and $\hat{y}_2[k]$ by using projection algorithm (y_2 P) and least squares algorithm (y_2 LS)

In Fig. 7 is presented the estimation result by using the projection algorithm (y_3 P) and least-squares algorithm (y_3 LS) for the level of tank 3. It can be seen that the output data are adequately approximated.

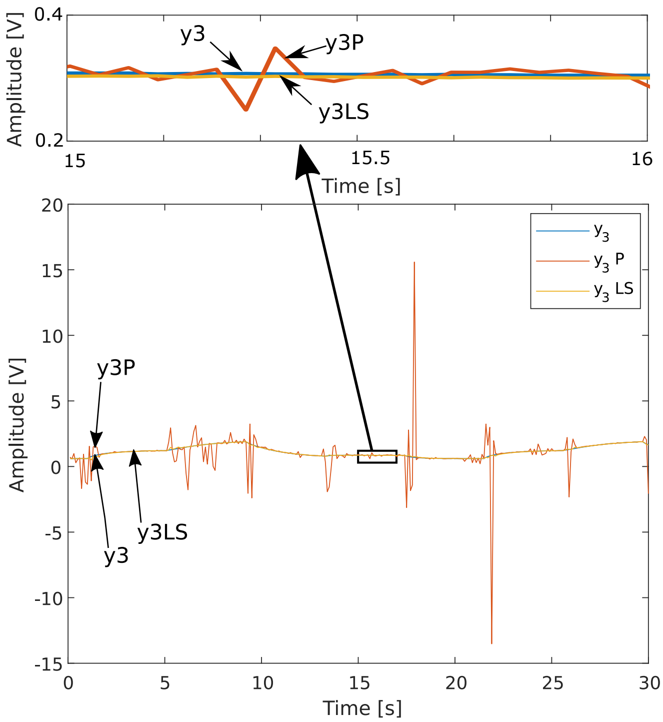

Fig. 7. Estimated output $y_3[k]$ and $\hat{y}_3[k]$ by using projection algorithm (y_3 P) and least squares algorithm (y_3 LS)

A detailed analysis of the estimation of Fig. 7 is presented in Fig. 8.

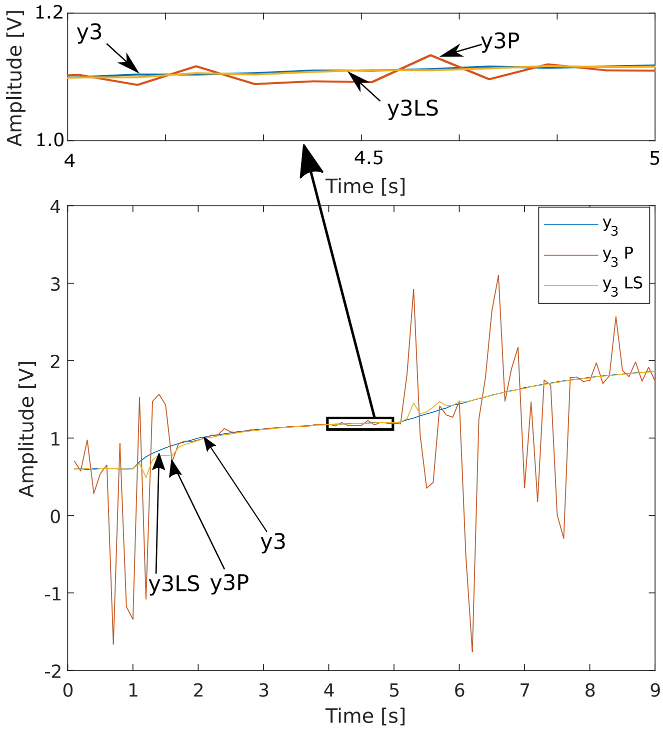

Fig. 8. Detailed analysis of estimated output $y_3[k]$ and $\hat{y}_3[k]$ by using projection algorithm (y_3 P) and least squares algorithm (y_3 LS)

In Fig. 9 is presented the estimation result by using the projection algorithm (y_4 P) and least-squares algorithm (y_4 LS) for the level of tank 4. It can be seen that the output data are adequately approximated.

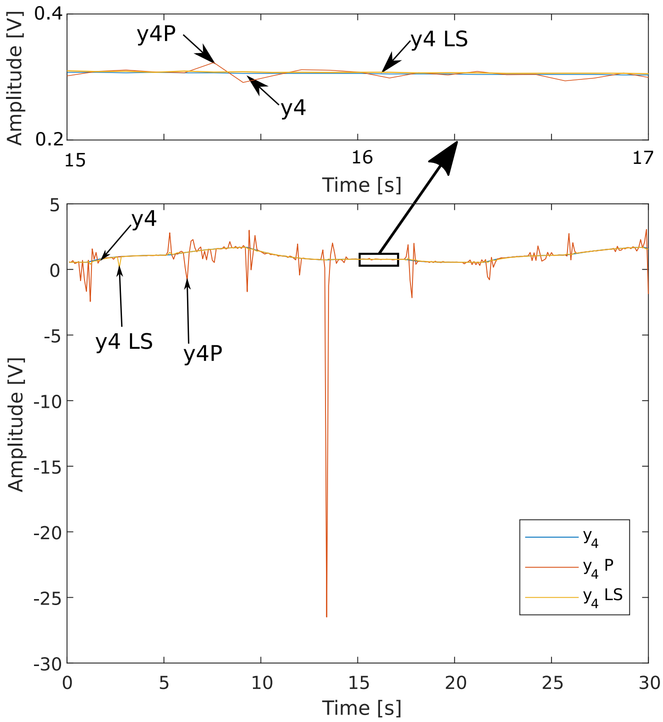

Fig. 9. Estimated output $y_4[k]$ and $\hat{y}_4[k]$ by using projection algorithm (y_4 P) and least squares algorithm (y_4 LS)

A detailed analysis of the estimation of Fig. 9 is presented in Fig. 10.

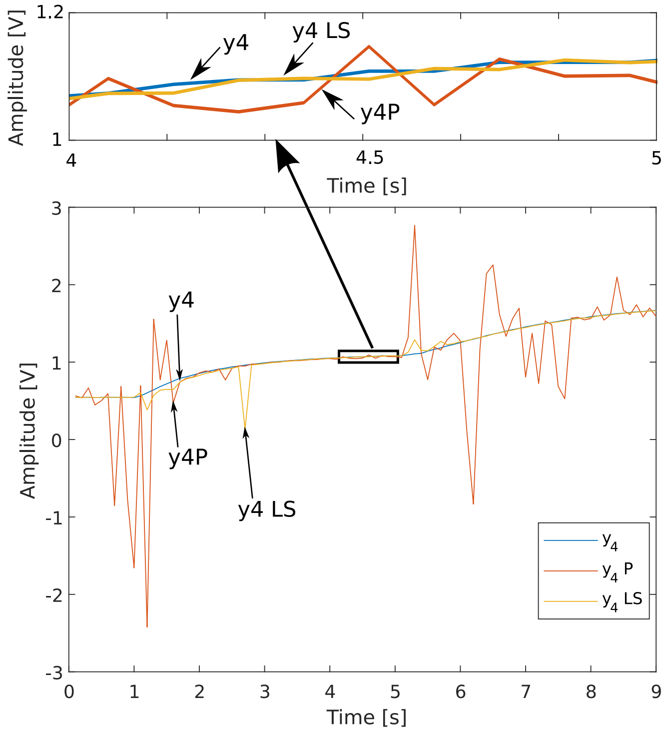

Fig. 10. Detailed analysis of estimated output $y_4[k]$ and $\hat{y}_4[k]$ by using projection algorithm (y_4 P) and least squares algorithm (y_4 LS)

In Fig. 11 is presented the estimation result by using the projection algorithm (y_5 P) and least-squares algorithm (y_5 LS) for the level of tank 5. It can be seen that the output data are adequately approximated.

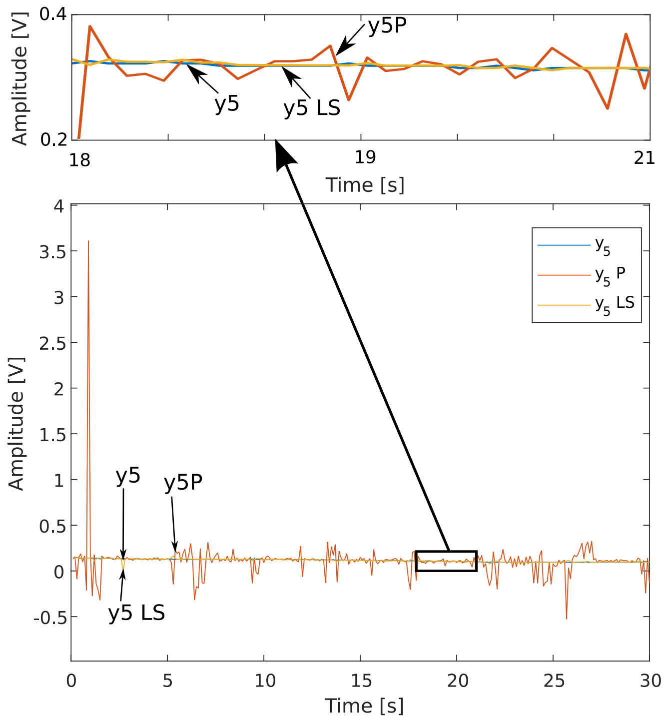

Fig. 11. Estimated output $y_5[k]$ and $\hat{y}_5[k]$ by using projection algorithm (y_5 P) and least squares algorithm (y_5 LS)

A detailed analysis of the estimation of Fig. 11 is presented in Fig. 12.

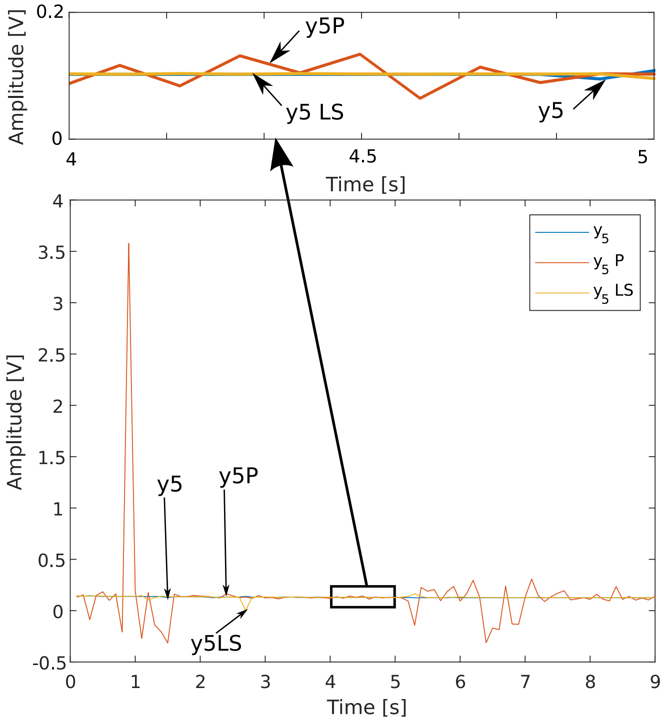

Fig. 12. Detailed analysis of estimated output $y_5[k]$ and $\hat{y}_5[k]$ by using projection algorithm (y_5 P) and least squares algorithm (y_5 LS)

In each figure from Fig. 3 to Fig. 12, the least squares method outperform the signal estimation of the projection method.

In Fig. 13 are presented the input signal u_1 used for the estimation algorithm.

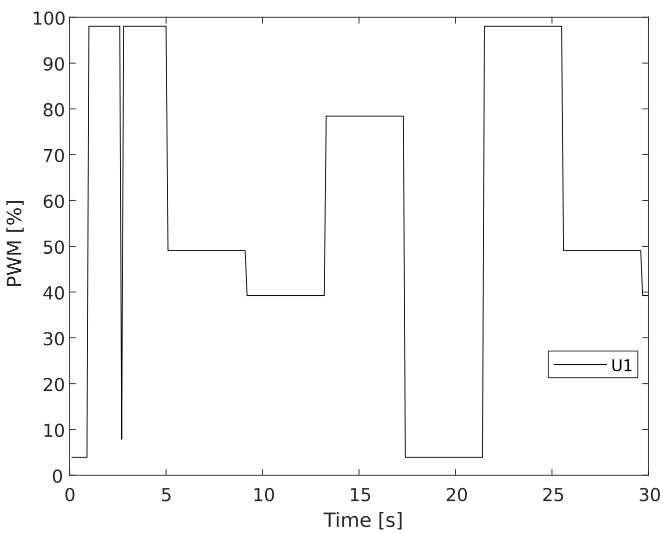

Fig. 13. Input signal u_1 used for the projection algorithm and least squares algorithm

In Fig. 14 are presented the input signal u_2 used for the estimation algorithm.

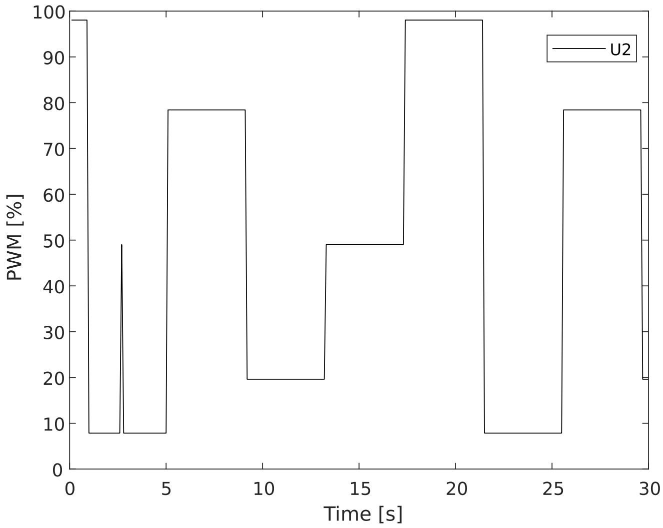

Fig. 14. Input signal u_2 used for the projection algorithm and least squares algorithm

In Fig. 15 are presented the input signal u_3 used for the estimation algorithm.

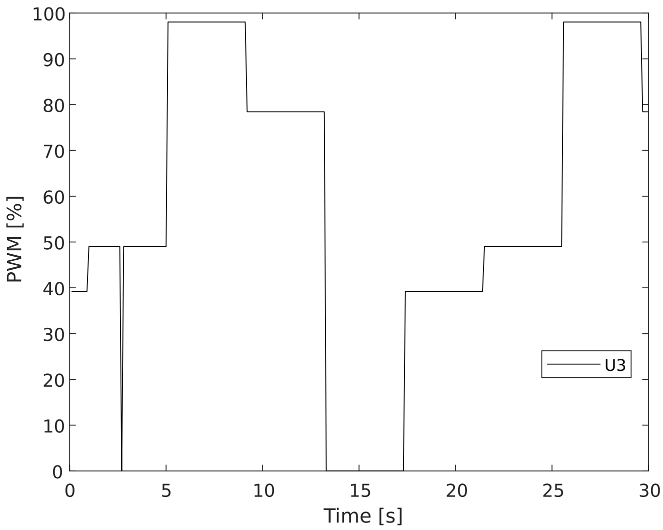

Fig. 15. Input signal u_3 used for the projection algorithm and least squares algorithm

In Fig. 16 are presented the input signal u_4 used for the estimation algorithm.

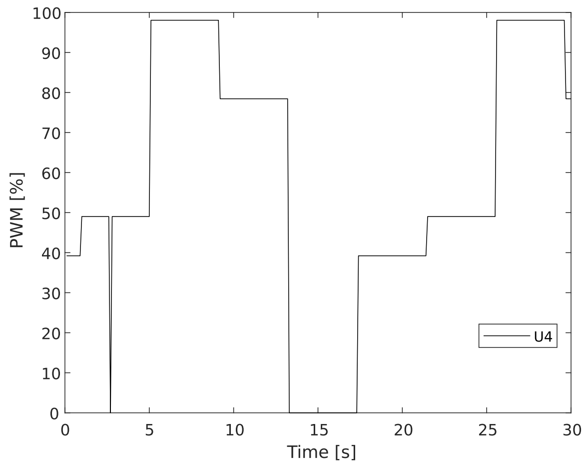

Fig. 16. Input signal u_4 used for the projection algorithm and least squares algorithm

In Fig. 17 are presented the input signal u_5 used for the estimation algorithm.

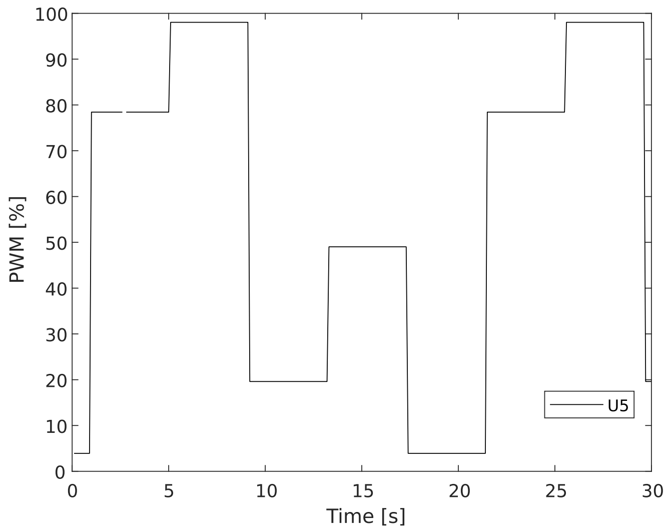

Fig. 17. Input signal u_5 used for the projection algorithm and least squares algorithm

In order to evaluate the closed-loop performance of the proposed approach, a comparison analysis is performed with a state-of-the-art adaptive PID controller. In this case, the identification stage is performed using a least-squares algorithm. In Fig. 18 is presented the closed-loop response for the level of tank 1, where a comparison between the PID and the proposed adaptive multivariable pole-placement (PP) approach are presented.

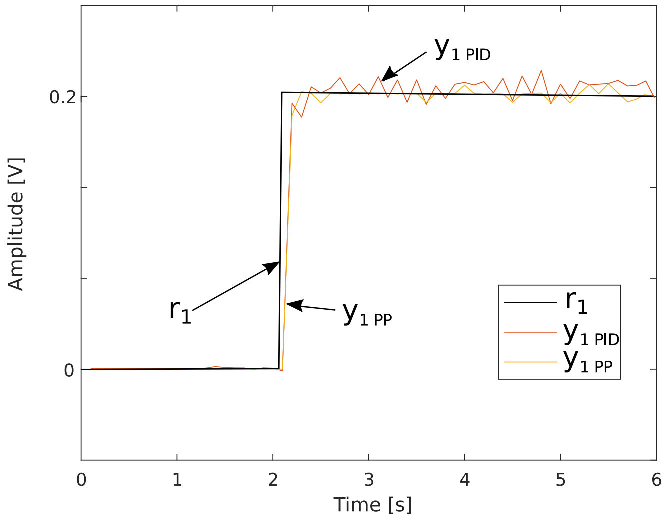

Fig. 18. Closed loop response for tank 1 level control by using the PID and the PP approach

In Fig. 19 is presented a comparison between the PID and the PP approach closed-loop response for the level of tank 2.

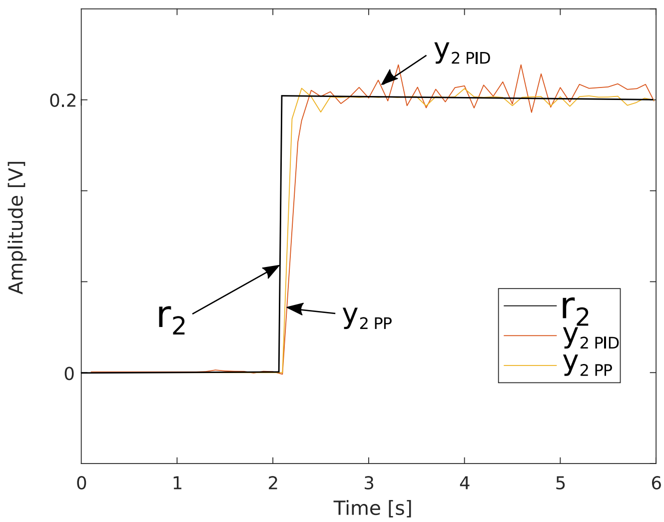

Fig. 19. Closed loop response for tank 2 level control by using the PID and the PP approach

In Fig. 20 is presented a comparison between the PID and the PP approach closed-loop response for the level of tank 3.

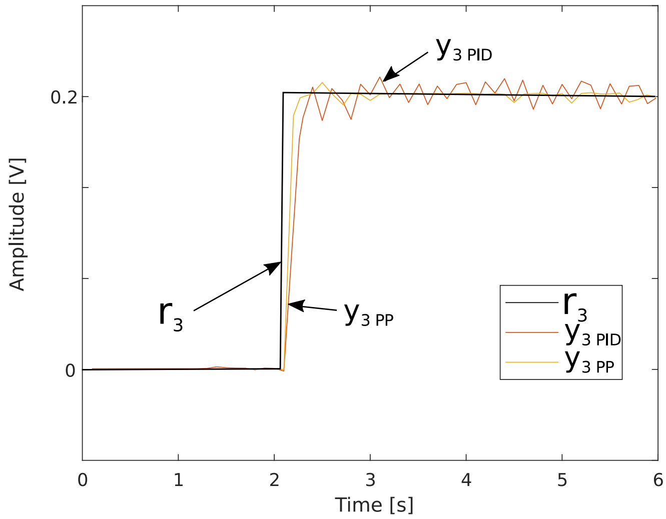

Fig. 20. Closed loop response for tank 3 level control by using the PID and the PP approach

In Fig. 21 is presented a comparison between the PID and the PP approach closed-loop response for the level of tank 4.

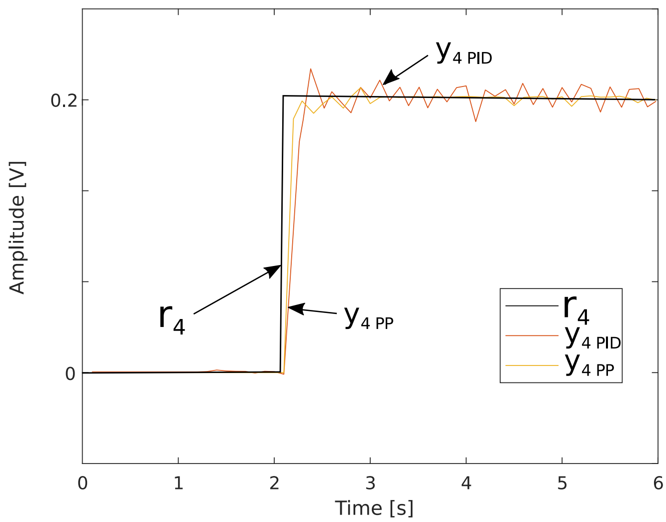

Fig. 21. Closed loop response for tank 4 level control by using the PID and the PP approach

In Fig. 22 is presented a comparison between the PID and the PP approach closed-loop response for the level of tank 5.

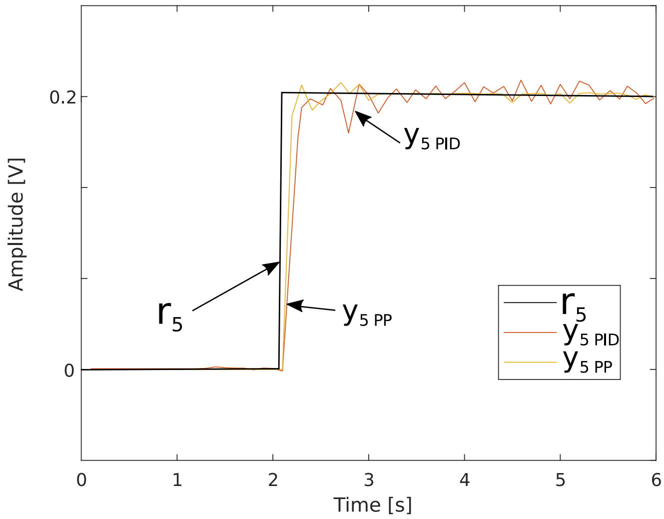

Fig. 22. Closed loop response for tank 5 level control by using the PID and the PP approach

An additional evaluation is performed in simulation for the second order electric circuit presented in (14). This circuit has an equivalent dynamic behavior to a two tank coupled system with two flow inputs u_1 and u_2 (which are equivalent to current sources) and two level outputs $y_1(t)$ and $y_2(t)$ (which are equivalent to the voltage at each capacitor).

$$\dot{x} = \begin{bmatrix} -\left(\frac{1}{C_1R_1}+\frac{1}{C_1R_{12}}\right) & \frac{1}{C_1R_{12}} \\ \frac{1}{C_2R_{12}} & -\left(\frac{1}{C_2R_2}+\frac{1}{C_2R_{12}}\right) \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \end{bmatrix} u(t) \tag{14}$$

being $R_1 = R_{12} = R_2 = 1000\Omega$, and $C_1 = C_2 = 100\mu F$, and being

$$y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) \tag{15}$$

with

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{16}$$

The performance of the PP proposed approach is presented in Fig. 23 for the output y_1 and in Fig. 24 for output y_2. It is worth noting that the references r_1 and r_2 are adequately tracked by each corresponding output.

In Fig. 25 are shown the control signals u_1 and u_2 of the closed-loop response. It is worth noting that even when the references r_1 and r_2 are fixed, the control signals are changing due to the inner coupling described in (14). The aforementioned behavior validate that the system dynamic is adequately identified and also controlled.

IV. Conclusions

A real-time multivariable adaptive control is applied over a waste-water treatment level plant. The system is identified in real-time by using an adaptive least-squares approach. The multivariable system is also controlled by using a pole placement multivariable polynomial controller designed by considering the identified model. It can be seen that the dynamical model is adequately approximated in

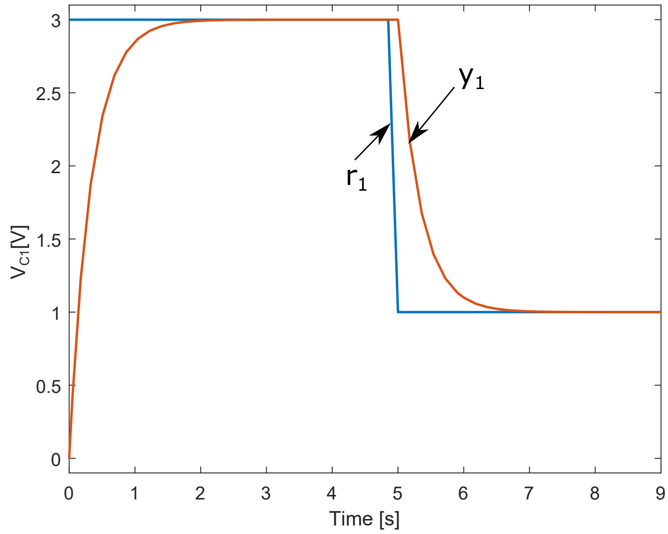

Fig. 23. Reference r_1 and Output y_1

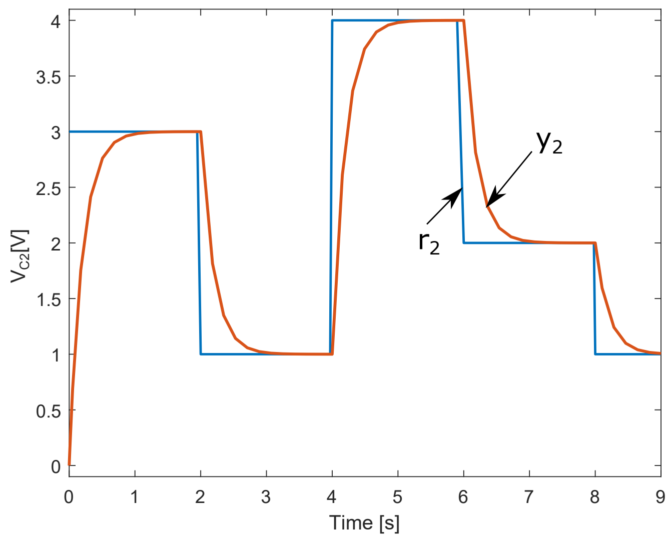

Fig. 24. Reference r_2 and Output y_2

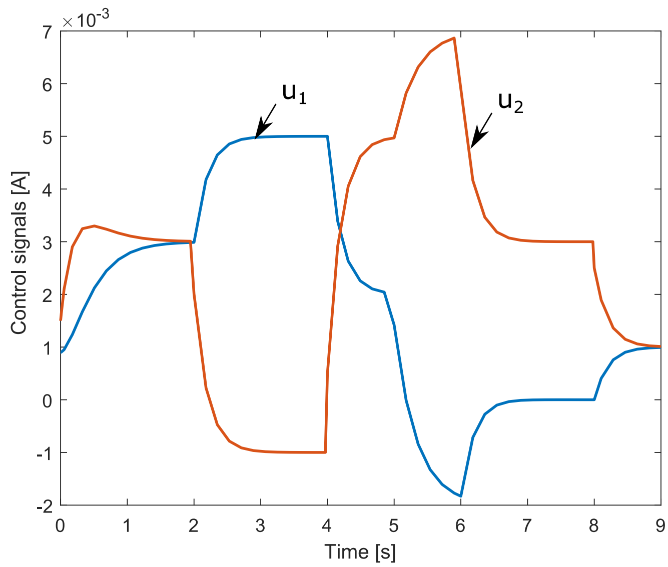

Fig. 25. Control signals for the simulated second order system

real-time by using the real-time least-squares identification method and the pole placement adaptive controller. The results show that the proposed adaptive identification and control pole-placement (PP) approach outperforms the adaptive PID controller performance. In future work, large-scale identification and pole-placement control method for large-scale multivariable systems will be developed for real-time implementation.

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