The Performance of Empirical Bayes Based on Weighted Squared Error Loss and K-Loss Functions in Skip Lot Sampling Plan with Resampling

Khanittha Tinochai and Katechan Jampachaisri

Abstract—**Skip lot sampling plans can be applied to manufacturing process to reduce sample size and inspection costs in the lot. In this paper, the objective is to propose the Empirical Bayes approach based on weighted squared error loss (WSEL) and K-loss (KL) functions in skip lot sampling plan with resampling (SkSP-R) on variables sampling plan for lot inspection with normally distributed data, assuming unknown mean and unknown variance. The proposed plans are also compared to traditional plans including skip lot sampling plan 2 (SkSP-2) and SkSP-R with single sampling plan (SSP) as a reference plan. The probability of acceptance (P_a), average sample number (ASN), and average total inspection (ATI) are considered as criteria for comparison. Afterwards, the proposed plan is applied to real data, amplified pressure sensor process. The results indicated that the proposed method yielded the smallest ASN and ATI but the highest P_a.**

Index Terms—**Empirical Bayes, WSEL function, KL function, SkSP-R**

I. Introduction

An acceptance sampling plan is a tool used for production inspection in the lot, which is an aspect of statistical quality control. The advantages include reduction of sample size and costs for inspecting products in monitoring processes. The types of acceptance sampling plan include attributes sampling plans and variables sampling plans. The quality characteristics for attributes are specified by defective and non-defective units, whereas the quality characteristics for variables are measured on continuous scales that often provide more information concerning production in the lots than attributes [1]. The types of plans include single sampling plan (SSP), double sampling plan (DSP), multiple sampling plans (MSP), sequential sampling plan (sequential SP) and skip lot sampling plan (SkSP). The variable sampling plan can be applied in several parts of industry. The research can also be seen as follows. Jun et al. [2] studied SSP and DSP for variables in Weibull distribution to aid decision making for sudden death testing. Lee [3] developed control chart for variables sampling plan based on process mean.

The SkSP is applied widely throughout industries for quality inspections of products in the lots since it can be done at lower cost compared to single sampling plan. SkSP is also more efficient than traditional methods [4]. Dodge [5] initially developed the SkSP, while Dodge and Perry [6] proposed the SkSP-2. Vijayaraghavan [7] illustrated the parameter selection of the SkSP-2 based on Poisson model. Balamurali and Subramani [8] developed optimal parameters for the SkSP-2 with DSP as the reference plan. Aslam et al. [9] presented optimal parameters for the SkSP-2 based on the truncated lifetime test. Koatpoothon and Sudasna-na-Ayudthya [10] compared the SkSP-2 based on P_a, ASN, ATI and average outgoing quality (AOQ), with by attributes skip lot sampling plan V (SkSP-V). Veerakumari1 and Kokila [11] considered the SkSP-2 for destructive testing of the production in the lot. Balamurali and Subramani [12] studied the SkSP-2 by variables with SSP by variables as a reference plan when data are normally distributed under both known and unknown variances, using P_a, ASN, ATI and AOQ as criteria of comparison. The research involving SkSP-R can also be seen as follows. Govindaraju and Ganesalingam [13] developed a resampling plan for attributes with SSP as a reference plan. Aslam et al. [14] studied the resubmitted lot in variables sampling plan based on process capability index (PCI), where data are normally-distributed with unknown mean and variance. Hussain et al. [15] considered the ASN of the SkSP-R which is compared with the SSP and the SkSP-2 with the SkSP-V. Kurniati et al. [16] developed resubmitted sampling plan for variables based on one-sided PCI. Aslam et al. [17] presented the optimal parameters of the SkSP-R for variables where data follow normal distribution under known and unknown variances. Balamurali et al. [18] considered the SkSP-R for destructive and non-destructive testing of cost items. Hussain et al. [19] developed the SkSP-R in PCI for optimal

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parameters based on symmetric and asymmetric proportions of defective units. Aslam et al. [20] studied accelerated life testing with the SkSP-R when data follow Weibull distribution. Balamurali and Mahalingam [21] developed resampling procedure in SkSP-2. Balamurali et al. [22] demonstrated the optimal parameters of the SkSP-R using DSP for attributes as a reference plan.

Bayesian statistical techniques are applied widely in various sampling plans which is an alternative to the classical approach. They are important for statistical inferences, such as the parameter estimation and hypothesis testing. In the Bayesian approach, the form of prior distributions and their parameters (hyper-parameters) are usually assumed to be known. In contrast, the Empirical Bayes (EB) approach assumes unknown hyper-parameters which are estimated from observable data [23]. This can be seen in Krutchkoff [24], Casella [25], Lu [26], Cui and George [27], Khaledi and Rivaz [28], Maswadah [29], Petrone et al. [30] and Lemon [31]. Ganesan et al. [32] developed Bayesian in sequential probability ratio test for monitoring processes of sensor data. Guure and Ibrahim [33] considered the estimation of Weibull parameters with maximum likelihood and Bayesian methods using interval-censored survival data. Gimlin and Breipohl [34] studied Bayesian approach which can be applied to non-sequential SP and sequential SP for acceptance sampling plan. The application for SkSP with the Bayesian method can be demonstrated as follows. Phelps [35] developed Bayesian method in SkSP for destructive testing based on Poisson distribution. Aslam et al. [36] exhibited the Bayesian method for attributes in resubmitted lots where data are assumed to be gamma-Poisson distribution. Suresh and Umamaheswari [37] developed the Bayesian method in SkSP-2 based on a conditional repetitive group sampling plan under the Poisson model. Suresh and Umamaheswari [38] presented the Bayesian approach in the SkSP for destructive testing based on the Poisson model. Rajeswari and Jose [39] developed the SkSP-2 with the Bayesian modified chain sampling plan as a reference plan. Nirmala and Suresh [40] studied Bayesian methods in SkSP-V with multiple deferred states (0, 2) as a reference plan. Seifi et al. [41] studied variables sampling plan with resubmitted lot based on PCI and Bayesian method. Veerakumari et al. [42] illustrated the Bayesian method in SkSP-V with conditional repetitive group sampling plan.

Rabie and Li [43] studied Burr-X distribution with hybrid censored data. The Bayesian and the expectation of the Bayesian estimate is considered under LINEX and squared error loss functions. Li et al. [44] proposed EB method using resampling in microarray data analysis which resulted in reduction of Type I and II error rate. Tinochai et al. [45] studied EB method for variables SkSP-V with normally distributed data in two cases of unknown mean but known variance and known mean but unknown variance. Tinochai et al. [46] developed EB approach in sequential sampling plan under a squared error loss (SL) and precautionary loss (PL) functions when data follow normal distribution under known mean and unknown variance. Jampachaisri et al. [47] considered EB approach in sequential sampling plan under SL and PL functions when data follow normal distribution, assuming unknown mean and unknown variance. In addition, EB method can be performed based on various loss functions, such as weighted squared error loss (WSEL) and K-loss (KL) functions, for making a decision about quality assurance of products in the lot. The WSEL function is a symmetric loss function which can be used for the parameter estimation with invariant property, whereas KL function is an asymmetric loss function and suitable for underestimated or overestimated circumstances [48]. Researches related to WSEL and KL functions can be seen as in Ali et al. [48], Ali et al. [49], Fan et al. [50], Rezaeian and Asgharzadeh [51].

Furthermore, the resampling procedure is utilized for quality inspections of products in the lots when decision making cannot be made on initial inspection and the quality of the lot is thus rejected by producer, which result in discarding the first inspected sample. Then, the decision making will depend on next inspection, which is time consuming and increased cost of inspection. The SkSP-R sampling plan can be used to reduce procedure of inspection when the history of products indicates good quality. Thus, if quality of products in previous lot is good, then the EB method in SkSP-R by variables sampling plan can be applied to estimate hyper-parameters using the observed data, yielding the acceptance probability obtained from the posterior distribution of defective proportion [52]. Therefore, EB in SkSP-R by variables sampling plan can reduce cost of inspection, producer risk and consumer risk in resampling step.

The aim of this paper is to utilize the EB approach in SkSP-R (EB in SkSP-R) based on WSEL and KL functions for variables sampling plan. The SkSP-R can also be applied to both continuous sampling plan and decision making about quality inspection of bulk products. Its advantages are to reduce the cost of product inspection in manufacturing process such as pressure sensor process. This paper mainly focuses on normal-distributed data, with unknown mean and unknown variance. The proposed plans are compared to SkSP-2 and SkSP-R with SSP as a reference plan. The P_a, ASN and ATI are considered as comparison criteria. The SSP by variables, SkSP-2 and SkSP-R by variables are given in Section 2, Section 3 and Section 4, respectively. Section 5 shows the use of EB in SkSP-R based on WSEL and KL functions by variables sampling plan. Section 6 covers the simulation and results, Section 7 expresses practical applications, while conclusions are drawn in Section 8.

II. Single Sampling Plan by Variables (SSP by Variables)

This plan depends on two parameters according to sample size (n) and acceptance criterion (K). The lot is accepted if $z \geq K$ and rejected if $z < K$ where $z = (USL - \bar{X})/\sigma$ using standard deviation (s) for unknown σ and USL representing an upper specification limit. The two parameters (n, K) can be specified as follows.

$$n = \left(1 + \frac{K^2}{2}\right)\left(\frac{Z_\alpha + Z_\beta}{Z_{p_1} - Z_{p_2}}\right)^2, \quad (1)$$

$$K = \frac{Z_{p_2} Z_\alpha + Z_{p_1} Z_\beta}{Z_\alpha + Z_\beta}. \quad (2)$$

The criteria for comparison are acceptance probability in the lots, denoted as P_a, and average sample number (ASN) inspected per lot which is used for making decision to accept or reject the lot. When the lot is rejected, the average number of items (ATI) inspected per lot is considered under 100% inspection of items [53]. Thus, criteria of comparison for SSP by variables can be defined as following:

$$P_a(p) = P(Z \leq z), \quad (3)$$

$$\text{ASN}(p) = n, \quad (4)$$

$$\text{ATI}(p) = n + (N - n)Q. \quad (5)$$

Suppose that $Q = 1 - P$ where P is the acceptance probability of a lot for the SSP by variables and p is the proportion of defective units in the lot.

III. Skip Lot Sampling Plan 2 (SkSP-2)

The SkSP-2 involving only two stages: a normal inspection and skipping inspection stage. It depends on two parameters, where i is consecutive lots on the reference plan and f is the proportion of lots ($0 < f < 1$). In this paper, the SkSP-2 is compared to the SSP by variables, regarded as the reference plan. The criteria for comparison of the SkSP-2 are the P_a, ASN and ATI [54] that can be given as follows.

$$P_a(p) = \frac{fP + (1-f)P^i}{f + (1-f)P^i}, \quad (6)$$

$$\text{ASN}(p) = \frac{nf}{f + (1-f)P^i}, \quad (7)$$

$$\text{ATI}(p) = \frac{f\left[n + (N-n)Q\right]}{f + (1-f)P^i}, \quad (8)$$

Let $Q = 1 - P$, where P is the acceptance probability of a lot with the SSP by variables and p is the proportion of defective units in the lot.

IV. Skip Lot Sampling Plan Resampling (SkSP-R)

The SkSP-R plan is based on three main stages for inspections as follows: normal inspection, skipping inspection and resampling stage which is different from SkSP-2. It consisted of considered by four parameters including i, f, k and r, where k is consecutive lots on skipping inspection when $k < i$, $k = i$, $k > i$ and r is the number of times in the lots which are submitted for resampling [55]. The procedures of the SkSP-R sampling plan are provided as follows (Fig 1).

1) Start at normal inspection; the lots are inspected one by one with the reference plan.
2) If i consecutive lots are continuously accepted from the normal inspection then switches to skipping inspection stage.
3) For the skipping inspection, a sampled lot is inspected randomly only a fraction (f) of the lots that continue until the sampled lot is rejected.
4) If a sampled lot is rejected from skipping inspection stage and k^{th} last sampled lots have been accepted consecutively and then go to the resampling for immediate next lot.
5) On the resampling stage, the lot is inspected with a reference plan, if the lot is accepted then switch to skipping inspection. However, if the lot is rejected then resampling r times. When the $(r\text{-}1)^{th}$ times have not been accepted, then the lot is rejected on resampling stage.
6) If the lot is rejected on resampling procedure then replace defective units with non-defective units and return to normal inspection.

The criteria for comparison of the SkSP-R are the P_a, ASN and ATI which can be calculated as follows.

$$P_a(p) = \frac{fP + (1-f)P^i + fP^k\left(P^i - P\right)\left(1 - Q^r\right)}{f\left(1 - P^i\right)\left[1 - P^k\left(1 - Q^r\right)\right] + P^i\left(1 + fQP^k\right)}, \quad (9)$$

$$\text{ASN}(p) = \frac{nf + nfQP^{i+k} - nfP^k\left(1 - P^i\right)\left(1 - Q^r\right)}{f\left(1 - P^i\right)\left[1 - P^k\left(1 - Q^r\right)\right] + P^i\left(1 + fQP^k\right)}, \quad (10)$$

$$\text{ATI}(p) = \frac{\left[n + (N-n)(1-P)\right]\left[f + fQP^{i+k} - fP^k\left(1 - P^i\right)\left(1 - Q^r\right)\right]}{f\left(1 - P^i\right)\left[1 - P^k\left(1 - Q^r\right)\right] + P^i\left(1 + fQP^k\right)}. \quad (11)$$

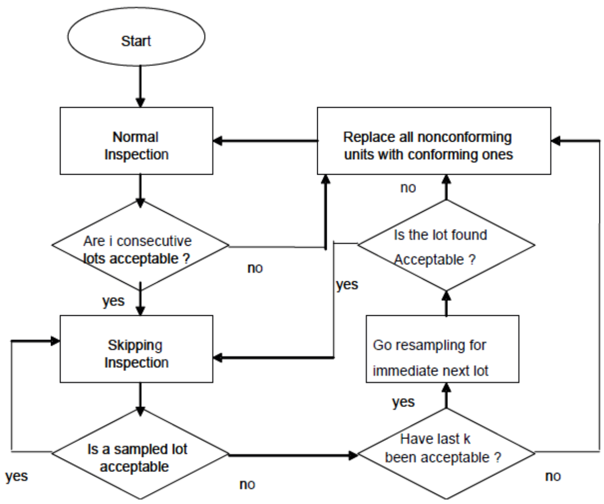

Fig. 1. The procedure of the SkSP-R sampling plan.

Let $Q = 1 - P$, P is the acceptance probability of a lot with the reference plan, proportion of defective units in the lot at

AQL, denoted as p_1 and at RQL, denoted as p_2. Furthermore, P_a is considered of two points, the producer's risk (α) and the consumer's risk (β), respectively. The P_a at the p_1 and p_2 are given by

$$P_a(p_1) \geq 1-\alpha, \quad (12)$$

$$P_a(p_2) \leq \beta. \quad (13)$$

V. Procedure for Paper Submission Empirical Bayes Based on WSEL and KL Functions in SkSP-R

In this paper, the EB in SkSP-R based on WSEL and KL functions is considered with data normally-distributed, $X \sim N(\mu, \sigma^2)$, with unknown mean (μ) and unknown variance (σ^2). The proportion of defective samples in a lot is given by

$$p = P(X > USL \mid \mu) = 1 - F[(USL-\mu)/\sigma], \quad (14)$$

where USL is an upper specification limit, $W = (USL-\mu)/\sigma$ and $F(W)$ is a cumulative distribution function of standard normal distribution [56].

The Bayesian approach consisted of an unknown parameters δ, and its distribution called prior probability density function. Assuming prior distribution, $\pi(\delta \mid \omega)$, and hyper-parameter ω are known. The likelihood function is written as $L(\delta)$. Therefore, the posterior distribution, $h(\delta \mid \underline{x})$ is given by

$$h(\delta \mid \underline{x}) = \frac{L(\delta)\cdot \pi(\delta \mid \omega)}{M(\underline{x} \mid \omega)} \propto L(\delta)\cdot \pi(\delta), \quad (15)$$

If the hyper-parameter (ω) is unknown, but can be estimated from the observed data, called EB approach [57]. The hyper-parameter can be specified from the marginal distribution of $\underline{x}$, given by

$$M(\underline{x} \mid \omega) = \int_{\delta} f(\underline{x} \mid \delta)\cdot \pi(\delta \mid \omega)\, d\delta, \quad (16)$$

where $M(\underline{x} \mid \omega)$ denotes the marginal distribution of $\underline{x}$.

A. Unknown mean (μ) and unknown variance (σ^2)

Let $X \sim N(\mu, \sigma^2)$, where μ and σ^2 are unknown and assuming informative priors on μ and σ^2: $\mu \sim N(\theta, \tau^2)$ and $\sigma^2 \sim IG(a,b)$ where the hyper-parameters $\theta,\ \tau^2,\ a$ and b are unknown.

The hyper-parameters can be estimated from the marginal likelihood distribution, written as

$$M(\underline{x} \mid \theta, \tau^2, a, b) = \int_0^{\infty}\int_{-\infty}^{\infty} f(\underline{x} \mid \mu, \sigma^2)\cdot \pi(\mu \mid \theta, \tau^2)\cdot \pi(\sigma^2 \mid a, b)\, d\mu\, d\sigma^2,$$

Then,

$$M(\underline{x} \mid \theta, \tau^2, a, b) = \int_0^{\infty}\int_{-\infty}^{\infty} \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} \cdot \frac{1}{(2\pi\tau^2)^{1/2}} e^{-\frac{1}{2\tau^2}(\mu-\theta)^2} \times \frac{b^a}{\Gamma(a)}(\sigma^2)^{-(a+1)} e^{-\frac{b}{\sigma^2}}\, d\mu\, d\sigma^2.$$

Thus,

$$M(\underline{x} \mid \theta, \tau^2, a, b) \propto \int_0^{\infty} \frac{(\sigma^2)^{-\left(a+1+\frac{n-1}{2}\right)}}{(n\tau^2+\sigma^2)^{1/2}} \times e^{-\frac{1}{2(n\tau^2+\sigma^2)}\left[\sum_{i=1}^{n}(x_i-\theta)^2 - \frac{n\tau^2}{\sigma^2}\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)\right] - \frac{b}{\sigma^2}}\, d\sigma^2. \quad (17)$$

Equation (17) does not have a closed form, the hyper-parameters thus cannot be estimated directly by classical method, such as maximum likelihood method (ML). Alternatively, the hyper-parameters $\theta,\ \tau^2,\ a$ and b are determined using Gibbs sampler [58]. After that, the estimators $\hat{\theta},\ \hat{\tau}^2,\ \hat{a}$ and $\hat{b}$ will be substituted into the posterior distribution function.

The posterior distribution function of μ and σ^2 is provided as

$$h(\mu, \sigma^2 \mid \underline{x}) \propto L(\mu, \sigma^2 \mid \underline{x})\cdot \pi(\mu \mid \theta, \tau^2)\cdot \pi(\sigma^2 \mid a, b).$$

Thus,

$$h(\mu, \sigma^2 \mid \underline{x}) \propto \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} \cdot \frac{1}{(2\pi\hat{\tau}^2)^{1/2}} e^{-\frac{1}{2\hat{\tau}^2}(\mu-\hat{\theta})^2} \times \frac{\hat{b}^{\hat{a}}}{\Gamma(\hat{a})}(\sigma^2)^{-(\hat{a}+1)} e^{-\frac{\hat{b}}{\sigma^2}}. \quad (18)$$

It can be seen that the joint posterior distribution functions of μ and σ^2 do not have a closed form, which can also be obtained using Gibbs sampler. Furthermore, the estimation of parameters μ and σ^2 for WSEL and KL function can be presented in the next section.

B. The EB based on WSEL Function for Estimation of μ and σ^2

The WSEL function provided as

$$L(T;\ \eta) = \frac{[\eta - T]^2}{\eta}, \quad (19)$$

where $\eta = (\mu, \sigma^2)$ and T is estimated values of μ and σ^2. Then, the EB estimators of μ and σ^2 with respect to WSEL function [49] can be obtained as

$$\hat{\mu}_{WSEL} = \frac{1}{E(\mu^{-1} \mid \underline{x})},$$

$$\hat{\sigma}^2_{WSEL} = \frac{1}{E\left[(\sigma^2)^{-1} \mid \underline{x}\right]}.$$

Let $u\left(\mu,\sigma^2\right)$ be any function of μ and σ^2, the posterior expectation can be shown as follows:

$$E\left[u\left(\mu,\sigma^2\right)|\underline{x}\right]=\int_0^\infty\int_{-\infty}^\infty u\left(\mu,\sigma^2\right)\cdot h\left(\mu,\sigma^2\,|\,\underline{x}\right)d\mu d\sigma^2. \quad (20)$$

Due to its complexity, Lindley's approximation procedure is used to estimate the parameters in [59]. The two parameters estimators of μ and σ^2 can be obtained from

$$E\left(\mu,\sigma^2\,|\,\underline{x}\right)=u+\frac{1}{2}\left(u_{11}\sigma_{11}+u_{22}\sigma_{22}\right)+P_1u_1\sigma_{11}+P_2u_2\sigma_{22}$$
$$+\frac{1}{2}\left[\sigma_{11}\sigma_{22}\left(u_1L_{12}+u_2L_{21}\right)+u_1\sigma_{11}^2L_{30}+u_2\sigma_{22}^2L_{03}\right]. \quad (21)$$

The EB estimator of μ with respect to the WSEL can be determined as follows.

Let $u\left(\mu,\sigma^2\right)=\mu^{-1}$ then

$$u_1=\frac{\partial u\left(\mu,\sigma^2\right)}{\partial\mu}=-\mu^{-2},\ u_{11}=\frac{\partial^2u\left(\mu,\sigma^2\right)}{\partial\mu^2}=2\mu^{-3},$$
$$u_2=\frac{\partial u\left(\mu,\sigma^2\right)}{\partial\sigma^2}=0,\ u_{22}=\frac{\partial^2u\left(\mu,\sigma^2\right)}{\partial\left(\sigma^2\right)^2}=0.$$

The likelihood function of μ and σ^2 is given by

$$L\left(\mu,\sigma^2\,|\,\underline{x}\right)=\frac{1}{\left(2\pi\sigma^2\right)^{n/2}}e^{-\frac{1}{2\sigma^2}\sum_{i=1}^n(x_i-\mu)^2}, \quad (22)$$

and joint prior distribution of μ and σ^2 can be provided from

$$\pi\left(\mu,\sigma^2\right)=\frac{1}{\left(2\pi\hat{\tau}^2\right)^{1/2}}e^{-\frac{1}{2\hat{\tau}^2}(\mu-\hat{\theta})^2}\cdot\frac{\hat{b}^{\hat{a}}}{\Gamma(\hat{a})}\left(\sigma^2\right)^{-(\hat{a}+1)}e^{-\frac{\hat{b}}{\sigma^2}}, \quad (23)$$

where

$$P=\ln\pi\left(\mu,\sigma^2\right)=-\frac{1}{2}\ln\left(2\pi\hat{\tau}^2\right)-\frac{1}{2\hat{\tau}^2}\left(\mu-\hat{\theta}\right)^2$$
$$+\hat{a}\ln\hat{b}-\ln\Gamma(\hat{a})-(\hat{a}+1)\ln\sigma^2-\frac{\hat{b}}{\sigma^2},$$

$$P_1=\frac{\partial P}{\partial\mu}=-\frac{1}{\hat{\tau}^2}\left(\mu-\hat{\theta}\right),\ P_2=\frac{\partial P}{\partial\sigma^2}=-\frac{(\hat{a}+1)}{\sigma^2}+\frac{\hat{b}}{\sigma^4},$$

$$\ln L=-\frac{n}{2}\ln\left(2\pi\sigma^2\right)-\frac{1}{2\sigma^2}\sum_{i=1}^n(x_i-\mu)^2,$$

$$L_{20}=\frac{\partial^2\ln L}{\partial\mu^2}=-\frac{n}{\sigma^2},\ L_{02}=\frac{\partial^2\ln L}{\partial\left(\sigma^2\right)^2}=\frac{n}{2\sigma^4}-\frac{1}{\sigma^6}\sum_{i=1}^n\left(x_i-\mu\right)^2,$$

$$L_{30}=\frac{\partial^3\ln L}{\partial\mu^3}=0,\ L_{03}=\frac{\partial^3\ln L}{\partial\left(\sigma^2\right)^3}=-\frac{n}{2\sigma^6}+\frac{3}{\sigma^8}\sum_{i=1}^n\left(x_i-\mu\right)^2,$$

$$L_{12}=\frac{\partial^3\ln L}{\partial\mu\,\partial\left(\sigma^2\right)^2}=\frac{2}{\sigma^6}\sum_{i=1}^n\left(x_i-\mu\right)^2,\ L_{21}=\frac{\partial^3\ln L}{\partial\mu^2\partial\sigma^2}=\frac{n}{2\sigma^4},$$

$$\sigma_{11}=-\frac{1}{L_{20}}=\frac{\sigma^2}{n},\ \sigma_{22}=-\frac{1}{L_{02}}=-\frac{2\sigma^6}{\left[n\sigma^2-2\left(\sum_{i=1}^n x_i^2-n\bar{x}^2\right)\right]}.$$

Refer to (21), the estimator of μ with respect to the WSEL reduce to

$$E\left(\mu^{-1}\,|\,\underline{x}\right)=u+\frac{1}{2}u_1\sigma_{11}+P_1u_1\sigma_{11}+\frac{1}{2}\left(\sigma_{11}\sigma_{22}u_1L_{12}+u_1\sigma_{11}^2L_{30}\right). \quad (24)$$

After that,

$$E\left(\mu^{-1}\,|\,\underline{x}\right)=\frac{1}{\hat{\mu}}\left[1+\frac{\hat{\sigma}^2}{n\hat{\mu}^2}+\frac{\left(\hat{\mu}-\hat{\theta}\right)\hat{\sigma}^2}{n\hat{\mu}\hat{\tau}^2}\right]+\left[\frac{2\hat{\sigma}^2\left(\sum_{i=1}^n x_i^2-n\bar{x}^2\right)}{n^2\hat{\mu}^2\hat{\sigma}^2-2n\hat{\mu}^2\left(\sum_{i=1}^n x_i^2-n\bar{x}^2\right)}\right]. \quad (25)$$

Thus, the EB estimator of μ based on WSEL can be provided as

$$\hat{\mu}_{WSEL}=\frac{1}{E\left(\mu^{-1}\,|\,\underline{x}\right)}.$$

Then,

$$\hat{\mu}_{WSEL}=\frac{1}{\frac{1}{\hat{\mu}}\left[1+\frac{\hat{\sigma}^2}{n\hat{\mu}^2}+\frac{\left(\hat{\mu}-\hat{\theta}\right)\hat{\sigma}^2}{n\hat{\mu}\hat{\tau}^2}\right]+\left[\frac{2\hat{\sigma}^2\left(\sum_{i=1}^n x_i^2-n\bar{x}^2\right)}{n^2\hat{\mu}^2\hat{\sigma}^2-2n\hat{\mu}^2\left(\sum_{i=1}^n x_i^2-n\bar{x}^2\right)}\right]}, \quad (26)$$

where $\hat{\mu}_{MLE}=\bar{x}$ and $\hat{\sigma}_{MLE}^2=\frac{\left(\sum_{i=1}^n x^2-n\bar{x}^2\right)}{n}$ are ML estimators.

The EB estimator of σ^2 with respect to WSEL can be determined as follows. Let $u\left(\mu,\sigma^2\right)=\left(\sigma^2\right)^{-1}$ then

$$u_1=\frac{\partial u\left(\mu,\sigma^2\right)}{\partial\mu}=0,\ u_{11}=\frac{\partial^2u\left(\mu,\sigma^2\right)}{\partial\mu^2}=0,$$
$$u_2=\frac{\partial u\left(\mu,\sigma^2\right)}{\partial\sigma^2}=-\left(\sigma^2\right)^{-2},\ u_{22}=\frac{\partial^2u\left(\mu,\sigma^2\right)}{\partial\left(\sigma^2\right)^2}=2\left(\sigma^2\right)^{-3}.$$

From (21), the EB estimator of σ^2 is shown as follow:

$$E\left[\left(\sigma^2\right)^{-1}|\,\underline{x}\right]=u+\frac{1}{2}u_{22}\sigma_{22}+P_2u_2\sigma_{22}+\frac{1}{2}\sigma_{11}\sigma_{22}u_2L_{21}. \quad (27)$$

Thus, the EB estimator of σ^2 based on WSEL is given as

$$E\left[\left(\sigma^2\right)^{-1}|\,\underline{x}\right]=\frac{1}{\hat{\sigma}^2}-\left(\frac{1}{A}\right)\left\{2-\frac{2\hat{b}}{\hat{\sigma}^2}+2(\hat{a}+1)-\frac{\hat{\sigma}^8}{2}\right\}. \quad (28)$$

where $A = n\hat{\sigma}^2 - 2\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$. Then, the estimators $\hat{\mu}_{WSEL}$ and $\hat{\sigma}^2_{WSEL}$ will be replaced into the posterior distribution function.

C. The EB based on KL Function for Estimation of μ and σ^2

The KL function is studied by Wasan [60] and it can be used to estimate a scale parameter of a distribution based on positive integer [45]. The form of KL function is provided by

$$L(T;\ \eta) = \left(\sqrt{\frac{T}{\eta}} - \sqrt{\frac{\eta}{T}}\right)^2, \tag{29}$$

where $\eta = \left(\mu, \sigma^2\right)$ and T is estimated value of μ and σ^2. Then, the EB estimators of μ and σ^2 respect to KL function [45] are specified by

$$\hat{\mu}_{KL} = \left[\frac{E\left(\mu \mid \underline{x}\right)}{E\left(\mu^{-1} \mid \underline{x}\right)}\right]^{1/2},$$

$$\hat{\sigma}^2_{KL} = \left[\frac{E\left(\sigma^2 \mid \underline{x}\right)}{E\left[\left(\sigma^2\right)^{-1} \mid \underline{x}\right]}\right]^{1/2}.$$

If $u\left(\mu, \sigma^2\right) = \mu$ then $\hat{\mu} = E\left(\mu \mid \underline{x}\right)$ and $u\left(\mu, \sigma^2\right) = \sigma^2$ then $\hat{\sigma}^2 = E\left(\sigma^2 \mid \underline{x}\right)$ [47]. Thus, the EB estimator of μ and σ^2 are the mean of posterior distribution based on squared error loss (SEL) function. It can be shown as follows.

$$\hat{\mu}_{SEL} = \hat{\mu} - \frac{\left(\hat{\mu} - \hat{\theta}\right)\hat{\sigma}^2}{n\hat{\tau}^2} - \frac{2\hat{\sigma}^2}{\left[n^2\hat{\sigma}^2 - 2n\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)\right]},$$

Then,

$$\hat{\sigma}^2_{SEL} = \hat{\sigma}^2 - \left(\frac{\hat{\sigma}^4}{A}\right)\left\{1 - 2\left(\hat{a}+1\right) + \frac{2\hat{b}}{\hat{\sigma}^2} - \frac{n}{\hat{\sigma}^4} + \frac{3\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}{\hat{\sigma}^6}\right\}.$$

If $u\left(\mu, \sigma^2\right) = \mu^{-1}$ and $u\left(\mu, \sigma^2\right) = \left(\sigma^2\right)^{-1}$ then the EB estimator of μ and σ^2 are refer to (25) and (28) which are the estimators of μ and σ^2 with respect to the WSEL function. Thus, the estimators of μ and σ^2 based on KL can be obtained as

$$\hat{\mu}_{KL} = \left\{\frac{\hat{\mu} - \frac{\left(\hat{\mu} - \hat{\theta}\right)\hat{\sigma}^2}{n\hat{\tau}^2} - \frac{2\hat{\sigma}^2}{\left[n^2\hat{\sigma}^2 - 2n\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)\right]}}{\frac{1}{\hat{\mu}}\left[1 + \frac{\hat{\sigma}^2}{n\hat{\mu}^2} + \frac{\left(\hat{\mu} - \hat{\theta}\right)\hat{\sigma}^2}{n\hat{\mu}\hat{\tau}^2}\right] + \left[\frac{2\hat{\sigma}^2\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}{n^2\hat{\mu}^2\hat{\sigma}^2 - 2n\hat{\mu}^2\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}\right]}\right\}^{1/2}, \tag{30}$$

and

$$\hat{\sigma}^2_{KL} = \left\{\frac{\hat{\sigma}^2 - \left(\frac{\hat{\sigma}^4}{A}\right)\left\{1 - 2\left(\hat{a}+1\right) + \frac{2\hat{b}}{\hat{\sigma}^2} - \frac{n}{\hat{\sigma}^4} + \frac{3\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}{\hat{\sigma}^6}\right\}}{\frac{1}{\hat{\sigma}^2} - \left(\frac{1}{A}\right)\left\{2 - \frac{2\hat{b}}{\hat{\sigma}^2} + 2\left(\hat{a}+1\right) - \frac{\hat{\sigma}^8}{2}\right\}}\right\}^{1/2}, \tag{31}$$

where $\hat{\mu}_{MLE} = \bar{x}$ and $\hat{\sigma}^2_{MLE} = \frac{\left(\sum_{i=1}^{n} x^2 - n\bar{x}^2\right)}{n}$ are ML estimators and $A = n\hat{\sigma}^2 - 2\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$. The estimators $\hat{\mu}_{KL}$ and $\hat{\sigma}^2_{KL}$ will then be replaced into the posterior distribution function.

Suppose that p is the proportion of defective units then the cumulative posterior distribution functions of μ and σ^2 can be defined as

$$F\left(p\right) = \int_{\eta}^{p} h(\eta \mid \underline{x})\, d\eta, \tag{32}$$

where $\eta = \left(\mu, \sigma^2\right)$. Therefore, the cumulative posterior probability distribution function of μ and σ^2 are used to obtain the P_a, ASN and ATI.

D. Criteria for comparison of the EB in SkSP-R based on WSEL and KL functions

The P_a, ASN and ATI of the EB in SkSP-R are given as follows.

$$P_a\left(p\right) = \frac{fP_l + \left(1-f\right)P_l^i + fP_l^k\left(P_l^i - P_l\right)\left(1 - Q^r\right)}{f\left(1 - P_l^i\right)\left[1 - P_l^k\left(1 - Q^r\right)\right] + P_l^i\left(1 + fQP_l^k\right)}, \tag{33}$$

$$\text{ASN}\left(p\right) = \frac{nf + nfQP_l^{i+k} - nfP_l^k\left(1 - P_l^i\right)\left(1 - Q^r\right)}{f\left(1 - P_l^i\right)\left[1 - P_l^k\left(1 - Q^r\right)\right] + P_l^i\left(1 + fQP_l^k\right)}, \tag{34}$$

$$\text{ATI}\left(p\right) = \frac{\left[n + (N-n)(1-P_l)\right]\left[f + fQP_l^{i+k} - fP_l^k\left(1 - P_l^i\right)\left(1 - Q^r\right)\right]}{f\left(1 - P_l^i\right)\left[1 - P_l^k\left(1 - Q^r\right)\right] + P_l^i\left(1 + fQP_l^k\right)}, \tag{35}$$

where l = 1 and 2 which denote the estimators of EB in SkSP-R based on WSEL and KL functions, $Q = 1 - P$ and

P is the acceptance probability of a lot with the reference plan. The P_a at the p_1 and p_2 are considered by $P_a(p_1) \geq 1-\alpha$ and $P_a(p_2) \leq \beta$.

VI. Simulation and Results

Data are generated from standard normal distribution which is considered unknown mean and variance. The number of iterations is given by t =1,000, α =0.05, β =0.10, the proportion of defects at p_1 = 0.01 and at p_2 = 0.02. The SSP by variables is considered with parameters N =1,000, n = 388 which is calculated from (1). The SkSP-2 is defined with parameters i = 5, 10, f = 1/5. The SkSP- R is specified with parameters i = 5, f = 1/5, k = 3, 5, r = 2, 3 and i = 10, f = 1/5, k = 5, 10, r = 2, 3. In this paper, the EB in SkSP-R is compared with traditional methods, SkSP-2 and SkSP-R with SSP as a reference plan. The P_a, ASN and ATI are considered as the criteria for comparison and the simulation results of four plans can be shown in Table I to Table III.

Table I shows a comparison of P_a for EB in SkSP-R based on WSEL and KL functions with classical methods; SkSP-2 and SkSP-R, at p_1 = 0.01 and p_2 = 0.02. Obviously, the EB in SkSP-R based on WSEL and KL functions when fixing at p_1 = 0.01 provides that all values of P_a are higher than those in classical methods, varied between 0.9980 and 0.9985. When fixing p_2 = 0.02, the result shows that all values of P_a for EB in SkSP-R based on WSEL and KL functions are smaller than those in SkSP-2 and SkSP-R.

Similarly, Table II and III provide the values of ASN and ATI for EB in SkSP-R based on WSEL and KL functions when compared with traditional approaches at fixing p_1 = 0.01 and p_2 = 0.02, respectively. The results indicate that the EB in SkSP-R for both cases at p_1 = 0.01 provide smallest values of ASN and ATI values in all cases. The ASN and ATI values are varied about 78-81 per lot and 79-83 per lot, respectively.

It is apparent that the results of the EB in SkSP-R based on WSEL and KL functions are similar, yielding the highest values of P_a and the smallest values for ASN and ATI when fixing p_2 = 0.02. In addition, two proposed plans result in similar result, yielding is the smallest values of P_a, ASN and ATI when fixing p_2 = 0.02.

As in Fig.2-7, the P_a, ASN and ATI of the EB in SkSP-R based on WSEL and KL functions are compared with the classical approaches; as SkSP-2 and SkSP-R, where the proportion of defective units is varied from 0.001 to 0.1 and i = 10, f = 1/5, k = 10, r = 3.

Fig.2-3 show that the values of P_a for EB in SkSP-R based on WSEL and KL functions are compared with SkSP-2 and SkSP-R. The results show that higher values of P_a occurred with EB in SkSP-R based on WSEL and KL functions than the SkSP-2 and SkSP-R and larger difference can be detected for EB in SkSP-R based on WSEL as the proportion of defective units is higher than 0.04. It is also evident that the values of P_a for EB in SkSP-R based on both loss functions are closer to those for SkSP-R than SkSP-2.

Furthermore, Fig.4-7 illustrate the values of ASN and ATI for EB in SkSP-R based on WSEL and KL functions in comparison with SkSP-2 and SkSP-R. It indicates that the two proposed plans provide smaller values of ASN and ATI than those in SkSP-2 and SkSP-R and larger difference of ASN and ATI can be seen with the use of EB in SkSP-R based on WSEL than KL. Likewise, the EB in SkSP-R based on both loss functions yield the values of ASN and ATI closer to SkSP-R than SkSP-2.

VII. Practical Applications

Real data about amplified pressure sensor process are observed (n = 136) with specification limits T = 2.0 V, USL = 2.1 V and LSL = 1.9 V [61]. Suppose that AQL(p_1) is 0.001, RQL(p_2) is 0.002, α = 0.05 and β = 0.10. The observations are provided as follows.

1.9422	1.9651	2.0230	1.9712	1.9975	2.0164	1.9927	1.9566
1.9738	1.9541	1.9800	1.9596	1.9811	2.0088	1.9858	1.9677
2.0001	1.9659	1.9955	1.9842	1.9909	1.9829	1.9684	1.9942
1.9897	1.9836	1.9891	1.9608	2.0109	1.9912	2.0077	1.9803
2.0106	1.9885	1.9704	1.9882	1.9689	1.9553	1.9741	1.9825
1.9640	2.0187	1.9616	1.9865	1.9556	1.9817	1.9774	1.9316
1.9841	1.9919	1.9737	1.9958	2.0121	2.0021	1.9665	1.9773
1.9841	1.9570	1.9610	2.0015	1.9750	1.9825	1.9758	1.9682
1.9668	1.9696	2.0334	1.9656	1.9819	2.0116	1.9754	1.9986
2.0114	1.9861	1.9743	1.9594	1.9712	1.9849	1.9711	1.9486
1.9837	1.9424	1.9744	1.9605	1.9719	1.9656	1.9549	2.0174
1.9779	2.0072	1.9875	1.9781	1.9834	1.9893	1.9276	1.9513
1.9971	1.9963	1.9375	1.9941	1.9763	2.0108	1.9687	1.9559
1.9611	1.9729	1.9992	1.9925	2.0073	1.9742	1.9557	1.9726
1.9964	1.9614	1.9768	1.9991	1.9832	1.9847	1.9849	1.9918
1.9748	1.9664	2.0035	1.9822	1.9882	1.9809	1.9920	1.9994
2.0030	1.9786	1.9720	1.9834	1.9726	2.0012	1.9557	1.9874

The sample mean and standard deviation of this data are 1.9807 and 0.0191, respectively. Let i = 10, f = 1/5, k = 10, r = 3. Results show that the P_a's of EB in SkSP-R based on WSEL and KL functions when fixing at p_1 = 0.001 are 0.9998 and 0.9980, respectively which provide higher the P_a's values than 0.9978 and 0.9979 obtained from SkSP-2 and SkSP-R approaches. When fixing at p_2 = 0.002 the P_a's of EB in SkSP-R based on WSEL and KL functions are 0.0039 and 0.0041 which is smaller than 0.0047 and 0.0045 which obtained from SkSP-2 and SkSP-R methods, when fixing at p_1 = 0.001, the ASN of EB in SkSP-R based on WSEL and KL functions are 27.2220 and 27.6152, while the SkSP-2 is 29.3090 and the SkSP-R is 28.2237, respectively. When fixing at p_1 = 0.001, the ATI of EB in SkSP-R based on WSEL and KL functions are 27.3950 and 29.3696, while the SkSP-2 is 31.3241 and the SkSP-R is 30.3966, respectively. Thus, it can see that the ASN and ATI of the proposed plans are smaller than those of classical plans. Similarly, when fixing p_2 = 0.002, the ASN and ATI of the proposed plans are smaller than those of traditional plans.

VIII. Conclusions

In this paper, we propose the EB approach in SkSP-R based on WSEL and KL functions when data follow normal distribution with unknown mean and variance. The study is performed as following: α =0.05, β =0.10, the proportions of defects fixing at p_1 = 0.01 and at p_2 = 0.02. The proposed plans are then compared with traditional approaches; SkSP-2 and SkSP-R with SSP as a reference plan, where P_a, ASN and ATI are considered as criteria for comparison. The results show that the EB in SkSP-R based on WSEL and KL functions outperforms SkSP-R and SkSP-2, providing higher values of P_a and smaller values of ASN and ATI than both classical methods. The P_a, ASN and ATI of the proposed plans based on WSEL and KL are closer to those obtained from SkSP-R than SkSP-2. It can see that the proposed plan can be applied to reduce cost and time for inspection of products, producer risk and consumer risk in manufacturing process. Furthermore, we applied the proposed plan to real data, amplified pressure sensor process, which yielded similar results with those in the simulation. In future research, the EB in the SkSP-R approach can be extended to non-normal distribution. Alternatively, other approach for parameter estimation, such as bootstrapping, can be implemented.

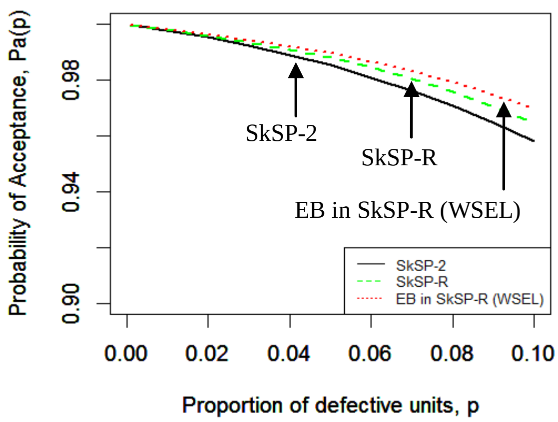

Fig. 2. The comparison of P_a for the SkSP-2, SkSP-R and EB in SkSP-R (WSEL).

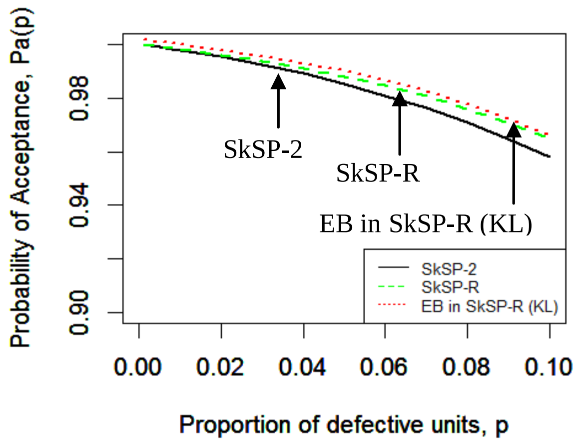

Fig. 3. The comparison of P_a for the SkSP-2, SkSP-R and EB in SkSP-R (KL).

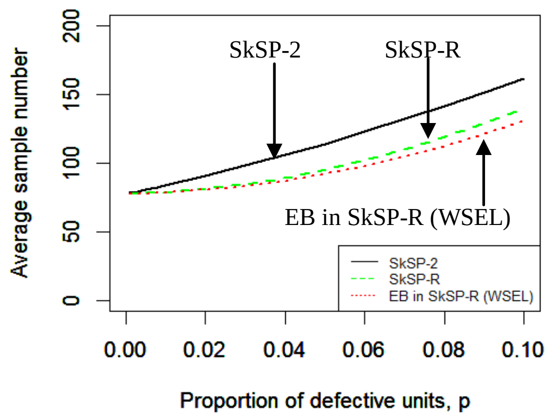

Fig. 4. The comparison of ASN for the SkSP-2, SkSP-R and EB in SkSP-R (WSEL).

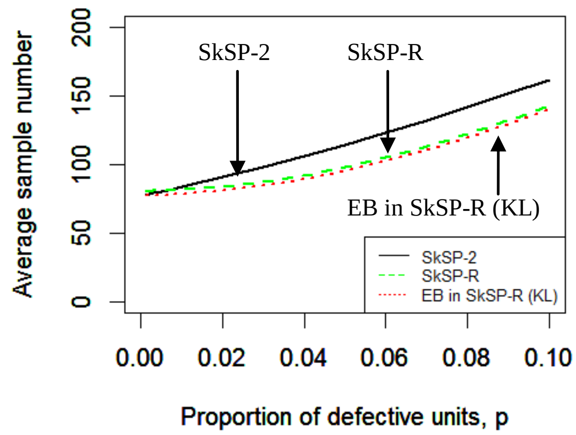

Fig. 5. The comparison of ASN for the SkSP-2, SkSP-R and EB in SkSP-R (KL).

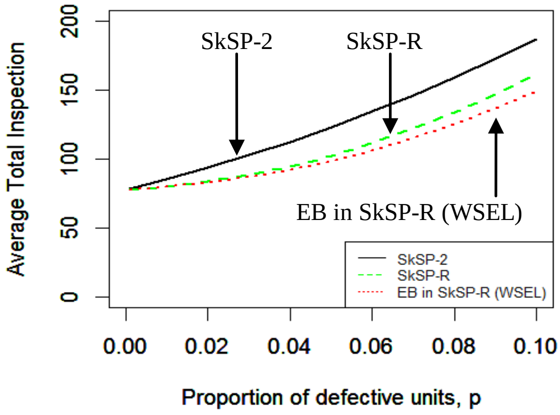

Fig. 6. The comparison of ATI for the SkSP-2, SkSP-R and EB in SkSP-R (WSEL).

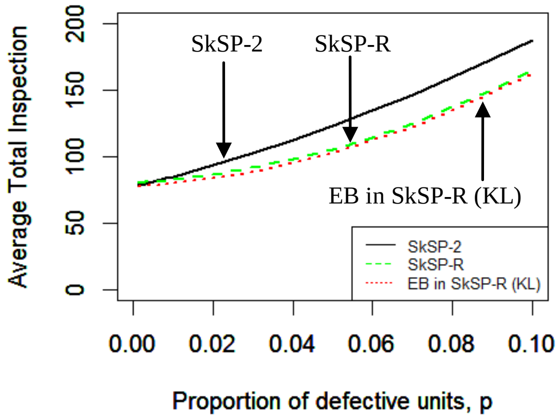

Fig. 7. The comparison of ATI for the SkSP-2, SkSP-R and EB in SkSP-R (KL).

TABLE I
COMPARISON OF P_A FOR THE SKSP-2, SKSP-R AND EB IN SKSP-R BASED ON WSEL AND KL FUNCTIONS AT $P_1 = 0.01$ AND $P_2 = 0.02$.

Parameters				P_a							
				$p_1 = 0.01$				$p_2 = 0.02$			
i	***f***	***k***	***r***	**SkSP-2**	**SkSP-R**	**EB in SkSP-R (WSEL)**	**EB in SkSP-R (KL)**	**SkSP-2**	**SkSP-R**	**EB in SkSP-R (WSEL)**	**EB in SkSP-R (KL)**
5	1/5	3	2	0.9979	0.9980*	0.9985	0.9984	0.0043	0.0042*	0.0041	0.0039*
			3	0.9979	0.9980*	0.9984	0.9982	0.0043	0.0042*	0.0039*	0.0042
		5	2	0.9979	0.9980*	0.9981*	0.9980*	0.0043	0.0042*	0.0042	0.0040*
			3	0.9979	0.9980*	0.9982	0.9981*	0.0043	0.0042*	0.0040	0.0041*
10	1/5	5	2	0.9978	0.9980*	0.9983	0.9983	0.0047	0.0041*	0.0037	0.0038
			3	0.9978	0.9979*	0.9981*	0.9981*	0.0047	0.0040*	0.0038	0.0039*
		10	2	0.9978	0.9979*	0.9980	0.9981*	0.0047	0.0042*	0.0044	0.0040*
			3	0.9978	0.9979*	0.9981*	0.9980*	0.0047	0.0042*	0.0039*	0.0041*

Note : * is the estimated value which is different at the 5th decimal position.

TABLE II
COMPARISON OF ASN FOR THE SKSP-2, SKSP-R AND EB IN SKSP-R BASED ON WSEL AND KL FUNCTIONS AT $p_1 = 0.01$ AND $p_2 = 0.02$.

ASN											
Parameters				***p*₁ = 0.01**				***p*₂ = 0.02.**			
i	***f***	***k***	***r***	**SkSP-2**	**SkSP-R**	**EB in SkSP-R (WSEL)**	**EB in SkSP-R (KL)**	**SkSP-2**	**SkSP-R**	**EB in SkSP-R (WSEL)**	**EB in SkSP-R (KL)**
5	1/5	3	2	80.6682	79.2961	78.2923	78.2866	83.9587	80.1513	79.1902	79.1304
			3	80.6682	79.2958	78.2822	78.3414	83.9587	80.1489	79.1191	79.2492
		5	2	80.6682	78.3458	78.4104	78.3757	83.9587	80.3487	79.5019	79.4199
			3	80.6682	78.3453	78.3524	78.3746	83.9587	80.3464	79.3630	79.4155
10	1/5	5	2	83.9251	81.5100	78.4322	78.4794	90.7920	83.0433	79.8257	79.9578
			3	83.9251	80.5094	78.4278	78.4633	90.7920	82.0384	79.8107	79.9099
		10	2	83.9251	80.7851	78.8973	78.7564	90.7920	82.1210	80.4597	81.0343
			3	83.9251	79.7845	78.7178	78.7883	90.7920	81.1167	80.9155	81.1284

TABLE III
COMPARISON OF ATI FOR THE SKSP-2, SKSP-R AND EB IN SKSP-R BASED ON WSEL AND KL FUNCTIONS AT $p_1 = 0.01$ AND $p_2 = 0.02$.

ATI											
Parameters				***p*₁ = 0.01**				***p*₂ = 0.02**			
i	***f***	***k***	***r***	**SkSP-2**	**SkSP-R**	**EB in SkSP-R (WSEL)**	**EB in SkSP-R (KL)**	**SkSP-2**	**SkSP-R**	**EB in SkSP-R (WSEL)**	**EB in SkSP-R (KL)**
5	1/5	3	2	81.9431	80.5311	79.5213	79.5063	86.6126	82.6483	81.7414	81.5988
			3	81.9431	80.5308	79.4953	79.6496	86.6126	82.6457	81.5750	81.8821
		5	2	81.9431	80.5844	79.7378	79.6543	86.6126	82.8519	82.1763	82.0032
			3	81.9431	80.5811	79.5982	79.6521	86.6126	82.8495	81.8849	81.9966
10	1/5	5	2	85.2515	83.7484	79.5885	79.6859	93.6619	85.5684	82.1939	82.4219
			3	85.2515	83.7477	79.5799	79.6534	93.6619	84.5634	82.1708	82.3426
		10	2	85.2515	81.0278	79.8097	79.9782	93.6619	84.6801	83.1753	83.5525
			3	85.2515	80.0271	79.9117	80.0338	93.6619	83.6756	83.3787	83.6928

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