Solving Multi-objective School Bus Routing Problem Using An Improved NSGA-II Algorithm

Yane Hou, Ning Zhao, and Lanxue Dang

***Abstract*—This paper deals with multi-objective school bus routing problem, which includes route balance, total number of school buses and total travel distance optimization objectives. An improved non-dominated sorting genetic algorithm (NSGA-II) is proposed to solve this problem. First, the definition of measurement indicator that denotes the degree of route balance is given based on the analysis of the solved problem. And then, the multi-objective optimization function is provided. In the proposed algorithm, the individuals are obtained by using the tournament selection, sequential crossover and inverse mutation. The 2-opt neighborhood operator is adopted to improve the best individuals obtained in each iteration. At the same time, the route selection rule based on the degree of route balance is applied to select the final optimal solution set. The solution with better balance degree will be taken as the best solution. Finally, some benchmark instances are used to test the effectiveness of proposed algorithm. The results reveal that the proposed algorithm outperforms the standard NSGA-II and Multi-objective Evolutionary Algorithm(MOEA). The experimental results also show that our algorithm has good stability.**

***Index Terms*—multi-objective optimization, school bus routing problem, NSGA-II, route balance, 2-opt.**

I. INTRODUCTION

SCHOOL bus route planning is an essential part of school bus operation service management. Reasonable school bus route planning scheme can not only reduce operation costs, but also improve the quality of school bus service. It is a kind of combinatorial optimization problem in operations research, also known as school bus routing problem (SBRP). SBRP seeks to find the optimal school bus routes to send students from the station to school or from the school to the station under certain constraints. SBRP belongs to an application category of vehicle routing problem (VRP) and it is also a NP-hard problem [1],[2]. Since SBRP was proposed, the model and algorithm of SBRP have been continuously concerned and studied. The latest research review is shown in literature[3].

The optimization objectives of SBRP usually include cost, quality and fairness[2]. Cost is usually the primary or only optimization objective. Cost-related objectives include total cost, total number of school buses, total travel distance, and total travel time. The quality goal is to measure the degree of service satisfaction, such as minimizing the student's riding time. The fairness goal is the length or load balance between different school bus routes. In the practice of school bus route planning, two or more optimization objectives usually need to be considered. Therefore, the study of school bus route planning with multi-objective optimization will be more in line with the needs of practical applications.

For the multi-objective school bus routing problem (MOSBRP), there are two methods in the existing literatures. One is the method of phased optimization, which divides the process of optimization into multiple stages and considers optimization objective in order of priority [4],[5],[6]. The other is the method of multi-objective linear weighting, which assigns a different weight to each objective [7],[8]. Although these two methods can solve the MOSBRP, it cannot meet the needs of practical application because of the strong subjective of the weight setting between multiple objectives and the difficulty of parameter setting.

In recent years, there have been some mature multi-objective algorithms in the field of VRP. For most of bi-objective algorithms, they commonly introduce special neighborhood operator or multi-heuristic technology into the conventional algorithm[10],[11],[12]. Since the multi-objective evolutionary algorithms based on Pareto set were proposed, different kinds of multi-objective optimization algorithms have been gradually applied to the field of VRP [13],[14]. For the bi-objective problem with different route balance criteria, an iteration flexible optimization algorithm was proposed in [13], which can get a better solution based on Pareto frontier. The fast non-dominated sorting genetic algorithm (NSGA) was used to solve the parallelization of VRP [15], and then it was continued to study the diversification of elite solutions and its parallelization [16],[17],[18].

The research results of multi-objective optimization in the field of VRP provide a direction for the multi-objective solution of SBRP. Some scholars have begun to use the method based on Pareto set to solve MOSBRP. A tabu search within the framework of multi-objective adaptive memory was proposed in [19] to solve the SBRP, which includes minimizing the longest driving route and the shortest total travel distance two objectives. A multi-objective ant colony optimization algorithm was designed in [20] to solve the fleet size and mix SBRP, considering the number of school bus and the average riding time of students. These two algorithms were both compared with the standard NSGA. Addressing the same problem as [20], the three objectives of total cost, total riding time of students and route length balance were taken into account by [21]. The iterated local search algorithm and path relinking techniques are introduced to solve the problem [21]. In the above multi-objective SBRP literatures, the standard NSGA used in [19] and [20] is just only employed to compare the experimental results. Although the mature non-dominated genetic algorithms have

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been widely applied in VRP, there are still only few applied researches in SBRP.

In view of this, we focus on the single-school SBRP, which the three optimization goals of route balance, total route number and total travel distance are considered. Then, we propose a MOSBRP optimization algorithm based on NSGA-II (denoted as H-NSGA-II) for this problem. The proposed algorithm uses 2-opt to improve solutions at each iteration. Meanwhile, the route selection rule with balance degree as preference is defined. Finally, the effectiveness of the algorithm is verified by the benchmark instances.

The paper is structured as follows: The MOSBRP is defined in Section II. Section III describes the proposed H-NSGA-II for the MOSBRP in details. Computational results and comparison analysis are presented in Section IV. Section V presents the concluding remarks of this work.

II. Problem Definition

This paper researches the multi-objective routing optimization problem for serving a single school. Assuming that the depot is at the school, let $G = (V, E)$ be the complete weighted graph, where the set $V = \{0, 1, 2, ..., n\}$ represents the school and the student bus station. 0 represents school S, and P is the set of bus stations, so $V = (S, P)$. $E = \{(i, j)|i, j \in V, i \neq j\}$ represents the set of edges between any stations. The driving distance between any two stations i and j is d_{ij} , and $d_{ij} = d_{ji}$. There are some homogenous school buses of number K at the depot, that is, all school buses have the same capacity. The decision variable x^k_{ij} ($x^k_{ij} \in \{0, 1\}$) is used to indicate whether a school bus drives from station i to station j. At the same time, the number of students at each student bus station is less than the capacity of the school bus. Each school bus departs from the depot, passes through the student station at a certain speed and sends the students to the school. The service time required for students at each station is related to the number of people at that station.

The constraints are described in the following. Each student station must be served and only be served by one school bus. The number of passengers on the school bus cannot exceed the capacity of the school bus at any time. The riding time of all students on the school bus cannot exceed the maximum riding time.

The mathematical model of the problem can be modeled by a mixed integer linear programming(MIP). Because of the constraints of the model is same as that in [6], we only give the optimization objectives of the research problem. The optimization objectives include route balance, the number of school buses and the total travel distance. Next, the three optimization objectives are defined.

1) route balance

The route balance goal is to make every school bus travel the same distance as possible. The equation (1) defines the average length of the route. The equation (2) gives the definition of the balanced degree, and the balance goal is measured by the value of the balanced degree. It is agreed that the smaller value of equation (2) means the better balance between the routes. If K equals 1, we think the degree of route balance is zero ($f_1 = 0$).

$$Bl = \frac{1}{K} \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} x^k_{ij} d_{ij} \tag{1}$$

$$f_1 = \sqrt{\frac{1}{K-1} \sum_{k\in K} \Big(\sum_{i\in V} \sum_{j\in V} x^k_{ij} d_{ij} - Bl\Big)^2} \tag{2}$$

2) number of school buses

The number of school buses goal is to use as few buses as possible to pass through all stations. The total number of vehicles is denoted by K. In other words, the smaller value of equation (3) means the better number of school buses.

$$f_2 = K \tag{3}$$

3) total travel distance

The total travel distance goal of is the total distance required for all school buses to travel through all student stations. Its definition is shown in equation (4).

$$f_3 = \sum_{k\in K} \sum_{i\in P} \sum_{j\in V, j\neq 0} x^k_{ij} d_{ij} \tag{4}$$

Based on the above definitions of the objective function, the multi-objective function of our problem is defined in equation (5).

$$min \quad F = (f_1, f_2, f_3)^T \tag{5}$$

III. Algorithm Design

A. Basic Description of NSGA-II

NSGA-II is one of the most widely used methods to solve multi-objective optimization problems [15]. The NSGA-II algorithm finds the solution set that makes each objective function value reach the optimal value as much as possible by coordinating the relationship between each objective function. The fast non-dominated sorting is to stratify the population according to the optimal solution level of the individual. Meanwhile, its function is to guide the search to the direction of the Pareto optimal solution set. This method improves the speed of the stratification of the dominated relationship between individuals. Then, the individuals in the same non-dominated front are selected by crowding distance. After fast non-dominated sorting and crowding distance assignment, the algorithm prefers to select the solution with lower rank value for the two solutions in different fronts. When the two solutions locate at the same front, the solution with the larger crowding distance is preferred to protect the diversity of the population. Therefore, NSGA-II can effectively find the Pareto set for each goal to reach the optimal value, and it has been widely used in vehicle routing problems[15],[16],[17],[18],[19].

B. Chromosome Coding

In this paper, sequential integer coding is used to describe the problem, which can directly reflect the route of the vehicle. Each chromosome represents a route scheduling scheme, and the genes in the chromosome denote student stations. The number of chromosome genes is the number of student stations. Firstly, these student stations are encoded by integer to generate the unique identification code of each station. Then, the station integer identification codes are randomly sorted to generate the initial population representing the scheduling routes.

The coding requirements for chromosomes are described as follows. The school node is represented by the integer 0, and student stations are represented by the sequence of integers, which starts from 1 and ends the number of student stations. There are no chromosome gene positions of the demarcation point. So the school station no longer appears in the chromosome. A chromosome must contain all stations that need to be served. The sequence of numbers in the gene sequence represents the access order of the school bus from one station to another.

C. Initial Population Construction

According to the coding method, the chromosome should be a full array of natural numbers. Not all chromosomes produced by coding are actually effective. According to the constraints determined in Section II, we need to determine the validity of chromosomes. In this paper, the initial population is randomly generated. In the initial population, each individual is generated by randomly sorting all student stations. The sequence of the chromosome gene is generated after the student stations are randomly sorted. These chromosomes could be divided into a set of sub-routes by the constraints.

For example, there are 17 student stations that need to be accessed, and student stations are represented by integers between 1 and 17. According to the constraints, the number of students on a bus passing through a station cannot exceed the school bus capacity and students' maximum riding time. If the constraint is violated, a new school bus is used to serve the station. Repeat the process until all stations in the sequence are served. Finally, a chromosome is divided into a set of several routes, and the number of routes is the number of vehicles. In this way, suppose the chromosome gene sequence is $\{$10, 1, 15, 6, 5, 13, 14, 17, 4, 2, 12, 8, 16, 3, 7, 9, 11$\}$. Then we split the chromosome to some sub-routes by the constraint conditions. The sub-routes may be denoted as $\{$0-10-1-15-6-0$\}$, $\{$0-5-13-14-17-4-0$\}$, $\{$0-2-12-8-0$\}$ and $\{$0-16-3-7-9-11-0$\}$. For every sub-route of this chromosome, it must be feasible. That is to say, each sub-route does not violate the constraints.

D. Selection

For the proposed algorithm in this paper, the selection method includes the selection of individuals after the combination of parent population and the offspring population, and the selection of optimal individuals.

Assuming that the size of initial population is N, the N individuals need to be selected from the individuals of size $2N$ when the parent population merges with the offspring population. Specifically, the non-dominated front of individuals is obtained through a fast non-dominated sorting. After sorting, the non-dominated individuals are selected as the offspring individuals. The individuals in the non-dominated front are selected based on the crowding distance when two individuals do not dominate each other.

For the optimal individual selection, we adopt the tournament selection method. Each time m individuals are extracted from the parent population, and the optimal individual among them is selected as the offspring. The process repeats until the initial population of the next generation is obtained. When these individuals of size m have a dominated relationship, the individuals who are not dominated are selected. When the individuals do not dominate each other, we select individuals based on the crowding distance. This select method has the

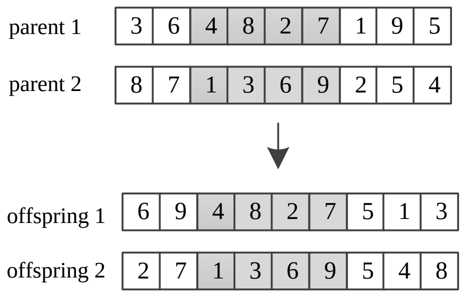


Fig. 1. Example of ordered crossover

advantage of lower calculation complexity. At the same time, the selected individuals have better fitness, which it is not easy to fall into the local optimum. It also can ensure that the high-quality individuals in the population are retained.

E. Crossover

This paper uses ordered crossover (OX) to obtain individuals with higher fitness. First, the same starting and ending crossover positions are randomly selected in the two parent chromosomes. Second, the gene segments of parent 1 are copied to the same position of offspring 1. Third, the unfilled genes in offspring 1 are filled in after its second crossover positions in order on parent 2. If the selected gene already exists in the offspring 1, skip the gene and select the next. The other offspring is obtained in the same way. This crossover method will not output an invalid and infeasible individuals, and there is no need for conflict detection. The procedure of Ordered crossover is shown in Fig. 1.

Seen from Fig. 1, there are two parents $\{$3, 6, 4, 8, 2, 7, 1, 9, 5$\}$ and $\{$8, 7, 1, 3, 6, 9, 2, 5, 4$\}$. The genes between the third and sixth positions of the two parents are selected as crossover segments. For offspring 1, the genes between the third and sixth, that is $\{$4, 8, 2, 7$\}$, are the same as those in parent 1. The remaining genes are obtained from parent 2 in order and start from the next gene after the crossover segments, that is, the seventh gene. If the last position is filled, fill the remaining gene from the first position to the next in order. The genes in parent 2 that have existed in offspring 1 will be skipped. Therefore, the genes list $\{$5, 1, 3, 6, 9$\}$ will be filled to offspring 1. The three genes 5, 1, 3 will be filled orderly in position 7 to 9 of offspring 1. The two genes 6 and 9 will be filled in first and second position of offspring 1. After the OX operator, the offspring 1 is $\{$6, 9, 4, 8, 2, 7, 5, 1, 3$\}$. While for offspring 2, the remained genes from parent 2 is $\{$1, 3, 6, 9$\}$. Like the same operation of offspring 1, the genes list $\{$5, 4, 8, 2, 7$\}$ from parent 1 will be filled to offspring 2. Similarly, the offspring 2 is $\{$2, 7, 1, 3, 6, 9, 5, 4, 8$\}$.

F. Mutation

In order to keep the diversity of individuals and prevent the occurrence of too fast convergence to obtain the local optimum, this paper uses reverse mutation to change the gene sequence of the chromosome. The genes at two different positions in the chromosome are randomly selected and then exchanged to generate a new individual as the offspring. The mutation probability adopts a fixed value, which is

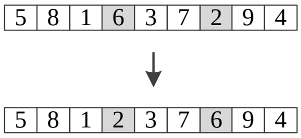


Fig. 2. Example of inverse mutation

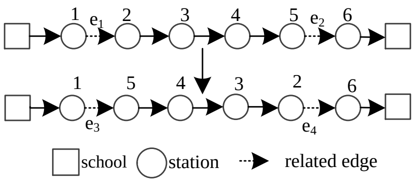


Fig. 3. Example of 2-opt

implemented in all iteration. The reverse mutation operation is shown in Fig. 2.

Suppose that the mutation positions are the fourth and seventh positions in Fig. 2. That is, the original gene 6 and 2 are reversed. The individual obtained after mutation is shown on the bottom of Fig. 2. The mutation process takes into account the adjacency relation with the original edge, which can better inherit the excellent gene performance on the route to the next generation, and improve the optimization speed.

G. 2-opt Operator

For the individuals obtained in each iteration, the 2-opt operator is used for optimization to reduce the total route length of the individuals. 2-opt is a local search operator that executes within a route. The principle is to remove two non-adjacent edges, reverse the sequence of stations between the two edges, and then add two edges to form a new route. The operation of 2-opt operator is shown in Fig. 3.

In Fig. 3, the edges $e1$ and $e2$ are removed from the original route firstly, and then the student stations from 2 to 5 between these two edges are reversed. Finally, adding two new edges $e3$ and $e4$ and then a new route is generated. The 2-opt operator is used to improve the excellent individuals in the population after the mutation operation, which helps to reduce the number of iterations required for convergence, thus improving the search efficiency.

H. Route Selection Rule

The SBRP solved in this paper has three objectives, which are in conflict and difficult to achieve the optimal at the same time. After the algorithm is executed, the final route plan is selected by this rule from a set of feasible Pareto solutions. We adopts a route selection rule with balance degree as the priority. As mentioned above in Section II, the three objectives functions are f_1, f_2 and f_3, which denotes the balance degree, total number of school buses and total travel distance respectively. For two solutions S_1 and S_2, the route selection rule is defined in the following.

(1) If $f_1(S_1) < f_1(S_2)$, the solution S_1 is selected.

(2) If $f_1(S_1) > f_1(S_2)$, the solution S_2 is selected.

(3) If $f_1(S_1) = f_1(S_2)$, we continue compare the value of second objective function f_2. The solution has lower the value of f_2 will be selected. If $f_2(S_1) = f_2(S_2)$, the value of third objective function f_3 will be compared, and then the solution with lower f_3 function value will be chosen as the final solution.

I. The Process of Designed Algorithm

The process of the improved algorithm based on NSGA-II proposed in this paper is shown in Fig. 4. The specific algorithm steps are described as follows.

(1) Initialize population size N, maximum iteration number $maxGen$, the number of selected parents m, crossover probability p_c and mutation probability p_m. Let initial iteration number t is set to 0. Generate the initial population P_0 of size N.

(2) Create the first generation offspring population Q_0 by the basic genetic operations of tournament selection, order crossover and inverse mutation for the individuals in P_0. Make the size of Q_0 is also N.

(3) Combine current parent population and offspring population into a new population R_t of size $2N$. Then R_t is sorted according to non-dominated sorting and crowded distance sorting. The new parent population P_{t+1} of size N is formed by the non-dominated rank and the crowding degree.

(4) After the basic genetic algorithm operations such as tournament selection, order crossover, inverse mutation and 2-opt local search operator, are executed, a new offspring population Q_{t+1} is generated.

(5) If $t < maxGen$, go to step (3) until the termination condition is satisfied.

IV. Computational Experiment

A. Experimental Environment and Parameters Setting

The proposed algorithm is implemented by python programming in PyCharm on a personal computer with 64-bit Windows 10 operating system, and the processor configuration is: Intel(R) Core(TM) i5-7500 CPU 3.40GHz, 16GB RAM. All the value of parameters are obtained through previous experiments. The parameters are be set as follows: the parameter N is set to 400, the parameter m of each parent selection is set to 4, the parameter $maxGen$ is set to 200, parameter p_c is set to 0.85, and the parameter p_m is set to 0.02.

B. Test Instances

The SBRP benchmark instances published by [2] are used to test the performance of our proposed algorithm. According to the distribution of stations and schools, instances are divided into two groups: Random Dispersion of Schools and Bus Stops (RSRB) and Cluster Dispersion of Schools and Bus Stops (CSCB). We choose RSRB01 and CSCB01 as 12 single school instances. These instances are denoted as R01$\sim$R06 and C01$\sim$C06, and the number of stations is 17$\sim$75. The description of the instances and the related settings as well as the calculation of the service time are detailed in [6]. The capacity of each school bus is 66. The distance between any two points is calculated by the Manhattan distance. The speed of school bus is 20 miles per hour, and the maximum riding time of student is set to 2700 seconds. At the same time, the total distance of each route is calculated from the first student station on the route to the target school, and the route length is expressed in mile.

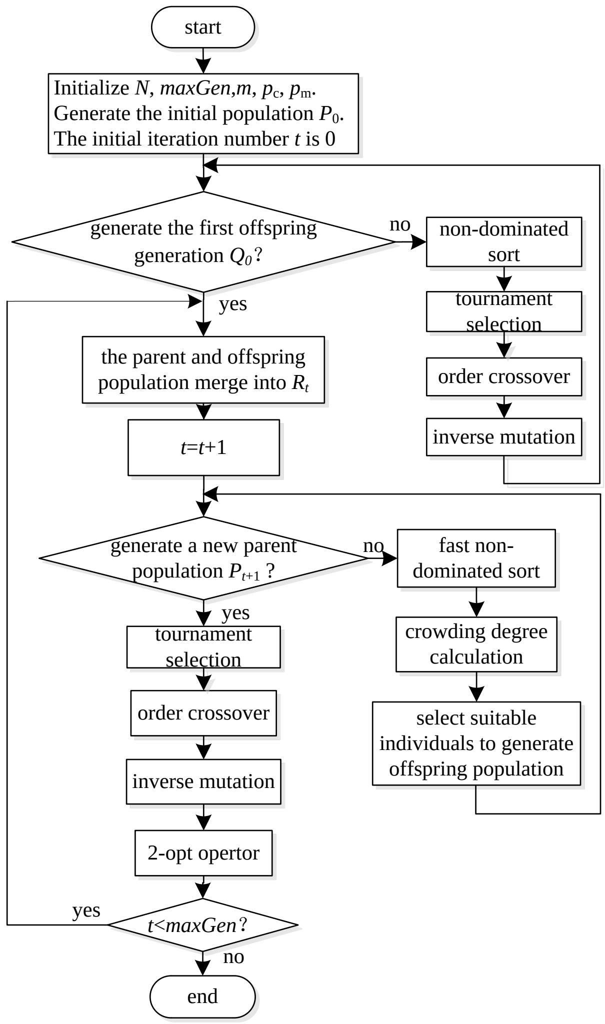


Fig. 4. Process of proposed H-NSGA-II algorithm

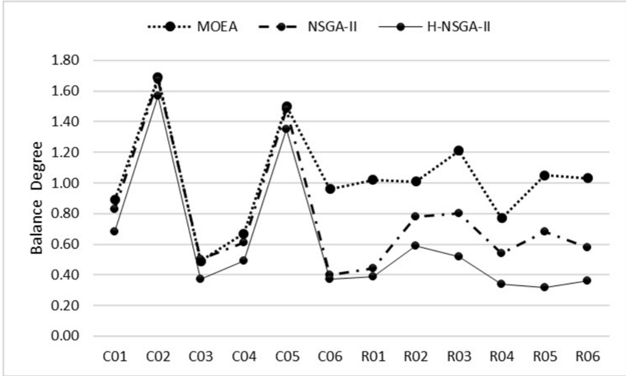


Fig. 5. Balance degree results comparison of three algorithms

C. Experimental Results and Comparison

In this section, we use our proposed algorithm to solve the twelve instances and compare it with the multi-objective evolutionary algorithm (MOEA) and the standard NSGA-II algorithm.

First of all, each instance was solved by MOEA, NSGA-II and H-NSGA-II separately under the same conditions. Each instance was executed 10 times, and then the optimal solution was selected according to the route decision rule. The optimal solutions obtained by these three algorithms are shown in TABLE I. The name and problem scale of each instance is given in columns *Instance* and *Stops*. The columns *BL*, *N*, and *TD* in TABLE I represent the values of the three objectives including balance degree, school bus number and total route distance in mile.

As shown in TABLE I, the H-NSGA-II algorithm can obtain better solutions than MOEA and NSGA-II on 12 instances. The H-NSGA-II find lowest average balance degree, average total number of buses and average total route distance. Because our solution selection rule is based on the balance degree, the H-NSGA-II algorithm reduce the balance degree by 40.20% and 21.05% on average when compared with MOEA and NSGA-II algorithms. For MOEA, the H-NSGA-II algorithm has fewer balance degree, total route number and total travel distance on all the instances. When compared with NSGA-II, there are five instances including C03, C04, R01, R02 and R03, whose the total route distance are increased slightly while the balance degree are decreased. This is because that the balance degree tends to make each route have same length as much as possible, which conflicts with the goal of total route distance. When choosing the Pareto optimal solutions, the route selection rule designed in this paper tends to accept the solution with a smaller balance degree.

Next, we analyze balance degree of each instance on different instance types in TABLE I. The trend of balance degree of all instances is shown in Fig. 5. As shown in Fig. 5, the H-NSGA-II algorithms can find the lowest value of balance degree. For different instance types, the decrease in the balance degree of the instances R01∼R06 is more obvious than that of the instances C01∼C06. For C01∼C06, the school and student stations are located in cluster, so their routes may be centralized and not easy to be changed owing to the strict constraints such as maximum ridding time of students.

D. Stability Analysis

We calculate the best solution (BS), the worst solution (WS), the average solution (AS) and standard deviation(STD) of each instances obtained by the three algorithms. We use BS, WS, AS and STD four indicators to evaluate the stability of these three algorithms. The statistical results are shown in TABLE II and TABLE III. The meanings of *BL*, *N* and *TD* in the two tables are the same as those in TABLE I. The columns *STD* represents the standard deviation value.

It can be seen from TABLE II and TABLE III, the H-NSGA-II algorithm outperforms the MOEA and NSGA-II algorithms. It gets lowest worst solution, best solution and average solution in general. Compare with the MOEA algorithm, the balance degree, the total number of routes and total distance of average solutions of C01∼C06 instances are reduced separately by 24.4%, 14.77% and 11.57% on average. The same three indicators of average solutions of R01∼R06 are decreased by 50.95%, 18.41% and 11.53% on average respectively. Form the results, we can find that the proposed algorithm is more competitive than MOEA algorithm. For NSGA-II algorithm, the balance degree of average solutions of C01∼C06 is also decreased separately by 5.06%, but the total route number and total distance of average solutions are increased slightly by 0.65% and

TABLE I
RESULTS OF THREE ALGORITHMS ON TWELVE INSTANCES

Instance	Stops	MOEA			NSGA-II			H-NSGA-II		
		BL	N	TD	BL	N	TD	BL	N	TD
C01	70	0.89	30	357.84	0.83	27	330.03	0.68	24	295.84
C02	35	1.69	16	181.34	1.67	14	156.57	1.57	14	154.92
C03	30	0.49	14	163.87	0.50	10	117.54	0.37	11	126.78
C04	23	0.67	10	109.26	0.61	8	101.11	0.49	8	104.95
C05	75	1.50	34	387.81	1.48	31	352.53	1.35	31	351.78
C06	17	0.96	7	71.11	0.40	6	67.07	0.37	6	66.94
R01	38	1.02	13	149.01	0.44	12	132.73	0.39	12	137.61
R02	40	1.01	16	187.14	0.78	12	139.27	0.59	12	140.88
R03	51	1.21	18	200.36	0.80	15	167.96	0.52	16	173.07
R04	35	0.77	12	146.02	0.54	12	145.99	0.34	12	145.48
R05	42	1.05	16	173.96	0.68	13	149.51	0.32	13	149.32
R06	44	1.03	16	184.34	0.58	13	157.27	0.36	12	142.59
Average	41.67	1.02	16.83	192.67	0.78	14.42	168.13	0.61	14.25	165.85

TABLE II
FOUR INDICATORS OF THREE ALGORITHMS ON C01$\sim$C06 INSTANCES

Instance	Indicators	MOEA			NSGA-II			H-NSGA-II		
		BL	N	TD	BL	N	TD	BL	N	TD
C01	WS	2.62	31	336.26	1.64	26	316.58	1.56	26	311.93
	BS	0.89	30	357.84	0.83	27	330.03	0.68	24	295.84
	AS	1.37	30.60	356.12	1.16	25.50	306.52	1.13	25.60	311.24
	STD	0.53	1.11	13.58	0.27	0.67	10.46	0.22	0.66	8.51
C02	WS	2.61	15	162.76	1.91	13	148.82	1.87	13	148.16
	BS	1.69	16	181.34	1.67	14	156.57	1.57	14	154.92
	AS	1.91	16.00	178.93	1.80	13.60	153.56	1.72	13.70	152.78
	STD	0.27	0.77	10.07	0.08	0.49	5.24	0.10	0.46	3.52
C03	WS	3.07	11	121.50	1.21	9	110.09	1.05	10	124.19
	BS	0.49	14	163.87	0.50	10	117.54	0.37	11	126.78
	AS	1.17	11.70	136.61	0.72	9.90	117.86	0.68	10.20	121.82
	STD	0.75	1.00	12.82	0.21	0.30	3.02	0.18	0.40	4.32
C04	WS	2.28	9	93.98	1.14	8	90.40	1.23	7	81.51
	BS	0.67	10	109.26	0.61	8	101.11	0.49	8	104.95
	AS	1.05	8.90	98.59	0.86	7.90	90.06	0.85	7.80	89.45
	STD	0.48	0.94	9.00	0.15	0.30	5.38	0.20	0.40	6.48
C05	WS	2.75	37	411.87	2.05	27	305.45	1.95	29	330.84
	BS	1.50	34	387.81	1.48	31	352.53	1.35	31	351.78
	AS	1.78	35.40	398.83	1.77	29.40	339.28	1.69	29.60	343.64
	STD	0.39	0.92	10.50	0.19	1.20	14.56	0.17	0.80	9.43
C06	WS	2.49	7	59.38	1.79	6	56.43	1.44	6	63.63
	BS	0.96	7	71.11	0.40	6	67.07	0.37	6	66.94
	AS	1.90	6.40	60.05	1.00	6.00	65.48	0.87	6.00	68.05
	STD	0.62	0.49	5.30	0.42	0.00	4.23	0.35	0.00	2.82

1.33%. While for R01$\sim$R06 instances, the results of first two indictors are also improved by 24.27% and 0.66% on average respectively. The total distance of average solutions are increased by 1.93%. In consideration of the priority of balance degree, the H-NSGA-II algorithm is more effective than MOEA and standard NSGA-II algorithms.

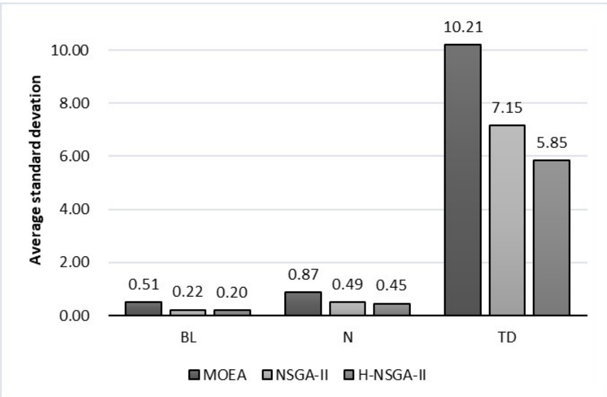
According to the TABLE II and TABLE III, we calculate the average standard deviation of all the instances and the results are shown in Fig. 6. As shown in Fig. 6, the H-NSGA-II algorithm have lowest average standard deviation on three optimization objectives, including balance degree (BL), total number of routes (N) and total travel distance(TD). The results demonstrate that our proposed algorithm is very stability. And we also find that average standard deviation of C01$\sim$C06 instances is smaller than that of R01$\sim$R02 instances. It can be explained that the solutions of C01$\sim$C06 may be located in a relatively small solution space, due to cluster distribution of school and bus stops.

E. Analysis of Pareto Solutions

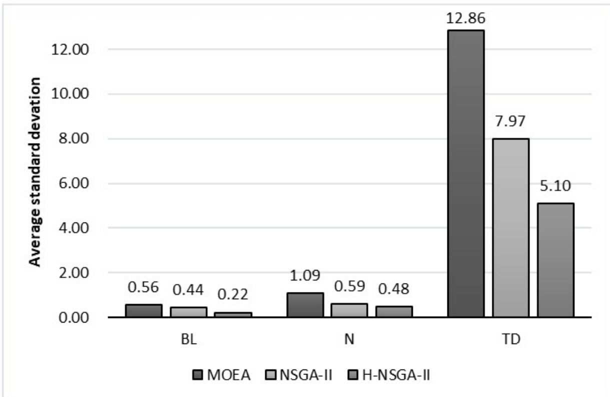
In this section, we compare and analyze the Pareto non-dominant solutions that obtained by MOEA, NSGA-II and

TABLE III
FOUR INDICATORS OF THREE ALGORITHMS ON R01∼R06 INSTANCES

Instance	Indicators	MOEA			NSGA-II			H-NSGA-II		
		BL	N	TD	BL	N	TD	BL	N	TD
R01	WS	2.57	13	136.12	2.07	11	124.08	1.4	11	134.13
	BS	1.02	13	149.01	0.44	12	132.73	0.39	12	137.61
	AS	1.62	13.60	145.75	1.00	11.60	129.92	0.80	11.40	133.65
	STD	0.49	0.92	9.10	0.47	0.49	6.90	0.26	0.49	3.21
R02	WS	3.34	16	154.42	1.84	12	133.9	1.21	12	141.42
	BS	1.01	16	187.14	0.78	12	139.27	0.59	12	140.88
	AS	1.82	15.80	171.71	1.12	12.30	143.41	0.85	12.40	145.73
	STD	0.76	1.25	16.42	0.30	0.74	8.66	0.18	0.49	6.78
R03	WS	3.26	17	168.66	2.61	15	153.57	1.36	15	165.54
	BS	1.21	18	200.36	0.80	15	167.96	0.52	16	173.07
	AS	1.97	17.80	186.71	1.28	15.60	168.32	0.91	15.30	172.47
	STD	0.60	0.60	9.96	0.64	0.49	6.24	0.24	0.46	3.06
R04	WS	1.73	14	156.31	1.62	11	127.85	1.10	12	145.58
	BS	0.77	12	146.02	0.54	12	145.99	0.34	12	145.48
	AS	1.13	14.40	163.45	0.78	11.60	140.54	0.72	11.80	141.94
	STD	0.32	1.50	15.07	0.32	0.66	8.37	0.18	0.40	4.42
R05	WS	3.48	15	139.11	1.96	13	132.83	1.18	12	144.38
	BS	1.05	16	173.96	0.68	13	149.51	0.32	13	149.32
	AS	1.92	15.90	164.37	1.14	12.90	144.06	0.76	12.70	148.13
	STD	0.80	1.22	14.44	0.46	0.70	9.88	0.22	0.64	5.45
R06	WS	2.10	14	155.61	1.82	12	139.29	1.14	13	152.14
	BS	1.03	16	184.34	0.58	13	157.27	0.36	12	142.59
	AS	1.53	15.40	172.08	1.15	12.30	145.20	0.86	12.20	146.37
	STD	0.36	1.02	12.16	0.46	0.46	7.77	0.24	0.40	7.65



(a) C01∼C06 instances



(b) R01∼R06 instances

Fig. 6. Average standard deviation of three algorithms on instances belonged to different types

H-NSGA-II algorithms. Every algorithm is executed 30 times under the same condition. We select small instance C06, medium instance R01 and large instance C01 to observe the distribution of non-dominant solutions, and then draw the distribution of non-dominant solutions in the coordinate axis.

Fig. 7 shows the distribution of Pareto non-dominant solutions of instance C06. For C06, the distribution image of the non-dominant solution sets is two-dimensional, because the number of school buses in the non-dominant solutions is the same. It can be seen from Fig. 7 that the image presented by the optimal solution tilts to the lower right, indicating that all solutions in the solution set are not dominated by each other. The H-NSGA-II algorithm has the most non-dominant solutions and the best target values, when compared with MOEA and NSGA-II algorithms.

For instances R01 and C01, their Pareto non-dominant solutions distributions shown in Fig. 8 are three-dimensional. The reason is that the numbers of school buses of their non-dominant solutions are different. The three objectives of non-dominant solution are shown in X, Y and Z axes respectively. Fig. 8 (a) shows the non-dominated solutions of instance R01. For the NSGA-II and H-NSGA-II algorithms, they both have 10 non-dominated solutions. But for the MOEA algorithm, it just has 7 non-dominated solutions. Form the view of optimization objectives, the H-NSGA-II algorithms have the better target values than the standard NSGA-II algorithm. When the instance scale reaches 70

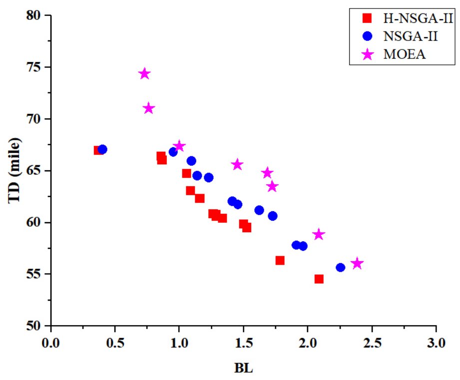
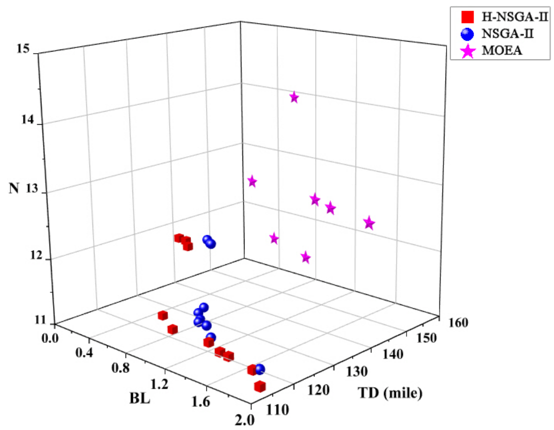
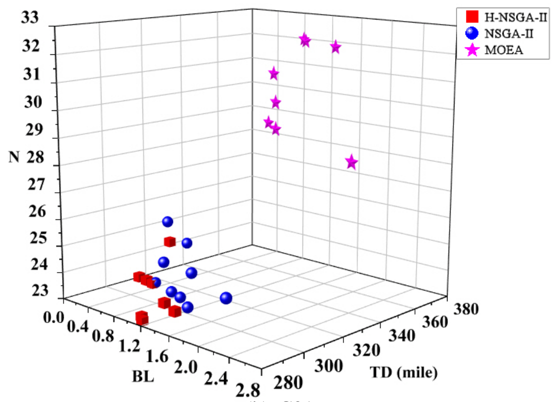


Fig. 7. Pareto solutions distribution of instance C06



(a) R01



(b) C01

Fig. 8. Pareto solutions distribution of instances R01 and C01

student stations, the non-dominant solutions distribution of C01 are shown in Fig. 8 (b). The H-NSGA-II algorithm and NSGA-II algorithm are both obviously better than the MOEA algorithm. Although the number of non-dominated solutions obtained by the H-NSGA-II algorithm is a little less than NSGA-II algorithm, the optimization objective values of it is the better than the NSGA-II algorithm.

In general, we can find that the H-NSGA-II algorithm has the best target values of three objectives and its dimension distribution of non-dominated solution set is the smallest. These results also reveal that the H-NSGA-II algorithm is very effective and reliable.

V. Conclusion

In this paper, a multi-objective optimization algorithm based on NSGA-II is proposed to optimize the three objectives of single-school SBRP, which minimizes route balance, total route number and total travel distance. Based on the original NSGA-II, the proposed algorithm(H-NSGA-II) uses 2-opt neighborhood operator to optimize the individuals after each iteration. The final solution is selected by a route decision rule, which consider that the balance degree of solution has the highest priority. For some SBRP benchmark instances, the H-NSGA-II algorithm, the standard NSGA-II algorithm and classical multi-objective evolutionary algorithm (MOEA) are executed respectively to evaluate their performance. The experimental results show that the proposed algorithm is more effective in performance than MOEA and standard NSGA-II. At the same time, the H-NSGA-II algorithm has a good stability.

In the future, we will improve the proposed algorithm and extend the application of it in other multi-objective school bus routing problems, such as heterogeneous SBRP, multi-depot SBRP or multi-school SBRP and so on.

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