A Mathematical Model of Risk Assessment on Airborne Infection in a Room with an Outlet Ventilation System

Wasu Timpitak, and Nopparat Pochai

***Abstract*—The airborne infection is spread through the air, especially in indoor spaces. Indoor spaces present a significant risk of infection, although this may be reduced by employing all methodologies to prevent infection via aerosols. TB, COVID-19, MERS, and SARS are all hazardous communicable diseases that spread from person to person through air or aerosol in a variety of ways, including coughing, spitting, sneezing, speaking, or through wounds. COVID-19, TB, MERS, and SARS are all risks, and the elevated risk of a lethal infection leads more patients to become infected in indoor spaces. We should also be notified about the recognition and prevention of these diseases. As a result, proper air quality control, such as carbon dioxide (CO2) concentrations, is needed to monitor and reduce the potential for infected air. It is difficult to assess and monitor carbon dioxide in a room with a ventilation system where the number of people in each room changes frequently. In this research, the numerical model of carbon dioxide concentration measurement in a space with an opened ventilation system is proposed. The model is used to calculate the concentration of carbon dioxide at any time when the number of persons and the rate of ventilation vary. The standard fourth-order Runge-Kutta method is employed to approximate the model solution. There are many scenarios for improving air quality in the suggested simulations. The proposed model for the air quality control system achieves a balance between the number of persons permitted to remain in the room and the air ventilation system's efficiency.**

***Index Terms*—airborne, infectious, diseases, ventilation system.**

I. Introduction

INFECTIOUS disease of airborne such as tuberculosis (TB) spread in several gathering locate areas with infectors and poor ventilation per person rates, [1],[2],[3],[4], and [5]. In [6],[7],[8], and [4], they proposed infectors could be dangerous if there no is high concentration of indoor rebreathed air because it could contain infector-borne infectious particles, which could lead to the spreading of airborne infectious illnesses like TB. In [6], and [9], they proposed carbon dioxide(CO_2) be used as an indicator of air quality indoor, built on the notion that people release CO_2 at a rate dictated by their body weight and bodily movement, and that levels of CO_2 indoor are measured by fresh air clearance. In [6],[10], and [9], they propose CO_2 concentration in the air of approximately 400 ppm in a area, but when people enter it, exhaled air concentration begins to rise, depending on the rate of ventilation each person, the length of the room, and the proportion of persons in the area, because of their oxygen intake, respiratory quotient, and bodily movement, the person in the area add to the rise in rebreathed air. In [4], and [2], they proposed that as the exhaled air concentration in a room rises in the presence of infectors, the probability of vulnerable individuals contracting infectious diseases transmitted by the air, this is because contaminated people's exhaled air also contains contagious airborne particles inside the nuclei with droplets that can stay airborne for extended periods and infect a susceptible person when inhaled. In [2] and [11], they proposed that the status of the immunological system of the host, host physiology, and the virulence of the Mycobacterium tuberculosis (Mtb) infectious strain are all important factors in the advancement of infection to TB disease.

In [12], they proposed that sneezing, laughing, singing, and crying would all contribute to the production of respiratory particles. In [13], and [14], they proposed when a susceptible person inhales airborne infectious particles, only a fraction of infective particles inhaled would successfully penetrate the respiratory tract infection's target site. In [15], they proposed a numerical model that can be used to describe the dynamic dispersion of airborne infectious diseases in an outpatient room. In [16], and [17], they proposed a critical scale range of infective particles of 1 Mm to 5 Mm, have a better risk of touching and deposition on the region of the alveoli than those with sizes larger than 5 Mm, which are stuck in the system of upper respiratory, according to the researchers. The respiratory concentration fraction airborne infective particles must also be considered when determining the possibility of airborne infectious disease. In [18], the main route is a droplet or an airborne transmission. Outside, where there is better ventilation, the risk of infection is considered to be much lower. As the northern hemisphere faces winter, opportunities for socialization and outdoor exercise are becoming more challenging and concerns about the increased risk of COVID-19 transmission are growing. In [19], they proposed developing a model for superspreading episodes of infectious diseases based on the SARS epidemic. In [20], they proposed the random forest method to fuse the WRF model, using the atmospheric pollutant concentration and fundamental meteorological parameters training model, and add the atmospheric thermal stability factor as an extra element to model and forecast the municipal PM2.5 concentration. In [21], they proposed that more secure indoor surroundings are required not simply to protect the unvaccinated and those for whom

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vaccines have failed but to prevent vaccine-resistant variations or novel airborne dangers from emerging anywhere at a time. Improved interior ventilation and air quality, especially in health, work, and educational institutions, would help us all stay healthy in the present and future. In this research, a mathematical model of risk assessment on airborne infection in a room with outlet ventilation system is introduced.

II. GOVERNING EQUATION

In general, the exhaled air output rate and ventilation per person determine the elevated concentration of indoor CO_2 [6],[8], and [9]. Since particles of infection are found in exhaled air from an infected human, exhaled air can be substituted with CO_2 levels [6],[7],[9],[4], and [22]. Exhaled air comprises around 40,000 ppm of CO_2, equivalent to about 400 ppm of CO_2 in the air of the environment [6],[4], and [3].

We suppose that an indoor area, such as a room with a volume of V, begins the day with an environment CO_2 concentration of C_E roughly 400 ppm and is inhabited by the number of people(n). Given the presence of infectors, the concentration of exhaled air that may include airborne contagious particles may tend to rise in the room, determined by the rate of ventilation (Q) and n. We simply assume that persons in the room make a significant contribution to the production of CO_2, which serves as an exhaled air marker. The general equation of the accumulation rate exhaled air concentration in a room with C_E, is equivalent to the exhaled air rate generated by inhabitants plus the rate of C_E, minus Q removes exhaled air:

$$V\frac{dC}{dt} = npC_a + QC_E - QC, \tag{1}$$

In this research, we focus on airborne infections generated by inhabitants. The fundamental equation of the accumulation rate exhaled air concentration in a room, is equivalent to the exhaled air rate generated by inhabitants minus ventilation rate removes exhaled air is introduced. If the value of Q is assumed by Q_{out}, then this value is named the outlet ventilation rate. In a simple scenario, a number of people are unstable and depend on the time assumed by $n(t)$. This study preferred to use Eq.(1) as follow:

$$V\frac{dC}{dt} = n(t)pC_a - Q_{out}C, \tag{2}$$

for all $0 \leq t \leq T$. Where C is the concentration of air exhaled indoors (ppm), p is the rate of respiration in the room (L/s) for each person and C_a is a fraction of the airborne infection concentration contained inbreathed air. t is the duration time and T is the stationery simulation time. Initial condition $C(0) = C_0$ where C_0 is the latent airborne infection concentration.

A. the volume fraction of exhaled air under unsteady-state

In Eq.(2), calculation of the volume fraction of exhaled air in a room with an outlet ventilation system under unsteady state conditions, we get

$$f(t) = \frac{C(t)}{C_a}. \tag{3}$$

B. The airborne infectious particles concentration

Any infectious particles that become lodged in the upper respiratory tract or other regions of the body may be impacted, even if the probability of infection is virtually nil. Assume that β is the infector's production rate of total released airborne infectious particles and μ is the rate of infectious particles death in the air caused by the infector that cannot be embedded in the alveoli layer. As a result, the rate of survival of airborne infectious particles generated by the infector that reaches its target the infected area of the person who is vulnerable to infection at a threshold value $(\beta - \mu)$ (particles per second) as illustrated in Figure 1.

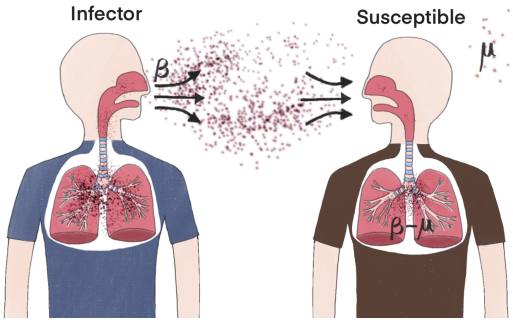

Fig. 1. Movement of airborne infectious particles.

The infection-causing concentration of infectious particles in the air, $N(t)$, is expressed as [23]:

$$N(t) = \frac{If(t)(\beta - \mu)}{np}, \tag{4}$$

where I is the number of people infected inside the room. So, the airborne infectious particles concentration that cause airborne infection unsteady-state conditions are derived by replacing Eq.(3) into Eq.(4):

$$N(t) = \frac{IC(t)(\beta - \mu)}{npC_a}. \tag{5}$$

C. The number of airborne infectious particles

Not that all infected particles can reach the alveolar cavity and deposit there; let θ be the proportion of airborne infected particles that penetrate and deposit at the host's location of the infected area. As a result, the number of airborne infectious particles(λ), inhaled by a susceptible individual and resulting in infection is expressed as [23]:

$$\lambda(t) = pt\theta N(t), \tag{6}$$

where t is the time consumed in the room got to the moment of infection in the room and $(0 < \theta < 1)$.

D. The probability of airborne infectors

In [2],[3] and [23], they proposed tuberculosis transmission follows a Poisson distribution, the probability of airborne infectors is expressed as:

$$P(T \leq t | I, Q, V, p, \theta, \mu, \beta) = 1 - e^{-\lambda(t)}, \tag{7}$$

where P is the probability of susceptible individuals with airborne infectors risk and $T \leq t$ are the random parameters that represent the infection risk for susceptible individuals up to around the time lived in a restricted location during an infected environment.

III. Numerical technique

There will be no continuous approximation to the solution $C(t)$; instead, approximations to C will be constructed at various values as in interval $[0, T]$, known as mush points.

Once the estimated solution at the points is determined, an approximation can be used to find the approximate solution at other points in the interval. We first make the stipulation that the mesh points are distributed equally over the interval $[0, T]$.

This condition is achieved by selecting a positive integer N and the mesh points $t_i = a+ih$, for each $i = 0, 1, 2, ..., N$. The general distance between the points $h = (T - 0)/N = t_{i+1} - t_i$ is called the step size.

A. Fourth-Order Runge-Kutta Method

The classical fourth-order Runge-Kutta method is expressed as [24]:

$$C \cong C_i \tag{8}$$

$$C_{i+1} = C_i + \frac{1}{6}(k_1 + 2k_2 + 2k3 + k_4)h \tag{9}$$

$$k_1 = f(t_i, C_i) \tag{10}$$

$$k_2 = f(t_i + \frac{1}{2}h, C_i + \frac{1}{2}k_1 h) \tag{11}$$

$$k_3 = f(t_i + \frac{1}{2}h, C_i + \frac{1}{2}k_2 h) \tag{12}$$

$$k_4 = f(t_i + h, C_i + k_3 h) \tag{13}$$

from Eq.(2), we get the classical fourth-order RK method

$$\frac{dC}{dt} = f(t_i, C_i) \tag{14}$$

$$f(t_i, C_i) = \frac{1}{V}(n(t)pC_a - Q_{out}C_i) \tag{15}$$

IV. Numerical experiments and results

Assuming that the respiration rate assumed by $p = 0.12$ (L/s) and a fraction of the Covid-19 concentration contained inbreathed air $C_a = 0.04$. By employing the classical fourth-order Runge-Kutta method Eqs.8-15.

A. Simulation 1 : The concentration measurement of exhaled air with difference number of people in a room.

The parameters are assumed in Table I. The number of people in the room is assumed in three cases by $n(t) = 5, 25,$ and 50. The approximated solutions are illustrated in Figure 2.

TABLE I
PARAMETER

C_E	C_0	V	Q_{out}
0.004	0.0025	75	4

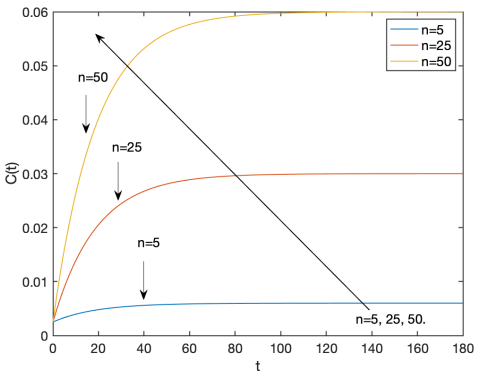

Fig. 2. The approximated air exhaled indoors concentration in a room with difference number of people in a room $\triangle t = 0.05$ $T = 180$.

B. Simulation 2 : The concentration measurement of exhaled air with difference primitive levels.

The parameters are assumed in Table II. The initial condition is assumed in three cases by $C_0 = 0.01, 0.005$ and 0.0025. The approximated solutions are illustrated in Figure 3.

TABLE II
PARAMETER

C_E	$n(t)$	V	Q_{out}
0.004	50	75	4

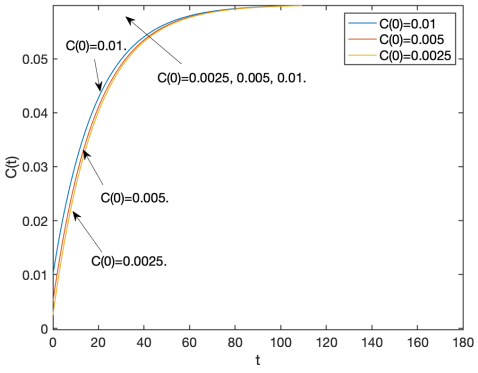

Fig. 3. The approximated air exhaled indoors concentration in a room with difference primitive levels $\triangle t = 0.05$ $T = 180$.

C. Simulation 3 : The concentration measurement of exhaled air when difference outlet ventilation levels.

The parameters are assumed in Table III. The outlet ventilation is assumed in three cases by $Q_{out} = 1, 4,$ and 8. The approximated solutions are illustrated in Figure 4.

TABLE III
PARAMETER

C_E	$n(t)$	V	C_0
0.004	50	75	0.0025

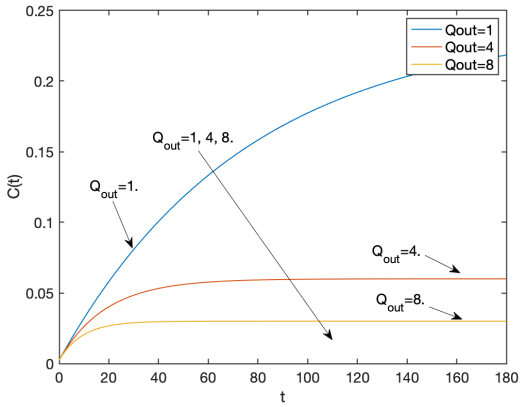

Fig. 4. The approximated air exhaled indoors concentration in a room when difference outlet ventilation levels $\triangle t = 0.05$ $T = 180$.

D. Simulation 4 : The concentration measurement of exhaled air with difference room sizes.

The parameters are assumed in Table IV. The class room of volume is assumed in three cases by $V = 50, 75$, and 100. The approximated solutions are illustrated in Figure 5.

TABLE IV
PARAMETER

C_E	$n(t)$	Q_{out}	C_0
0.004	50	4	0.0025

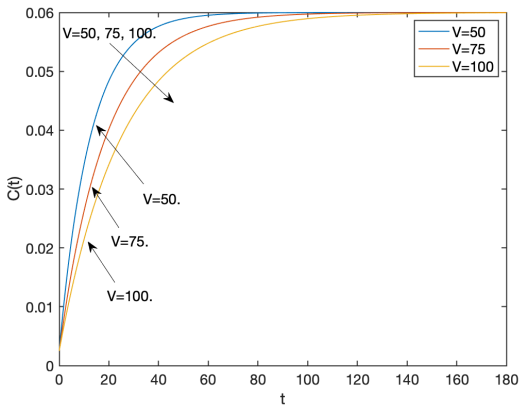

Fig. 5. The approximated air exhaled indoors concentration in a room with difference room sizes $\triangle t = 0.05$ $T = 180$.

E. Simulation 5 : The concentration measurement of exhaled air with varied numbers of people.

The parameters are assumed in Table V. As assumed in Table VI, the number of people changes over time. The approximated solutions are illustrated in Figure 6.

TABLE V
PARAMETER

C_E	V	Q_{out}	C_0
0.004	50	8	0.0025

TABLE VI
A NUMBER OF PEOPLE $n(t)$

t	0	20	40	60	80	100	120	140	160	180
$n(t)$	5	10	15	30	45	50	45	30	20	10

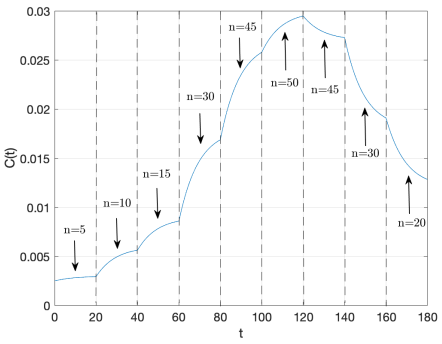

Fig. 6. The approximated air exhaled indoors concentration in a room with varied numbers of people $\triangle t = 0.05$ $T = 180$.

F. Simulation 6 : The concentration measurement of exhaled air when the outlet ventilation levels depend on the stayed number of people.

The parameters are assumed in Table VII. As asumed in Tables VIII-IX, the number of people and the rate of ventilation change over time. The approximated solutions are illustrated in Figure 7.

TABLE VII
PARAMETER

C_E	Q_{out}	C_0
0.004	8	0.0025

TABLE VIII
A NUMBER OF PEOPLE $n(t)$

t	0	20	40	60	80	100	120	140	160	180
$n(t)$	5	10	15	30	45	50	45	30	20	10

TABLE IX
THE RATE OF VENTILATIONS

t	0-60	61-140	141-180
$Q_{out}(t)$	4	8	4

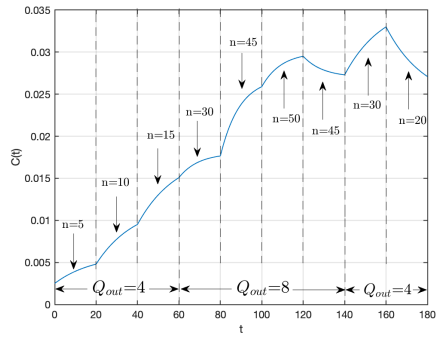

Fig. 7. The approximated air exhaled indoors concentration in a room when the outlet ventilation levels depend on the stayed number of people $\triangle t = 0.05$ $T = 180$.

G. Simulation 7 : The risk of normal peoples who staying in a room with infectors.

The parameters are assumed in Table X. The number of infected people in the room is assumed in three cases by $I = 1, 2,$ and 4. The probability of normal people being infected is illustrated in Figure 8.

TABLE X
PARAMETER

C_E	$n(t)$	Q_{out}	C_0	V	β	μ
0.004	50	4	0.0025	50	30	20

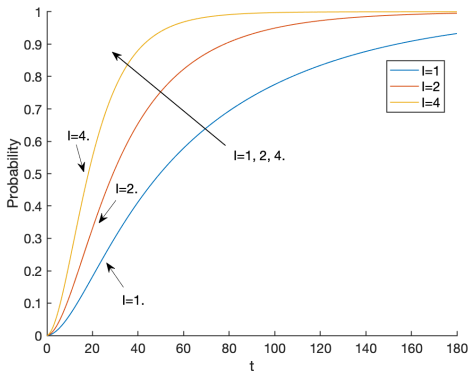

Fig. 8. The risk of normal peoples who staying in a room with infectors $\triangle t = 0.05$ $T = 180$.

H. Simulation 8 : The risk of vaccinated peoples who constantly staying in a room with an infectors.

The parameters are assumed in Table XI. The survival rate of airborne infectious particles is assumed in three cases by $\beta - \mu = 20, 10,$ and 2. The probability of normal people being infected is illustrated in Figure 9.

TABLE XI
PARAMETER

C_E	$n(t)$	Q_{out}	C_0	V	I
0.004	50	4	0.0025	50	1

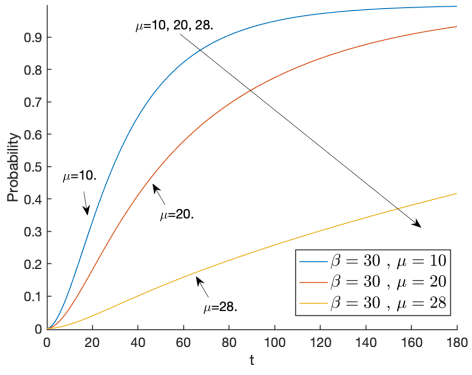

Fig. 9. The risk of vaccinated peoples who constantly staying in a room with an infectors $\triangle t = 0.05$ $T = 180$.

I. Simulation 9 : The risk of vaccinated peoples who varied staying in a room with infectors.

The parameters are assumed in Table XII. As assumed in Table XIII, the number of people changes over time. The probability of normal people being infected is illustrated in Figure 10.

TABLE XII
PARAMETER

C_E	Q_{out}	C_0	V	β	μ	I
0.004	4	0.0025	50	30	20	1

TABLE XIII
A NUMBER OF PEOPLE $n(t)$

t	0	20	40	60	80	100	120	140	160	180
$n(t)$	5	10	15	30	45	50	45	30	20	10

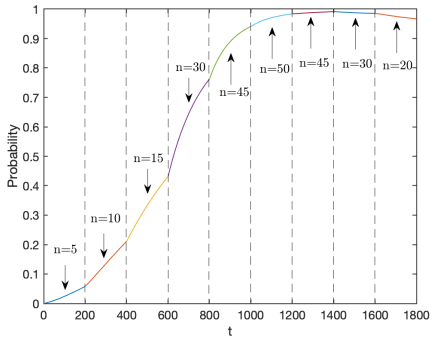

Fig. 10. The risk of vaccinated peoples who varied staying in a room with infectors $\triangle t = 0.05$ $T = 180$.

V. Discussion

In simulation 1, the exhaled air concentration the simulation depends on the number of persons in a room. If the number of persons in the room increases, the exhaled air concentration will increases. Figure 2 illustrates the approximate RK4 solutions when the number of persons is different.

In simulation 2, the exhaled air concentration along with the starting and the middle of the simulation depends on the potential concentration level. The exhaled air concentration for each case becomes close to 0.060 around 1.5 hours.Figure 3 illustrates the approximate RK4 solutions when $C(0)$ is divided by half for each case.

In simulation 3, the exhaled air concentration the simulation depends on the outlet ventilation level. If the outlet ventilation level is increased, the exhaled air concentration is reduced. Figure 4 illustrates the approximate RK4 solutions when the outlet ventilation level is different.

In simulation 4, the exhaled air concentration along with the starting and the middle of the simulation depends on the room sizes. If the room sizes are increased, the exhaled air concentration is reduced. Figure 5 illustrates the approximate RK4 solutions when the different room sizes.

In simulation 5, the exhaled air concentration of the simulation depends on the various number of persons in time. The exhaled air concentration of interval $0 - 2$ hours

is increased and the exhaled air concentration interval $2-3$ hours are reduced. Figure 6 illustrates the approximate RK4 solutions when $n(t)$ is varied in time.

In simulation 6, the exhaled air concentration when the number of persons is varied with the out ventilation level depends on the staying number of persons in the room. If we change the outlet ventilation level varies on the number of persons, the exhaled air concentration values will change. Figure 7 illustrates the approximate RK4 solutions when $n(t)$ is varied with the out ventilation level.

In simulation 7, the risk of infectors depends on the number of infectors in the room. If we change the number of infectors in the room, the risk of infection will change. We can see that the more time spent in the room, the greater the risk of infection. Figure 8 illustrates the risk of normal people staying in a room with infectors.

In simulation 8, the risk of infectors depends on the number of vaccinated persons constantly staying in a room. In case $\beta-\mu=2$, the risk of infection is very low, when compared to cases $\beta-\mu=20$ and $\beta-\mu=10$. When a person gets an effective vaccine, the risk of infection is low. Figure 9 illustrates the risk of vaccinated people who constantly stay in a room with infectors.

In simulation 9, the risk of vaccinated people remaining in a room containing infectors is related to the number of persons staying. If we change the number of vaccinated persons in the room, the risk of infection will change. At intervals of 2.4 – 3.0 hours, the number of vaccinated persons is reduced, and the risk of infection is reduced. Figure 10 illustrates the risk of vaccinated persons who vary their time in a room containing infectors.

VI. Conclusion

This research can use mathematical analysis to simulate the exhaled air concentration in a space with an outlet ventilation system and the risk of infection while normal people and vaccinated people remain in the same room as infectors. As a result, the exhaled air concentration and infection risk are affected by the actual concentration level, the number of users, and the rate of ventilation. Using the classical fourth-order RK method, we show that the proposed method applies to real-world situations. In the air quality management process, the proposed model achieves a balance between the number of people permitted to stay in the room and the potential of the air ventilation system.

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