Dynamical Properties of a Modified Chaotic Colpitts Oscillator with Logarithmic Function

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***Abstract*—In this paper, two logarithmic non-linearities are proposed for a new four-dimensional chaotic system. The phase portrait, Lyapunov exponent, bifurcation, stability, and other dynamical features of the new chaotic system are all discussed. The multi-stability of the new chaotic system with coexisting attractors has been established. The adaptive backstepping control approach with proper Lyapunov functions is used in the control application to retrieve the unknown parameters of the system. To synchronise the states between the drive-response system, non-linear feedback control is used, as well as backstepping control to synchronise the states on the system's error dynamics. Op-amp circuits are used to create the electronic circuit design for a new chaotic system. The system's efficiency is confirmed using MATLAB numerical simulation.**

***Index Terms*—chaos, Colpitts oscillator, diffusion, Lyapunov exponent, logarithmic function non-linearity, stability, synchronization.**

I. Introduction

In recent years, researchers have become interested in studying chaotic dynamical systems. The study of a chaotic system with a chaotic attractor presents various obstacles, making it a fascinating subject.

A chaotic system [1] is a non-linear dynamical system that exhibits complicated and unpredictable behaviour. The sensitivity depends on the initial conditions and the parameter values fluctuate with range. These are chaotic systems' exceptional properties [2]. Chaotic systems are sometimes deterministic [3], [4] and exhibit long-term unpredictable behaviour [5], [6].

Even though chaotic systems are extremely sensitive, the sensitivity of these systems is depend on the initial conditions. The chaotic character is one of the qualitative [7], [8] properties of a dynamical system [9], [10], [11], [12].

Stabilization [13], [14] of unstable periodic motion "contained" in the chaotic set, inhibition of chaotic behaviour by external force like periodic noise, periodic parametric perturbation, and algorithms of various automatic control like feedback [15], [16], [17], [18], backstepping [19], [20], [21], [22], sample feedback, time delay feedback and so on can all be used to control chaotic systems.

In a chaotic system, there are two methods for applying controls. The first is a change in the system's attractor. The second is the change in the system's point position in the phase space, which is a constant parameter value.

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An oscillator is a device that produces a continuous, repeated, and alternating wave without any input. One of the main functions of oscillators is to convert power into an alternating current signal. A Colpitts oscillator [23], [24] is the signal produced by a feedback system with two coils and an inductive divider in the server. Chaotic behaviour may emerge in Colpitts oscillators due to changes in settings and input variations.

A novel chaotic Colpitts oscillator is proposed in this paper. It is a modified version of Colpitts oscillators from the past. The modified form of Colpitts oscillator [25], [26] is described in section 2 along with the mathematical model formulation. Furthermore, the system's invariant property, equilibrium point, and Lyapunov exponents [27], [28], [29], [30] are examined. The adaptive backstepping technique [31] is discussed in section 3 for the proposed system. A non-linear feedback system is established in section 4. In section 5, the backstepping control approach is used to examine the non-linear feedback system. Finally, the numerical simulation [32], [33], [34], [35] shows that the hypothetical outcomes are consistent.

II. The Mathematical Model of a Chaotic Colpitts Oscillator

Figure 1 shows a simplified illustrative diagram for a modified Colpitts oscillator. The Colpitts oscillator is widely used in communication systems, in addition to electronic gadgets. A sinusoidal oscillator with a single transistor implementation.

The following are the hypotheses for simplifying the extensive simulation of the complete circuit model.

1) The base-emitter(B-E) driving point(V-I) characteristic of the R_E with logarithmic function is

$$I_E = f\left(V_{BE}\right) = I_S\left[\log(x_3) - 1\right]$$
$$\text{and } I_E = f\left(V_{BE}\right) = I_S\left[\log(x_1) - 1\right]$$

where I_S is the emitter current (inverse saturation current), a is amplitude and p is period of the B-E junction.

2) The state space is schematically represented in Figure 1.

$$\begin{aligned}
R_C C_1 \frac{dV_{C_1}}{dt} &= V_0 - V_{C_1} - V_{C_2} + R_C I_L - R_C f(V_{BE}) \\
R_C C_2 \frac{dV_{C_2}}{dt} &= V_0 - V_{C_1} - V_{C_2} - R_C I_0 + R_C I_L \\
C_3 \frac{dV_{C_3}}{dt} &= I_L - (1-\alpha) f\left(V_{BE}\right) \\
L \frac{dI_L}{dt} &= -R_b I_L - V_{C_1} - V_{C_2} - V_{C_3}
\end{aligned}$$

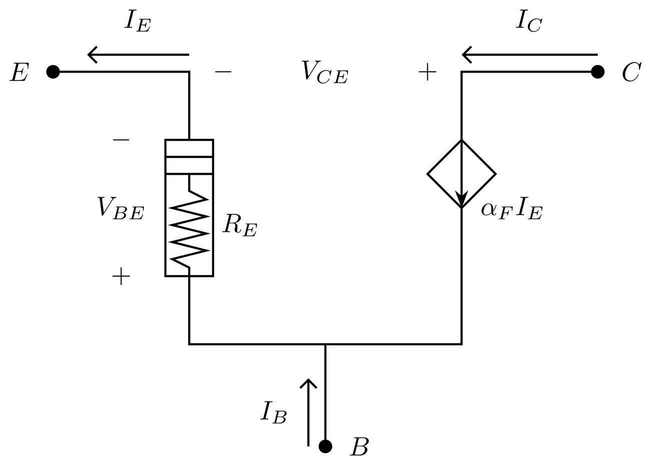


Fig. 1. The circuit diagram

The following is the proposed new system with Colpitts oscillator:

$$\begin{array}{rcl} \dot{x}_1 & = & \sigma_1(-x_1 - x_2) + x_4 - \gamma\phi_1(x_3) \\ \dot{x}_2 & = & \varepsilon_1\sigma_1(-x_1 - x_2) + \varepsilon_1 x_4 \\ \dot{x}_3 & = & \varepsilon_2(x_4 - (1-\alpha)\gamma\phi_2(x_1)) \\ \dot{x}_4 & = & -x_1 - x_2 - x_3 - \sigma_2 x_4 \end{array} \tag{1}$$

where $\phi_1(x_3) = \log(x_3) - 1$, $\phi_2(x_1) = \log(x_1) - 1$.

In system (1), the state variables are assumed as x_1, x_2, x_3 and x_4 along with six positive parameters, σ_1, γ, ε_1, ε_2, σ_2 and α. The system (1) is an autonomous system to which a logarithmic function expression is associated.

With the modification of coordinates provided by the scheme $(x_1, x_2, x_3, x_4) \mapsto (-x_1, -x_2, -x_3, -x_4)$, the system (1) is found to be invariant.

The mathematical system of the Colpitts oscillator when equated to zero gives the equilibrium points of the system as specified below:

$$\begin{array}{rcl} \sigma_1(-x_1 - x_2) + x_4 - \gamma\phi_1(x_3) & = & 0 \\ \varepsilon_1\sigma_1(-x_1 - x_2) + \varepsilon_1 x_4 & = & 0 \\ \varepsilon_2(x_4 - (1-\alpha)\gamma\phi_2(x_1)) & = & 0 \\ -x_1 - x_2 - x_3 - \sigma_2 x_4 & = & 0 \end{array} \tag{2}$$

Solving the system (2), it is seen that the new chaotic system (2) has a unique equilibrium at the origin.

$$x_1 = \frac{\gamma\phi_1(x_3) - x_4 + x_2\sigma_1}{-\sigma_1}$$
$$x_2 = \frac{-\varepsilon_1 x_4 + \varepsilon_1\sigma_1 x_1}{-\varepsilon_1\sigma_1}$$
$$x_3 = \text{any value} \neq 0$$
$$x_4 = \frac{x_1 + x_2 + x_3}{-\sigma_2}$$

The Jocobian matrix of the system (1) at the equilibrium point E is given by

$$J_E = \begin{bmatrix} -\sigma_1 & -\sigma_1 & -\dfrac{\gamma}{x_3} & 1 \\ -\varepsilon_1\sigma_1 & -\varepsilon_1\sigma_1 & 0 & \varepsilon_1 \\ -\varepsilon_2(1-\alpha)\dfrac{\gamma}{x_1} & 0 & 0 & \varepsilon_2 \\ -1 & -1 & -1 & -\sigma_2 \end{bmatrix} \tag{3}$$

The corresponding characteristic equation of the Colpitts oscillator system (1) with respect to E is given by the relation

$$\Delta_1\lambda^4 + \Delta_2\lambda^3 + \Delta_3\lambda^2 + \Delta_4\lambda + \Delta_5 = 0 \tag{4}$$

where

$\Delta_1 = 1$

$\Delta_2 = \varepsilon_1\sigma_1 + \sigma_1 + \sigma_2$

$$\Delta_3 = \frac{\left[\begin{array}{l} \alpha\varepsilon_2\gamma^2 + \varepsilon_1\sigma_1\sigma_2 x_1 x_3 + \varepsilon_1 x_1 x_3 - \varepsilon_2\gamma^2 \\ +\varepsilon_2 x_1 x_3 + \sigma_1\sigma_2 x_1 x_3 + x_1 x_3 \end{array}\right]}{x_1 x_3}$$

$$\Delta_4 = \frac{\left[\begin{array}{l} \alpha\varepsilon_1\varepsilon_2\gamma^2\sigma_1 + \alpha\varepsilon_2\gamma^2\sigma_2 + \alpha\varepsilon_2\gamma x_3 - \varepsilon_1\varepsilon_2\gamma^2\sigma_1 \\ +\varepsilon_1\varepsilon_2\sigma_1 x_1 x_3 - \varepsilon_2\gamma^2\sigma_2 - \varepsilon_2\gamma x_1 + \varepsilon_2\sigma_1 x_1 x_3 \end{array}\right]}{x_1 x_3}$$

$$\Delta_5 = \frac{\left[\begin{array}{l} \alpha\varepsilon_1\varepsilon_2\gamma^2\sigma_1\sigma_2 + \alpha\varepsilon_1\varepsilon_2\gamma^2 - \varepsilon_1\varepsilon_2\gamma^2\sigma_1\sigma_2 \\ -\varepsilon_1\varepsilon_2\gamma^2 \end{array}\right]}{x_1 x_3}$$

The system is unstable for all values of the parameters at the equilibrium point E, according to the Routh-Hurwitz stability criterion [36].

From the Jacobian matrix (3), among the states x_1, x_2, x_3 and x_4, if x_1 and x_3 are both positive or negative or of opposite signs, it implies "*Hopf bifurcation*". This phenomenon is also known as "*Poincaré–Andronov–Hopf bifurcation*". This bifurcation leads to a local birth of "*chaos*" nature in modified Colpitts oscillator (1).

Interestingly, the system (1) is chaotic for the parameters

$$\begin{aligned} \varepsilon_1 &= 1,\ \varepsilon_2 = 20, \\ \sigma_1 &= 1.49,\ \sigma_2 = 0.872, \\ \gamma &= 2.2001, 25.80, 200.00, 250.00, 5500.00, \\ \alpha &= \frac{255}{256} \end{aligned}$$

Lyapunov exponents are one of the essential factors in determining whether a system is chaotic, hyperchaotic, stable, or periodic.

Table I gives the details of the quasi-periodic and chaotic nature of the system. For the evaluation of the system, the observation time (T) is considered as $100, 8500$ and the sampling time (Δt) is taken as 0.5. For various initial conditions, the system (1) exhibits chaotic nature.

$\log(0)$ value is undefined. The defined non-linearity function is not working for zero initial condition in x_1 state and x_3 state.

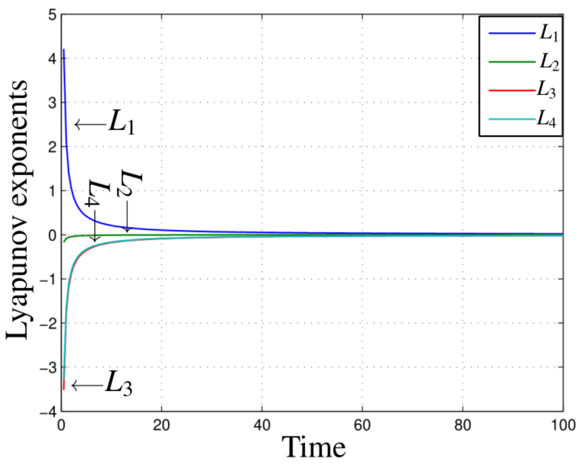
By applying Wolf algorithm [37], the Lyapunov exponents(LEs) corresponding to the new chaotic system (1) are obtained as detailed below:

The Lyapunov exponential dimension is obtained from Table I. The new system's attractor is discovered to be a strange attractor with fractal dimensions. Figure 3 shows the chaotic attractor of the system (1) found through numerical simulation.

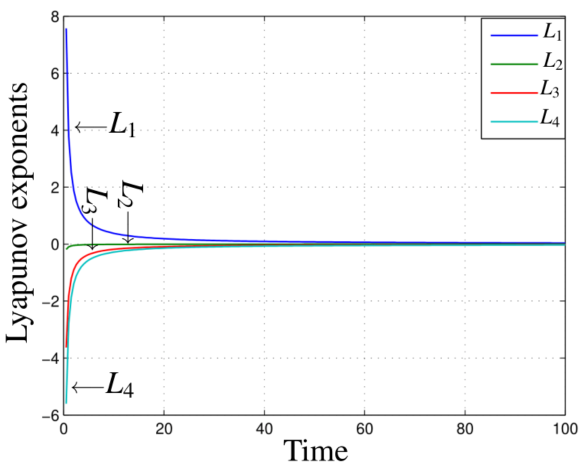
The Lyapunov exponents of the modified Colpitts oscillator are shown in Figure 2 and the chaotic nature and Poincar'e Map of the modified Colpitts oscillator are shown in Figure 3.

TABLE I
LEs OF SYSTEM (1) FOR OBSERVATION TIME $(T) = 100, 8500$, SAMPLING TIME $(\Delta t) = 0.5$, $\varepsilon_1 = 1$, $\varepsilon_2 = 20$, $\sigma_1 = 1.49$, $\sigma_2 = 0.872$, $\alpha = \frac{255}{256}$, $\gamma = 2.2001, 25.80, 200.00, 250.00, 5500.00$ WITH VARIOUS SAMPLING AND OBSERVATION TIMES USING WOLF ALGORITHM.

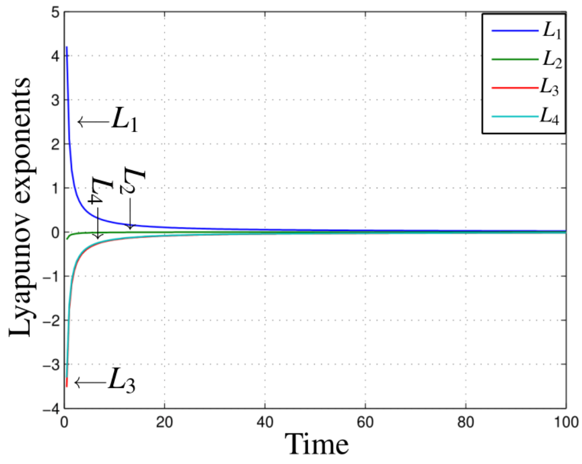
Sl. No.	Parameter γ, time t	Initial condition	LEs	Sign of the LEs	Nature
1	$\gamma = 2.2001$, $t = 0$, $\Delta t = 0.5$, $T = 100$	0.3453, 0.5467, 0.9834, 0.1736	0.021060, −0.000842, −0.017594, −0.016507	+, ≈ 0, −, −	Chaotic
2	$\gamma = 25.80$, $t = 0$, $\Delta t = 0.5$, $T = 100$	0.3453, 0.5467, 0.9834, 0.1736	0.037866, −0.000934, −0.018158, −0.028019,	+, ≈ 0, −, −,	Chaotic
3	$\gamma = 200.00$, $t = 0$, $\Delta t = 0.5$, $T = 100$	0.3453, 0.5467, 0.9834, 0.1736	0.047162, −0.000875, −0.019369, −0.029810	+, ≈ 0, −, −	Chaotic
4	$\gamma = 250.00$, $t = 0$, $\Delta t = 0.5$, $T = 100$	0.3453, 0.5467, 0.9834, 0.1736	0.047538, −0.000769, −0.020563, −0.028571	+, ≈ 0, −, −	Chaotic
5	$\gamma = 5500.00$, $t = 0$, $\Delta t = 0.5$, $T = 8500$	0.3453, 0.5467, 0.9834, 0.1736	0.000562, −0.000000, −0.000530, −0.000033	+, 0, −, −	Chaotic



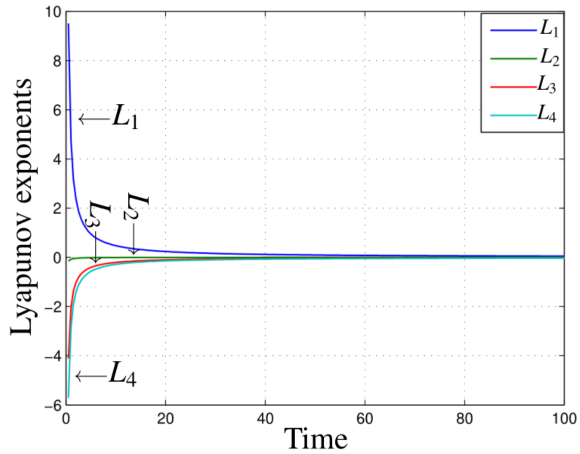
(a) The Lyapunov exponents for modified Colpitts oscillator with $\gamma = 2.2001$



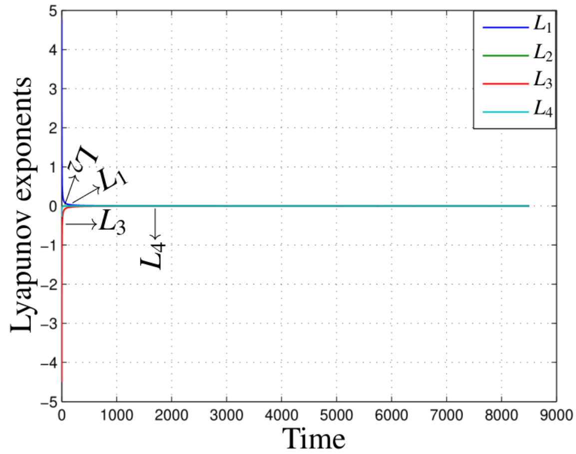
(b) The Lyapunov exponents for modified Colpitts oscillator with $\gamma = 25.80$



(c) The Lyapunov exponents for modified Colpitts oscillator with $\gamma = 200.00$

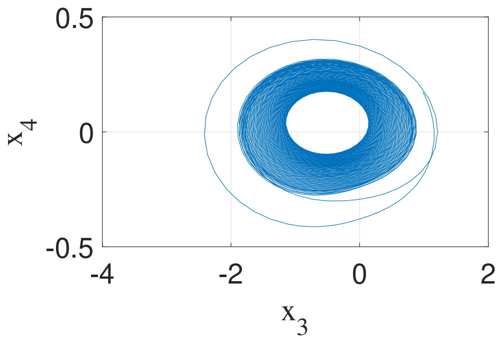


(d) The Lyapunov exponents for modified Colpitts oscillator with $\gamma = 250.00$

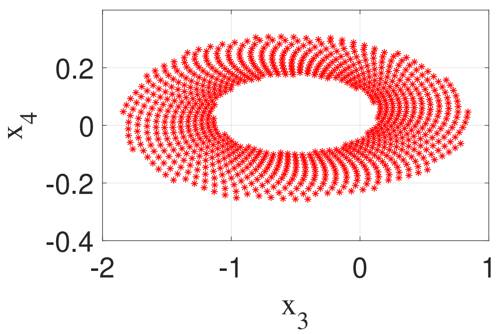


(e) The Lyapunov exponents for modified Colpitts oscillator with $\gamma = 5500.00$

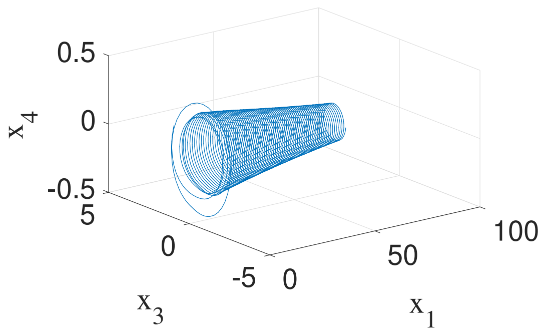
Fig. 2. Lyapunov exponents of the modified Colpitts oscillator with $\varepsilon_1 = 1$, $\varepsilon_2 = 20$, $\sigma_1 = 1.49$, $\sigma_2 = 0.872$, $\alpha = \frac{255}{256}$ with initial condition $(x_1, x_2, x_3, x_4) = (0.3453, 0.5467, 0.9834, 0.1736)$



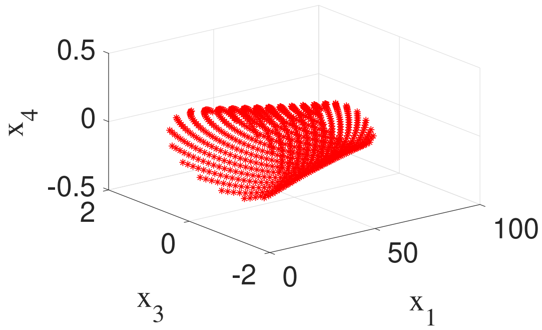
(a) Quasi-periodic nature between x_3 and x_4



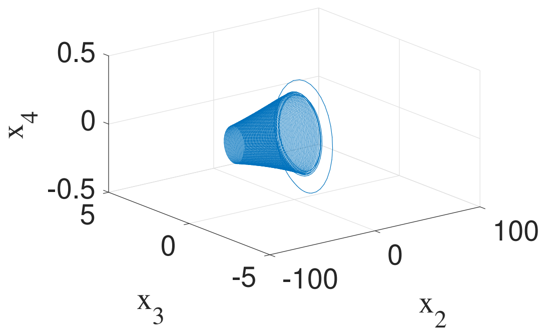
(b) Poincaré Chaotic nature between x_3 and x_4



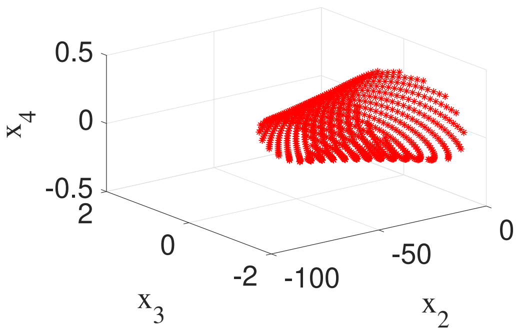
(c) Quasi-periodic nature between x_1, x_3 and x_4



(d) Poincaré Map between x_1, x_3 and x_4



(e) Quasi-periodic nature between x_2, x_3 and x_4



(f) Poincaré Map between x_2, x_3 and x_4

Fig. 3. Portrait of Colpitts

One of the advantages of this paradigm is the examination of qualitative qualities. Some qualitative properties worth noting are stability control, limit cycle, periodicity and chaos.

Only one state of the system, state 1, exhibits exponential non-linearity. Logarithmic nonlinearity exists in the suggested four-dimensional chaotic system. State 1 and state 3 of the proposed system introduce logarithmic non-linearities. The existence of strange attractors and bifurcation nature, as well as the chaotic character shown by Lyapunov exponent and Poincar'e map, are all seen in the suggested system.

The following theorems bring out the local stability properties of the modified Colpitts oscillator.

Theorem 1: The interior equilibrium point E is locally asymptotically stable in the positive octant.

Proof: By *divergence criterion* theorem, assume

$$\theta(x_1, x_2, x_3, x_4) = \frac{1}{x_1 x_2 x_3 x_4} \tag{5}$$

where $\theta(x_i, i = 1, 2, 3, 4) > 0$ if $x_i > 0, i = 1, 2, 3, 4$.

Now consider

$$\begin{aligned} p_1 &= \sigma_1(-x_1 - x_2) + x_4 - \gamma\phi_1(x_3) \\ p_2 &= \varepsilon_1\sigma_1(-x_1 - x_2) + \varepsilon_1 x_4 \\ p_3 &= \varepsilon_2(x_4 - (1-\alpha)\gamma\phi_2(x_1)) \\ p_4 &= -x_1 - x_2 - x_3 - \sigma_2 x_4 \end{aligned} \tag{6}$$

where $\phi_1(x_3) = \log(x_3) - 1$, $\phi_2(x_1) = \log(x_1) - 1$.

Define

$$\nabla = \frac{\partial}{\partial x_1}(p_1\theta) + \frac{\partial}{\partial x_2}(p_2\theta) + \frac{\partial}{\partial x_3}(p_3\theta) + \frac{\partial}{\partial x_4}(p_4\theta) \tag{7}$$

We have to determine ∇ given by (7) along with the trajectories provided by (5) and (6). We obtain

$$\begin{aligned} \nabla = &-\frac{\{[\sigma_1]\,x_1 + [\sigma_1(-x_1 - x_2) + x_4 - \gamma\phi_1(x_3)]\}x_2x_3x_4}{x_1^2x_2^2x_3^2x_4^2} \\ &-\frac{\{\varepsilon_1\sigma_1 x_2 + [\varepsilon_1\sigma_1(-x_1 - x_2) + \varepsilon_1 x_4]\}x_1x_3x_4}{x_1^2x_2^2x_3^2x_4^2} \\ &-\frac{\varepsilon_2\,[x_4 - (1-\alpha)\gamma\phi_2(x_1)]\,x_1x_2x_4}{x_1^2x_2^2x_3^2x_4^2} \\ &-\frac{\{\sigma_2 x_4 + (-x_1 - x_2 - x_3 - \sigma_2 x_4)\}x_1x_2x_3}{x_1^2x_2^2x_3^2x_4^2} \end{aligned}$$

which is less than zero.

According to the *Benedixon-Dulac criterion*, the first octant does not contain any limit cycles.

Consequently, the equilibrium provided by E is found to be locally asymptotically stable.

The local asymptotic stability can be seen in the relationship between the limit cycle and closed trajectories. The following theorem is concerned with the stability of a closed trajectory employing Bendixson's theorem's conditions. ■

Theorem 2: There is no closed trajectory for the interior equilibrium point.

Proof: Define

$$\Psi(x_i, i=1,2,3,4) = \frac{\partial p_1}{\partial x_1} + \ldots + \frac{\partial p_4}{\partial x_4}. \tag{8}$$

Find Ψ along with the trajectories associated with (8). It follows that

$$\Psi = -\sigma_1 - \varepsilon_1\sigma_1 - \sigma_2 \neq 0. \tag{9}$$

By using *Bendixson's criteria* to (9), it becomes clear that there is no closed trajectory surrounding the point E.

Hence, limit cycle does not exist encompassing E.

Therefore, it follows that the point E is locally asymptotically stable.

It refers to a special form of oscillator solution that exhibits a stable periodic orbit. The non-trivial periodic solution is emphasised in the following theorem. ■

Theorem 3: The modified Colpitts oscillator given by (1) has a non-trivial periodic solution.

Proof: Define

$$\begin{aligned}\Phi &= \frac{d}{dt}\left(\frac{x_1^2+x_2^2+x_3^2+x_4^2}{2}\right)\\ &= x_1\frac{dx_1}{dt} + x_2\frac{dx_2}{dt} + x_3\frac{dx_3}{dt} + x_4\frac{dx_4}{dt}\\ &= x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 + x_4\dot{x}_4\\ &= \sum_{i=1}^{4} x_i\frac{dx_i}{dt}.\end{aligned} \tag{10}$$

Find Φ from (10) along the trajectories of (1). We see that

$$\begin{aligned}\Phi &= x_1[\sigma_1(-x_1-x_2) + x_4 - \gamma\phi_1(x_3)]\\ &\quad + x_2[\varepsilon_1\sigma_1(-x_1-x_2) + \varepsilon_1 x_4]\\ &\quad + x_3[\varepsilon_2(x_4 - (1-\alpha)\gamma\phi_2(x_1))]\\ &\quad + x_4[-x_1 - x_2 - x_3 - \sigma_2 x_4]\\ &= -\sigma_1 x_1^2 - \sigma_1 x_1 x_2 + x_1 x_4 - \gamma x_1\phi_1(x_3)\\ &\quad - \varepsilon_1\sigma_1 x_1 x_2 - \varepsilon_1\sigma_1 x_2^2 + \varepsilon_1 x_2 x_4\\ &\quad + \varepsilon_2 x_3 x_4 - \varepsilon_2(1-\alpha)x_3\gamma\phi_2(x_1)\\ &\quad - x_1x_4 - x_2x_4 - x_3x_4 - \sigma_2 x_4^2\\ &= -(\sigma_1 x_1^2 + \varepsilon_1\sigma_1 x_2^2 + \sigma_2 x_4^2)\\ &\quad - \sigma_1 x_1 x_2(1+\varepsilon_1) - (1-\varepsilon_1)x_2x_4\\ &\quad - (1-\varepsilon_2)x_3x_4\\ &\quad - x_1\gamma\phi_1(x_3) - x_3\varepsilon_2(1-\alpha)\gamma\phi_2(x_1)\\ &= -(\nabla_1 + \nabla_2)\end{aligned} \tag{11}$$

where $\nabla_1 = \sigma_1 x_1^2 + \varepsilon_1\sigma_1 x_2^2 + \sigma_2 x_4^2$,

$$\begin{aligned}\nabla_2 &= \sigma_1 x_1 x_2(1+\varepsilon_1) + (1-\varepsilon_1)x_2x_4\\ &\quad + (1-\varepsilon_2)x_3x_4 + x_1\gamma\phi_1(x_3)\\ &\quad + x_3\varepsilon_2(1-\alpha)\gamma\phi_2(x_1).\end{aligned}$$

It is observed that $\nabla_1 + \nabla_2$ is positive for $x_1^2 + x_2^2 + x_3^2 + x_4^2 < a$ and negative for $x_1^2 + x_2^2 + x_3^2 + x_4^2 > b$, where a, b are positive constants.

This implies that any solution $x_i(t)$ of (1) will be in the annulus $a < \sum_{i=1}^{4} x_i^2 < b$.

Hence, by *Poincaré-Bendixson* theorem, there exists at least one periodic solution $x_i(t), i = 1,2,3,4$ of (1) lying in this annulus.

Hence, the modified Colpitts oscillator (1) has a non-trivial periodic solution. ■

The act of changing an oscillator's behaviour to achieve a desired purpose, primarily through the use of feedback control, is known as control theory. When parameter values are uncertain, backstepping control is described in the next section.

III. Adaptive Backstepping control of the modified Colpitts oscillator with unknown parameters

A. Proposed system

The modified Colpitts oscillator system is given by the dynamics with controllers

$$\begin{aligned}\dot{x}_1 &= \sigma_1(-x_1-x_2) + x_4 - \gamma\phi_1(x_3) + u_1\\ \dot{x}_2 &= \varepsilon_1\sigma_1(-x_1-x_2) + \varepsilon_1 x_4 + u_2\\ \dot{x}_3 &= \varepsilon_2(x_4 - (1-\alpha)\gamma\phi_2(x_1)) + u_3\\ \dot{x}_4 &= -x_1 - x_2 - x_3 - \sigma_2 x_4 + u_4\end{aligned} \tag{12}$$

where $\phi_1(x_3) = \log(x_3) - 1$, $\phi_2(x_1) = \log(x_1) - 1$.

In system (12), x_1, x_2, x_3 and x_4 are state variables and u_1, u_2, u_3 and u_4 are adaptive controllers.

The synchronization error is defined as $e_i = y_i - x_i$, $i = 1,2,3,4$.

The unknown parameters are updated by

$$\begin{aligned}e_{\sigma_1} &= \sigma_1 - \widehat{\sigma}_1(t), \quad e_{\sigma_2} = \sigma_2 - \widehat{\sigma}_2(t)\\ e_{\varepsilon_1} &= \varepsilon_1 - \widehat{\varepsilon}_1(t), \quad e_{\varepsilon_2} = \varepsilon_2 - \widehat{\varepsilon}_2(t)\\ e_\alpha &= \alpha - \widehat{\alpha}(t), \quad e_\gamma = \gamma - \widehat{\gamma}(t)\end{aligned} \tag{13}$$

By differentiating (13) with respect to 't', one obtains

$$\begin{aligned}\dot{e}_{\sigma_1} &= -\dot{\widehat{\sigma}}_1(t), \quad \dot{e}_{\sigma_2} = -\dot{\widehat{\sigma}}_2(t)\\ \dot{e}_{\varepsilon_1} &= -\dot{\widehat{\varepsilon}}_1(t), \quad \dot{e}_{\varepsilon_2} = -\dot{\widehat{\varepsilon}}_2(t)\\ \dot{e}_\alpha &= -\dot{\widehat{\alpha}}(t), \quad \dot{e}_\gamma = -\dot{\widehat{\gamma}}(t)\end{aligned}$$

At this stage, the state of the system is considered as

$$\dot{x}_1 = \sigma_1(-x_1-x_2) + x_4 - \gamma\phi_1(x_3) + u_1 \tag{14}$$

where x_2 is regarded as a virtual controller.

In order to stabilize the system, the suitable Lyapunov function is defined as

$$V_1(x_1) = \frac{1}{2}x_1^2 + \frac{1}{2}e_{\sigma_1}^2 + \frac{1}{2}e_\gamma^2.$$

By differentiating V_1 with respect to t,

$$\begin{aligned}\dot{V}_1 &= x_1\dot{x}_1 + e_{\sigma_1}\dot{e}_{\sigma_1} + e_\gamma\dot{e}_\gamma\\ &= x_1[\sigma_1(-x_1-x_2) + x_4 - \gamma\phi_1(x_3) + u_1]\\ &\quad + e_{\sigma_1}\left(-\dot{\widehat{\sigma}}_1\right) + e_\gamma\left(-\dot{\widehat{\gamma}}\right)\end{aligned} \tag{15}$$

where x_2 is regarded as a virtual controller and is defined as

$$x_2 = \beta_1(x_1) \text{ and } \beta_1(x_1) = 0.$$

The controller u_1 is assumed as

$$u_1 = -x_1 + \widehat{\sigma}_1 x_1 - x_4 + \widehat{\gamma}\phi_1(x_3) \quad (16)$$

and the unknown parameters $\widehat{\sigma}_1$ and $\widehat{\gamma}$ are updated by

$$\begin{aligned} \dot{\widehat{\sigma}}_1 &= -x_1^2 + e_{\sigma_1} \\ \dot{\widehat{\gamma}} &= -x_1\phi_1(x_3) + e_{\gamma}. \end{aligned} \quad (17)$$

On substitution of (16) and (17) into (15), we get

$$\dot{V}_1 = -x_1^2 - e_{\sigma_1}^2 - e_{\gamma}^2$$

which is found to be a negative definite function.

Hence, by Lyapunov stability theory, the system is globally asymptotically stable.

Now define the relation between β_1 and x_2 by

$$\omega_2 = x_2 - \beta_1.$$

Consider the subsystem (x_1, ω_2). We have

$$\begin{aligned} \dot{x}_1 &= -e_{\sigma_1}x_1 - \sigma_1\omega_2 - e_{\gamma}\phi_1(x_3) - x_1, \\ \dot{\omega}_2 &= -\varepsilon_1\sigma_1x_1 - \varepsilon_1\sigma_1\omega_2 + \varepsilon_1x_4 + u_2. \end{aligned}$$

Define V_2 by the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}\omega_2^2 + \frac{1}{2}e_{\varepsilon_1}^2.$$

On differentiating V_2 with respect to t, we get

$$\dot{V}_2 = x_1\dot{x}_1 + e_{\sigma_1}\left(-\dot{\widehat{\sigma}}_1\right) + e_{\gamma}\left(-\dot{\widehat{\gamma}}\right) + e_{\varepsilon_1}\left(-\dot{\widehat{\varepsilon}}_1\right) + \omega_2\dot{\omega}_2. \quad (18)$$

The controller u_2 is assumed as

$$u_2 = \sigma_1x_1 + \widehat{\varepsilon}_1(\sigma_1x_1 + \sigma_1\omega_2 - x_4) + x_3 - \omega_2. \quad (19)$$

Let x_3 be the virtual controller. It is defined as $x_3 = \beta_2(x_1, \omega_2)$ with the assumption that $\beta_2(x_1, \omega_2) = 0$.

The parameter ε_1 is estimated as

$$\dot{\widehat{\varepsilon}}_1 = -\omega_2(\sigma_1x_1 + \sigma_1\omega_2 - x_4) + e_{\varepsilon_1}. \quad (20)$$

Substituting (19) and (20) into (18), we get

$$\dot{V}_2 = -x_1^2 - e_{\sigma_1}^2 - e_{\gamma}^2 - w_2^2 - e_{\varepsilon_1}^2$$

which is a negative definite function.

Hence, by Lyapunov stability theory, the system is globally asymptotically stable.

The relation between x_3 and β_2 is defined by

$$\omega_3 = x_3 - \beta_2.$$

Consider the subsystem $(x_1, \omega_2, \omega_3)$. We have

$$\begin{aligned} \dot{x}_1 &= -e_{\sigma_1}x_1 - \sigma_1\omega_2 - e_{\gamma}\phi_1(x_3) - x_1, \\ \dot{\omega}_2 &= -e_{\varepsilon_1}(\sigma_1x_1 + \sigma_1\omega_2 - x_4) - \omega_2 + \sigma_1x_1 + \omega_3, \\ \dot{\omega}_3 &= \varepsilon_2(x_4 - (1-\alpha)\gamma\phi_2(x_1)) + u_3. \end{aligned}$$

Now consider the Lyapunov function

$$V_3 = V_2 + \frac{1}{2}\omega_3^2 + \frac{1}{2}e_{\varepsilon_2}^2 + \frac{1}{2}e_{\alpha}^2$$

The derivative of V_3 with respect to t is obtained as

$$\dot{V}_3 = \dot{V}_2 + \omega_3\dot{\omega}_3 + e_{\varepsilon_2}\dot{e}_{\varepsilon_2} + e_{\alpha}\dot{e}_{\alpha} \quad (21)$$

where $u_3 = -\omega_2 - \omega_3 + \widehat{\varepsilon}_2\gamma\phi_2(x_1) - \varepsilon_2\widehat{\alpha}\gamma\phi_2(x_1)$. (22)

Let us denote the virtual controller by x_4. It is defined as $x_4 = \beta_3(x_1, \omega_2, \omega_3)$ and we assume that $\beta_3(x_1, \omega_2, \omega_3) = 0$.

The parameters are estimated as

$$\begin{aligned} \dot{\widehat{\varepsilon}}_2 &= -\omega_3\gamma\phi_2(x_1) + e_{\varepsilon_2}, \\ \dot{\widehat{\alpha}} &= \omega_3\varepsilon_2\gamma\phi_2(x_1) + e_{\alpha}. \end{aligned} \quad (23)$$

Substitute (22) and (23) into (21). Then we get

$$\dot{V}_3 = -x_1^2 - e_{\sigma_1}^2 - e_{\gamma}^2 - w_2^2 - e_{\varepsilon_1}^2 - w_3^2 - e_{\varepsilon_2}^2 - e_{\alpha}^2$$

which is a negative definite function.

Hence, by the theory of Lyapunov, it follows that the system provided by (12) is stable.

Now the relation between x_4 and β_3 is defined by

$$\omega_4 = x_4 - \beta_3.$$

Consider the subsystem $(x_1, \omega_2, \omega_3, \omega_4)$ provided by

$$\begin{aligned} \dot{x}_1 &= -e_{\sigma_1}x_1 - \sigma_1\omega_2 - e_{\gamma}\phi_1(x_3) - x_1, \\ \dot{\omega}_2 &= -e_{\varepsilon_1}(\sigma_1x_1 + \sigma_1\omega_2 - x_4) - \omega_2 + \omega_3 + \sigma_1x_1, \\ \dot{\omega}_3 &= \varepsilon_2\omega_4 - e_{\varepsilon_2}\gamma\phi_2(x_1) + e_{\alpha}\varepsilon_2\gamma\phi_2(x_1) - \omega_2 - \omega_3, \\ \dot{\omega}_4 &= -x_1 - x_2 - x_3 - \sigma_2\omega_4 + u_4. \end{aligned}$$

Now consider the Lyapunov function

$$V_4 = V_3 + \frac{1}{2}\omega_4^2 + \frac{1}{2}e_{\sigma_2}^2.$$

The derivative of V_4 with respect to t is obtained as

$$\dot{V}_4 = \dot{V}_3 + \omega_4\dot{\omega}_4 + e_{\sigma_2}\dot{e}_{\sigma_2}. \quad (24)$$

where $u_4 = -\varepsilon_2\omega_3 + x_1 + x_2 + x_3 + \widehat{\sigma}_2\omega_4 - \omega_4$. (25)

By working backward, the parameter is estimated as

$$\dot{\widehat{\sigma}}_2 = e_{\sigma_2} - w_4^2. \quad (26)$$

Substitute (25) and (26) into (24). Then we are led to

$$\dot{V}_4 = -x_1^2 - e_{\sigma_1}^2 - e_\gamma^2 - w_2^2 - e_{\varepsilon_1}^2 - w_3^2 - e_{\varepsilon_2}^2 - e_\alpha^2 - w_4^2 - e_{\sigma_2}^2$$

which is a negative definite function.

By the stability theory due to Lyapunov, it is seen that the Colpitts oscillator provided by (1) is asymptotically stable.

B. Numerical simulation

For the numerical simulation, the initial conditions of the parameters are taken as

$$\widehat{\sigma}_1(0) = 10.9546, \quad \widehat{\sigma}_2(0) = 5.9353,$$
$$\widehat{\alpha}(0) = 3.8765, \quad \widehat{\gamma}(0) = 2.1654,$$
$$\widehat{\varepsilon}_1(0) = 7.8762, \quad \widehat{\varepsilon}_2(0) = 9.9876$$

with the initial conditions for the modified Colpitts oscillator $x_1(0) = 1.9124$, $x_2(0) = 1.3942$, $x_3(0) = 1.3125$ and $x_4(0) = 1.9873$.

Figure 4 depicts the parameter estimation of the modified Colpitts oscillator.

Figure 5 depicts the stability of the modified Colpitts oscillator.

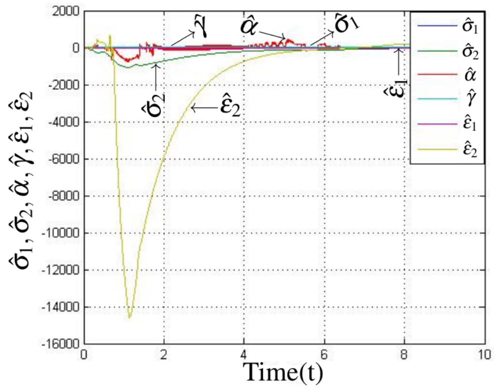


Fig. 4. The parameter estimation of the modified Colpitts oscillator

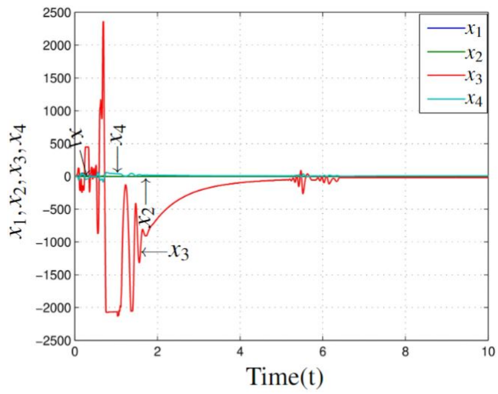


Fig. 5. The stability of the modified Colpitts oscillator

IV. SYNCHRONIZATION OF MODIFIED CHAOTIC COLPITTS OSCILLATOR

Another technique to illustrate sensitivity depending on initial conditions is the synchronisation of a chaotic system. *Master-slave* or *drive-response* coupling between the two chaotic systems must be designed so that the temporal evolution is optimum.

The master and slave systems are the two dynamic systems that are engaged in the synchronisation in general. A well-designed controller will align the slave system's trajectory with the master system's, resulting in synchronisation between the two systems.

The synchronisation procedure for the modified Colpitts oscillator employing non-linear control is discussed in the next sub-section.

A. Synchronization of modified chaotic Colpitts oscillator using non-linear feedback method

The modified Colpitts oscillator is now being synchronised. The drive-response approach is used. The modified Colpitts oscillators have a synchronisation that is identical.

The chaos synchronization basically requires the global asymptotic stability of the error dynamics

$$\text{i.e., } \lim_{t\to\infty} \|e(t)\| = 0.$$

The modified Colpitts oscillator is taken as drive system, which is described by

$$\begin{aligned} \dot{x}_1 &= \sigma_1(-x_1 - x_2) + x_4 - \gamma\phi_1(x_3), \\ \dot{x}_2 &= -\varepsilon_1\sigma_1 x_1 - \varepsilon_1\sigma_1 x_2 + \varepsilon_1 x_4, \\ \dot{x}_3 &= \varepsilon_2 x_4 - \varepsilon_2(1-\alpha)\gamma\phi_2(x_1), \\ \dot{x}_4 &= -x_1 - x_2 - x_3 - \sigma_2 x_4, \end{aligned} \tag{27}$$

where x_1, x_2, x_3 and x_4 are state variables, σ_1, σ_2, ε_1, ε_2, γ, α are positive parameters, $\phi_1(x_3) = \log(x_3) - 1$ and $\phi_2(x_1) = \log(x_1) - 1$.

The modified Colpitts oscillator is also taken as the response system which is described by

$$\begin{aligned} \dot{y}_1 &= \sigma_1(-y_1 - y_2) + y_4 - \gamma\phi_1(y_3) + u_1, \\ \dot{y}_2 &= -\varepsilon_1\sigma_1 y_1 - \varepsilon_1\sigma_1 y_2 + \varepsilon_1 y_4 + u_2, \\ \dot{y}_3 &= \varepsilon_2 y_4 - \varepsilon_2(1-\alpha)\gamma\phi_2(y_1) + u_3, \\ \dot{y}_4 &= -y_1 - y_2 - y_3 - \sigma_2 y_4 + u_4, \end{aligned} \tag{28}$$

where $\phi_1(y_3) = \log(y_3) - 1$, $\phi_2(y_1) = \log(y_1) - 1$.

The synchronization error occurring in the system is defined by

$$e_i = y_i - x_i, i = 1, 2, 3, 4. \tag{29}$$

The resulting error dynamics of the system is governed by the set of equations

$$\begin{aligned} \dot{e}_1 &= -\sigma_1 e_1 - \sigma_1 e_2 + e_4 - \gamma\phi_1(y_3) + \gamma\phi_1(x_3) + u_1, \\ \dot{e}_2 &= -\varepsilon_1\sigma_1 e_1 - \varepsilon_1\sigma_1 e_2 + \varepsilon_1 e_4 + u_2, \\ \dot{e}_3 &= \varepsilon_2 e_4 - \varepsilon_2(1-\alpha)\gamma(\phi_2(y_1) - \phi_2(x_1)) + u_3, \\ \dot{e}_4 &= -e_1 - e_2 - e_3 - \sigma_2 e_4 + u_4, \end{aligned} \tag{30}$$

where $u = (u_1, u_2, u_3, u_4)^T$ is the non-linear controller to be designed so as to synchronize the states of identically modified Colpitts oscillator.

Now the objective is to find the control law u_i, $i = 1, 2, 3, 4$ for stabilizing the error variable of the system (30) at the origin.

Let the energy source function Lyapunov be chosen as

$$V = \frac{1}{2}\sum_{i=1}^{4} e_i^2. \tag{31}$$

The derivative of (31) with respect to t is provided by

$$\dot{V} = \sum_{i=1}^{4} e_i \dot{e}_i. \tag{32}$$

Substituting (29) and (30) into (32) we are led to the relation

$$\begin{aligned}\dot{V} = {} & e_1\left(-\sigma_1 e_1 - \sigma_1 e_2 + e_4 - \gamma\phi_1(y_3) + \gamma\phi_1(x_3) + u_1\right) \\ & + e_2\left(-\varepsilon_1\sigma_1 e_1 - \varepsilon_1\sigma_1 e_2 + \varepsilon_1 e_4 + u_2\right) \\ & + e_3\left(\varepsilon_2 e_4 - \varepsilon_2(1-\alpha)\gamma\left(\phi_2(y_1) - \phi_2(x_1)\right) + u_3\right) \\ & + e_4\left(-e_1 - e_2 - e_3 - \sigma_2 e_4 + u_4\right)\end{aligned}$$

The controllers are defined by

$$\begin{aligned} u_1 &= \sigma_1 e_2 - e_4 + \gamma\left(\phi_1(y_3) - \phi_1(x_1, x_3)\right), \\ u_2 &= \varepsilon_1\sigma_1 e_1 - \varepsilon_1 e_4, \\ u_3 &= \varepsilon_2(1-\alpha)\gamma\left(\phi_2(y_1) - \phi_2(x_1)\right) - \varepsilon_2 e_4 - e_3, \\ u_4 &= e_1 + e_2 + e_3. \end{aligned}$$

Therefore the relation (32) becomes

$$\dot{V} = -\sigma_1 e_1^2 - \varepsilon_1\sigma_1 e_2^2 - e_3^2 - \sigma_2 e_4^2$$

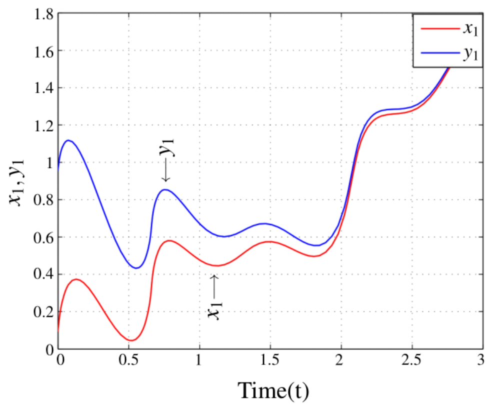
which is a negative definite function.

Thus, by Lyapunov stability theory, the error dynamics provided by (30) is found to be globally asymptotically stable for all initial conditions $e(0) \in R^4$.

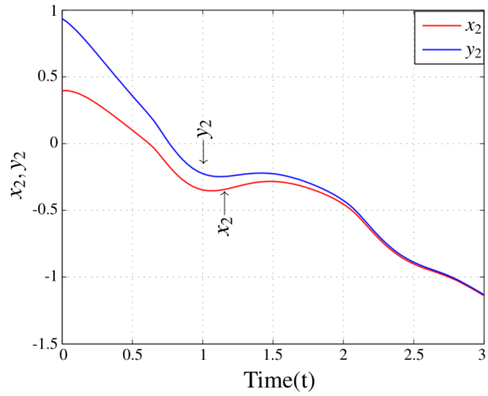
Consequently, it is seen that the states of the drive and response system synchronize globally and asymptotically.

B. Numerical simulation

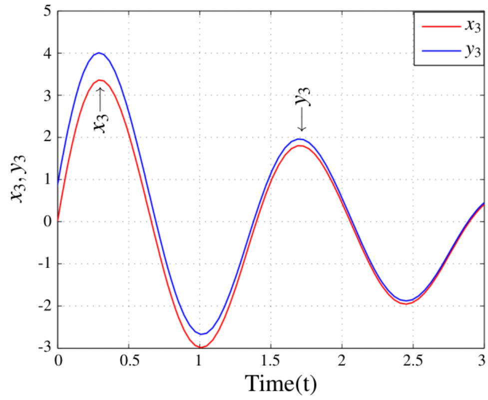
For numerical simulation, the initial conditions of the drive system are chosen as 0.09124, 0.3942, 0.0125, 0.9823 and the initial conditions for the response system are taken as 0.9546, 0.9353, 0.8765, 0.1654.



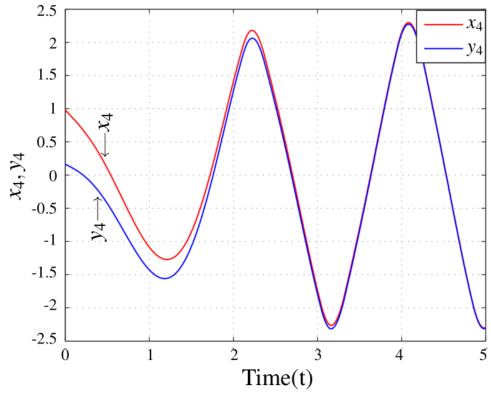
(a) Synchronization between x_1 and y_1



(b) Synchronization between x_2 and y_2



(c) Synchronization between x_3 and y_3



(d) Synchronization between x_4 and y_4

Fig. 6. Synchronization of the modified Colpitts oscillator

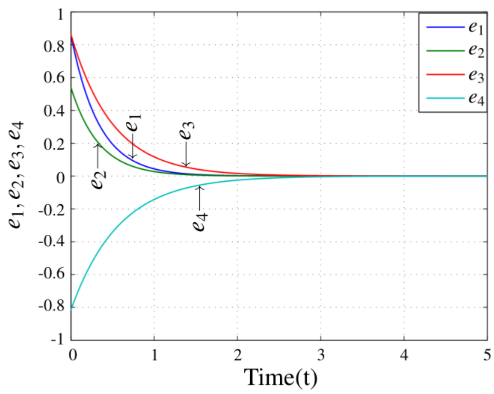


Fig. 7. Error Dynamics of Chaotic Colspitts oscillator

V. THE SYNCHRONIZATION OF COLPITTS OSCILLATOR VIA BACKSTEPPING CONTROL

The backstepping approach is a cyclic procedure that uses a feedback controller and an appropriate Lyapunov function. It causes the strict feedback chaotic systems to achieve global stability synchronisation. The proposed system is implemented using the backward backstepping method in this section.

A. Analysis of the error dynamics

The error dynamics system is taken as

$$\begin{aligned}\dot{e}_4 &= -e_1 - e_2 - e_3 - \sigma_2 e_4 + u_1,\\ \dot{e}_3 &= \varepsilon_2 e_4 - \varepsilon_2 (1-\alpha)\gamma(\phi_2(y_1) - \phi_2(x_1)) + u_2,\\ \dot{e}_2 &= -\varepsilon_1\sigma_1 e_1 - \varepsilon_1\sigma_1 e_2 + \varepsilon_1 e_4 + u_3,\\ \dot{e}_1 &= -\sigma_1 e_1 - \sigma_1 e_2 + e_4 - \gamma(\phi_1(y_3) - \phi_1(x_3)) + u_4.\end{aligned} \tag{33}$$

Now the objective is to find the control laws $u_i (i = 1, 2, 3, 4)$ for stabilizing the error variables of the system (33) at the origin.

First, consider the stability of the system

$$\dot{e}_4 = -e_1 - e_2 - e_3 - \sigma_2 e_4 + u_1 \tag{34}$$

where e_3 is considered as a virtual controller provided by

$$e_3 = \beta_1(e_4) \text{ and } \beta_1(e_4) = 0.$$

The Lyapunov function is defined as

$$V_1 = \frac{1}{2}e_4^2. \tag{35}$$

The derivative of V_1 with respect to t is obtained as

$$\dot{V}_1 = e_4\dot{e}_4 \tag{36}$$

If $\beta_1 = 0$ and $u_1 = e_1 + e_2$, then we obtain

$$\dot{V}_1 = -\sigma_2 e_4^2 \tag{37}$$

which is a negative definite function.

Hence, the system (34) is globally asymptotically stable.

The function $\beta_1(e_4)$ is an estimator when e_3 is considered as virtual controller.

The relation between e_3 and β_1 is defined by

$$\omega_2 = e_3 - \beta_1 = e_3.$$

Consider the subsystem (e_4, ω_2) given by

$$\begin{aligned}\dot{e}_4 &= -\omega_2 - \sigma_2 e_4,\\ \dot{\omega}_2 &= \varepsilon_2 e_4 - \varepsilon_2(1-\alpha)\gamma(\phi_2(y_1) - \phi_2(x_1)) + u_2.\end{aligned} \tag{38}$$

Let e_2 be a virtual controller in system (38).

Assume that when $e_2 = \beta_2(e_4, \omega_2)$, the system (38) is rendered globally asymptotically stable.

Consider the Lyapunov function defined by

$$V_2 = V_1 + \frac{1}{2}\omega_2^2.$$

The derivative of V_2 with respect to t is

$$\dot{V}_2 = e_4\dot{e}_4 + \omega_2\dot{\omega}_2$$

If $\beta_2 = 0$ and $u_2 = -(\varepsilon_2 - 1)e_4 + \varepsilon_2(1 - \alpha)\gamma(\phi_2(y_1) - \phi_2(x_1)) + e_2 - \omega_2$, then we obtain

$$\dot{V}_2 = -\sigma_2 e_4^2 - \omega_2^2$$

which is a negative definite function.

Hence by Lyapunov stability theory, the system is stable.

Let us consider the relation between e_2 and β_2 defined by

$$\omega_3 = e_2 - \beta_2 = e_2.$$

Now the subsystem $(e_4, \omega_2, \omega_3)$ is considered as

$$\begin{aligned}\dot{e}_4 &= -\omega_2 - \sigma_2 e_4,\\ \dot{\omega}_2 &= e_4 + \omega_3 - \omega_2,\\ \dot{\omega}_3 &= -\varepsilon_1\sigma_1 e_1 - \varepsilon_1\sigma_1\omega_3 + \varepsilon_1 e_4 + u_3.\end{aligned} \tag{39}$$

Consider the function V_3 due to Lyapunov function defined by

$$V_3 = V_2 + \frac{1}{2}\omega_3^2.$$

On differentiating V_3 with respect to t, we get

$$\dot{V}_3 = e_4\dot{e}_4 + \omega_2\dot{\omega}_2 + \omega_3\dot{\omega}_3.$$

If $\beta_3 = 0$ and $u_3 = -\omega_2 - \varepsilon_1 e_4$, then we obtain

$$\dot{V}_3 = -\sigma_2 e_4^2 - \omega_2^2 - \varepsilon_1\sigma_1\omega_3^2$$

which is a negative definite function.

Now the relation between e_1 and β_3 is defined as

$$\omega_4 = e_1 - \beta_3 = e_1.$$

Let us consider the subsystem $(e_4, \omega_2, \omega_3, \omega_4)$ provided by

$$\begin{aligned}\dot{e}_4 &= -\omega_2 - \sigma_2 e_4,\\ \dot{\omega}_2 &= e_4 + \omega_3 - \omega_2,\\ \dot{\omega}_3 &= -\varepsilon_1\sigma_1\omega_4 - \varepsilon_1\sigma_1\omega_3 - \omega_2,\\ \dot{\omega}_4 &= -\sigma_1\omega_4 - \sigma_1\omega_3 + e_4 - \gamma(\phi_1(y_3) - \phi_1(x_3)) + u_4.\end{aligned} \tag{40}$$

Consider the Lyapunov function

$$V_4 = V_3 + \frac{1}{2}\omega_4^2.$$

The derivative of V_4 with respect to t is

$$\dot{V}_4 = e_4\dot{e}_4 + \omega_2\dot{\omega}_2 + \omega_3\dot{\omega}_3 + \omega_4\dot{\omega}_4.$$

If $\beta_4 = 0$ and $u_4 = \varepsilon_1\sigma_1\omega_3 + \sigma_1\omega_3 - e_4 + \gamma(\phi_1(y_3) - \phi_1(x_3))$, then we obtain

$$\dot{V}_4 = -\sigma_2 e_4^2 - \omega_2^2 - \varepsilon_1\sigma_1\omega_3^2 - \sigma_1\omega_4^2$$

which is a negative definite function.

Hence by Lyapunov stability theory, the system is stable.

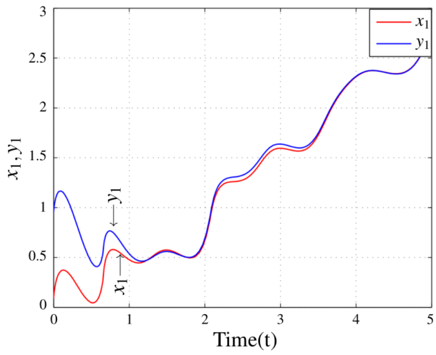
B. Numerical simulation

For solving the system of differential equations (33) with the backstepping controls u_1, u_2, u_3 and u_4, the fourth-order Runge–Kutta method is used and numerical simulation is carried out. We have

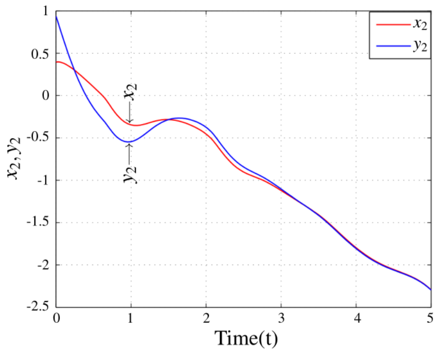
$$\begin{aligned} u_1 &= e_1 + e_2, \\ u_2 &= -(\varepsilon_2 - 1)e_4 + \varepsilon_2(1-\alpha)\gamma\left(\phi_2(y_1) - \phi_2(x_1)\right) \\ &\quad + e_2 - \omega_2, \\ u_3 &= -\omega_2 - \varepsilon_1 e_4, \\ \text{and } u_4 &= \varepsilon_1\sigma_1\omega_3 + \sigma_1\omega_3 - e_4 + \gamma\left(\phi_1(y_3) - \phi_1(x_3)\right). \end{aligned}$$

The initial values of the drive system (27) are chosen as $x_1(0) = 0.09124$, $x_2(0) = 0.3942$, $x_3(0) = 0.0125$, $x_4(0) = 0.9873$. The initial values of the response system (28) are taken as $y_1(0) = 0.9546$, $y_2(0) = 0.9353$, $y_3(0) = 0.8765$, $y_4(0) = 0.1654$.

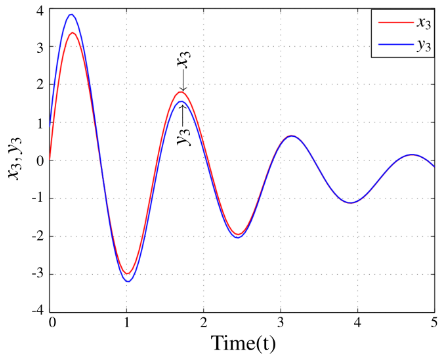
Figure 8 portrays the chaos synchronization of identical drive and response systems provided by (27) and (28), respectively.



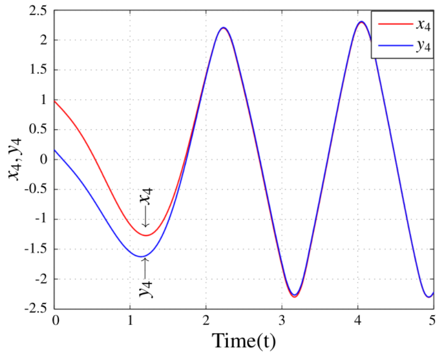
(a) Synchronization between x_1 and y_1



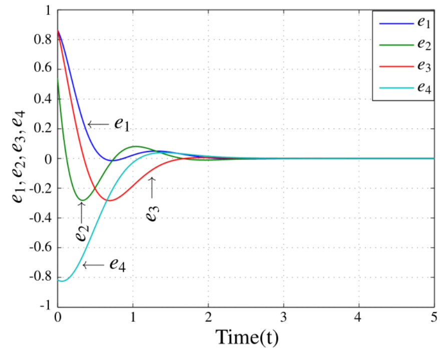
(b) Synchronization between x_2 and y_2



(c) Synchronization between x_3 and y_3



(d) Synchronization between x_4 and y_4



(e) Error Dynamics of modified Colpitts oscillator

Fig. 8. Synchronization of identical modified Colpitts oscillator, error plot for identical modified Colpitts oscillator

VI. Circuit Implementation

An operational amplifier circuit is developed in accordance with in order to verify the dynamical properties of the modified Colpitts oscillator (1). Linear resistance and linear capacitors are used to build the circuit. The operational amplifiers permitted voltage range determines the appropriate variables proportional compression transformation for the system's state variables. The oscillation circuit equation is as follows in terms of circuit diagrams.

$$\begin{aligned} \dot{x}_1 &= \sigma_1(-x_1 - x_2) + x_4 - \gamma\phi_1(x_3), \\ \dot{x}_2 &= \varepsilon_1\sigma_1(-x_1 - x_2) + \varepsilon_1 x_4, \\ \dot{x}_3 &= \varepsilon_2(x_4 - (1-\alpha)\gamma\phi_2(x_1)), \\ \dot{x}_4 &= -x_1 - x_2 - x_3 - \sigma_2 x_4, \end{aligned}$$

where $\phi_1(x_3) = \log(x_3) - 1$, $\phi_2(x_1) = \log(x_1) - 1$ and the parameter values are

$$\begin{aligned} \sigma_1 &= \frac{R_2(R_5 + R_8)}{R_5 R_1 C_1 R_3 (R_6 + R_7)} = \frac{R_{36}(R_{31} + R_{32})}{R_{37} R_{31} (R_{34} + R_{35})}, \\ \sigma_2 &= \frac{R_{64} R_{76} R_{78}}{R_{63} C_4 R_{65} R_{75} R_{77}}, \\ \varepsilon_1 &= \frac{R_{28} R_{37}}{R_{27} C_2 R_{29} R_{36}}, \\ \varepsilon_2 &= \frac{R_{42} R_{46}}{R_{41} C_3 R_{43} R_{45}}, \\ \gamma &= \frac{R_2 R_{20}}{R_1 C_1 R_3 R_{17}} = \frac{R_{58}}{R_{55}}, \\ \alpha &= \frac{R_{46} - R_{45}}{R_{46}} \end{aligned}$$

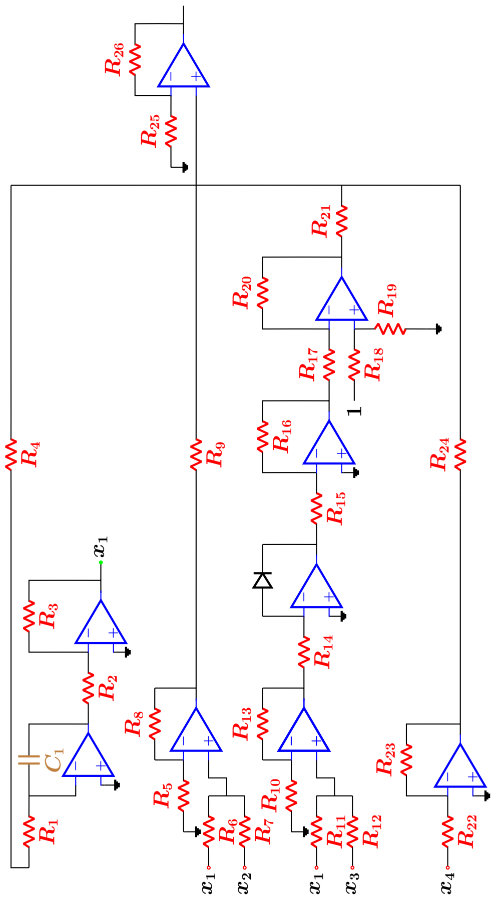


Fig. 9. Op-Amp circuit diagram of chaotic variable x_1

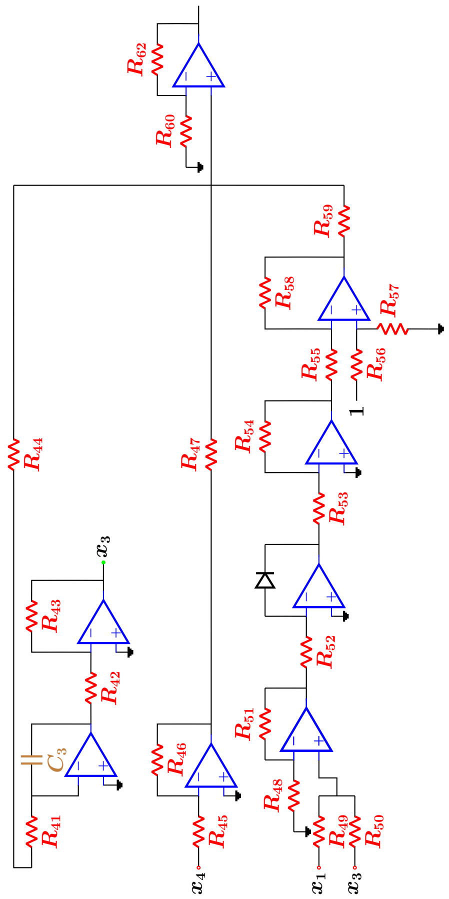


Fig. 11. Op-Amp circuit diagram of chaotic variable x_3

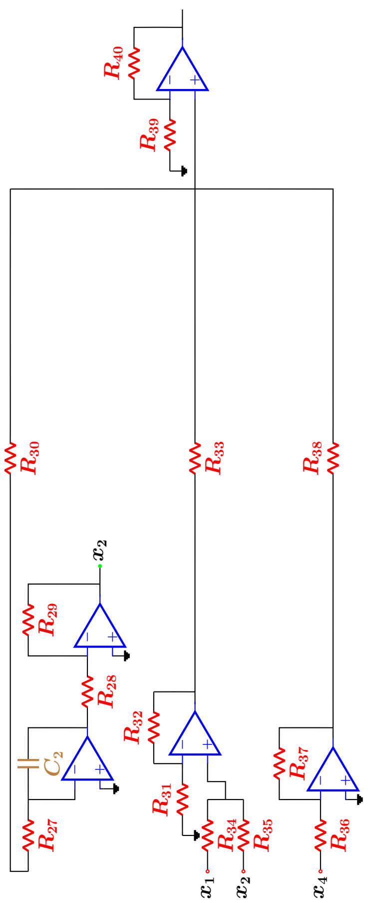


Fig. 10. Op-Amp circuit diagram of chaotic variable x_2

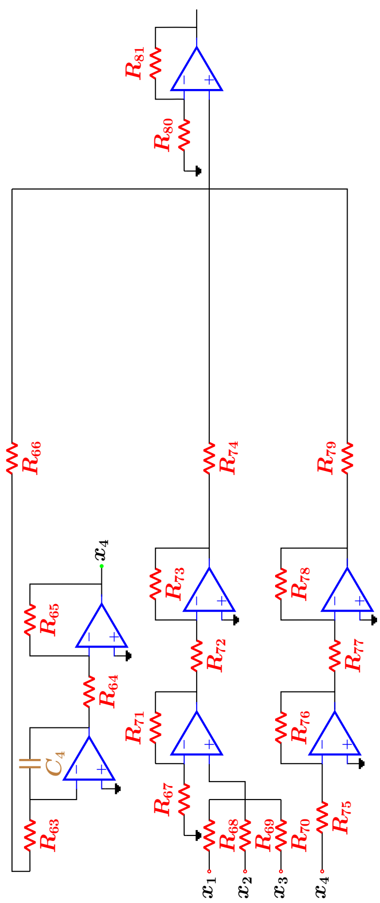


Fig. 12. Op-Amp circuit diagram of chaotic variable x_4

VII. Conclusion

The Colpitts oscillator with logarithmic function non-linearity is investigated in this paper. The modified Colpitts oscillator's qualitative qualities are being studied. For some initial conditions and parameter values, the proposed system exhibits chaotic behaviour. The Lyapunov exponent is determined using the Wolf technique. The system is dissipative under certain beginning conditions. The system is controlled using the adaptive backstepping control approach. Backstepping control, synchronisation, and non-linear control are all used. The theoretical results are supported by numerical simulations conducted in MATLAB for this investigation. Backstepping control and the non-linear feature are used. The findings are supported by numerical simulations. For numerical simulation, MATLAB is employed.

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