Optimal Control of the SEIR Model of Online Game Addiction Using Guidance and Counseling

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***Abstract*—The purpose of this research is to develop the SEIR model to deal with online game addiction, identify optimal control strategies through guidance and counseling for students addicted to online games, as well as to analyze and simulate models to predict the proportion of students who manage their online game addiction and those who do not. This research is a theoretical and applied research that determined the equilibrium point, stability, and basic reproduction number (R_0) of the model. The Pontryagin principle was used to optimize the control of the SEIR model, while Maple was used to simulate the online game addiction model. The results show that the SEIR model can be used to deal with online game addiction. The analysis of the model showed that the basic reproduction number before and after control was $R_0 = 0.2221$ and $R_0 = 0,1342$, respectively, which indicates that the problem of online game addiction can be overcome. The simulation results of the SEIR model showed that optimal control through guidance and counseling can reduce the number of students addicted to online games.**

***Index Terms*—Guidance and counselling, Online game, Optimal control, Pontryagin principle, and SEIR model.**

I. Introduction

ONLINE games can be addictive and detrimental to the health status of individuals. Furthermore, it can cause forgetfulness, negligence towards studies, and even distraction during study hours, specifically when played uncontrollably. Gaming can lead to death when individuals spend too much time in front of the computer [1]. Addiction is defined as a pleasurable activity that is performed in such a way that it causes people to lose track of time and renders them incapable of self-control [2]. Indonesia has the largest population of gamers in Southeast Asia at 6 million people, where 40% are teenagers who suffer negative consequences due to their inability to stop gaming. Boys and girls between the ages of 12 and 22 can be addicted to online games at 64.45% and 47.85%, respectively [3]. The World Health Organization (WHO) officially classified game addiction as a mental health disorder in the 11th edition of the International Classification of Diseases [4].

A mathematical model approach can be utilized to solve the problem of online game addiction. Several research have been conducted based on the mathematical model in order to predict the incidence of online game addiction [5], [6].

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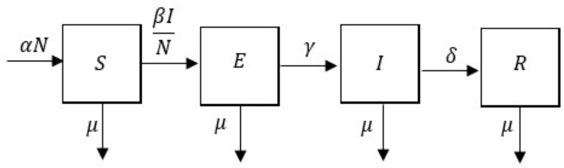

Fig. 1. SEIR Model Schematic for Online Game Addiction Problems

The mathematical model of online game addiction can be designed in the same way as the mathematical model of infectious disease prevalence by classifying online game addiction as a social disease. The review of the mathematical models for the spread of infectious diseases, such as dengue fever, hepatitis, tuberculosis, and Covid-19, has been conducted by [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. Also, the research on optimal control, focusing on infectious disease spread was carried out by [19], [20], [21]. Therefore, this research develops a mathematical model of infectious disease based on online game addiction, to provide optimal control through guidance and counselling.

This research establishes a model of online game addiction by examining those who played online games without getting addicted (E). The factor of counselling and guidance in population class E was incorporated into the modifications. Furthermore, the model was analyzed by employing control parameters, such as optimal control to reduce the number of students who were addicted to online games.

II. Research Method

This is a pure and applied research that determines the equilibrium point, stability, and basic reproduction number of the model [5]. The optimal control of the SEIR model was conducted using the Pontryagin principle [22], while the Maple and secondary data were used to simulate the online game addiction model [6]. The steps involved in this research include the determination of assumptions concerning the development of the SEIR model of online game addiction, incorporation of control parameters to the model, determination of the Lagrangian and Hamiltonian equations, as well as the solution (t) in order to achieve optimal conditions, and lastly numerical simulation and analysis of the results using the optimal control analysis.

III. Results and Discussion

A. *The SEIR Model for the Problem of Online Game Addiction*

The population changes for the SEIR model of online game addiction are interpreted in Figure 1, while the variables and parameters are presented in Table 1.

TABLE I
DEFINITION OF THE VARIABLES AND PARAMETERS OF THE SEIRS MODEL FOR ONLINE GAME ADDICTION

Variable & Parameters	Definition
αN	Proportion of total student population
S	Group of students who are prone to online game addiction
E	Group of students who play online games
I	Group of students who are addicted to online games
R	Group of students who are not addicted to online games
β	The rate at which students who are prone to online game addiction (S) start to play online games (E)
γ	The rate at which students who play online games (E) become addicted (I)
δ	The rate at which students who are addicted to online games (E) overcome their addiction (R)
β	The rate of students leaving all groups

Based on the scheme in Figure 1, a mathematical model is obtained in a system of differential equations as seen in Equation (1).

$$\begin{cases} \frac{dS}{dt} = \alpha N - \frac{\beta I}{N} S - \mu S \\ \frac{dE}{dt} = \frac{\beta I}{N} S - \gamma E - \mu E \\ \frac{dE}{dt} = \gamma E + \delta I - \mu I \\ \frac{dR}{dt} = \delta I + \mu R \end{cases} \tag{1}$$

B. Analysis of the SEIR Model for Online Game Addiction

1) Online Game Addiction-Free Equilibrium Point: The term "Addiction-free" assumes that there is no addiction to online games within the human population. Therefore, the addiction-free equilibrium point can be obtained when I = 0, then

$$S, E, I, R = 1, 0, 0, 0$$

2) Equilibrium Point for Not Free of Online Game Addiction: The determination of the equilibrium point of online game addiction can be conducted using the S, E, I, R, when the system (1) is zero. Therefore, the equilibrium point of online game addiction based on (S^*, E^*, I^*, R^*) is:

$$S^* = \frac{\delta\gamma + \delta\mu + \gamma\mu + \mu^2}{\beta\gamma}, E^* = \frac{(\alpha - \mu)(\delta + \mu)}{\beta\gamma},$$

$$I^* = \frac{\alpha - \mu}{\mu}, R^* = \frac{\delta(\alpha - \mu)}{\mu}$$

Furthermore, the Generation Matrix method is used to determine the basic reproduction number (R_0) , as presented in Equation (2).

$$F = \begin{pmatrix} \beta SI \\ 0 \end{pmatrix} \ or \ F' = \begin{pmatrix} 0 & \beta S_0 \\ 0 & 0 \end{pmatrix} \tag{2}$$

Afterward, the value of V^{-1} is presented in Equation (3).

$$V = \begin{pmatrix} (\delta + \mu)E \\ -\delta E + (\alpha + \mu)I \end{pmatrix} \ or \ V^{-1} = \begin{pmatrix} \frac{1}{\delta + \mu)} & 0 \\ \frac{\delta}{(\delta+\mu)(\sigma+\mu)} & \frac{1}{\alpha+\mu} \end{pmatrix} \tag{3}$$

It was found:

$$FV^{-1} = \begin{pmatrix} 0 & \beta \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\delta + \mu)} & 0 \\ \frac{\delta}{(\delta+\mu)(\sigma+\mu)} & \frac{1}{\alpha+\mu} \end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix} \frac{\delta\beta}{(\delta+\mu)(\alpha+\mu)} & \beta \\ 0 & 0 \end{pmatrix}$$

Therefore, the basic reproduction number was obtained before the given control and presented in Equation (4).

$$R_0 = \frac{\delta\beta}{(\delta + \mu)(\alpha + \mu)} \tag{4}$$

where $u(t)$ is the control with guidance and counseling, and the SEIR model of online game addiction is obtained after the control setup as in Equation (2).

3) The SEIR Model Optimal Control Analysis for Online Game Addiction Problem: Equation (1) shows the modification of the model's control analysis by optimal control, which was supported by the Pontryagin Maximum Principle [22].

Given control variable

$$U = \{u(t) | 0 \leq u(t) \leq 1, \forall t \in [0, T]\},$$

where is the control with guidance and counseling, and the SEIR model of online game addiction is obtained after the control setup as in Equation (2).

$$\begin{cases} \frac{dS}{dt} = \alpha N - \frac{\beta I}{N} S - \mu S \\ \frac{dE}{dt} = \frac{\beta I}{N} S - (1 - u(t))\gamma E - \mu E \\ \frac{dE}{dt} = (1 - u(t))\gamma E + \delta I - \mu I \\ \frac{dR}{dt} = \delta I + \mu R \end{cases} \tag{5}$$

The coefficient $1 - u(t)$ is an attempt to limit the likelihood of students becoming addicted to online games. The efficiency level of the control treatment based on the counseling of the students addicted to online gaming is given by u(t).

IV. PONTRYAGIN MINIMUM PRINCIPLE

The objective function of the system in Equations (5) is presented in Equation (6).

$$J(u) = \int_0^T (E(t) + \frac{1}{2} CU^2) dt \tag{6}$$

with

$$U = \{u(t) | 0 \leq u(t) \leq 1, \forall t \in [0, T]\},$$

Where T is the end time and C is the cost factor of guidance and counseling.

The Lagrangian equation of the optimal control is presented in Equation (7).

$$\mathcal{L}(I, U) = E(t) + \frac{1}{2} CU^2 \tag{7}$$

Furthermore, the Hamilton equation is formed, which is the sum of Equation (7) and the constraint function multiplied by the Lagrange multiplier, as shown in Equation (8).

$$\lambda^T = (\lambda_1 \ \lambda_2 \ \lambda_3 \ \lambda_4) \tag{8}$$

Therefore, the Hamilton function is obtained as in Equation (9).

$$\begin{aligned}\mathscr{H} &= \{S(t), E(t), I(t), R(t), u(t), \lambda_1(t), \lambda_2(t), \lambda_3(t)\, \lambda_4(t)\}\\ \mathscr{H} &= E(t) + \frac{1}{2}CU^2 + \lambda_1(t)(\alpha N - \frac{\beta I}{N}S - \mu S) +\\ &\lambda_2(t)(\frac{\beta I}{N}S - (1-u(t))\gamma E - \mu E) +\\ &\lambda_3(t)((1-u(t))\gamma E + \delta I - \mu I) + \lambda_4(t)(\delta I + \mu R)\end{aligned} \tag{9}$$

The stationary conditions in Equation (10) must be satisfied in order to obtain the optimal conditions of $\mathscr{H}$.

$$\frac{\partial \mathscr{H}}{\partial U} = cU + \lambda_2\gamma E - \lambda_3\gamma E = 0$$

$$U^* = \frac{\lambda_2\gamma E - \lambda_3\gamma E}{c} \tag{10}$$

Based on equation (10) the range of the control variable is $0 \leq u(t) \leq 1$, while the optimal solution $u(t)$ is presented in Equation (11).

$$U^* = \begin{cases} 0, & if \quad \frac{\lambda_2\gamma E - \lambda_3\gamma E}{c} \leq 0 \\ \frac{\lambda_2\gamma E - \lambda_3\gamma E}{c}, & if \quad 0 \leq \frac{\lambda_2\gamma E - \lambda_3\gamma E}{c} \leq 1 \\ 1, & if \quad \frac{\lambda_2\gamma E - \lambda_3\gamma E}{c}, \geq 1 \end{cases} \tag{11}$$

It can also be written in Equation (12).

$$U^* = min\{1, max(0, \frac{\lambda_2\gamma E - \lambda_3\gamma E}{c})\} \tag{12}$$

The optimal conditions can be obtained by solving the state and co-state equations, which are presented in Equation (13) and (14). The initial conditions are shown in Equation (15).

State Equation

$$\begin{cases} \frac{\partial \mathscr{H}}{\partial \lambda_1} &= \alpha - \beta IS - \mu S \\ \frac{\partial \mathscr{H}}{\partial \lambda_2} &= \beta IS - \lambda E + \lambda EU - \mu E \\ \frac{\partial \mathscr{H}}{\partial \lambda_3} &= \lambda E - \lambda EU - \delta I - \mu I \\ \frac{\partial \mathscr{H}}{\partial \lambda_3} &= \delta I - \mu R \end{cases} \tag{13}$$

Co-State Equation

$$\begin{cases} \frac{\partial \mathscr{H}}{\partial S} &= -(-\lambda_1\beta I - \lambda_1\mu + \lambda_2\beta I) \\ \frac{\partial \mathscr{H}}{\partial E} &= -(-1 - \lambda_2\gamma + \lambda_2\gamma U - \lambda_2\mu + \lambda_3\gamma - \lambda_2\gamma U) \\ \frac{\partial \mathscr{H}}{\partial I} &= -(\lambda_1\beta S + \lambda_2\beta S - \lambda_3\delta - \lambda_3\mu - \lambda_4\delta) \\ \frac{\partial \mathscr{H}}{\partial R} &= -(\lambda_4\mu) \end{cases} \tag{14}$$

With initial condition

$$\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = 0 \tag{15}$$

Equation (16) shows the basic reproduction number formula R_0 , which is obtained after applying the control using the Generation Matrix method in Equation (2) and (3).

$$R_0 = (1-u^*)\frac{\delta\beta}{(\delta+\mu)(\alpha+\mu)} \tag{16}$$

A. Numerical Simulation of SEIR Model for Online Game Addiction Problem

The state and co-state equations are differentiated using the 4th order Runge-Kutta method [23] before the simulation. Table 2 shows the initial values of the variables and parameters used in solving the state and co-state equations using a forward and backward scheme, respectively.

TABLE II
INITIAL VALUE OF VARIABLES AND PARAMETERS OF SEIR MODEL FOR ONLINE GAME ADDICTION PROBLEMS

Variable & Parameters	Initial Value	Source
S_0	0.62	[6]
E_0	0.22	[6]
I_0	0.023	[6]
R_0	0.05	[6]
μ	0.097	[6]
α	0.438	[6]
β	0.102	[6]
γ	0.051	[6]
δ	0.0612	Assumption
C	125	Assumption

B. The Stability of the SEIR Model for Online Game Addiction Problem

The stability of the model was analysed using the equilibrium point, while the basic reproduction number of junior high school students addicted to online games was determined by substituting the parameters in Table 2 into the Equation (1) system from Equation (17).

$$\begin{cases} \frac{dS}{dt} = 0.341S - 0.102IS \\ \frac{dE}{dt} = 0.102IS - 0.148E \\ \frac{dE}{dt} = 0.051E - 0.1582I \\ \frac{dR}{dt} = 0.0612I - 0.097R \end{cases} \tag{17}$$

Furthermore, while the system of Equation (17) is equal to zero, then the equilibrium points of the SEIR model for the online game addiction problem are obtained, namely:

$$(S, E, I, R) = (0.50088427, 0.37028, 0.34313, 0.10927)$$

While the parameter values in Table 2 are substituting into Equation (5) and Equation (16), then the basic reproduction number value is $R_0 = 0.2221$ before the control and $R_0 = 0.1342$ after the control is given in the form of guidance and counseling. This shows that control in the form of guidance and counseling can help to reduce the rate of increase in the number of students addicted to online games.

The results of numerical simulations of suspected students, starting to play, addicted and stop playing online games with and without control are shown in Figures 2 to Figure 5.

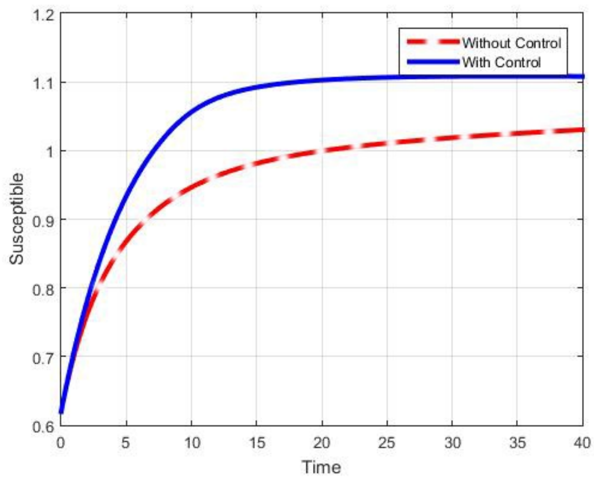

Fig. 2. Simulation Results of Numerical Game of Students who are suspected to Online Game Addiction

Figure 2 shows that there is a significant difference between students who are prone to online game addiction before and after intervention with guidance and counseling. Also, there was an increase in the number of students who are prone to addiction after intervention with guidance and counseling.

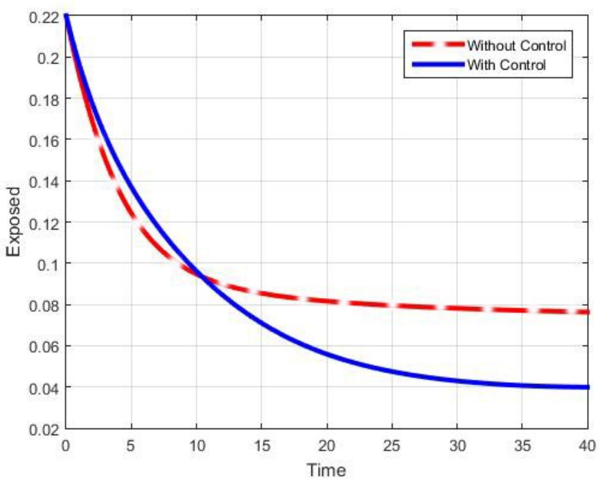

Fig. 3. Numerical Simulation Results of Students who Start Playing Online Games

Figure 3 shows that there was a significant difference in the population of students who started playing online games before and after the intervention with guidance and counseling. Also, there was a decrease in the number of students addicted to online games after the intervention through guidance and counselling.

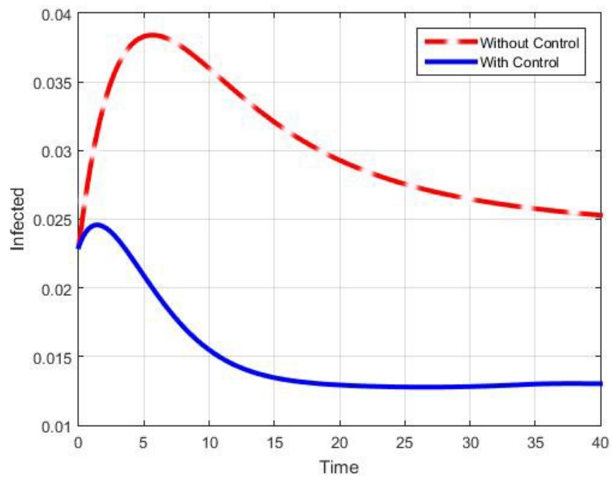

Fig. 4. Results of the Numerical Simulation of Students Addicted to Online Games

Figure 4 shows that there was a significant difference in the population of students addicted to online games before and after the intervention through guidance and counseling. Also, there was a decrease in the number of students addicted to online games after the intervention through guidance and counseling.

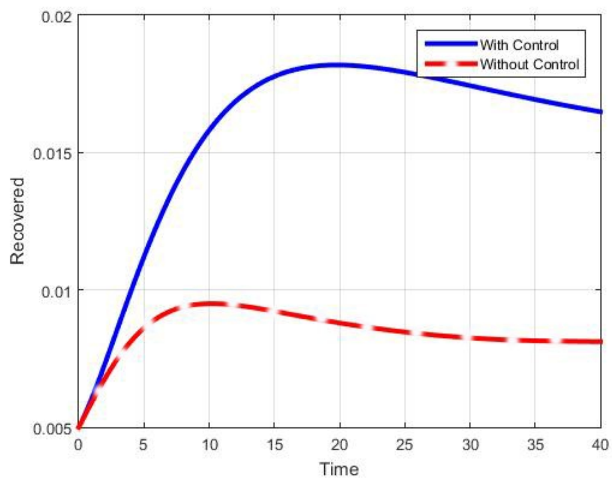

Fig. 5. Numerical Simulation Results of Students who Stop Playing Online Games

Figure 5 shows that there was a significant difference in the population of students who started playing online games before and after the intervention through guidance and counseling. Also, there was an increase in the number of students who stop playing online games after the intervention through guidance and counseling.

The results of the numerical simulation show that guidance and counseling as the control variable can cause changes in the population dynamics as expected, such as a decrease in the number of students addicted to online games and an increase in the number of students who stopped playing online games.

V. Discussion

The case of online game addiction investigated by [6] demonstrated that guidance and counseling by teachers can reduce the number of students addicted to online games. According to previous research by [6], [9] the results showed that optimal control can reduce the number of students addicted to online games. Also, the result of previous research by [5] show that the number of students addicted to online games increases by 6 every year at a state of equilibrium. Meanwhile, the simulation results in Table 2 show the value of the basic reproduction number $R_0 = 0.2221$, which indicates that online game addiction in Junior High School students in Makassar City can still be overcomed through the necessary control measures. The simulation results showed that there was a decrease in the basic reproduction number$R_0 = 0.1342$, which indicates that providing control through guidance and counseling can help students overcome their addiction to online games.

VI. Conclusion

Therefore, it can be concluded from this research that the SEIR model can be used as a reference with optimal control to identify online game addiction. The analysis of the model provided an overview of the stability of individuals who are addicted to online games. Also, optimal control through guidance and counseling can reduce the number of junior high school students who are addicted to online games, while increasing the number of students who stop playing online games

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