Optimization Study of M/M/c Machine Repairable System with Two Service Rates

Shengli Lv, Tong Li

***Abstract*—In a machine repairable system, when the number of failure machines in the system reaches to a certain threshold, in order to maintain the machines' production efficiency, repairers will accelerate the repair rate, although generally this comes at a higher cost. Therefore, this paper discusses the M/M/c machine repairable systems with two service rates. Using Markov process theory, the steady-state reliability and queueing indexes are obtained. Through building an optimization model, the system is optimized according to the model analysis results. Finally, the numerical experimental analysis of the system steady-state indexes and optimization model is given.**

***Index Terms*—machine repairable system, steady-state availability, variable repair rate, system optimization.**

I. Introduction

REPAIRABLE queueing systems have been used widely in computer systems[1], flexible manufacturing systems, production management, and fluid model[2]. The study of machine repairable models started in the 1940s. Initially, Palm[3] studied machine repair models with a single repairer. Tian et al.[4] analyzed the complex queueing system model using quasi birth and death process and matrix geometric solution method. Gross et al.[5] systematically studied and elaborated the classical machine repairable model M/M/c with the application of finite source queueing, and deduced the distribution law of the number of system failure machines and waiting for repair time in a stable state through the method of the classical birth and death process. Lv et al.[6] studied the M/M/1 repairable queueing system. In this article, the repair rate varies when the system's customer count approaches a specific threshold, which provides a theoretical foundation and data source for the optimal design of related service systems in real life.

The machine repairable queue phenomenon is more prominent in the application of production and manufacturing areas. Working closely with each component in industrial production lines is essential to ensuring that industrial production runs smoothly. Once one of the links goes wrong, the production line's efficiency will be reduced even when it continues to run normally. To ensure productivity, how to set up machine production operations and repairs is practical problems related to queueing theory. The N-strategy of machining system is an effective measure to improve economic performance in the machine repairable system. The repairer can perform auxiliary tasks, which not only improves system efficiency but also saves costs. Chen et al.[7] studied the reliability of the repair system of a single maintenance server with N-strategy, M operating units and S warm spare units. When a server fails, the repairer provides service at a lower service rate, and through a large number of numerical experiments, the influence of each system parameter change on the system is measured. Li et al.[8] mainly studied the replacement of the N-strategy based on the number of failures of component 1 in the system, further gave the expression of the average cost of the system, and found the optimal strategy of the system through numerical analysis. Liu et al.[9] considered the M/M/1 queueing model under the N-strategy, constructed the social income function with the benefit maximization as the starting point, and analyzed the impact of each parameter on the equilibrium income.

To improve productivity, the system can stock a portion of spare parts. Yue et al.[10] studied repairable queueing systems with spare parts and obtained reliability indicators such as the steady-state probability of the system using Markov process theory and an iterative approach. Jain et al.[11] studied the reliability index of machine-repairable systems with M operating units and group standby units, derived the explicit expression of reliability function and system average failure time by using Laplace transform technology, and analyzed the influence of various system parameters on system reliability index by numerical results.

Queueing theory and reliability mathematical theory are used widely in the optimal allocation of resources[12], [13], [14], [15]. Meng[16] established a multi-criteria optimization model for the optimization of the number of repairers and illustrated the effectiveness and practicality of the optimization model in enterprise management with examples. Jain et al.[17] studied the queueing model with single server and single working vacation under the policy, analyzed the system performance index value under the optimal control policy, and discussed the maximum revenue under the relevant constraints, which provided a theoretical reference for the transportation system and manufacturing system. Rational allocation of the number of repair tools to minimize the cost and time of repair is a typical optimization problem in the random service process. In short, production is plagued by repair and queueing issues, making it increasingly crucial to research more realistic and reasonable queueing and dependability issues related to machine repair.

II. Model Description

In the initial state, the system has m running machines, s warm standby machines and c repairers.

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1) It is assumed that the lifetimes of the running and standby machines obey exponential distributions with parameters λ and α $(0 < \alpha < \lambda)$, respectively.

2) When the running machine fails, if there are some available standby machines, the failure machine is immediately replaced by a standby machine; If no standby machines are available, the system will start degraded operation until all the machines fail and the system stops working.

3) The system contains c repairers. When the number of failure machines is more than the number of repairers, the failure machines obey the queuing rule of first-come first-served in the waiting queue. Each repairer can only repair one machine at a time, and the repaired machines are restored as new.

4) When the number of failure machines n in the system satisfies $0 \leq n \leq c$, the repair rate of each repairer obeys the exponential distribution of parameter μ_a. When the number of failure machine is $n > c$, the repair rate of each repairer will be accelerated, which follows the exponential distribution with parameter μ_b, where $\mu_a < \mu_b$.

Let $N(t)$ represents the number of failure machines in the system at the time t, $N(t) = 0, 1, 2 \cdots m+s$. Then $\{N(t)\}$ is a Markov process with state-space $\Omega = \{0, 1, 2 \cdots, m+s\}$. The state transition rate matrix Q of the system is shown as follows:

$$Q = \begin{pmatrix} -\lambda_0 & \lambda_0 & & & & & \\ \mu_1 & -(\lambda_1+\mu_1) & \lambda_1 & & & & \\ & \ddots & \ddots & \ddots & & & \\ & & \mu_n & -(\lambda_n+\mu_n) & \lambda_n & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & \mu_{m+s} & -\mu_{m+s} & \end{pmatrix}$$

where

$$\lambda_n = \begin{cases} m\lambda + (s-n)\alpha, & n = 0, 1 \cdots s, \\ (m+s-n)\lambda, & n = s+1, s+2 \cdots m+s-1, \end{cases} \tag{1}$$

$$\mu_n = \begin{cases} n\mu_a, & n = 1, 2 \cdots c, \\ c\mu_b, & n = c+1, c+2 \cdots m+s. \end{cases} \tag{2}$$

III. Steady-state indicators of the system

Define the steady-state probability of the system as

$$\pi_n = \lim_{t\to\infty} P\{N(t) = n\}, n \in \Omega.$$

Then $\{\pi_n, n \in \Omega\}$ satisfies the following system of linear relationship equations

$$\begin{cases} \pi Q = 0, \\ \pi e = 1. \end{cases} \tag{3}$$

where $\pi = (\pi_0, \pi_1, \pi_2 \cdots \pi_{m+s})$ is a $(m+s+1)$ dimensional row vector, $e = (1, 1, \cdots, 1)^T$ is a $(m+s+1)$ dimensional column vector.

Theorem 1 The steady-state probability of the system is

$$\pi_n = \frac{\lambda_{n-1}}{\mu_n}\pi_{n-1} = \prod_{i=1}^{n} \frac{\lambda_{i-1}}{\mu_i}\pi_0, n = 1, 2 \cdots m+s.$$

where

$$\begin{aligned} \pi_0 = [1 + \sum_{n=1}^{c} \tfrac{A_n}{n!{\mu_a}^n} + \tfrac{1}{c!{\mu_a}^c} \sum_{n=c+1}^{s} \tfrac{A_n}{(c\mu_b)^{n-c}} + \\ K \sum_{n=s+1}^{m+s} \tfrac{m!}{(m+s-n)!} (\tfrac{\lambda}{c\mu_b})^{n-s}]^{-1}. \end{aligned} \tag{4}$$

$$\pi_n = \begin{cases} \dfrac{\prod_{i=0}^{n-1}[m\lambda+(s-i)\alpha]}{n!{\mu_a}^n}\pi_0, n = 1, 2 \cdots c, \\ \dfrac{\prod_{i=0}^{n-1}[m\lambda+(s-i)\alpha]}{c!{\mu_a}^c(c\mu_b)^{n-c}}\pi_0, n = c+1 \cdots s, \\ \dfrac{\prod_{i=0}^{s-1}[m\lambda+(s-i)\alpha]}{c!{\mu_a}^c(c\mu_b)^{s-c}} \dfrac{m!}{(m+s-n)!}(\dfrac{\lambda}{c\mu_b})^{n-s}\pi_0, \\ n = s+1 \cdots m+s. \end{cases} \tag{5}$$

Prove The equations (3) are written in component form as follows

$$-\lambda_0\pi_0 + \mu_1\pi_1 = 0, \tag{6}$$

$$\begin{aligned} \lambda_n\pi_n - (\lambda_{n+1}+\mu_{n+1})\pi_{n+1} + \mu_{n+2}\pi_{n+2} = 0, \\ n = 0, 1, 2 \cdots m+s-2, \end{aligned} \tag{7}$$

$$\lambda_{m+s-1}\pi_{m+s-1} - \mu_{m+s}\pi_{m+s} = 0, \tag{8}$$

$$\sum_{i=0}^{m+s} \pi_i e = 1. \tag{9}$$

From equation (6), we have

$$\pi_1 = \frac{\lambda_0}{\mu_1}\pi_0.$$

From equation (8), we have

$$\pi_{m+s} = \frac{\lambda_{m+s-1}}{\mu_{m+s}}\pi_{m+s-1}.$$

From equation (7), we obtain the following iterative formula

$$\begin{aligned} \lambda_n\pi_n - \mu_{n+1}\pi_{n+1} = \lambda_{n+1}\pi_{n+1} - \mu_{n+2}\pi_{n+2}, \\ n = 0, 1, 2 \cdots m+s-2. \end{aligned} \tag{10}$$

From equation (10) we obtain

$$\pi_n = \frac{\lambda_{n-1}}{\mu_n}\pi_{n-1} = \prod_{i=1}^{n} \frac{\lambda_{i-1}}{\mu_i}\pi_0. \tag{11}$$

Substitute equations (1) and (2) into equations (9) and (11), we obtain equations (4) and (5).

To facilitate the calculation, we let

$$A_n = \prod_{i=1}^{n} [m\lambda + (s-i+1)\alpha] = \prod_{i=0}^{n-1} [m\lambda + (s-i)\alpha].$$

and

$$K = \frac{A_s}{c!{\mu_a}^c(c\mu_b)^{s-c}}.$$

The steady-state indicators of the system are given as follows:

1) The average number of failure machines in the system

$$\begin{aligned} E(L) = \sum_{n=0}^{m+s} n\pi_n = K \sum_{n=s+1}^{m+s} n \tfrac{m!}{(m+s-n)!} (\tfrac{\lambda}{c\mu_b})^{n-s}\pi_0 \\ + \sum_{n=1}^{c} \tfrac{A_n}{(n-1)!{\mu_a}^n}\pi_0 + \tfrac{1}{c!{\mu_a}^c} \sum_{n=c+1}^{s} \tfrac{nA_n}{(c\mu_b)^{n-c}}\pi_0. \end{aligned}$$

2) The average number of machines waiting for repair of the system

$$E(L_q) = \sum_{n=c+1}^{m+s} (n-c)\pi_n$$
$$= E(L) - c + c\pi_0 + \sum_{n=1}^{c} \tfrac{(c-n)A_n}{n!{\mu_a}^n}\pi_0.$$

3) The average waiting time for a failure machine of the system

$$T = \frac{E(L_q)}{\Lambda},$$

where

$$\Lambda = \sum_{n=0}^{m+s-1} \pi_n\lambda_n$$
$$= -(\lambda-\alpha)K\sum_{n=s+1}^{m+s} \tfrac{(n-s)m!}{(m+s-n)!}(\tfrac{\lambda}{c\mu_b})^{n-s}\pi_0$$
$$+\ m\lambda + s\alpha - \alpha E(L).$$

4) The average number of machines in normal operation of the system

$$E(L_n) = \sum_{n=0}^{s} m\pi_n + \sum_{n=s+1}^{m+s} (m+s-n)\pi_n$$
$$= m - K\sum_{n=s+1}^{m+s} \tfrac{(n-s)m!}{(m+s-n)!}(\tfrac{\lambda}{c\mu_b})^{n-s}\pi_0.$$

5) The Probability that the failure machine in the system does not need to wait for repair

$$P = \sum_{n=0}^{c}\pi_n = \pi_0 + \sum_{n=1}^{c}\frac{A_n}{n!{\mu_a}^n}\pi_0.$$

6) Steady-state availability of the system

$$A = 1 - K(\frac{\lambda}{c\mu_b})^m m!\pi_0.$$

IV. Numerical experiments

Through the above analysis, we have obtained the steady-state average value $E(L)$ of failure machines in the system, the steady-state average value $E(L_n)$ of the machine in the normal operating state, and the steady-state value $E(L_q)$ of the average number of failure machines waiting to be repaired in the system. Considering the influence of system parameters λ, α, μ_a and μ_b on system performance index, the sensitivity analysis of parameters is carried out through numerical experiments. In the experiment, we fixed the system parameters as $\alpha = 0.7$, $\lambda = 1.5$, $\mu_a = 2.5$, $\mu_b = 3.5$, $m = 10$, $s = 6$ and $c = 4$.

Table 1 shows that the effect of the failure rate α of the standby machine on each system indicators is relatively small. This is because when the failure rate α of the standby machine is low, the probability of damage to the standby machine is small. Moreover, the damage of the standby machine only increases the number of failure machines, which does not affect the system operation. Therefore, the failure of the standby machine has less impact on the system.

The data in Table 2 shows that $E(L)$ and $E(L_q)$ increase as the failure rate λ of the running machines increases, the average value $E(L_n)$ of normal operating machines decreases as λ increases. This is due to the fact that the higher the λ, the greater the likelihood of failure of the running machine. At this time, the number of failure machines will increase, correspondingly the number of normal opretaing machines decreases, as a result, the system's overall performance will decrease.

According to Table 3, we find that the change in the primary repair rate μ_a has a little impact on all indicators. This is because there are relatively few repairers in relation to the total number of running machines. After a period

Table 1. System indicators with varying α, m and c.

(m,c)	α	$E(L)$	$E(L_q)$	$E(L_n)$
(10,2)	0.4	8.826	6.826	7.123
	0.7	8.834	6.834	7.119
	1	8.840	6.841	7.115
(10,4)	0.4	6.963	3.112	8.371
	0.7	7.049	3.177	8.336
	1	7.124	3.235	8.304
(15,4)	0.4	11.679	7.686	9.271
	0.7	11.687	7.693	9.266
	1	11.694	7.700	9.261

Table 2. System indicators with varying λ, m and c.

(m,c)	λ	$E(L)$	$E(L_q)$	$E(L_n)$
(10,2)	1.2	8.313	6.315	7.551
	1.5	8.834	6.834	7.119
	1.8	9.120	7.121	6.860
(10,4)	1.2	5.659	1.959	9.173
	1.5	7.049	3.177	8.336
	1.8	8.306	4.357	7.397
(15,4)	1.2	9.531	5.567	11.256
	1.5	11.687	7.693	9.266
	1.8	13.223	9.224	7.766

Table 3. System indicators with varying μ_a, m and c.

(m,c)	μ_a	$E(L)$	$E(L_q)$	$E(L_n)$
(10,2)	2.2	8.834	6.834	7.118
	2.5	8.834	6.834	7.119
	2.8	8.833	6.834	7.119
(10,4)	2.2	7.120	3.225	8.311
	2.5	7.049	3.177	8.336
	2.8	6.972	3.127	8.362
(15,4)	2.2	11.694	7.699	9.262
	2.5	11.687	7.693	9.266
	2.8	11.680	7.6875	9.270

Table 4. System indicators with varying μ_b, m and c.

(m, c)	μ_b	$E(L)$	$E(L_q)$	$E(L_n)$
(10,2)	3.2	8.988	6.988	6.982
	3.5	8.834	6.834	7.119
	3.8	8.666	6.666	7.263
(10,4)	3.2	7.634	3.728	7.907
	3.5	7.049	3.177	8.336
	3.8	6.550	2.713	8.678
(15,4)	3.2	12.471	8.474	8.505
	3.5	11.687	7.693	9.266
	3.8	10.924	6.936	9.992

of time, the system begins secondary repair since there are more failure machines, thus the primary repair rate μ_a has less impact on various indicators. If the number of repairers is relatively reasonable, such as when $m = 10, c = 4$, $E(L)$ decreases with the increase of the primary repair rate μ_a, $E(L_q)$ decreases with the increase of μ_a, and $E(L_n)$ increases with the increase of μ_a. At this point, the larger the value of μ_a indicates that the system has fewer failure machines and more normal running machines, indicating that the repairer is making repairs more quickly.

In Table 4, $E(L)$ decreases with the increase of secondary repair rate μ_b, $E(L_q)$ decreases with the increase of μ_b, and $E(L_n)$ increases with the increase of μ_b, the reasons are the same as those analyzed in Table 3. Additionally, the less repairers there are in the system, the more likely it is that they are repairing at the secondary repair rate, and the more visible this phenomena will be while all other system parameters remain constant.

According to the above four tables, when the number of repairers is less than the number of running machines, the number of failure machines will increase, and increasing the number of repairers will improve the system efficiency.

V. Optimization analysis

Through the above analysis, we find that the change of m and c has a significant impact on the system. To further analyze the economic applicability, the unit time benefit function W is introduced below to analyze the maximization of system benefits. The following are the system benefit parameters:

1) The revenue per unit time per normally running machine is C_1.

2) The stop work loss per unit time per failed machine is C_2.

3) When the number of failure machines in the system $E(L) \leq c$, the repair cost per machine per unit of time is U_1. When the number of failure machines in the system $E(L) > c$, the repair cost per machine per unit of time is U_2.

4) When the machine enters the queue, the waiting cost per machine per unit time is C_3.

5) Multiple machines working at the same time can produce joint benefits, that is, the system has $E(L_n)$ machines in normal working condition, then the benefit of each machine is $C_1(1+\frac{E(L_n)}{km})$, where k is the proportionality coefficient, and the value of k can be determined by hypothesis testing.

To facilitate the calculation, let $B_n = C_1(1+\frac{E(L_n)}{km})$,

Then the revenue function of the system per unit time is

$$W = E(L_n)B_n - E(L)C_2 - E(L)U - C_3E(L_q)$$
$$= \begin{cases} E(L_n)B_n - E(L)C_2 - E(L)U_1, E(L) \leq c, \\ E(L_n)B_n - E(L)C_2 - cU_2 - C_3E(L_q), E(L) > c. \end{cases}$$

In the experiment, we fixed the system parameter as $\alpha = 0.7$, $\lambda = 1.5$, $\mu_a = 2.5$, $\mu_b = 3.5$, $m = 10$, $s = 6$ and $c = 4$. Suppose $C_1 = 100$, $C_2 = 20$, $U_1 = 30, U_2 = 50$, $C_3 = 8$ and $k = 10$. In practice, the values of C_1 and C_2 can be obtained by hypothesis testing. The change of benefit function is shown in the following figure:

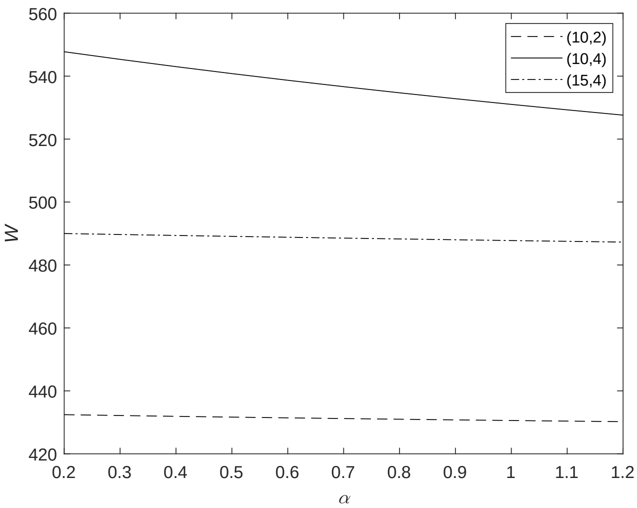

Figure 1. The variation of W with α.

According to the observation in Figure 1, it can be seen that the change scope of W with α is limited, since there are fewer repairers compared to running machines, and the probability of accumulation of failure machines in the system is large. However, since the failure rate of standby machines is low, the number of failure machines is only increased with a small probability, and the impact on system benefits is small. Secondly, longitudinally, the benefit W is jointly determined by m and c. Clearly, the higher the value of m, the more revenue the system can generate per unit time. However, if the value of c is too small, there will be an excessive number of failure machines in the system, which will not only lose its own benefits, but also bring higher repair costs. Therefore, in Figure 1, when $(m, c) = (10, 4)$, the system gains the highest value.

According to the observation in Figure 2, it can be found that the benefit W decreases with the increase of λ. Because with the increasing value of λ, $E(L)$ will become larger and $E(L_n)$ will become smaller, the system can create less revenue per unit time, the repairers' repair cost increases, so the system benefit becomes smaller. And when m is larger and c is smaller, as shown in Figure 2, when $(m, c) = (15, 4)$, the change of system benefit is more obvious.

According to the observation in Figure 3, it can be found that the change of benefit W with μ_a is not instantly obvious.

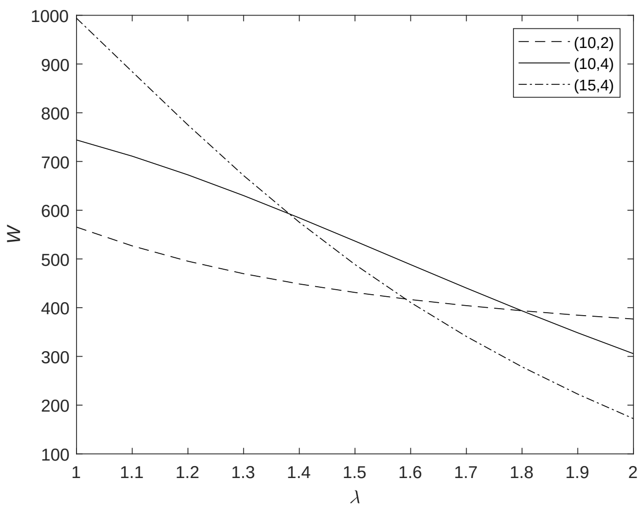

Figure 2. The variation of W with λ.

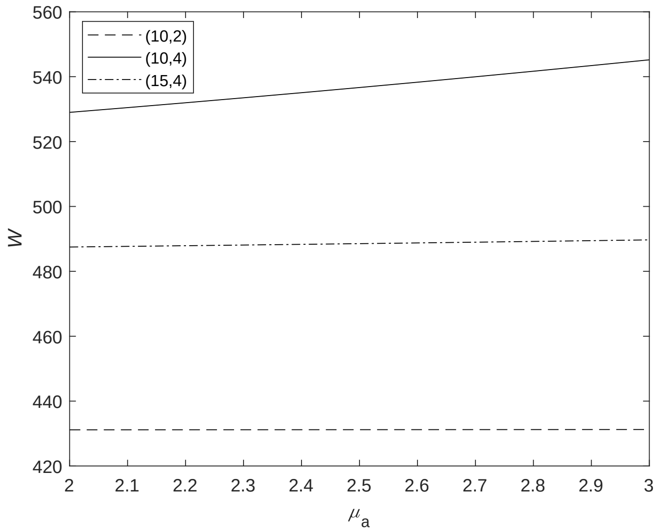

Figure 3. The variation of W with μ_a.

This is because the secondary repair rate will begin more quickly for the system the smaller the value of c, whereas the primary repair rate μ_a has no effect on the system. When $(m, c) = (10, 4)$, the number of repairers is relatively large, the variation of the primary repair rate μ_a will prolong the time when the number of failure machines is less than the number of repairers, thus the system is less likely to start the secondary repair rate μ_b. At this point, the larger μ_a is, the faster the repairers' repair rate will be, the more normal machines in the system will be, and the higher the benefit will be, at the same time, the number of machines waiting for repair will become less, the waiting time will become shorter, the corresponding waiting cost will be lower, and the system efficiency will be higher.

According to the observation in Figure 4, it can be found that W increases with the increase of μ_b, indicating that the number of failure machines is more than the number of repairers at this time, the system starts the secondary repair rate. In this way, the larger μ_b, the faster the overall repair speed of the system, the more machines can operate normally, and the machine can create the greater profit per unit time. To a certain extent, it reduces the probability of

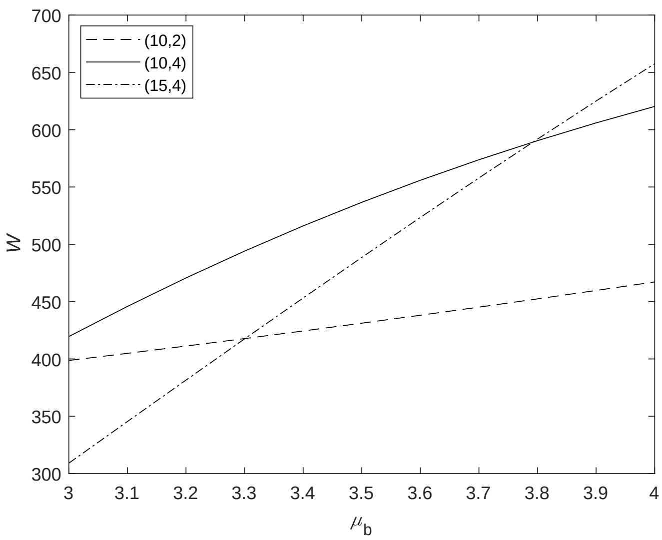

Figure 4. The variation of W with μ_b.

increasing the number of failure machines, which will offset some of the high repair costs and lost profits. In addition, when $(m, c) = (10, 2)$, μ_b has little influence on W, because m is small, in a steady state, it can create less revenue per unit time. Secondly, because the value of c is small, even if μ_b increases, the system's total repair speed will remain somewhat slow, and the system generates more loss revenue, etc. Therefore, the change of the system benefit is slow.

The conclusion of the numerical analysis show that it is not wise to simply increase the number of machines in running or decrease the number of repairer in order to minimize costs. High repair rates can only result in high profits when the system's running machine number is suitably balanced with the number of repairers.

VI. Conclusion

This paper studies the M/M/c machine repairable system with two service rates. Firstly, the steady-state performance index of the system is obtained by using the steady-state equilibrium equation. Through numerical experiments, the impacts of system parameters on the steady-state average value $E(L)$ of the failure machine, the steady-state average value $E(L_n)$ of the machine under normal operating conditions, and the average value of waiting machines $E(L_q)$ are analyzed. Finally, the optimization model is established by introducing the benefit function, and the influence of some system parameters on the benefit function are analyzed by numerical experiments. The experimental results show that the increase of primary repair rate μ_a and secondary repair rate μ_b will increase the system efficiency, which is consistent to the accustomed understanding. Therefore, in practice, when deciding the optimal strategy of the system, factors such as the number of machines, the number of repairers and the repair rate should be considered comprehensively. The research of this paper provides theoretical and technical support for the design and optimization of machine repairable system with two service rates.

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