Robust Optimization of Mixed-Load School Bus Route Based on Multi-Objective Genetic Algorithm

Minghui Zhao, Yongpeng Zhao, Changxi Ma, and Xu Wu

***Abstract*—Various optimization methods for non-mixed-load school bus routes based on certain environments have been applied to practical problems. In practice, school buses can transport students from different schools; however, the travel time is uncertain. This study accounts for the interests of both the students and school bus company using the robust discrete optimization theory of Bertsimas. A robust optimization model of the multi-school mixed-load school bus route under the condition of uncertain road section time impedance is developed considering the minimum travel time of all students and total operating cost of all school buses as optimization goals. Moreover, a two-stage insertion method, NSGA-II, is designed to solve the problem. Then a certain area of Baotou City is considered as an example to conduct empirical research. The study results show that when the robust control parameters are 0, 30, and 60, a vehicle driving route resulting in excellent performance can be rapidly generated. The total travel time of students and total vehicle operational cost were reduced by 12.89% and 8.45%, respectively, compared with those of the multi-objective non-mixed-load school bus route plan. Compared with the multi-objective bat algorithm, NSGA-II saves 27.51% of the solution time. This indicates that the robust optimization of the multi-objective mixed-load school bus route examined in this study provides certain advantages and considerably improves the operating efficiency of the school buses.**

***Index Terms*—Mixed-load school bus, Robust optimization, NSGA-II algorithm, Route optimization**

I. Introduction

The school bus is a kind of public transportation specially for students. With the acceleration of people's life rhythm and the enlargement of the urban framework. The school bus has become the main means for some students to commute. Since the students commute to and from school at the same time as the morning and evening rush hours. That makes the traffic congestion around the school particularly serious. For the school bus, the most important thing is to plan the school bus route. The School Bus Routing Problem (SBRP) mainly studies how to reasonably plan the school bus route under established constraints. Transport students from the student stop to the school (or from the school back to the student stop). SBRP belongs to the category of Vehicle Routing Problem (VRP), which is a kind of combinatorial optimization problem that is extremely difficult to solve.

By researching and analyzing the problem of optimizing school bus routes, a reasonable school bus route is achieved. On the one hand, it can reduce the number of school buses and save costs. On the other hand, it can reduce the waiting time of students and improve the quality of service. In addition, avoid traffic-prone routes by reducing the number of school bus routes. This contributes to the development of urban transport and reduces carbon emissions.

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II. Literature Review

A. Non-Mixed-Load School Bus

Many scholars have conducted in-depth research on the problem of non-mixed-load school buses. Schittekat et al. [1] defined the school bus routing problem (SBRP) as a variant of the vehicle routing problem, and developed an efficient non-parameter meta-heuristic algorithm. By comparing the meta-heuristic method with the sequential method, the experiment proves that the algorithm exhibits excellent performance. Chen et al. [2] defined the school bus routing problem as a demand-splitting school bus routing problem (SDSBRP) to solve, and constructed a dual-objective SDSBRP mathematical model for the first time. In order to solve the issue of routing school buses for students with special needs. Caceres et al. [3] developed a greedy heuristic algorithm and column generation method to obtain an approximate solution of the benchmark example. The school bus routing problem (SBRP) solved by Euchi and Mraihi [4] is a variant of the vehicle routing problem. They developed a hybrid algorithm based on artificial ant colony and variable neighborhood local search algorithm. Dejan et al. [5] developed a heuristic algorithm based on route planning. The optimization goals include minimizing vehicle costs and the total travel time of all students. The experimental results show that the total mileage of all vehicles is significantly reduced compared to the previously unoptimized situation. Avilés-González et al. [6] used the simulated annealing algorithm (SA) to solve the optimization of the school bus route problem, and realized an experimental design technique to provide statistical support for parameter selection. The results of many experiments show that the meta-heuristic algorithm is better in quality and time. Arias-Rojas et al. [7] used the ant colony algorithm to study the SBRP of a school in Bogotá, Colombia. The results showed that the utilization of school buses has increased, the transportation time has been reduced, and the students can be delivered to the school on time. Dang et al. [8] proposed an iterative local search (ILS) and set partition process (SP) hybrid meta-heuristic algorithm for the dual-objective problem of reducing the number of school buses and total driving distance. Beatha et al. [9] developed an SBRP heuristic algorithm based on taboo search and tested it with data from Tusiime Nursery and Primary School in Dar es Salaam, Tanzania. The results showed that student travel time was reduced by 19.24%. Bektaş and Elmastaş [10] proposed a related integer linear programming to solve the real-life school bus route problem. Compared with the current algorithm, the total travel cost can be saved by 28.6%. Parvasi et al. [11] developed a two-layer mathematical model for the school bus route. Two hybrid meta-heuristic methods based on the Location Allocation Path (LAR) strategy are proposed to solve the problem of 50 random instances and perform sensitivity analysis. Babaei et al. [12] proposed a method to simultaneously solve the school bus route and scheduling problem with random time-dependent travel time, and combined the ant colony algorithm and the path decomposition heuristic algorithm to solve it. For the multi-objective school bus routing problem that takes into account the number of vehicles and operating mileage, Hou et al. [13] first established its mathematical model, and then proposed a variable neighborhood search (VNS) algorithm to solve it in stages. Kong et al. [14] tried to design an ant colony optimization (ACO) algorithm to solve the dual-objective SBRP problem. Under the constraints of school bus capacity and the maximum travel time of students, reducing the number of paths is the first goal, and reducing the total length of the paths is the second goal, and an ACO algorithm is designed. ÜNSAL and YİĞİT [15] have developed a software for SBRP optimization using clustering and artificial intelligence technology, which supports GPS, GIS tools and mobile applications. The experimental results show that the developed method can effectively optimize the school bus route. Samadi-Dana et al. [16] developed a robust optimization model for SBRP, which has time uncertainty between stations. And they use the simulated annealing algorithm to solve the real case. Manumbu et al. [17] developed a simulated annealing heuristic algorithm to solve the mathematical model of the student bus route problem. The goal of this model is to minimize the time it takes for students to take the bus from the pick-up point to the school. Finally, the model and algorithm are applied to examples, and the results are well-performed. Eldrandaly and Abdallah [18] proposed a new decision-making framework based on geographic information system (GIS). The framework combines GIS, clustering technology, network cutting technology, and hybrid ant colony optimization meta-heuristics and iterative Lin-Kernighan local improvement heuristics to solve the SBRP.

B. Mixed-Load School Bus

Some scholars have also made research on the optimization of mixed-load school bus route problem(MLSBRP). Mokhtari and Ghezavati [19] proposed a two-objective mixed integer linear programming (BO-MILP) formula to simulate MLSBRP. The hybrid multi-objective ant colony optimization (h-MOACO) algorithm is used to solve the associated BO-MILP. And the sensitivity analysis of the main parameters of MLSBRP. The calculation results show that the h-MOACO algorithm has strong applicability. In order to solve the problem of large-scale mixed-load school buses, Douglas Moura Miranda et al. [20] combined iterative local search with neighborhood search to solve the new model. The results obtained show that the multi-loading method can greatly reduce the cost of the solution. Hang [21] constructed a multi-school mixed-load SBRP model with both soft and hard time windows aiming to minimize the total cost. And designed a genetic algorithm to solve this problem, and further demonstrated the effectiveness of the constructed multi-school mixed-load SBRP model and the operability of the design algorithm. Pan et al. [22] proposed a time-discretized multi-commodity network flow model based on the time-space network of student load status. By implementing the augmented Lagrangian relaxation method, the original SBRP is reformulated as a quadratic 0-1 programming model with linear flow balance constraints. Park et al. [23] present a new mixed-load improvement algorithm and measured its impact on the number of vehicles required. Finally, compared with the existing algorithm, the result shows that the improved algorithm can reduce the number of vehicles required. Ellegood et al. [24] reviewed 64 new SBRP research papers. Research shows that in recent years, SBRP researchers are studying more complex real-world problem settings, using evolution-based and trajectory-based meta-heuristic solution methods, and considering the uncertainty of passengers and travel time. Finally, the direction and hope of future SBRP research are put forward. Park and Kim [25] reviewed the school bus route problem comprehensively, summarized the assumptions, constraints and solutions used in the literature on SBRP, and proposed further research directions.

Summarized from the studies above, it can be found that there are still many problems in the study of school bus route optimization. The research on the school bus route focuses more on the level of big cities and single school route, while the research on mixed-load and multi-school level school bus is relatively rare. Most papers regard the school bus travel time as a fixed value when constructing the mixed-load school bus model, ignoring the actual conditions of road congestion and accidents caused by weather and artificial influences. This is inconsistent with the actual situation, so the obtained target value has little reference significance. Most school bus route optimization modelling considers only a single objective. Although a few of them consider multi-objective, the multi-objective is linearly weighted to become a single objective. In view of this, this paper aims at the SBRP problem considering the station service time and the number of school bus load restrictions. A robust optimization model of multi-school mixed-load paths is constructed with the minimum total operating cost of all school buses and the minimum travel time of all students. NSGA-II is used to solve the problem to obtain a school bus route optimization scheme with better robustness.

III. Robust Optimization Modeling of Mixed-load School Bus Route

A. Description of the Problem

Within a certain area, there are multiple schools and several stops for students to get on and off the bus, the same stop has the same location for getting on and off the bus. Each school stop has a corresponding arrival time requirement. Mixed-load SBRP allows students from different schools to ride on the same school bus. There may be multiple school stops on a route, passing the bus stop and school stop alternately. The problem to be solved is to determine the specific visiting route of each school bus. The goal is to minimize the total operating cost of all school buses and minimize the travel time of all students.

The mixed-load SBRP schematic diagram is shown in Figure 1.

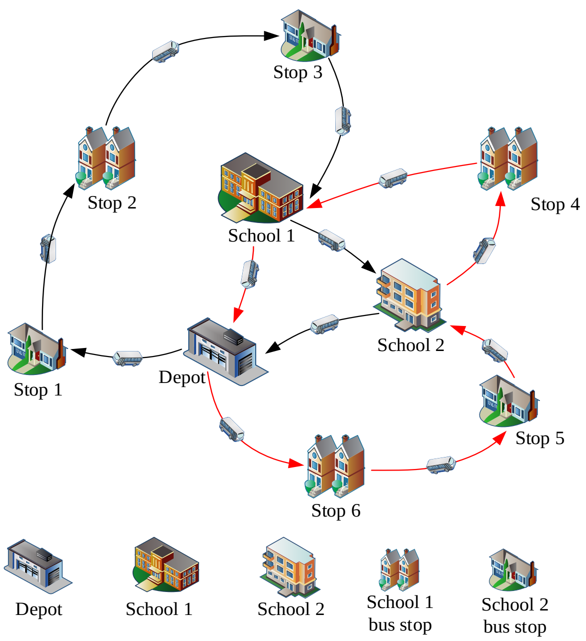

Fig. 1. Schematic diagram of the mixed-load school bus route

B. Model Hypothesis

(1) Each student stop is served by only one school bus and can only be served once;

(2) The school buses depart from a fixed depot and returns to the depot after completing the transportation task;

(3) The school bus cannot be overloaded;

(4) The time for providing services to students should meet the time window requirements of their school;

(5) The depot has an equal number of school buses with sufficient numbers to meet the travel needs of all students in the road network;

(6) The distance between node and node is known;

(7) The departure time of the school bus from the depot is time 0;

(8) The transportation cost of school buses is only related to distance, not to the number of passengers.

C. Symbol Description

Related symbol definitions are shown in Table I.

TABLE I
DEFINITION OF RELATED SYMBOL

Symbol	Type	Definition
S_1	set	Student stop collection
S_2	set	School stop collection
S	set	$S = S_1 \cup S_2$, Union of student stops and school stops
P	set	Depot collection
D	set	$D = S \cup P$, Collection of all stops
K	set	School bus collection
$s(i)$	parameter	Student stop i corresponding school stop, $i \in S_1$, $s(i) \in S_2$
Q	parameter	School bus rated load capacity
C	parameter	School bus unit transportation cost coefficient
α	parameter	Fixed usage fee for school bus
N	parameter	The total number of school buses required to complete the shuttle task
v	parameter	School bus travel speed
d_{ij}	parameter	Distance from stop i to stop j , $(i, j) \in D$
t_{ij}	parameter	Nominal value of travel time from stop i to stop j , $t_{ij} = \frac{d_{ij}}{v}$
$\hat{t}_{ij}$	parameter	The deviation value of the travel time from stop i to stop j , $\hat{t}_{ij} > 0$ ， $(i, j) \in D$
$\tilde{t}_{ij}$	parameter	Variable travel time from stop i to stop j , $\tilde{t}_{ij} \in \left[t_{ij}, t_{ij} + \hat{t}_{ij}\right]$
J_i	set	The set of subscripts j of all uncertain data $\tilde{t}_{ij}$ in the i row of variable time matrix $\tilde{t}_{ij}$, where $\lvert J_i \rvert \le n$
Γ_i	parameter	Parameter for controlling time robustness
$\lfloor \Gamma_i \rfloor$	parameter	Largest integer less than Γ_i
Ψ_i	set	The set of the subscript j of the uncertain data $\tilde{t}_{ij}$ in the i row of the variable time matrix $\tilde{t}_{ij}$
q_i	parameter	Number of people getting on the bus at student stop i , $i \in S_1$
t_i	parameter	Service time required at stop i , $i \in S$
$[e_i, l_i]$	parameter	Acceptable time window for site i , $i \in D$
x_{ijk}	variable	$x_{ijk} = \begin{cases} 1, \text{School bus } k \text{ passes through stop } i \text{ to reach } j \\ 0, \text{Otherwise} \end{cases}$
y_{ik}	variable	$y_{ik} = \begin{cases} 1, \text{ School bus } k \text{ serves stop } i \\ 0, \text{Otherwise} \end{cases}$
T_{ik}	variable	The time when the k school bus arrived at stop i
U_{ijk}	variable	The number of people on the bus when the k school bus travels from stop i to stop j

D. Model construction

$$\min Z_1 = \sum_{i\in D}\sum_{j\in D}\sum_{k\in K}(U_{ijk}\cdot x_{ijk}\cdot t_{ij} + y_{ik}\cdot t_i)$$
$$+ \max_{\{\Psi_i\cup\{m\}|\Psi_i\subseteq J_i|,|\Psi_i|=\lfloor\Gamma_i\rfloor, m\in J_i\setminus\Psi_i\}} \left\{ \sum_{i\in D}\sum_{j\in\Psi_i}\sum_{k\in K} U_{ijk}\cdot\hat{t}_{ij}\cdot x_{ijk} \right. \quad (1)$$
$$\left. + \sum_{i\in D}\sum_{m\in J_i\setminus\Psi_i}\sum_{k\in K}(\Gamma_i - \lfloor\Gamma_i\rfloor)\cdot U_{imk}\cdot \hat{t}_{im}\cdot x_{imk}\right\}$$

$$\min Z_2 = C\sum_{i\in D}\sum_{j\in D}\sum_{k\in K}(x_{ijk}\cdot d_{ij}) + \alpha\cdot N \quad (2)$$

subject.

$$\sum_{k\in K} y_{ik} = 1, i\in S_1 \quad (3)$$

$$\sum_{j\in D} x_{ijk} - \sum_{j\in D} x_{js(i)k} = 0, i\in S_1, s(i)\in S_2, k\in K \quad (4)$$

$$e_i \le T_{ik} \le l_i, i\in S, k\in K \quad (5)$$

$$x_{ijk}(U_{ijk} + q_j) \le Q, i\in D, j\in S_1, k\in K \quad (6)$$

$$\sum_{i\in S}\sum_{j\in D} x_{jik} = \sum_{i\in S}\sum_{j\in D} x_{ijk}, k\in K \quad (7)$$

$$U_{ijk} = 0, i\in P, j\in S, k\in K \quad (8)$$

$$\sum_{i\in D} x_{ijk} = \sum_{i\in D} x_{jik}, j\in D, k\in K \quad (9)$$

Equation (1) is the first objective function, which indicates minimizing the total travel time of all students. Equation (2) is the second objective function, which indicates minimizing the operating cost of school buses. Equation (3) ensures that any student stop can only be served by one school bus and can only be served once. Equation (4) ensures that vehicles passing through the student stop must pass through its corresponding school stop. Equation (5) is the time window limit, and the time to reach any stop must be within the time window. Equation (6) is the number limit. It indicates that the sum of the number of people on the bus after vehicle k has served the current stop and the number of people at the next stop to be served shall not exceed the vehicle load capacity. Equation (7) ensures that all vehicles depart from the depot and finally return to the depot. Equation (8) stipulates that the capacity of the school bus is 0 when it departs from the depot. Equation (9) ensures that the school bus enters stop i and then leaves stop i.

The first objective function in the multi-objective robust model of the mixed-load school bus route corresponds to parameter Γ. The purpose of which is to control the degree of the conservativeness of the solution. In this study, Bertsimas' robust optimization theory is used to carry out peer transformation of the mixed-load school bus model. The adjacency matrix representing the uncertain transport time between nodes is transformed into a one-dimensional matrix. Let $m=(i-1)n+j(1\le i\le n, 0\le j\le n)$, then the decision variable $x_{ij}=x_m$, uncertain travel time $\tilde{t}_{ij}=\tilde{t}_m$. Other corresponding basic data are also changed into the form of subscript m. Some uncertain parameters and parameter sets are subjected to corresponding one-dimensional transformations. According to the literature [26], the first objective function can be converted to solve the following problem:

$$T(t) = \Gamma\hat{t}_l + \min\left(\sum_{m=1}^{n^2} U_{ijk}\cdot t_m\cdot x_m + \sum_{m=1}^{l} U_{ijk}(\hat{t}_m - \hat{t}_l)\cdot x_m\right)$$

From this, the optimal value of the total travel time of all students can be obtained as $Z_1 = \min_{l=1,\cdots,n^2+1} T(l)$.

IV. Algorithm Design

The model established in this paper contains two objective functions. The first objective is to start from the interests of students and require the students to take the minimum time on the school buses. The second objective is to start from the interests of the school bus company and require the minimum total operating cost of the school buses. The two objective functions restrict each other. If one of the sub-objective values is improved, the other sub-objective value will be reduced. At the same time, the model contains robust control parameters. So this study uses the two-stage insertion method NSGA-II to solve the mixed-load school bus model.

A. Algorithm Flow

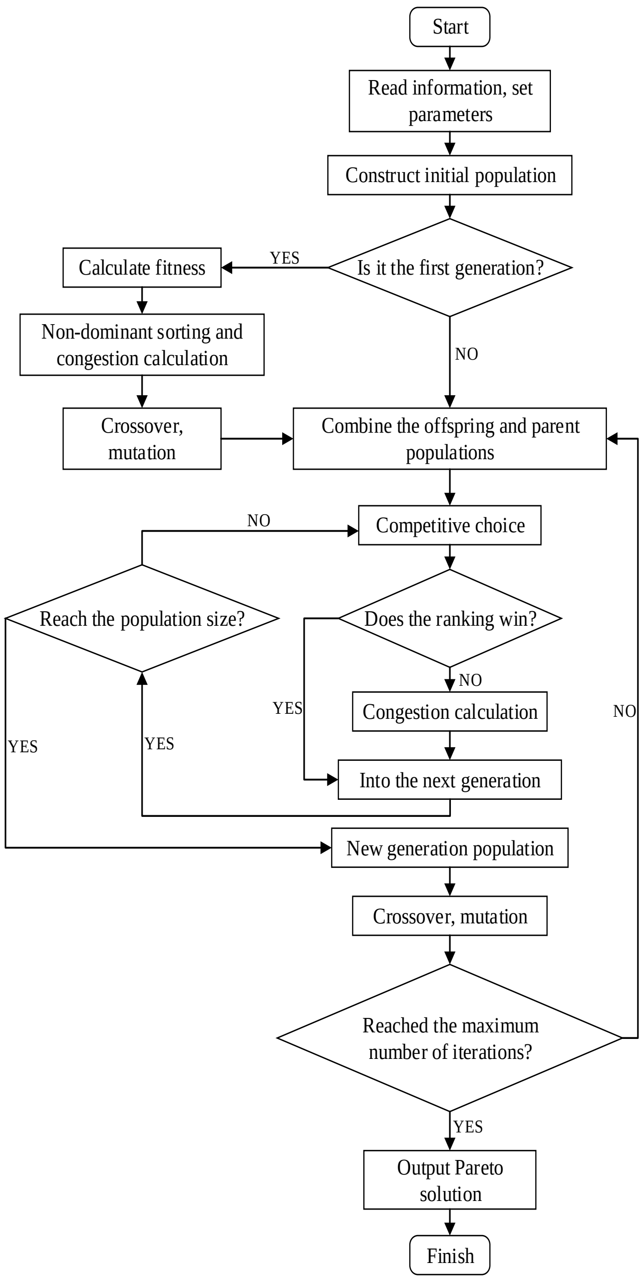

Fig. 2. Algorithm flowchart

Specific steps are as follows:

Step1: Read the stop number, the number of people who gets on (off), time window, service time and other information. Set chromosome length, population size, the maximum number of iterations, crossover probability, mutation probability and other related parameters.

Step2: The insertion method is used to generate the initial population. (1) Randomly arrange all student stops. (2) According to the time window and the number of load requirements, insert the school stops corresponding to the student stops into the random order of the student stops. (3) Generate an initial chromosome. (4) Repeat the above operations to generate the initial population.

Step3: Decode the chromosomes of the initial population, and calculate fitness values. Then perform non-dominated sorting and crowding calculations. Moreover, select appropriate individuals to enter the next generation.

Step4: Combine the newly generated child population with the parent population, further rank them according to the Pareto rank from low to high. Put the whole layer of the population into the new parent population according to the Pareto level. Until a certain layer of individuals cannot be put into the new parent population. Individuals in this layer sort in descending order according to the degree of crowding. In turn, individuals are placed into the new parent population until the new parent population is filled.

Step5: Perform genetic algorithm operations on the new parent population, including crossover and mutation.

Step6: Judge whether the maximum number of iterations is reached. If it is reached, output the Pareto solution, otherwise return to Step4.

B. Chromosome Encoding and Decoding

This paper adopts two-stage real number coding. In the first stage, randomly arrange the student stops and generate the first stage chromosomes. In the second stage, the school stops corresponding to the student stops are inserted into the random sequence chromosome generated in the first stage. The insertion basis is the time window constraint and the school bus load number constraint. If the time to arrive at a school stop exceeds the right time window of the school, another school bus will depart from the depot again. Until all the stops are completely traversed, the chromosomes of the second stage are generated.

The specific coding operation is as follows: Suppose there are 1 depot (numbered 0), 10 student spots (numbered 1 ~ 10), and 3 school stops (numbered a, b, c) in a certain area. The corresponding stops of school a are 1, 4, and 8, and the time window is from 7:00 to 7:30. The corresponding stops of school stop b are 2, 5, and 7, and the time window is from 7:20 to 7:50. The corresponding stops of school stop c are 3, 6, 9, and 10, and the time window is from 7:15 to 7:45. The number of students at the 10 student stops is 12, 15, 9, 8, 11, 10, 18, 14, 11, and 13. The school bus loads 30 people. Randomly generate a series of numbers for the student stops:

9-4-8-2-5-10-3-1-7-6

Insert the school stop into the chromosome of the first stage according to the steps of the second stage:

9-4-c-8-a-a-2-5-b-b-10-c-3-1-c-a-7-b-6-c

At this point, the second stage of the chromosome and coding step are completed. A school bus route plan is generated.

In the decoding process, the above-mentioned compiled chromosome is divided into several segments according to the time window and the number of school bus load constraints. Further, each segment is a route served by a school bus. The results of the above-mentioned mixed chromosome decoding are shown in Table II.

TABLE II
DECODING RESULT

School Bus	Service Route
1	0-9-4-c-8-a-a-0
2	0-2-5-b-b-10-c-0
3	0-3-1-c-a-0
4	0-7-b-6-c-0

C. Cross Operation

Crossover operation is the main way to generate new chromosomes in genetic algorithm. The genetic algorithm in this paper adopts the OX crossover method. Each student stop corresponds to a school, thus there is a potential corresponding relationship between the student stop and the school stop. In cross operation and mutation operation, we use one pair as the basic unit to operate. First, select two parent chromosomes according to the crossover probability. Then randomly generate two crossover points in the parent chromosomes. Second, move the student stops and their corresponding schools in the cross segment of the parent 2 to the front of the parent 1. Move the student stops and their corresponding schools in the cross segment of the parent 1 to the front of the parent 2. Finally, delete the second duplicate student stops and their corresponding schools from front to back of the offspring chromosomes. Assuming that the two randomly generated crossover points are 7 and 11 respectively, the specific crossover process is shown in Table III.

TABLE III
CROSS PROCESS

Before Crossing	Chromosome1	9-4-c-8-a-a-2-5-b-b-10-c-3-1-c-a-7-b-6-c
	Chromosome2	5-1-a-b-8-a-6-9-c-c-2-4-b-a-10-3-c-c-7-b
After Crossing	Chromosome1	2-5-b-b-10-c-1-a-8-a-6-9-c-c-4-a-3-c-7-b
	Chromosome2	6-9-c-c-2-b-4-8-a-a-5-b-10-c-3-1-c-a-7-b

D. Mutation Operation

Mutation operation is also the main operation in genetic algorithm. This paper uses order-based mutation in real-valued mutation. First, randomly generate two student stops, then exchange the locations of the two student stops and the corresponding schools.

Assume that the two randomly generated student stops are 8 and 3 respectively. Their corresponding schools are a and c respectively. The specific mutation operation is shown in Table IV.

TABLE IV
MUTATION PROCESS

Before Mutation	9-4-c-8-a-a-2-5-b-b-10-c-3-1-c-a-7-b-6-c
After Mutation	9-4-c-3-a-c-2-5-b-b-10-c-8-1-a-a-7-b-6-c

V. CASE STUDY

Research is carried out in a certain area of Baotou City to verify the model and algorithm. There are 1 depot (numbered 0), 18 student stops (numbered 1-18), and 3 school stops (numbered a, b, c) in this area. There are a total of 8 school buses available for deployment in the depot. Each school bus has the same model and has a load of 40 people. The proposed unit transportation cost coefficient is 2 yuan/km. The school bus travel speed is 30km/h. The fixed use cost of the school bus is 50 yuan/vehicle. The nominal value of road travel time is $t_{ij} = d_{ij} / v$ .Set the travel time deviation $\hat{t}_{ij} (0 < \hat{t}_{ij} \leq 0.5 t_{ij})$ to be randomly generated real numbers.

A. Data preparation

The information of each school stop is shown in Table V.

TABLE V
SCHOOL STOPS INFORMATION

Stop Number	Left Time Window	Right Time Window	Service Time (min)
a	7:15	7:45	2
b	7:25	7:55	2
c	7:30	8:00	2

The information of each student stop is shown in Table VI.

B. Case Solving

The algorithm is solved in Visual Studio 2016. Set the following parameters respectively: the population size is 100, the maximum number of iterations is 200, the crossover probability is 0.8, and the mutation probability is 0.1. Set the time robust control parameters as $\Gamma = 0, 30, 60$ respectively. The results of running the program several times are shown in Table VII to Table IX.

TABLE VI
STUDENT STOPS INFORMATION

Stop Number	Corresponding School	Number of Passengers	Service Time (min)
1	a	8	1
2	a	11	1
3	a	7	1
4	a	13	1
5	c	9	1
6	a	11	1
7	c	10	1
8	c	14	1
9	c	10	1
10	b	9	1
11	b	14	1
12	b	12	1
13	c	7	1
14	b	16	1
15	b	14	1
16	b	13	1
17	a	12	1
18	c	11	1

TABLE VII
PARETO SOLUTION SET WHEN $\Gamma = 0$

Vehicle	Via the Stop			
	Pareto Solution 1	Pareto Solution 2	Pareto Solution 3	Pareto Solution 4
1	0-17-16-15-b-a-5-c-0	0-11-10-12-b-18-16-b-c-0	0-1-2-3-4-a-0	0-17-18-1-a-c-7-c-0
2	0-10-11-b-9-7-c-0	0-7-6-5-a-c-15-b-0	0-10-11-13-b-c-18-c-0	0-11-12-15-b-13-c-0
3	0-4-3-2-a-12-14-b-0	0-1-2-3-4-a-0	0-6-17-a-5-c-0	0-3-2-a-16-b-0
4	0-8-6-a-18-c-0	0-13-9-8-c-0	0-7-8-9-c12-b-0	0-4-5-6-a-c-0
5	0-13-1-a-c-0	0-14-17-a-b-0	0-15-16-b-14-b-0	0-8-9-10-c-b-14-b-0
Travel Time (min)	2596.66	2801.76	2668.32	2758.82
School Bus Cost (yuan)	497.26	477.46	492.56	484.86

TABLE VIII
PARETO SOLUTION SET WHEN $\Gamma = 30$

Vehicle	Via the Stop		
	Pareto Solution 1	Pareto Solution 2	Pareto Solution 3
1	0-10-12-14-b-9-c-0	0-4-3-6-a-7-c-0	0-16-15-b-14-13-b-c-0
2	0-4-3-a-7-8-c-0	0-11-13-14-b-c-0	0-17-12-a-b-0
3	0-6-5-a-13-c-0	0-16-15-17-b-a-18-c-0	0-9-10-11-b-7-8-c-0
4	0-11-2-1-a-b-0	0-5-1-2-a-c-0	0-4-2-1-a-0
5	0-16-17-15-b-a-18-c-0	0-9-10-12-b-8-c-0	0-6-5-3-a-18-c-0
Travel Time (min)	3196.64	3253.36	3301
School Bus Cost (yuan)	501.9	492.62	489.28

TABLE IX
PARETO SOLUTION SET WHEN $\Gamma = 60$

Vehicle	Via the Stop		
	Pareto Solution 1	Pareto Solution 2	Pareto Solution 3
1	0-4-6-8-a-c-0	0-6-5-3-a-7-c-0	0-14-17-1-a-b-0
2	0-12-13-15-b-9-c-0	0-4-2-1-a-15-b-0	0-18-12-10-b-c-0
3	0-5-14-16-b-c-0	0-11-13-14-b-18-c-0	0-11-13-b-9-8-c-0
4	0-11-10-7-b-c-0	0-10-16-17-a-b-0	0-3-5-6-a-7-c-0
5	0-2-3-1-18-a-c-0	0-12-9-b-8-c-0	0-4-2-a-15-16-b-0
Travel Time (min)	3945.85	3873.96	3794.8
School Bus Cost (yuan)	494.48	499.08	505.5

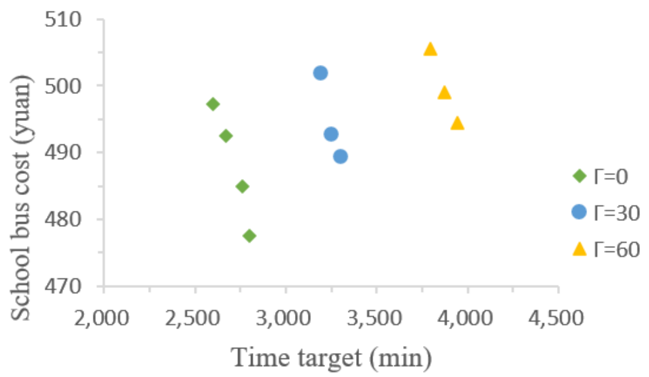

Fig. 3. Pareto optimal solution distribution under different Γ values

It can be seen from Table VII to Table IX that when the time robust control parameters are 0, 30 and 60 respectively. The constructed multi-objective model can rapidly obtain 4, 3 and 3 groups of robust optimal solutions respectively. It can be seen from Figure 3 that when the value of Γ is different. The target value of the solution increases as the time risk value increases, and the vehicle cost also increases. In addition, the enhancement of robustness will affect the optimality of the solution to a certain extent.

C. Case Analysis

In order compare the difference between the target value of mixed-load and non-mixed-load. The total operating cost of all school buses and the travel time of all students under the condition of multi-school non-mixed-load are calculated. The solution is shown in Table X and Figure 4. Although each route takes less time, the total cost is the highest in the case of non-mixed-load. Compared with the non-mixed-load, the travel time of all students has been reduced by 12.89% and the total operating cost of all school buses has been reduced by 8.45% in the multi-school mixed-load. The data obtained by using the two-stage insertion method NSGA-II is generally better.

TABLE X
THE RESULTS OF MULTI-SCHOOL MIXED-LOAD AND NON-MIXED-LOAD ARE COMPARED WHEN $\Gamma = 0$

	Multi-school Mixed-load	Multi-school Non-mixed-load
Number of School Buses	5	6
Travel Time (min)	2758.82	3167.04
School Bus Cost (yuan)	484.86	529.62

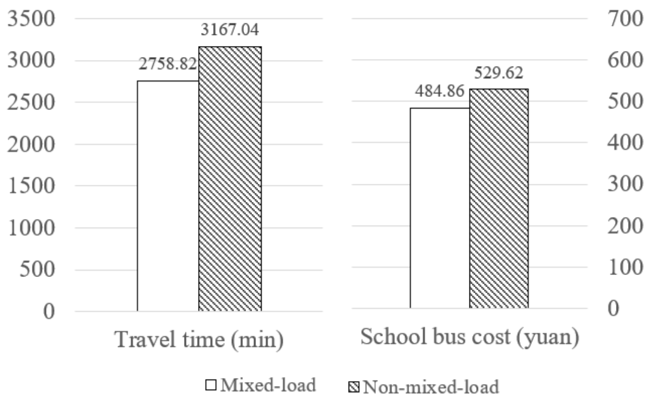

Fig. 4. The target value comparison between mixed-load and non-mixed-load when $\Gamma = 0$

Combined with the data of the calculation example, when $\Gamma = 0$, the multi-objective bat algorithm (MOBA) and NSGA-II are used to solve the model. The comparison of the solution results is shown in Table XI and Figure 5. It can be seen that the use of NSGA-II to solve the problem can reduce the school bus by one. The total travel time of students and total vehicle operational cost were reduced by 5.90% and 9.42%, respectively. MOBA is very easy to fall into a local optimal situation, so it may lose part of the optimal solution. The calculation time of the solution method in this study can save 27.51%, which is more suitable for multi-school mixed-load school bus route optimization problem. Robust optimization model of the multi-school mixed-load school bus route can more effectively complete the multi-stop and multi-school school bus route optimization. This indicates that the robust optimization of the multi-objective mixed-load school bus route examined in this study provides certain advantages and considerably improves the operating efficiency of the school buses.

TABLE XI
COMPARISON OF TARGET VALUES UNDER THE TWO ALGORITHMS

	NSGA-II	MOBA
Number of school buses	5	6
Travel time (min)	2758.82	2931.67
School bus cost (yuan)	484.86	535.28

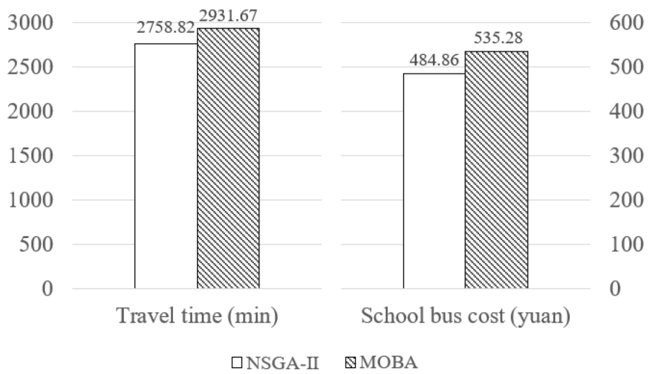

Fig. 5. Comparison of target values under the two algorithms

VI. CONCLUSIONS

In this study, a robust optimization model for multi-school mixed-load SBRP is established with the goal of minimizing the travel time of all students and the total operating cost of all school buses. A corresponding two-stage insertion method, NSGA-II, is also designed for the model. Then the model and algorithm are applied to practical cases. Furthermore, a set of optimal solutions is obtained using different robust control parameters. The solutions indicate that the travel time of all students is negatively correlated with the total operating cost of all school buses.

The case study results show that when the robust control parameters are varied, the travel time of all students significantly increases, and the total operating cost of school buses slightly increases with the parameters. The reason for this phenomenon is that the driving distance of the school bus only slightly varies, but the change in driving speed is considerable due to a number of reasons, such as traffic jams. This study is significant in view of the following. It provides a reasonable and scientifically determined route for school buses that can effectively reduce their operating cost. Moreover, the travel time of students is reduced, and the travel comfort is improved.

The model considers the uncertainty of the travel time through road sections, realistically resolves the SBRP, and effectively compensates for the absence of a fixed driving speed. Further, in comparing the mixed-load and non-mixed-load school bus schemes, the former is found to considerably reduce each target value. In addition, the algorithm used in the model compared with MOBA has a shorter computing time and yields better results. This demonstrates the advantages of the proposed model and its suitability to real situations.

Due to the complexity of the routing problem, certain assumptions and limitations were established to construct the model of mixed-load SBRP in this study. Considering the variety of models, making the mixed-load school bus route scheme more realistic will be the focus of the next step of research.

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