Wave Resonance Phenomena in 2Dxy Rectangular Basin

Ikha Magdalena, Daniel Rivaldo Gunawan, and Hany Qoshirotur Rif'atin,

***Abstract*—In this research, we apply a mathematical model to investigate resonance in a 2Dxy rectangular basin for non-homogeneous waves. The shallow water equations are used as our governing equations. Analytical and numerical solutions to the mathematical model are used to determine the natural wave period responsible for resonance. Additionally, we explore resonance that happens when various wave forms interact. The numerical model are validated using comparisons with analytical solutions. Both procedures give comparable results. Additionally, we analyze the effect of modes and dimensions on resonance.**

***Index Terms*—Wave resonance, Natural resonant period, 2D Shallow water equations, finite volume method**

I. INTRODUCTION

SEICHE motions in lakes can be harmful when the period of an external force on the water coincides with the lake's natural period. This phenomenon is called resonance, and it can cause minor disruptions to the ecosystem surrounding the lake. Extreme versions of such occurrences result in property damage and/or human fatalities. Therefore it is essential to analyze this phenomenon and comprehend its characteristics in order to avoid undesirable consequences. Several studies have been conducted to explore the presence of resonance in lakes [1], [2], [3]. Meanwhile, several academics have examined the resonance phenomenon itself, concentrating on the one-dimensional case where the wave mode is homogeneous in the y direction. For the 1D approach, the available literature includes field measurements explained briefly in [4], [5], experimental studies [6], [7], analytical explorations [8], [9], and studies combining experimental and analytical approaches [10]. Using a multiple-scale perturbation method, Wu and Liu [9] have shown that initial ocean wave groups can cause a small-frequency resonant. In addition, linear models have been used to investigate basins with regular forms, constant depths, and friction-less bottoms are [11], [12], [13], [14]. Furthermore, since resonant waves can be highly dangerous to the surroundings, engineers and researchers in related fields are particularly interested in establishing the natural resonant period and magnitude of resonant waves. Several scholars have used analytical approaches to predict the resonant periods for various kinds of topographies. For example, the Linear Shallow Water Equations model was used to investigate seiches and harbor oscillations in basins of various one-dimensional geometric forms, including their resonant periods [15], [16], [17], [18], [19]. However, all of the studies cited have only undertaken a 1D approach, implying that the wave disruption that triggers resonance occurs only in the x direction. Meanwhile, wave disturbance on lakes may not be uniform in either the x or y direction. Therefore, a two-dimensional analysis is required to better model resonance and determine a more realistic resonant wave period.

Previously, the 2D approach was used to identify the presence of two kinds of seismic wave resonant modes [20]. Others have applied a 2D potential-flow theory to investigate the resonance modes of sloshing waves using a time-independent finite difference method and the Boundary Element Method [21], [22]. On the other hand, an experimental study has performed to record the dynamic of hydraulic jumps in an oscillating rectangular container [23]. Another study employs various iteration method to construct periodic wave and solitary wave solutions for the long–short wave resonance equations [24]. Few other scholars have dived deeper into the subject, focusing not only at the resonance phenomenon, but also at the natural frequency (period) that corresponds to its occurrence. For example, Jung et al. [25] assessed the effect of given natural frequencies on homogeneous sloshing waves in a 2D rectangular tank using level set method. Furthermore, Cueva et al. [3] have studied a more specific case in which the natural frequencies of seiches in Lake Chapala, Mexico are analyzed using the HAMSOM model. Despite the fact that studies have been done to explore the resonance phenomena of non-homogeneous waves and their natural frequency (period), nearly none of them derive the natural periods analytically, especially by using a 2D model. One of the very few works addressing the derivation of the natural period of seiches in a 2D rectangular basin is presented by Rabinovich [26] who has derived the periods in question using a Potential Theory. These results are commonly utilized by engineers as practical guidelines for estimating a basin's resonant periods. However, the values of natural periods stated in the cited study are only relevant to a 2D rectangular basin when the width is half the length, which means that the values are not applicable if the length and width of the basin differ from that value. This study extends Rabinovich's findings by deriving a general formula for the natural period of a 2D rectangular basin of any length. A numerical scheme is formulated to simulate resonance in 2D rectangular basins of arbitrary dimensions with minimal computational cost.

In our current research, we shall concentrate on determining the natural periods of seiches in a lake analytically. In this scenario, the lake will be represented by the simplest

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Ikha Magdalena is a Lecturer and researcher in Faculty of Mathematics and Natural Sciences, Bandung Institute of Technology, Indonesia (corresponding author; e-mail: ikha.magdalena@math.itb.ac.id).

Daniel R. Gunawan is a research assistant in the Faculty of Mathematics and Natural Sciences, Bandung Institute of Technology, Indonesia (e-mail: rivaldo.danielg@gmail.com).

Hany Qoshirotur Rif'atin is a research assistant in the Faculty of Mathematics and Natural Sciences, Bandung Institute of Technology, Indonesia (e-mail: hany.qoshirotur@gmail.com).

2D domain, which is a rectangular closed basin. To further explore the issue, study will be undertaken on a variety of wave mode types. The 2D shallow water equations model, a relatively simple model, will be used. Several prior studies have employed shallow water equations to evaluate wave propagation in various circumstances, particularly in 1D cases. For instance, Magdalena et al. [27] used the equations to model wave shoaling. while others extended this work by considering rigid and porous obstacles in modelling fluid flows [28], [29]. Rif'atin and Magdalena [30] have also applied the two-layer shallow water equations to study internal wave propagation over submerged breakwaters. Andadari and Magdalena [31] have implemented non-linear shallow water equations to simulate wave run-up. In the case of 2D Shallow Water Equations, the implementation can be found in [32], [33], [34] to model different wave phenomena, such as wave attenuation by porous structures, dam-break simulation, wave refraction, and wave shoaling. Aside from shallow water equations, there are a number of other models that may be used to simulate traveling waves under various situations. A study using regularized boundary integral method by Tsao shows a significant affect at the dynamic characteristics of the sloshing behavior at the presence of porous media in a water tank [35]. Another study shows the use of a new viscous-inviscid interaction (VII) method on local behavior of the sloshing jet induced by the fluid impact [36]. Meanwhile, Kumar deals with the (2+1)-dimensional potential Kadomtsev-Petviashvili (pKP) equation, which is used to describe the dynamics of a wave of small but finite amplitude in two dimensions [37]. However, for this work, the linear shallow water equations were selected since they are simpler and much easier to solve, both analytically and numerically. This is a significant benefit, given that we intend to model the resonance phenomenon and derive the resonant periods analytically and numerically. Moreover, numerous previous studies for 1D cases have demonstrated their ability to properly model resonance [15], [16], [38]. The shallow water equations are therefore considered appropriate for this study. Apart from obtaining the analytical solution for the natural periods, we also solved the model numerically using a staggered finite volume method. Aside from this one, a study shows the use of finite volume element method in space to precisely conserve the global mass and energy at the discrete level [39]. Another study using the finite volume element method (FVEM) in space and the discrete variational derivative method (DVDM) in time by Yan derives schemes for Gardner equation [40]. As the finite volume method is devoid of damping error, it is ideal for investigating changes in wave amplitude caused by physical phenomena. It is used to formulate a highly efficient and accurate numerical model. Thus, this paper presents a novel and complete study of resonance in a generalized rectangular two-dimensional basin by analytically and numerically solving the Shallow Water Equations.

Furthermore, this paper is organized as follows. Section 2 introduces the two-dimensional Shallow Water Equations used to represent the physical movements of seiches, while Section 3 discusses the derivation of the analytical natural resonant period for each basin. In Section 4, we describe a computational scheme based on the staggered finite volume method. In Section 5, the numerical findings are presented and compared to the analytical solutions. Section 6 concludes with a succinct conclusion.

II. Mathematical Model

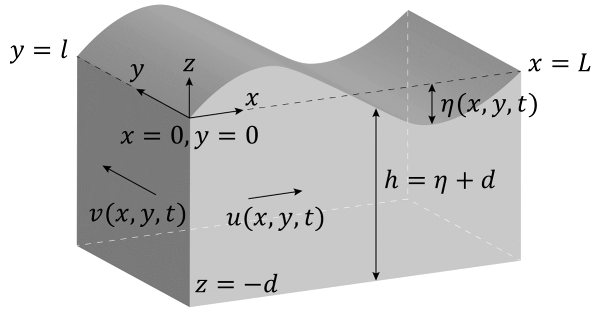


Fig. 1. The illustration of wave propagation over a 2D rectangular basin

In this section, a mathematical model to investigate the resonance phenomenon is discussed. We consider waves propagating in a lake with wave elevation η, horizontal velocity u, and vertical velocity v as depicted in Figure 1. The approaching wave flows from the lake's edge to its centre, entering the domain from the outside. The term $h = \eta + d$ denotes the total water depth. We assume that the value of η is much smaller than h, so we can rewrite the total depth as $h \approx d$, which is essentially the water depth measured at the undisturbed water condition.

The 2Dxy Shallow Water Equations consist of one mass conservation equations and two equations for momentum on the $x-$axis and momentum on the $y-$axis, respectively.

$$\eta_t + (hu)_x + (hv)_y = 0, \tag{1}$$

$$u_t + g\eta_x = 0, \tag{2}$$

$$v_t + g\eta_y = 0, \tag{3}$$

where $g = 9.81m/s^2$ denotes acceleration due to gravity. This paper aims to solve Equations (1) - (3) analytically and numerically for 2D rectangular lakes for various waves modes.

III. Analytical Solutions

From the equations (1), (2), and (3) above, we will then solve the analytical solutions in this segment. The solution allows us to discover the natural resonant frequency of a wave as it propagates over a rectangular 2D basin.

The wave is assumed to be monochromatic. As such, the functions η, u, and v are defined below.

$$\eta(x, y, t) = Ae^{-ik_x x}e^{-ik_y y}e^{-i\omega t}, \tag{4}$$

$$u(x, y, t) = \alpha Ae^{-ik_x x}e^{-ik_y y}e^{-i\omega t}, \tag{5}$$

$$v(x, y, t) = \beta Ae^{-ik_x x}e^{-ik_y y}e^{-i\omega t}. \tag{6}$$

These assumed solutions allow us to define a dispersion relation, from which the analytical resonant frequency of a wave propagating in a rectangular basin is obtained.

First, we substitute Equations (4) and (5) into (2) will yield

$$i\alpha\omega + igk_x = 0, \tag{7}$$

Thus,

$$\alpha = \frac{-gk_x}{\omega}. \tag{8}$$

Substituting (4) and (6) to (3) yields

$$i\beta\omega + igk_y = 0, \tag{9}$$

which may also be written as

$$\beta = \frac{-gk_y}{\omega}. \tag{10}$$

After we obtained α and β, then we proceed to substitute them into Equation (1), hence

$$\omega - \frac{ghk_x^2}{\omega} - \frac{ghk_y^2}{\omega} = 0, \tag{11}$$

$$\omega^2 - gh(k_x^2 + k_y^2) = 0, \tag{12}$$

$$\omega = \sqrt{gh(k_x^2 + k_y^2)}. \tag{13}$$

From the dispersion relation above (13), we could define the natural wave period as follows

$$T = \frac{2\pi}{\sqrt{gh}}(k_x^2 + k_y^2)^{-\frac{1}{2}}. \tag{14}$$

Assume a rectangular basin with length $L(x = (0, L))$ and width $l(y = (0, l))$. As $k_x = \frac{m\pi}{L}$ and $k_y = \frac{n\pi}{l}$, the natural wave period can be written as

$$T = \frac{2\pi}{\sqrt{gh}}\left((\frac{m\pi}{L})^2 + (\frac{n\pi}{l})^2\right)^{-\frac{1}{2}} \tag{15}$$

Where m and n denote the number of nodal lines across and along the basin, respectively. When $m = 1$ and $n = 0$, we can consider the domain to be a 1D closed rectangular domain. In this case, the natural period would be equal to $T = \frac{2L}{\sqrt{gh}}$. This is the formula for the natural resonant period of a 1D rectangular closed basin [15]. The formula for the natural period obtained using our 2D model confirms the result from a 1D model. Additionally, this implies that the derived formula is general; it is equally applicable in 1D and 2D domains.

IV. Numerical Method

Using a staggered finite volume method, we will solve Equations (1) - (3) numerically. First, consider the computational domain $x = [0, L]$ and $y = [0, l]$ with the observation time $t = [0, T]$. Figure 2 illustrates the numerical domain that has been divided into rectangular grids of $\Delta x * \Delta y$. We also divided the time interval t into N_t time steps.

After setting up the domain, we then continue to approximate the mass conservation equation (1) at units centered on points labeled $\eta_{i,j}$, while the units centered on points labeled by $u_{i-\frac{1}{2},j}$ (the blue square) and $v_{i,j-\frac{1}{2}}$ (the red square) are respectively used for calculating the momentum equations in x-direction (u) (2) and in y-direction (v) (3), with $i = 0, 1, 2, ..., Nx$ and $j = 0, 1, 2, ..., Ny$. The values of the surface elevation (η) and water depth (h) are computed at full-grid points $x_{i,j}$ using the mass conservation equation (1). Meanwhile, the values of the horizontal velocity in x-direction (u) and y-direction (v) are computed at half-grid points $x_{i-\frac{1}{2},j}$ and $x_{i,j-\frac{1}{2}}$ using the momentum equation (2) and (3), respectively.

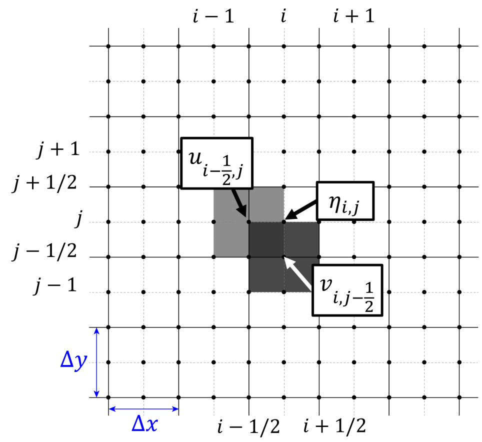


Fig. 2. Illustration of 2D staggered grid discretization.

Here, we consider wave resonance in a rectangular basin, thus the water depth is a constant. The numerical scheme is written below. The approximation of η at spatial partition point $x_{i,j}$ and time partition point t^n represented by $\eta_{i,j}^n$, with $n = 1, 2, ..., N_t$.

$$\frac{\eta_{i,j}^{n+1} - \eta_{i,j}^n}{\Delta t} + \frac{(hu)_{i+\frac{1}{2},j}^n - (hu)_{i-\frac{1}{2},j}^n}{\Delta x} + \frac{(hv)_{i,j+\frac{1}{2}}^n - (hv)_{i,j-\frac{1}{2}}^n}{\Delta y} = 0, \tag{16}$$

$$\frac{u_{i+\frac{1}{2},j}^{n+1} - u_{i+\frac{1}{2},j}^n}{\Delta t} + g\frac{\eta_{i+1,j}^{n+1} - \eta_{i,j}^{n+1}}{\Delta x} = 0, \tag{17}$$

$$\frac{v_{i,j+\frac{1}{2}}^{n+1} - v_{i,j+\frac{1}{2}}^n}{\Delta t} + g\frac{\eta_{i,j+1}^{n+1} - \eta_{i,j}^{n+1}}{\Delta y} = 0. \tag{18}$$

Equations (16), (17), and (18) allow a numerical solution to be obtained for the case of waves propagating in a two-dimensional basin with a flat bottom.

V. Results and Discussion

In this section, we will implement the numerical scheme from the previous section to simulate resonance in a 2D-rectangular basin. Numerous simulations are conducted to reproduce the resonance of each mode parameter (n or m) in a rectangular basin of uniform depth. All simulations presented in this section are performed for $T = 30$ s using several computational domains, which are $[20, 10]$m, $[20, 20]$m, and $[10, 30]$m with a water depth of $d = 10$ m, uniformly. To fulfill the stability condition, we set the time step for each iteration as $\Delta t = \frac{0.5\Delta x}{\sqrt{gd}}$. Each domain is then partitioned into smaller sections with a length of $\Delta x = 0.2$ m and a width of $\Delta y = 0.2$ m. All the initial velocities along the x- and y-axes equal zero. The boundaries at $x = 0$ and $y = 0$ are open. All other boundaries are hard walls. A harmonic wave will then enter from one or both open boundaries with an amplitude of 0.1 m and an angular frequency of $\omega = \frac{2\pi}{T}$, with a period of T as written in Equations (15).

Figure 3 shows resonance (indicated by the gradual increase in wave amplitude) in a 2D rectangular basin with mode of $m = 1$ and $n = 0$, compared to the same

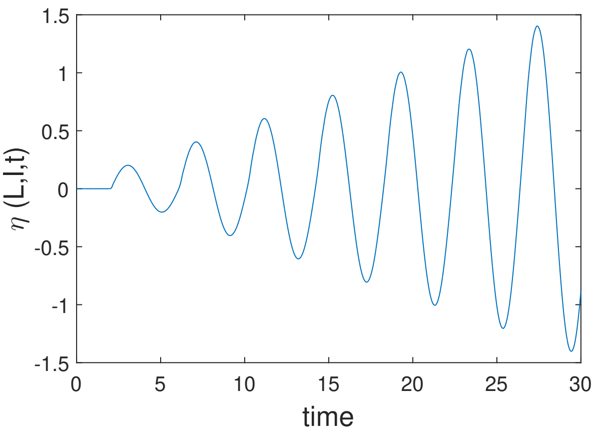


Fig. 3. Simulation results of the resonance phenomenon in a rectangular basin ($l = 0.5L$)with mode $m = 1$ and $n = 0$.

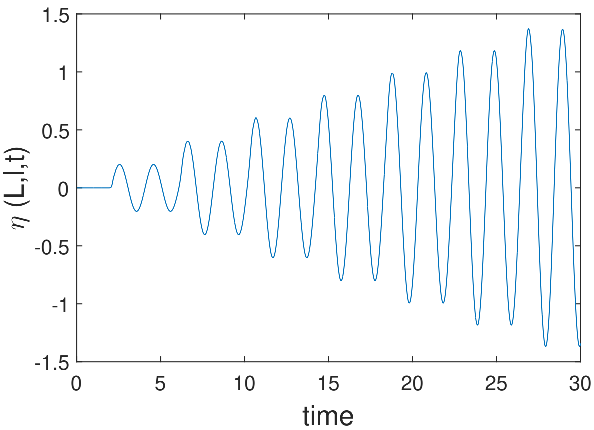


Fig. 4. Simulation results of the resonance phenomenon in a square basin ($l = L$)with mode $m = 0$ and $n = 2$.

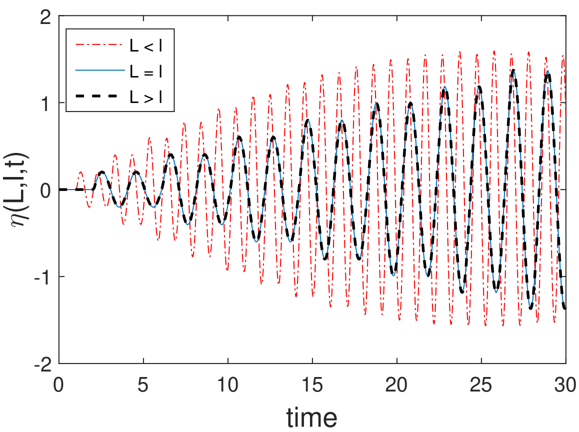


Fig. 5. Simulation results of the resonance phenomenon in a rectangular ($l < L$ and $l > L$)and square basin ($l = L$)with mode $m = 2$ and $n = 0$.

occurrence in a 1D closed rectangular basin. The comparison is acceptable since this mode configuration of $m = 1, n = 0$ represents the 1D closed rectangular basin presented in [15]. The natural period of a wave in a 1D basin is exactly the same as that of a wave in a 2D basin (see Section 3). The profiles of both waves are also identical (Figure 3). In addition, take a look at Figure 4, where resonance in a square basin of mode $m = 0, n = 2$ is shown. Notice that the resonant wave profile is similar to the one presented in Figure 3. However, there is a slight difference. The wave amplitude fluctuates throughout the observation period instead of undergoing a gradual increase. Despite the fact that the wave amplitude fluctuates during the simulation, we can still see an increasing trend dominating the plot. Therefore, the captured phenomenon is still considered resonance.

Furthermore, in order to further explore the resonance in a 2D domain, simulations also conducted for different configurations of L and l as well as various values of mode m and n. To analyze wave profile differences caused by differing L and l configurations, we can take a look at Figure 5. It is shown in the figure that for $m = 2, n = 0$, when $L = l$ (square basin), the resonant wave amplitudes are much smaller compared to that in rectangular case ($L < l$), and the periods much larger. In cases where $L > l$, the resonant wave profile and period are found to be exactly the same with the one for square basin case. The results provided in Figure 5 are explained by the effect of varied modes and length configurations on the resonant wave's maximum amplitude. The results of the simulations with varied modes and length configurations are summarized in Table I.

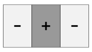
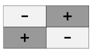
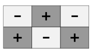
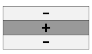
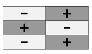
TABLE I
AMPLITUDES FOR EACH MODE OF EVERY BASINS FOR FLAT BOTTOM CASE MEASURED WHEN $T = 30$S.

Mode No.		Rectangular	Square	Rectangular
m	n	($L > l$)	($L = l$)	($L < l$)
1	0	1.403	1.403	2.469
2	0	1.368	1.368	1.542
0	1	2.469	1.403	1.008
1	1	3.275	2.806	3.149
2	1	1.370	2.430	2.527
0	2	1.542	1.368	1.004
1	2	2.753	2.430	3.467

From Table I, we can draw some conclusions about the effect of modes and length configurations on resonant wave's maximum amplitudes. To begin, when the mode parameters are set as $m > n$, a basin with the length configuration $L > l$ will have a smaller amplitude than a basin where L is smaller than l. On the other hand, when $m < n$, basins where $L > l$ have larger amplitudes than basins where L is smaller than l. Second, most of the basins with non-zero m and n values have greater amplitudes than those with at least one parameter equal to zero. This phenomena occurs in rectangular as well as square basins. Thirdly, for cases where one mode has a value of zero ($m = 0$ or $n = 0$), the resonant wave amplitude increases if the value of L (for $n = 0$ case) or l (for $m = 0$ case) decreases, resulting in the pattern demonstrated in Figure 5. This can be explained by a decrease in period when a wave enters a narrow channel and vice versa. Since the period is much smaller in a narrow basin, the wave amplitude increases more quickly. This is because a wave in a narrow channel possesses more energy compared to a wave entering a wider channel, resulting in a faster increase in wave amplitude. The part of the basin that produces this effect corresponds with the non-zero mode of the rectangular basin or channel. In this case, we have explicitly established that the mode m corresponds with length L and n corresponds with l. Therefore, if $m \neq 0, n = 0$, the value of L is the one that we need to assess. When $m = 0, n \neq 0$, we need to observe l. The wave resonant amplitudes will be inversely proportional to changes in L or l.

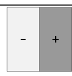
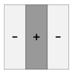
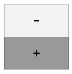
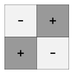
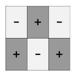
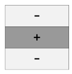
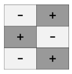
Now, to observe the differences on the resonant wave profile in each mode and each length configuration further, we have conducted several simulations with various parameter values. First, we will evaluate the relative wave period, which is the ratio between the resonant period for a mode (m, n) and the resonant period of the fundamental mode $(m = 1, n = 0)$, in a rectangular basin with a length of $L = 20$ m and a width of $l = 0.5L = 10$ m. The relative wave periods are denoted by T_{mn}/T_{10}. Comparisons between analytically derived solutions, numerically calculated periods, and Rabinovich's results [26] are listed in Table II.

TABLE II
COMPARISON OF ANALYTICALLY-DERIVED, NUMERICALLY-OBTAINED, AND RABINOVICH'S RATIOS OF NATURAL RESONANT PERIODS FOR EACH MODE OF RECTANGULAR BASIN ($l = 0.5L$) FOR FLAT BOTTOM CASE.

Mode No.		Mode Forms	Relative Period T_{mn}/T_{10}		
m	n		Analytical	Rabinovich	Numerical
1	0		1.000	1.000	1.0049
2	0		0.500	0.500	0.5024
0	1		0.500	0.500	0.5054
1	1		0.447	0.447	0.4705
2	1		0.354	0.354	0.423
0	2		0.250	0.250	0.252
1	2		0.243	0.243	0.253

In Table II, it can be seen the comparison between the natural relative periods we derived, analytically and numerically, and the ones obtained by Rabinovich [26] using a Potential Theory. Evidently, our analytical solutions are exactly the same as those derived by Rabivonich using a different mathematical model. Comparison between the analytical and numerical wave relative periods shows that the values are quite similar. Therefore it is also acceptable to say that our numerical model confirms the analytical solutions fairly well. In addition, we have also undertaken the comparisons for length configurations $L = l$ and $L < l$. The comparisons for a 2D closed square basin are presented in Table III for $L = l = 20$ m. Meanwhile, the comparisons for $L < l$ scenario are displayed in Table IV for $l = 30$ m and $L = 10$ m.

TABLE III
COMPARISON OF ANALYTICALLY-DERIVED AND NUMERICALLY-OBTAINED RATIO OF NATURAL RESONANT PERIODS FOR EACH MODE OF SQUARE BASIN ($l = L$) FOR FLAT BOTTOM CASE.

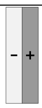
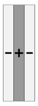
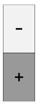
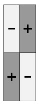
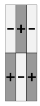
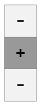
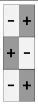
Mode No.		Mode Forms	Relative Period T_{mn}/T_{10}	
m	n		Analytical	Numerical
1	0		1.000	1.0049
2	0		0.500	0.5024
0	1		1.000	1.0049
1	1		0.7071	0.8545
2	1		0.4472	0.4978
0	2		0.500	0.5024
1	2		0.4472	0.4978

For Tables III and IV, we did not compare our analytical solutions and numerical results to those obtained by Rabinovich, since only data for the case of $l = 0.5L$ are available. Therefore, Table III and IV will only present and compare our analytical and numerical derived natural relative periods. From both tables, it is quite clear how different the wave natural relative periods in square and rectangular basins ($L < l$). In general, the relative periods in rectangular basins with $L < l$ are considerably larger than those in square basins. These are also generally larger than or equal to those in rectangular basins with $L > l$. In addition, the numerically derived relative periods are also quite similar to the analytical ones. This indicates that our numerical scheme can successfully approximate the analytical solutions for all variations of modes and length configurations.

VI. CONCLUSION

2Dxy Shallow Water Equations are able to accurately simulate resonance phenomena in a 2Dxy rectangular basin. The model investigated the oscillations in different uniform-depth-basins with various parameters (m and n). The model was solved analytically to determine the general formula for the natural resonant period for waves propagating in a lake, expressed in terms of the natural resonant period of waves propagating in a lake with no bottom friction. A numerical

TABLE IV
COMPARISON OF ANALYTICALLY-DERIVED AND NUMERICALLY-OBTAINED RATIO OF NATURAL RESONANT PERIODS FOR EACH MODE OF RECTANGULAR BASIN ($L < l$) FOR FLAT BOTTOM CASE.

Mode No.		Mode Forms	Relative Period	
m	n		T_{mn}/T_{10}	
			Analytical	Numerical
1	0		1.000	1.0108
2	0		0.500	0.5049
0	1		3.000	3.0117
1	1		0.9487	1.0116
2	1		0.4932	0.5047
0	2		1.500	1.5051
1	2		0.8320	0.9453

scheme is then constructed using a finite volume method on a staggered grid. The numerically derived natural resonant periods are then compared against the analytically derived and Rabinovich's [26] solutions to validate the numerical scheme. Moreover, simulations of the resonance phenomena in other types of basins, such as square and different rectangular shapes, are done to investigate the trend. For closed basins with rectangular type, if the wave parameters fulfill $m > n$, waves in basins with $L > l$ have lower amplitudes than waves in basins with $L < l$, and vice versa for $m < n$. Furthermore, waves in basins with parameters of $m \neq 0$ and $n \neq 0$ have higher amplitudes compared to waves in basins with at least one of their parameters equal to zero.

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Ikha Magdalena is a Lecturer at Institut Teknologi Bandung's Department of Mathematics. She conducts research on fluid dynamics, wave models, numerical modeling, and coastal protection. Her research has been published in Q1 Journal, and other reputable mathematics and engineering Journals. Since the beginning of her career as a researcher, she has been awarded numerous grants from the Indonesian Ministry of Research, Technology, and Higher Education as well as from Institut Teknologi Bandung. She received an Award in Industrial and Academia Partnership from Royal Academy of Engineering, the Individual Researcher Links Award from the British Council and a grant from the Newton Fund. She is also the creator of Wave Interaction with Vegetation, a copyrighted fluid modeling software, and she actively manages ITB's Research Center for Coastal and Ocean. She believes in the power of collaboration in research, and works on research problems with colleagues from Indonesia, the UK, the Philippines, China, India and the Netherlands. Aside from joint projects, she maintains active participation in mathematical societies. Since 2014, she has been heavily involved in the management of the Indonesian Mathematical Society (IndoMS) and now serves as its Coordinator of International Collaborations. She is a member of the Silkroad Mathematics Center's Steering Committee and the Executive Committee of the Society for Industrial and Applied Mathematics (SIAM). She is the faculty advisor of SIAM Student Chapter ITB.