Mathematical Model of Predictive Indicators of Violence: Limit of Fatality

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Abstract—**This article provides an estimation of the likelihood of fatality of a victim at the hands of her perpetrator in a situation of maximum domestic violence, based on the hypothesis that there must be a real limit to predict the fatality scenario under such circumstances. Simulations are performed considering the initial status obtained from the risk questionnaire for two different cycles of violence. From these simulations (graphs of violent behavior), the victim's violence limits are observed over a period of twelve months. The fatality scenario of intimate partner violence is an improvement to the predictive mathematical model of violence indicators proposed by Leal-Enríquez (2018). Such model shows that the first risk questionnaire applied to the victim is only an approximation to the real level of violence that the victim would be experiencing in the near future.**

Index Terms—**intimate partner violence, predictive mathematical model, questionnaire, fatality limit.**

I. Introduction

Violence ("violence is the intentional use of threatened or actual physical force or power, against oneself, another person, a community or a group, that may result in a high likelihood of psychological harm, death, injury, deprivation or maldevelopment" [24]) is a major public health problem that afflicts all societies worldwide [13]. Specifically, violence against women is of significant importance to international organizations and leading human rights groups (violence against women is defined by the United Nations as "any act of gender-based abuse that results in, or is likely to result in, physical, sexual or psychological harm or suffering to women, including threats of such acts, coercion or arbitrary deprivation of liberty, whether occurring in public or in private life" [21]) [8]. Violence against women does not respect social class, race, age or religious beliefs [23]. Statistics show that, in 75% of the cases, a man is identified as the perpetrator of such violent acts and a woman as the victim. In most countries, programs to monitor cases of violence and to assist the victim have been established. These programs are estimated to last between one and two years [8].

Once a health worker in a government or private institution identifies a woman as the recurrent victim of Intimate Partner Violence (IPV), assistance is provided through the application and monitoring of a program that assesses the risks to the victim. The average duration of these assistance programs ranges from one to two years (the average level of violence has been measured in monthly intervals, establishing an observation period of one year [8]) [8], [18]. However, the percentage of teenagers who do not consider the alternative of seeking help is approximately 79% [1]. Hence, in view of the small number of victims seeking for help, emotional support, advice, and information for finding a solution to the problems that antagonize them with their partners (a variable percentage of aggressors and victims seeking for help has been observed, and this varies depending on factors such as age, race, social class and other traits (as an example, it can be stated that the percentage of adolescents seeking for help is around 21% [1])), adequate care of the victims by the health worker is relevant. Thus, in order to help the victim in the decision-making issue that may lead her to find the solution to her problems of violence at the hands of her intimate partner, being able to have both, quantitative and qualitative tools at hand, is crucial (the three main stages of the domestic violence cycle are: occurrence of violent incidents (physical / sexual / emotional), stage of tension accumulation (violent outburst) and *honeymoon phase* [10]).

With the purpose of detecting whether a woman is a victim trapped in the cycle of domestic violence, predictive tools are currently being developed. Additionally, *mathematical models are being created to calculate the likelihood of violence that a woman could suffer within the following twelve months* [4], [11], [14], [5].

Such predictive tools could provide several benefits to the victims. They would show women the likelihood of violence that they may experience if they are not willing to be assisted and/or to be treated in order to find relief to their afflictions [14], [1]. Particularly, the likelihood of IPV (Intimate Partner Violence) that could arise within a period of twelve months is calculated through the mathematical model proposed by Leal-Enríquez E. [14], using as data/input the initial diagnosis of the level of violence at the exact moment when the victim arrives at the attention center (a variety of questionnaires are applied in order to determine the initial level of violence that the victim is suffering (statistics). These questionnaires consider the frequency of such violent events [22], [27]).

Nevertheless, this mathematical model does not specifically show how a social worker or individual should use it to calculate the likelihood of violence against women, on the basis of a questionnaire. Indeed, it is not simple (it is indeed possible to estimate the initial level of violence by applying questionnaires, where a numerical weight of severity is assigned to each consultation provided to the victim [20]). The difficulty to use the studies on predictive mathematical

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models from which the indicators of violence are obtained, derives from the fact that they do not show their practical usefulness in clinical applications. This is because the authors do not indicate, in a simple way or through specific examples, the way in which the violence risk questionnaires should be used along with the given mathematical model. In addition, the model does not establish a real fatal violence limit level and the mere application of the risk questionnaire to the victim does not explain how the violence levels are reached.

Therefore, this study presents an improvement and potential applications of the proposed mathematical model [14], with the aim of improving the adequacy of social work and the attention to the victims of violence. The use of this model is proposed to simulate the likelihood of violence levels that a victim of domestic violence could experience over a one-year period.

One of the premises assumed in this model is an increasing probability of loss of control by the aggressor, which occurs monthly, in addition to the fact that the loss of control occurs in acts of domestic violence towards the partner in cycles of abuse [25], [14], [9]. One of the bases of this model is the presumption that the aggressor has psychological limitations to control his violent impulses [7].

Several key variables are used in this mathematical model to simulate the likelihood of violence that the victim may suffer in different scenarios. One of them is the loss of control of the aggressor (treatments to attend loss of control problems are a factor that helps to reduce violence levels (studies have reported that violence levels have been reduced to 53%. This is true provided that perpetrators find support through aid programs for a period of one year [7])). Specific weights are assigned to each item of the applied questionnaire as well as the frequency of such violent acts (various statistical studies and meetings with experts in the IPV field have been conducted in order to be able to assign the specific weights and frequencies to each question. An example case may be found in [22] and [27]).

II. Questionnaire

One questionnaire, which was found to be helpful to evaluate the dimensions of the psychological, sexual, physical, and severe physical violence (methodological proposal to measure intimate partner violence in women [20]), is used in this section to connect the mathematical model proposed by Leal-Enríquez E., [14] with a questionnaire used to make a quantitative calculation of the initial level of domestic violence faced by a victim (either the social worker or the care center may use the questionnaires and/or protocols placed at their disposal to be able to estimate the initial condition of violence of the victim, see for example [27], [20]).

The questionnaire to be used is presented in Table I, where 18 items selected from two instruments are incorporated (“Index of Spouse Abuse (ISA)” and “Severity of Violence Against Women Scale (SVAWS)” [12], [16]). This questionnaire has proven to be useful to quantify the level of violence used by men against women within an intimate relationship. In this questionnaire, it is possible to observe the variables ω_n (weight assigned to the violent actions in each of the items the method for assigning these weights uses the judgement of experts [20], [27]) as well as the frequency in which violent actions towards the victims have taken place recently f_n, therein, frequencies are assigned the following values: 0=never, 1= sometimes, 2=several times and 3=many times. The following factors are considered in this questionnaire: I=psychological violence, II=physical violence, III=severe physical violence and IV=sexual violence.

A numerical value between $[0 \quad 1]$ is used together with a factorial analysis to assign the corresponding factor to each item, (see [20] for more details on the analysis).

After that, both frequency and weight must be assigned to each one of the items f_n, ω_n. Consequently, the level of violence Ω_n can be calculated by using the following equation [14], [20]:

$$\Omega_n\left(0\right) = \omega_n\left(0\right) \times f_n(0), \tag{1}$$

where $f_n(0) = 0, 1, 2, 3$ and $\omega_n(0)$ are respectively given values from Table I. For instance, given the case of a woman, presumably being a victim of violence, who is reported as having been repeatedly burned with cigarettes ($n = 11$) or any other burning substance ($f_n = 2$ and $\omega_n = 6$), the following is obtained [20]:

$$\Omega_{11}\left(0\right) = \omega_{11}\left(0\right) \times f_{11}(0) = 6 \times 2 = 12. \tag{2}$$

Therefore, the calculation of the level of initial violence of the victim $\Omega(0)$, considering factors I, II, III and IV (where the corresponding dimension has already been assigned to every single question a factor analysis to identify the grouping of the variables that best explain each of the dimensions is usually conducted, obtaining the relevant factors for each type of violence and also assigning a dimension to each question [20].), is calculated by using the equation [20], [14]:

$$\Omega\left(0\right) = \sum_{1}^{n} \omega_n\left(0\right) f_n(0). \tag{3}$$

In equation (3) each of the four factors contained in the questionnaire to be given to the victims is separated. As an example, the following is noticed in in the questionnaire displayed in Table I:

$$\begin{aligned}\Omega\left(0\right) \quad &= \textstyle\sum_{n=1}^{5} \omega_n\left(0\right) f_n(0) + \sum_{n=6}^{10} \omega_n\left(0\right) f_n(0) \\ &+ \textstyle\sum_{n=11}^{15} \omega_n\left(0\right) f_n(0) + \sum_{n=16}^{18} \omega_n\left(0\right) f_n(0) \\ &= \Omega_I(0) + \Omega_{II}(0) + \Omega_{III}(0) + \Omega_{IV}(0). \end{aligned} \tag{4}$$

The indicator of violence considered in (3), is a general index that gives an indication of the initial condition of violence suffered by a woman, victim of violence, upon her arrival at the care center (it is important to highlight that this indicator considers the psychological, physical, severe physical and sexual dimensions [20]). The calculation of the initial global condition of violence is displayed in Table I, (see (3)) and as far as the dimensions thereof are concerned (see (4)) for homogeneous frequencies [20].

The figures corresponding to the calculation of the initial global condition of violence are presented in Table I. For this calculation see (3) and for its dimensions see (4) for homogeneous frequencies. This is indicated in [20].

TABLE I: INITIAL CONDITION OF VIOLENCE $\Omega(0)$ BY FACTORS: I=PSYCHOLOGICAL VIOLENCE, II=PHYSICAL VIOLENCE, III=SEVERE PHYSICAL VIOLENCE AND IV=SEXUAL VIOLENCE BY FREQUENCY F_N, AND WEIGHT ω_N.

n	Question	Factor	Weight	$\Omega_n(0)$			
			ω_n	f_n 0	f_n 1	f_n 2	f_n 3
1	Has he ever told you that you are not attractive or that you are ugly?	I	4.5	0	4.5	9	13.5
2	Has he ever displayed jealousy towards you or become suspicious of your friends?	I	4	0	4	8	12
3	Has he ever rejected you?	I	5	0	5	10	15
4	Has he ever offended you?	I	4	0	4	8	12
5	Has he ever made you feel worthless in front of other people?	I	5.5	0	5.5	11	16.5
	Indicator of psychological violence	$\Omega_I(0)$		0	23	46	69
6	Has he ever kicked you?	II	8	0	8	16	24
7	Has he ever pushed you intentionally?	II	5	0	5	10	15
8	Has he ever beaten you or slapped you on your face?	II	7	0	7	14	21
9	Has he ever twisted your arm?	II	6.5	0	6.5	13	19.5
10	Has he ever pulled you forcefully?	II	5	0	5	10	15
	Indicator of physical violence	$\Omega_{II}(0)$		0	31.5	63	94.5
11	Has he extinguished a cigarette on your body or burned you with any other item or substance?	III	6	0	6	12	18
12	Has he ever threatened you with a gun or any other type of firearm?	III	6.5	0	6.5	13	19.5
13	Has he ever shot at you with a gun or any other type of firearm?	III	9.5	0	9.5	19	28.5
14	Has he ever threatened you with a knife?	III	7	0	7	14	21
15	Has he ever tried to drown you or suffocate you?	III	9.5	0	9.5	19	28.5
	Indicator of severe physical violence	$\Omega_{III}(0)$		0	38.5	77	115.5
16	Has he ever forced you to engage in sexual intercourse?	IV	6	0	6	12	18
17	Has he ever used physical force to have sex?	IV	9	0	9	18	27
18	Has he ever threatened you with leaving you for other women if you do not agree to engage in sexual intercourse?	IV	4	0	4	8	12
	Indicator of sexual violence	$\Omega_{IV}(0)$		0	19	38	57
	Initial condition of global violence	$\Omega(0)$		0	112	224	336

A. Categorization

In order to categorize the initial IPV condition in a verbal fashion in [20], the following is proposed (the initial condition of violence is categorized based on the minimum, maximum and average values of the data obtained for each factor of domestic violence that has been considered. For more details, see [20]), see Table I:

- Factor I: the case is classified as ”not a case of psychological violence”.
- Factor II: the case is classified as ”a case of physical violence”.
- Factor III: the case is classified as ”a case of severe physical violence”.
- Factor IV: the case is classified as ”a case of sexual violence”.

After the violence questionnaire has been applied and categories have been assigned, the simulation of the probable levels of violence that the victim could experience during the following twelve months can be performed. The initial condition of global violence or the dimensions assigned are a possible starting point, $\Omega(0)$ (see Table I and equations (1)-(4)). For this to be achieved, it is advisable to use the model proposed by Leal-Enríquez E. [14]. This model is composed as follows:

$$\Omega(k) = \Omega(0) \prod_{1}^{k} \alpha(k) \tag{5}$$

It is important to note that, the index k uses the values from 1, 2, 3, to 12. In other words, it considers the values of a violence cycle within a period of twelve months, where $k = 0$, is the initial IPV condition observed after the arrival of the victim at the care center. This is given by:

$$\Omega(0) = \sum_{1}^{n} \omega_n(0) f_n(0). \tag{6}$$

The indicator $\Omega(0)$ expresses the initial condition of violence (IPV) (refer to (1) and (4)). The factors $\omega_n(0)$ and $f_n(0)$ represent the assigned weight and the frequency of each one of the n items included in the questionnaire which is respectively given to the victim upon arrival at the care center (for more details of the methods used to determine the values presented in Table I, see [22]). The proportionality factor $\alpha(k)$ considers the amount accumulated as a consequence of the probable loss of control of the perpetrator in the following months. This is expressed by:

$$\alpha(k) = \sum_{1}^{k} \beta(k), \tag{7}$$

with

$$\beta(k) = \xi(k) \times \sigma_{-}(k), \tag{8}$$

and

$$\begin{bmatrix}\sigma_{+}(k) & \sigma_{-}(k)\end{bmatrix} = \begin{bmatrix}\sigma_{+}(0) & \sigma_{-}(0)\end{bmatrix} \begin{bmatrix}(1-\lambda) & \lambda \\ \mu & (1-\mu)\end{bmatrix}^{k}. \tag{9}$$

The factor $\beta(k)$ (for $k = 1, 2, \ldots,$ 12) presents a proportionality corresponding to the probable states of loss of control by the aggressor $\sigma_{-}(k)$, for a specified period of one month k. $\xi(k) \in [0 \quad 1]$ represents the proportion that considers the percentage of loss of control by the perpetrator, which can be reflected in the appearance of injuries or in the execution of violent acts towards the victim (see Table I-Dimensions I, II, III, IV). The elements of the vector $[\sigma_{+}(k) \quad \sigma_{-}(k)]$ express the respective odds that the aggressor is either in a state of self-control or loss of it. The components $(1-\lambda)$ and $(1-\mu) \in [0 \quad 1]$ represent the parameters associated to the probability that the perpetrator finds himself in a state of self-control or in one manifesting the loss of it [14]. The vector $[\sigma_{+}(0) \quad \sigma_{-}(0)] \in [0 \quad 1]$ expresses the probabilities that the aggressor is in either a state of self-control or loss of it. These values are assigned at the beginning of the assessment of risk that the victim may face at a given time (a finite Markov chain is used to model the level of self-control and the states of loss of control by the perpetrator. For further information, see [14]).

B. Proposal to improve the model

In the model of $\Omega(k)$ which is being analyzed and is given by (5), it is observed that the indicator of violence against women in the months following completion of the first risk questionnaire (see Table I), is proportional to the initial condition of violence that was identified $\Omega(0)$. This is the initial condition of violence of the victim upon arrival at the care center. Nonetheless, from the data obtained in equation (6) it is established that $\Omega(k) = \sum_{1}^{n} \omega_n(k) f_n(k)$. This means that, when the month k of violence prediction level is reached, the victim would have to answer the violence questionnaire once again with equation (3). The violence indicator for month k would have to be recalculated to update the figures. Consequently, the equality equation which is obtained would be the following:

$$\Omega(k) = \Omega(0) \prod_{1}^{k} \alpha(k) \quad = \sum_{1}^{n} \omega_n(k) f_n(k) \tag{10}$$

A product, $\prod$, is observed in (10) and also an addition $\sum$. These are associated to the concepts of accumulation

TABLE II: INDICATORS OF MAXIMUM LEVELS OF VIOLENCE: TWELVE-MONTH MONITORING, k.

k	σ_{-}	ξ	β	α	Ω
1	1	1	1	1	336
2	1	1	1	2	672
3	1	1	1	3	2016
4	1	1	1	4	8064
5	1	1	1	5	40320
6	1	1	1	6	241920
7	1	1	1	7	1693440
8	1	1	1	8	13547520
9	1	1	1	9	121927680
10	1	1	1	10	1219276800
11	1	1	1	11	13412044800
12	1	1	1	12	1.60945E+11

of violence by the aggressor $\alpha(k)$ in which the frequency $f_n(k)$ as well as its severity weight $\omega_n(k)$ are related to the risk questionnaire (refer to Table I). All this leads to the performance of a maximum value analysis which results from the equations (3) and (5) individually, one at a time respectively.

In order to calculate the maximum values of (5), the parameters of loss of control by the aggressor must be selected per month. In other words, $\sigma_{max}(k) = 1$; it is necessary to consider the proportion of loss of control which may be reflected in the form of violence towards the victim $\xi_{max}(k) = 1$ for $k = 1, \ldots, 12$ and the maximum value of initial condition must be considered $\Omega(0)$. This represents the violence that the victim may confront after the questionnaire is applied (refer to Table I), to provide an example in a practical form for this analysis, $\Omega_{max}(0) = 336$ (this represents the maximum value that should be reached and is represented on the right-hand side of (3) and (10)).

Once these maximum values have been obtained, the parameters can be calculated $\beta_{max}(k)$, $\alpha_{max}(k)$ y $\Omega_{max}(k)$. This is for every month, from $k = 1$ to $k = 12$ (refer to equations (5), (7) and (8)). The maximum values that have been calculated for these parameters are shown in Table II.

It is important to note that the violence level prediction of this model (see equation (5)) expresses that the victim may find a fatal fate when $\Omega(k) = 1.6E + 11$ for $k = 12$. This would mean that to reach such level of fatality, a number of violent acts towards the victim that were not initially considered in the questionnaire should have occurred (see Table I). This means that for the eighteen questions ($n = 18$), there was a maximum level of violence of $\Omega(0) = 336$ and this would lead to infer the number of violent acts needed to reach a fatal scenario of $\Omega(0) = 1.6E + 11$; **From this questioning, as well as the equation (10), it is possible to establish the hypothesis that the model, as proposed by Enríquez, must be represented as**:

$$\mathbf{\Omega}\,(\mathbf{k}) = \mathbf{\Omega}\,(\mathbf{0})\,\alpha(\mathbf{k}). \tag{11}$$

Equation (11) in conjunction with the values indicated in table II, gives a maximum value of $\Omega_{max}(12) = 4032$. This implies that, employing a risk questionnaire for violent acts of 141 where questions take a value of $\omega_n = 10$ y $f_n = 3$ for $p = 19, \ldots, 141$, an approximate value of $\Omega_{max}(12) = 4056$ would be calculated. This calculation is the product considered from the logics established in Table

I and equation (3). This means that, to a new term which considers p, it is necessary to add the probable violent acts that were not considered when the original questionnaire was applied. Therefore,

$$\Omega\left(k\right)=\Omega\left(0\right)\alpha(k) \quad =\sum_{1}^{n}\omega_n(k)f_n(k)+\Theta(\omega_p,f_p) \quad (12)$$

where

$$\Theta(\omega_p,f_p)=\sum_{p=n+1}^{m}\omega_p(k)f_p(k). \quad (13)$$

The term $\Theta(\omega_p, f_p)$, includes the p likely violent acts that were not taken into consideration at the moment of the application of the initial risk questionnaire to the victim of violence (refer to Table I) at the time the evaluation of the initial condition of violence is conducted, $\Omega(0)$. This means that the term $\Theta(\omega_p, f_p)$ expresses the violent acts that might probably occur, which are predicted by the mathematical model, (11) within twelve months following the initial risk assessment.

It may also be noted that when $k = 0$, the parameter $\alpha(0) = 1$. Consequently, (12) this implies that:

$$\Omega\left(0\right)=\sum_{1}^{n}\omega_n(0)f_n(0)+\Theta(\omega_p,f_p). \quad (14)$$

Equation (14) shows that the first time the risk questionnaire was applied to calculate the initial condition of violence $\Omega(0)$ (refer to (3) and Table I), which actually constitutes an approximation to the initial risk of violence, not all acts of violence which are likely to turn into injuries to the victim are included. Thus, (14) an explanation is provided regarding the reasons why a variety of questionnaires with a wide range of questions asked to establish the likely risk of violence that the victim may experience upon her arrival at the care center, are applied around the world (the following references present a variety of questionnaires of violence for victims, [28], [29]).

In conclusion, once the initial condition of violence of the victim is determined $\Omega(0)$ (see (14)), it is essential to consider that the violent acts represented by $\Theta(\omega_p, f_p)$ are not being taken into consideration (refer to (16)). Nonetheless, model (11), which is the subject of this article, will be useful to conduct the simulation of the likelihood of violent scenarios that a victim may suffer during a one-year period (see equations (11)-(14)).

III. IPV MODEL: $\Omega(k)$

On account of what has been presented in section II of this article, the model to calculate the risk of intimate partner violence towards a victim at the hands of the perpetrator is established as:

$$\Omega\left(k\right)=\Omega\left(0\right)\alpha(k) \quad =\sum_{1}^{n}\omega_n(k)f_n(k)+\Theta(\omega_p,f_p) \quad (15)$$

where

$$\Theta(\omega_p,f_p)=\sum_{p=n+1}^{m}\omega_p(k)f_p(k) \quad (16)$$

$$\alpha(k)=\sum_{1}^{k}\beta(k), \quad (17)$$

$$\beta(k)=\xi(k)\times\sigma_-(k), \quad (18)$$

$$\begin{bmatrix}\sigma_+\left(k\right) & \sigma_-\left(k\right)\end{bmatrix}=\begin{bmatrix}\sigma_+\left(0\right) & \sigma_-\left(0\right)\end{bmatrix}\begin{bmatrix}(1-\lambda) & \lambda\\ \mu & (1-\mu)\end{bmatrix}^k. \quad (19)$$

and

$$\Omega\left(0\right)\cong\sum_{1}^{n}\omega_n(0)f_n(0) \quad (20)$$

$\Omega(0)$ represents the initial condition of violence that the victim experiences, which may be approximately (20) applying a risk questionnaire including n questions (refer to Table I where $n = 18$). $\alpha(k)$ expresses the accumulated level of violence displayed by the perpetrator. $\beta(k)$ constitutes the probable proportion of violence which may cause injuries to the victim. $\xi(k)$ represents the amount of loss of control that the perpetrator displays in a given cycle of violence. $\sigma_-(k)$ y $\sigma_+(k)$ accounts for the respective loss of control of the aggressor and his self-control level. $\Theta(\omega_p, f_p)$ are the p number of questions which were not included in the initial questionnaire applied to the victim when she arrives at the care center for the first time, in order to assess the risk of violence. ω_n and f_n represent the weight of severity and the frequency which is correlated to every single question included in the risk questionnaire (refer to Table I). 1-λ and 1-μ comprise the respective prevalence shown by the perpetrator when preserving a state of self-control or loss of control.

For the application of equations (15)-(20) to be presented, the data of indicator $\Omega_{III}(0) = 77$ will be used in the scenario of severe physical violence (the value of $(1-\mu)$ approximated to the pervasiveness of physical violence which has been reported as 23.4%; in a study which was applied to the workers of IMSS in Morelos, a state in México. The value of the probability of loss of control by the aggressor is calculated as 1 due to the fact that the victim is already afflicted by episodes of domestic violence [19], [14]), in which a frequency of "many times" is considered, $f_n = 2$ (refer to Table I. The data proposed by Leal-Enríquez E., constitute a complement of these calculations. (see [14]). Thus, what is obtained is: $\sigma_-(0) = 1$, $\sigma_+(0) = 0$, $1-\mu = 0.234$, $\mu = 0.7660$, $\Omega_{III}(0) = 77$ and $\lambda = 0.5$ random.

Due to the fact that the arrival of the victim at the attention center occurs after the violent acts have already increased, the following are the values which are considered to determine the proportion of loss of control by the perpetrator $\xi(k)$. These can result in injuries or violent acts towards the victim during a period of twelve months (data distribution for $\xi(k)$ is determined by considering a cycle of violence occurring

TABLE III: INDICATORS OF VIOLENCE: SCENARIO 1, TWELVE-MONTH MONITORING.

Month	σ_-^1	β^1	α^1	Ω_{III}^1
1	0.2340	0.073359	0.073359	5.6486
2	0.43776	0.033401	0.106760	8.2205
3	0.38356	0.076826	0.183586	14.1361
4	0.39797	0.260912	0.444498	34.2263
5	0.39414	0.365446	0.809944	62.3657
6	0.39516	0.332171	1.142114	87.9428
7	0.39489	0.251070	1.393184	107.2752
8	0.39496	0.135234	1.528418	117.6882
9	0.39494	0.347666	1.876084	144.4585
10	0.39495	0.017773	1.893857	145.8270
11	0.39494	0.024447	1.918304	147.7094
12	0.39494	0.031359	1.949663	150.1240

between a perpetrator of violent acts and the victim [10], [14]):

$$\begin{aligned}\xi(k) = \; & [0.3135 \quad 0.0763 \quad 0.2003 \quad 0.6556 \\ & 0.9272 \quad 0.8406 \quad 0.6358 \quad 0.3424 \\ & 0.8803 \quad 0.0450 \quad 0.0619 \quad 0.0794]. \end{aligned} \qquad (21)$$

By substituting these values in (19), and establishing a random probability of control prevalence of $\lambda = 0.3$ for $i = 1, 2$, the result is (18). The factors $\alpha(k)$ for $k = 1$ to $k = 12$ are determined afterwards. In the end, once the values from (17) and the initial condition of violence have been calculated $\Omega_{III}(0)$ (refer to Table I), the indicator of IPV $\Omega(k)$ (see (15)) for $k = 1, 2$, to 12 is established. The results after the application of this methodology are presented in Table III.

A. Simulations

Simulations have been performed considering mathematical models, which are deemed to be transcendental tools because they can be used in the prediction of scenarios based on given hypotheses [2]. Particularly, one scenario that must be expected when the model to predict domestic violence is used $\Omega(k)$ (refer to equations (15)-(20)) is the potential violent nature of an aggressor (clinical studies conducted by experts are useful to assess the probability of finding evidence of violent behavior in any male individual. For an example of this refer to [17]) It is also assumed that on average, the perpetrator, whose victim is a submissive woman not strong enough to repel the aggressions, changes his state of self-control month after month. It is then implied that for $k = 0$ the corresponding value of $\sigma_-(0) = 1$ [6], [3], [4], [14].

The simulations (specifically, those for the programming of the mathematical model (15)-(20) were performed at MatLab [26]. Even so, those calculations may be executed by making use of an appropriate mathematical software as Excel) were executed considering the corresponding values of section III, for five probable prevalence rates connected to the state of control of the perpetrator (the task of assigning the values, depending on the number of scenarios to be simulated to calculate the probable levels of violence to which the victim of domestic violence could be exposed, corresponds to the care center health worker [14]) $(1-\lambda) = [0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8]$ which is given randomly. Thus, by applying the methodology presented in the equations (15)-(20), the values of $\Omega_{III}^n(k)$ are calculated, where n=1,2,3,4,5 corresponds to every simulated scenario (a heuristic procedure was used to select the five probable scenarios; and also taking into consideration the behavior expected to be shown by the aggressor when remaining in a state of self-control).

B. Simulation: tension-outburst-honeymoon

Fig. 1(a) shows the possible scenarios associated to the loss of control by the perpetrator $\sigma_-(k)$ (refer to (19)). Fig. 1(b) presents $\xi(k)$ for a period of twelve months. This period is divided into three parts, all of them being a derivation from the cycle of violence: tension-outburst-honeymoon [14]. The coefficients $\beta(k)$ corresponding to the proportion of loss of control by the perpetrator, which could later turn into violent actions towards the victim (refer to (18)) are shown in Fig. 1(c). In fig. 1(d), the likely values associated to the indicator of IPV $\Omega_{III}^n(k)$ are graphed; therein, it is possible to observe their evolution in three, six, nine and twelve months (lines were used to connect the points corresponding to the values in Figures 1 and 2 so as to ease their monitoring throughout the forthcoming months).

C. Simulation: outburst-honeymoon-tension

At last, a simulation which considers the following given values of $\xi(k)$ is calculated:

$$\begin{aligned}\xi(k) = \; & [1 \quad 0.6 \quad 0.4 \quad 0.7 \quad 0.8 \quad 0.35 \\ & 0 \quad 0.01 \quad 0.02 \quad 0.2 \quad 0.3 \quad 0.1] \end{aligned} \qquad (22)$$

These values represent the fact that the victim is in the cycle of violence: outburst, honeymoon, tension [14].

Fig. 2 shows the likely scenarios of loss of control by the aggressor $\sigma_-(k)$ (refer to (9)). Additionally, the factor $\xi(k)$ for a period of twelve months is displayed. This is derived from the cycle of violence: outburst, honeymoon, tension [14]. The coefficients $\beta(k)$ of the proportion of loss of control by the perpetrator which might appear as violent acts towards the subject victim of violence are also set out (refer to (18)) as well as the accumulation factor corresponding to these acts $\alpha(k)$ (see (17)).

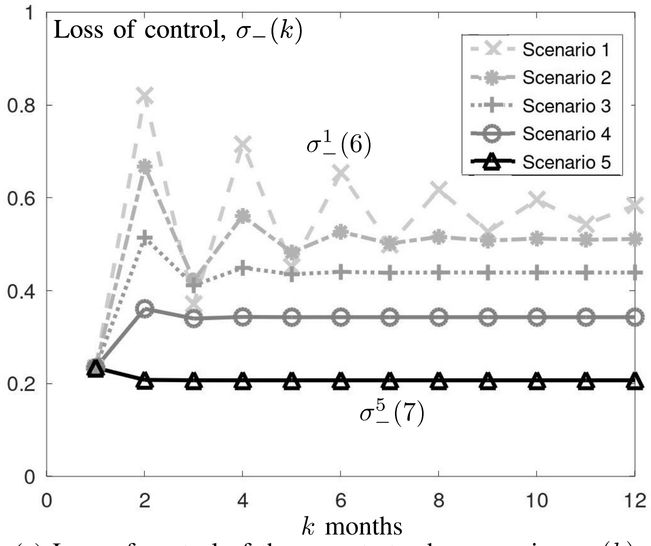
Fig. 4, shows the probable values related to the indicator of domestic violence $\Omega_{III}^n(k)$. The behavior of the indicator for three, six, nine and twelve months[20] is observed.

The simulations were performed only for two probable cycles of violence. The reason for this is that the victim resorts to the care centers after she has been experiencing some kind of incident in which violence is displayed ([7], [15], [20]).

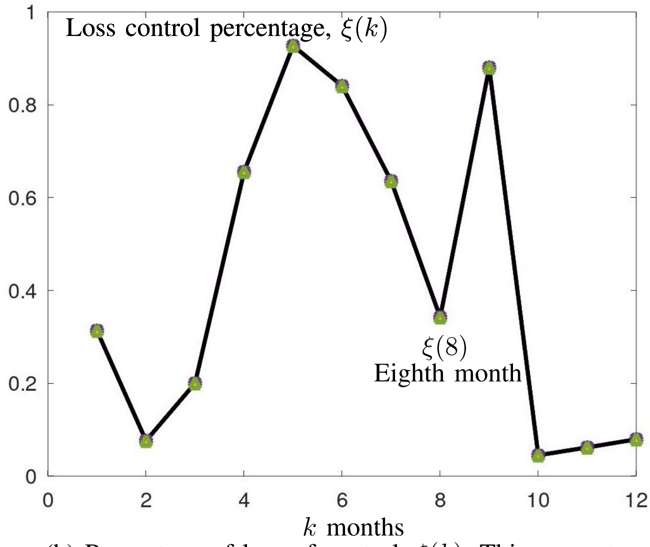
IV. Discussion and Conclusions

In this article, a proposal is made to improve the model designed by Leal-Enriquez E. [14], so that it can explain and establish limits to the risk of violence towards the victim at the hands of the perpetrator.

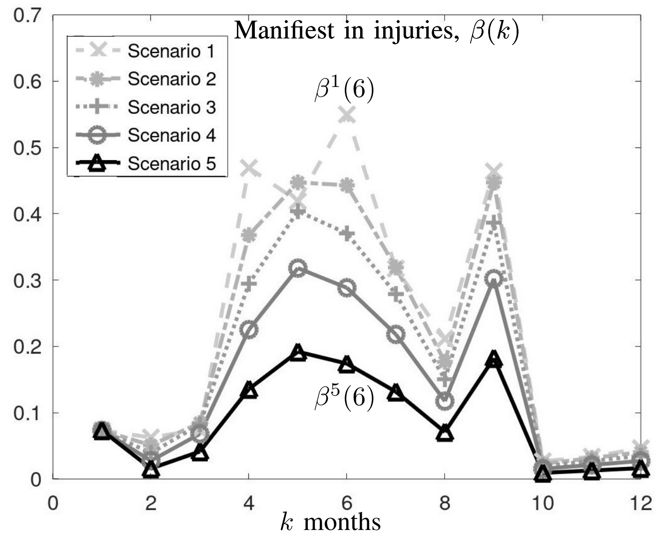
The model proposed in this work establishes an upper limit numerical) of fatality (likely scenario of femicide) based on the risk questionnaire applied to the victim in relation to severity weight and assigned frequency (see



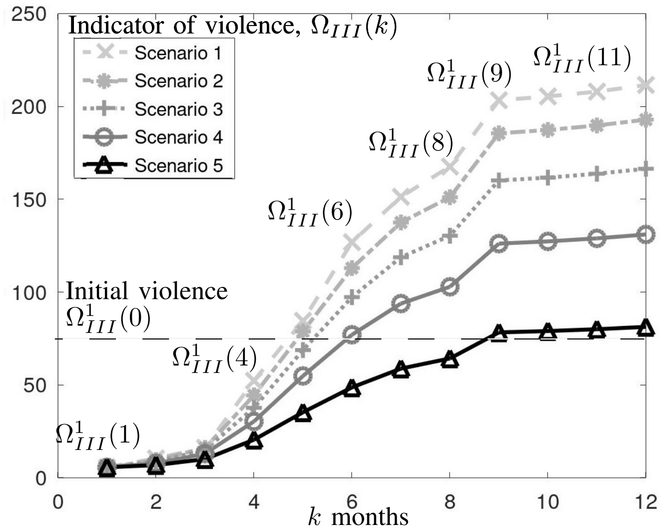
(a) Loss of control of the perpetrator by scenario, $\sigma_{-}(k)$. It can be seen that in scenario 5, the perpetrator has an almost constant loss of control (almost straight line).



(b) Percentage of loss of control, $\xi(k)$. This percentage is the same for each one of the scenarios of loss of control by the perpetrator, shown in Fig. 1(a).



(c) Calculation of the proportion of loss of control, $\beta(k)$ that can be manifiested in injuries towards the victim. Note that the lowest level is that of scenario $k = 5$, $\beta^{5}(k)$.



(d) Indicator of intimate partner violence, $\Omega_{III}(k)$ of the victim. It is observed that scenario 1, is the most dangerous for the victim.

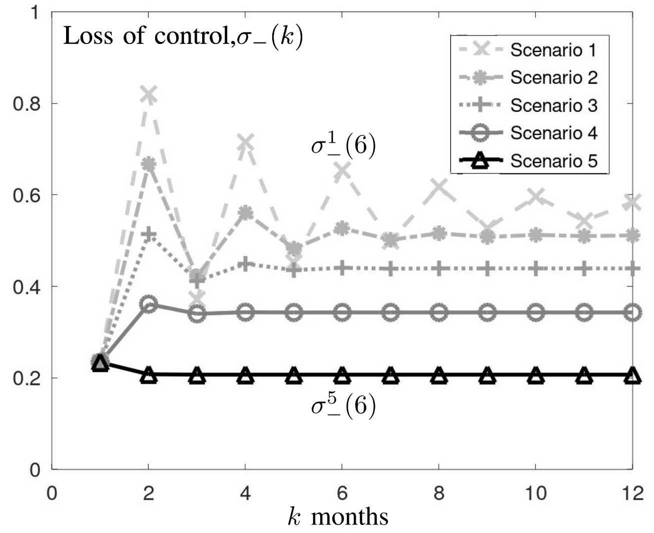
Fig. 1: Simulation of five scenarios:tension-outburst-honeymoon. The connection of values, $\circ$, by means of lines is for monitoring purposes.

Table I). Note that this value of maximum fatality is: $\Omega_{max}(k) = \Omega_{max}(0) \times 12$ (see equation (11)). In other words, this proposed model indicates that the maximum value of violence will be twelve times the one corresponding to the initial condition of maximum violence that the victim may suffer, for this specific work $\Omega_{max}(k) = 4032$. With this maximum value of risk of fatality, it is possible to estimate, approximately, how many violent acts the victim may experience over a period of twelve months.

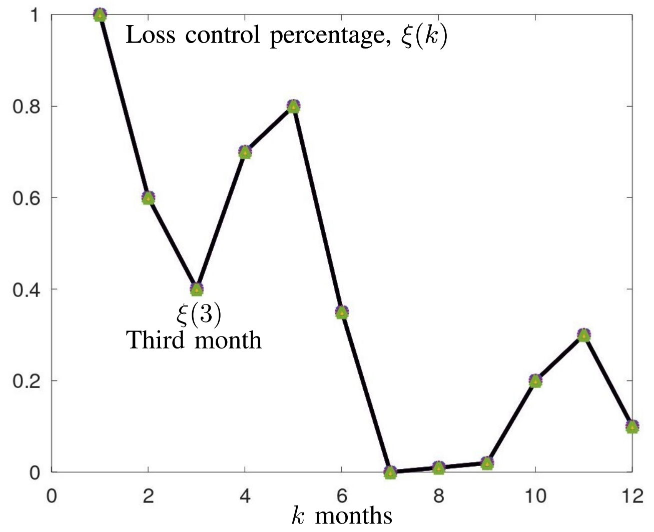
This improvement to the model allows the performance of simulations where it is possible to analyze month by month the way in which the risk of violence evolves for the victim, both graphically and numerically (see Table III as well as Figures 1(d) and 2(d)).

The expert in violence will be able to observe each of the likely scenarios that the victim may experience, always taking into consideration the maximum level of fatality, and therewith, based on the graphs and numerical values, analyze whether there is a likely scenario that approaches that risk limit. The purpose of this is to decide, subsequently, if action should be taken following the expected protocols to help the victim and prevent the risk of femicide.

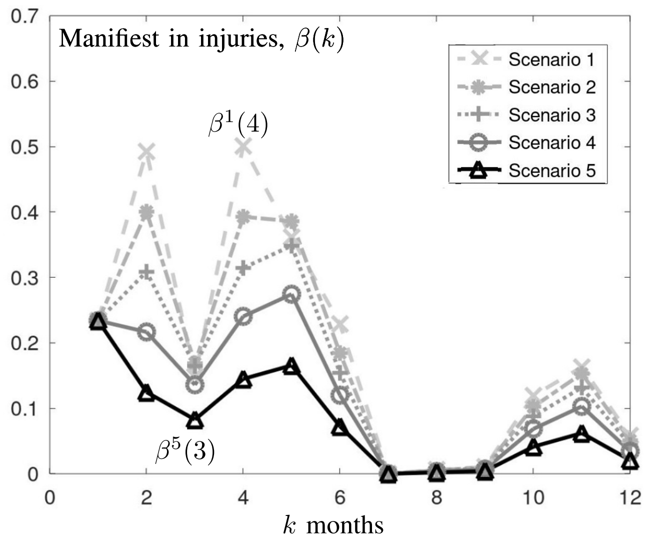
The expert must analyze the graphs of likely behaviors of loss of control by the perpetrator (see Fig. 2(a)), which can be used to show the victims their possible scenarios of violence. The violence expert must select the most likely scenario or scenarios that the victim may face over the following months to be considered. With this information, the expert would have an idea of the probable danger that the victim may suffer. See for example Table III, which shows that there is a scenario of loss of control by the perpetrator that could result in a certain level of violence ($\Omega_{III}^{1}(12) \simeq 150$), a value that exceeds the initial condition $\Omega(0)$ with which the victim was evaluated in the initial interview. This could be



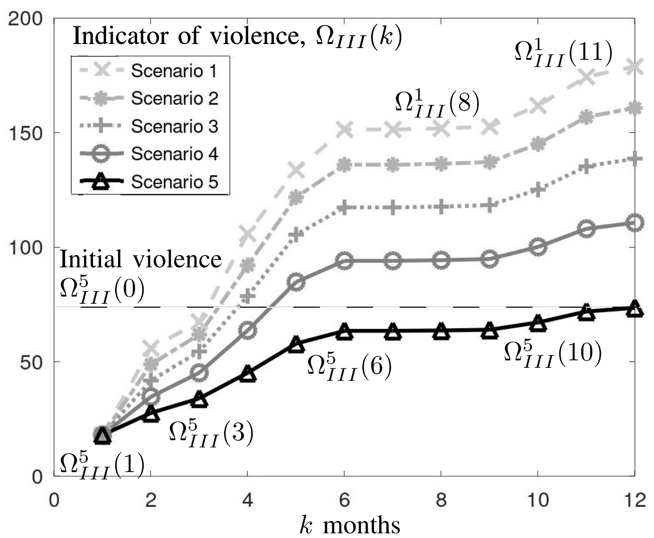
(a) Loss of control of the perpetrator by scenario, $\sigma_-(k)$. It can be seen that in scenario 5, the perpetrator has an almost constant loss of control (almost straight line).



(b) Percentage of loss of control, $\xi(k)$. This percentage is the same for each one of the scenarios of loss of control by the perpetrator, shown in Fig. 1(a).



(c) Calculation of the proportion of loss of control, $\beta(k)$ that can be manifiested in injuries towards the victim. Note that the lowest level is that of scenario $k = 5$, $\beta^5(k)$.



(d) Indicator of intimate partner violence, $\Omega_{III}(k)$ of the victim. It is observed that scenario 1, is the most dangerous for the victim.

Fig. 2: Simulation of five scenarios:tension-outburst-honeymoon. Values have been linked, $\circ$, through lines for monitoring purposes.

interpreted as a probable high-risk scenario that is not yet to reach the level of fatality. (The assigned value in this exercise is 4032).

From Table III, it can be seen that in the seventh month, violence begins to escalate to dangerous levels. That is, if the health expert detects this behavior of loss of control by the perpetrator, he/she must provide assistance to the victim for the first five months so as to decrease the levels of violence.

The model (see equations (15)-(20)) is very general because it does not explicitly take into consideration some specific factors that may alter the state of loss of control by the perpetrator, such as alcoholism and drug addiction, to mention a few. Another factor to consider is that it is limited to a model in which the victim is completely submissive. In addition, the model assumes that a series of factors build up, resulting in the perpetrator's loss of control, which manifests in injuries and/or violent acts towards his partner. However, this model can serve as an aid for care centers, and especially for the victim, because the risk she is facing, as well as the likely scenarios of violence that she may experience in the forthcoming months, if adequate assistance to break the cycle of violence is not provided to her, can be observed and measured.

After all the simulations, it is observed that most of the scenarios do not end in a situation of fatality towards the victim (this conclusion does not intend to pose a generalization), see Figures 1(d) and 2(d) as well as Table III.

It is important to state that the author expects this work may be useful for care centers and researchers with the aim of contributing to the study and knowledge of the complex phenomenon of violence, which affects all societies worldwide.

In general, this model is expected to help care centers and victims to break the cycle of violence. For instance, the violence expert can show victims an interpretation of Fig. 1(d) on a predictive risk bar graph display. Fig. 3, shows risk monitoring for the five likely violence scenarios that the victim may experience in a tension-outburst-honeymoon cycle of violence at the end of a period of twelve months. This, taking into account that the initial condition of the victim was $\Omega_{III}(0) = 77$ calculated after the application of the initial risk questionnaire (see Table I). From Fig. 3, it can be seen that the indicator of violence $\Omega^n_{III}(12)$, states there are four scenarios of violence (n=1,2,3,4), and the risk of violence in the twelfth month exceeds the initial condition of violence $\Omega_{III}(0)$ which could imply a probable risk of fatality for the victim at the end of this cycle of violence.

Finally, to graphically show the operation of the proposed model as well as its usefulness for specialized care centers in gender violence as well as for victims of Intimate Partner Violence, Fig. 4 shows a scenario of violence that can be suffered by the victim in a cycle of violence, in which everything begins with a violent man $\sigma_0 = 1$, who accumulates levels of tension, $\alpha(k)$, which can be reflected in a level of violence towards the victim in different types of violent acts such as: blows to the body, cigarette burns, suffocation, threats with a weapon and shooting in any part of the body without causing death, whose total violence indicator is measured by $\Omega(k)$ (see equations (15)-(20)).

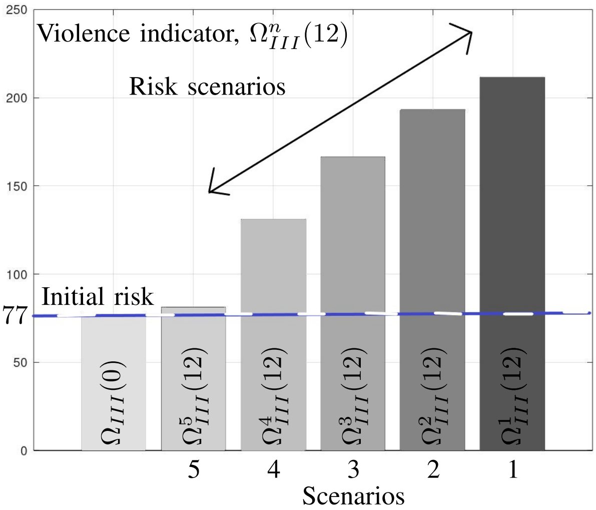


Fig. 3: Predictive violence risk indicator in the twelfth month. $\Omega^5_{III}(12) \approx 81$, $\Omega^4_{III}(12) \approx 131$ $\Omega^3_{III}(12) \approx 166$, $\Omega^2_{III}(12) \approx 192$, $\Omega^1_{III}(12) \approx 211$.

V. Forthcoming work

Since the proposed model (see equations (15)-(20)) does not consider external aid, social controllers are to be developed with the aim of reducing the loss of control by the perpetrator as a percentage, which would likely result in reducing the violence indicator $\Omega(k)$. Likewise, clinical validations conducted by experts, will be performed in relation to the level of violence that a victim of IPV presents. This is in order to compare it to the likely scenarios of violence generated from the model herein proposed, and consequently be able to contribute in assisting victims to prevent future IPV related injuries or other types of violence. This experimental study is intended to conduct data mining for the validation of a mathematical model, as well as the running of demonstration clinics of the usefulness of this mathematical model, so as to show how it helps the victims of IPV.

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The violent man, $\sigma_{-}(0) = 1$, accumulates levels of violence, $\alpha(k)$ (see (17)), from the proportion of loss of control that can become in acts violent forward her victim $\beta(k)$ (ver (18)), in a cycle of violence, $\xi(k)$ (see (22)).

This accumulated violence $\alpha(k)$ is reflected in different violent acts, $\Omega(k)$ (see (15)),for example physical violence: blows to the body, cigarette burns, suffocation, threats with a weapon and firing it somewhere in the body without causing death, in some month k of the cycle of violence.

The initial measurement of the level of violence is obtanied using a risk measuring instrument (see TABLE I). The proposed model $\Omega(k)$ in this work, it intends to help the violence expert to make decisions showing a forecast of the levels of violence that the victim could reach in the coming months in a cycle of violence (see TABLE III).

Fig. 4. Story board of a likely scenario of intimate partner violence that the victim may experience at the hands of the perpetrator, these levels of violence and scenarios can probably be predicted by the model proposed in this work by $\Omega(k)$, (see TABLE III).

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