Competition Strategy of CSR Input for Low-carbon and Ordinary Products under Consumers' Dual Preference

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Abstract—**Based on the dual influence of consumers' willingness to pay for low-carbon and ordinary products and CSR preferences, this paper studies the corporate social responsibility (CSR) investment of low-carbon and ordinary product manufacturers. The competition and cooperation investment strategies of the two manufacturers and the impact of different CSR inputs on product prices, market demand and manufacturer profits are investigated. It is found that: 1) CSR investment of manufacturers is not always conducive to the improvement of supply chain profits; 2) The cooperative investment mode is the best strategy choice for the two products, and the cooperative investment mode can improve the competitive efficiency loss. 3) When CSR is applied to the same product, the supply chain profit in the cooperative input mode is higher than that in the competitive input mode, and the revenue sharing-bidirectional cost sharing contract can effectively coordinate the competitive input mode. Finally, this paper verifies the establishment of the corresponding conclusions by example analysis.**

Index Terms—**Low-carbon Products, CSR Input, Consumer Utility, Pricing**

I. Introduction

A. Motivation

WITH the gradual improvement of residents' living standards and awareness of environmental protection, more and more attention have been paid to the attributes of products, such as their greenness, carbon emissions, recyclability, sustainability, etc. in the process of purchasing. At the same time, the sensitivity of the enterprise's response to market changes also determines whether the enterprise can win the first opportunity to a certain extent. At present, not only consumers but also the government and social parties pay attention to the carbon emissions of products and require enterprises to reduce emissions. The United Nations Climate Conference in Poland in 2018 is an important opportunity to achieve the goals of the Paris Agreement. China are striving to achieve carbon neutrality before 2060 by using measures such as "increasing income and reducing cost", which focus on promoting the development and utilization of new energy and reducing carbon emissions (Xi(2021) [1]). It is of vital importance for enterprises who is aiming at maximizing profits to pay attention to consumer demand, reduce carbon emissions in the process of transformation and maximize their own interests at the same time. Henri et al. (2013) [2] found that CSR input is beneficial to increase the value of an enterprise under certain conditions. Apple maintains active partnerships with two aluminum companies and actively uses "carbon-free" raw materials to reduce carbon emissions (Lin (2021) [3]). Apple's production behavior not only shows that enterprises actively invest in corporate social responsibility, but also conforms to the global trend of carbon reduction. Enterprises can not only establish a good corporate image but also improve demand and profits by CSR input at the same time. Therefore, the hot issue that enterprises care about currently in the low-carbon supply chain will be how to better combine the emission reduction target of carbon peaking and carbon neutralization with social responsibility investment of the enterprises.

This paper takes the competition and cooperation of manufacturers of low-carbon and common products to invest in CSR as the entry point, and considers the influence of consumers on the willingness to pay for the two types of products and CSR preference. Under the joint action of the two, this paper analyzes product price, market demand and manufacturer's profit, and provides reasonable opinions for manufacturing enterprises to improve CSR investment efficiency.

B. Literature Review

This part mainly focuses on low-carbon supply chain and corporate social responsibility.

Many scholars have conducted in-depth research on carbon emission reduction (Lin (2021) [3]), government policy (Zhu et al. (2014) [4], Huang et al. (2019) [5], and Ding et al. (2020) [6]), carbon emission trading (2020) [6] and other issues in the low-carbon supply chain. In the study of this paper, product differences in the low-carbon supply chain are firstly considered. Xia et al. (2014) [7] studied the effects of carbon trading on product price, market demand, profit and consumer surplus under different products. Based on consumers' preference for low-carbon and ordinary products, Zhu et al. (2014) [4] analyzed the influence of different government subsidies for low-carbon products and enterprises on optimal decision-making of supply chain. The results showed that direct government subsidies for low-carbon production enterprises could better promote the

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development of low-carbon enterprises. Taking the same problem as the background, Xu et al. (2014) [8] studied Pareto optimization of product pricing and supply chains with and without government subsidies for low-carbon products. Based on carbon trading and government subsidies, Miu and Shen (2020) [9] analyzed the impact of the two policies on the prices, market demand and corporate profits of low-carbon products and ordinary products. Under the background of carbon tax mechanism, Zhang et al. (2020) [10] analyzed the price and demand of two products and their influence on manufacturers' profits under different competition and cooperation models.

Secondly, it is the aspect of the consumer preference. In addition to literature Huang(2014) [4], Zhou et al. (2020) [11] considered the impact of consumers' environmental awareness on the supply chain when studying government subsidies and carbon taxes, and found that consumers' environmental awareness and low-carbon subsidies are complementary to each other. Sun Jiayi et al. (2021) [12] introduced fairness concerns into the supply chain based on consumer utility function to analyze the impact of consumer behavior on manufacturers. Zhang and Wang (2021) [13] used consumer utility function to consider the purchase demand of three different products, and discussed the influence of consumer behavior on manufacturer's production mode. Li and Li (2019) [14] analyzed the impact of consumer preference for product's performance and its impact on the environment on sales channels under different circumstances whether the government subsidizes manufacturers or not. In current scholars' research, the literature analyzing the price difference between low-carbon products and ordinary products is seldom to analyze from the perspective of consumer utility function, and most of them only consider one product when analyzing consumers' preference for certain performance of products. Few literatures which analyzed the price difference between low-carbon products and ordinary products are analyzed from the perspective of consumer utility function, while only considered one type of product when considering customers preferences for a certain performance.

Another problem related to this paper is enterprise social responsibility. Although there is no unified definition about enterprise social responsibility at present, many scholars have studied and analyzed the investment of corporate social responsibility (CSR) from the perspective of supply chain. Cao et al. (2020) [15] studied the impact of government subsidies and enterprise social responsibility input on supply chain efficiency under different power structures. Li and Li (2020) [16] mainly analyzed the problems of investors under the background of enterprise social responsibility input. The results show that there is no absolute correlation between the quality of CSR investment and the participants. Hossein et al. (2021) [17] introduced the greenness and transparency of products when analyzing the investment in corporate social responsibility. Studies have found that CSR investment can effectively improve levels of both factors above. Liu et al. (2021) [18] constructed three game models of whether two retailers should engage in CSR investment or not, and the analysis showed that the supply chain system was optimal when two retailers engaged in CSR investment simultaneously. Johari et al. (2019) [19] used evolutionary game method to analyze manufacturers' dynamic evolution strategy when considering whether two manufacturers invest in CSR. As in literature [18], the scale of CSR input has dual influence on the demand function of different products. Shu et al. (2018) [20] studied the relationship between carbon emission reduction coefficient of different enterprises and government incentive and punishment policies when the awareness of enterprise social responsibility exist. Wang et al. (2021) [21] used consumers' different preferences for new products and remanufacturing to study the impact of fairness concerns and government subsidies on supply chains. Based on the above analysis, it can be found that few scholars have introduced enterprise CSR investment into the consumer utility function from the perspective of the difference between the two products. At the same time, only one situation that consumers' preference for CSR investment had been considered in the aspect of influence on consumers, while the analysis of interaction of product differences and scale of expenditure preferences from the perspective of consumer utility function has been ignored. In real life, consumers' willingness to pay for products will also affect the effect of CSR input.

Based on the above research, this paper introduces the CSR investment scale into the consumer utility function, and discusses the influence of the difference of consumers' willingness to pay for products and the CSR preference coefficient on the optimal decision of supply chain. In order to analyze the influence of competition and cooperation on manufacturers' CSR investment scale, this paper discusses the differences of product price, market demand and manufacturer profit between low-carbon product manufacturers and ordinary product manufacturers under different CSR investment models, so as to put forward more reasonable management opinions.

The innovation points of this paper are as follows: 1) Under the dual effect of consumers' willingness to pay for products and CSR preference, whether manufacturers' CSR input can always improve corporate profits; 2) The impact of different CSR investment modes on manufacturers' profits; 3) What is the impact of CSR input from competition and cooperation on the supply chain, and how to coordinate to improve the supply chain.

II. Model Assumptions and Symbol Description

This paper studies the main content by constructing two supply chain systems. One is the low carbon products manufacturers and ordinary products manufacturers there is competition of CSR into low carbon supply chain (hereinafter referred to as "Model D") model, the second is two manufacturers cooperation of CSR into low carbon supply chain (hereinafter referred to as "Model C") model. Both manufacturers sell their products directly through their own channels, determine the retail price of their products, and decide whether to invest in CSR. As consumers have different preferences, they will have different willingness to pay for different products, so they will decide whether to buy the product through their own utility function.

A. Basic Assumptions

The basic assumptions of the model are as follows:

Assumption 1. In the model composed of two manufacturers, namely Model D, there does not exist information asymmetry, which is a complete information game. Consumers in the market are divided into low-carbon product preferences and ordinary product preferences, and there exists demand for both kinds of products.

Assumption 2. Consumers' willingness to pay for ordinary products will be less than that for low-carbon products affected by the improvement of environmental protection awareness and other factors. In this case, compared to low-carbon products, consumers' willingness to pay for ordinary products is λv, and consumers' utility functions for the two kinds of products are respectively: $U_L = v - p_L$, $U_H = \lambda v - p_H$.

Assumption 3. In order to compare the influence of CSR input from competition and cooperation on product price, market demand and profit, it is assumed that the preference coefficient of consumers for Manufacture's CSR input is the same in different competition-cooperation models, which is: $U_L = v - p_L + \theta d_L, U_H = \lambda v - p_H + \theta d_H$.

Assumption 4. When there exists CSR input in the different Model, referring to the research of Yao et al. (2021) [22], it is assumed that the manufacturer's CSR input cost is $\frac{1}{2} k d_\alpha^2, \alpha = L, H$, and $k > 0$ is the scale parameter of supply chain or manufacturer's CSR input. In order to ensure the existence of the solution results in this paper and referring to the research of Yao et al. (2021) [22], it is assumed that k is large enough.

B. Symbol Description

In the model constructed in this paper, the corresponding variables and specific meanings are listed in TABLE I.

TABLE I
PARAMETERS SETTING AND THEIR MEANINGS

Decision Variables	Meaning
p_L	Retail price of low-carbon products
p_H	Retail price of ordinary products
d_L	Scale of CSR investment for low-carbon products
d_H	Scale of CSR investment for ordinary products
Other Variables	**Meaning**
λ	Consumers' preference coefficient of ordinary products relative to low-carbon products, $0 < \lambda < 1$
θ	Consumers' CSR preferences, $0 < \theta$
k	Scale coefficient of manufacturers' CSR input, $0 < k$
A	Upper limit of consumers' willingness to pay
q_L	Market demand for low-carbon products
q_H	Market demand for ordinary products
π_{CS}^{Cj}	Supply chain profit of model C under different CSR input j, $j = \mathrm{LH, L, H}$
π_i^{Dj}	Under different CSR input j, the manufacturer/supply chain profit of model D is i $i = \mathrm{LM, HM, CS}$, $j = \mathrm{LH, L, H}$

III. THE SOLUTION AND ANALYSIS OF THE MODEL

Based on the consumer utility function, this paper compares the dual effects of CSR input and consumer preference on the optimal decision of supply chain due to product differences. When different manufacturers invest in CSR, the changes of product price, market demand and manufacturer profit in D model and C model are investigated, and the differences between the two models are analyzed and coordinated.

A. The Construction and Solution of D Model

In model D, low-carbon manufacturers and ordinary manufacturers compete to invest in CSR. Under different CSR investment modes, the two manufacturers are in equal status and there is no order of decision-making, so Nash game is carried out. When a manufacturer has CSR input, the two product manufacturers decide the retail price of the product at the same time, and then decide the CSR input scale.

(1)Manufacturers have no CSR input (hereinafter referred to as "DN Model")

When manufacturers of low-carbon and ordinary products do not invest in CSR, the utility functions of consumers for ordinary products and low carbon products respectively are:

$$U_L = v - p_L \tag{1}$$

$$U_H = \lambda v - p_H \tag{2}$$

When there is a demand for both low carbon and ordinary products in the market, the demand functions of the two products are:

$$q_L = 1 - \frac{p_L - p_H}{(1-\lambda)A} \tag{3}$$

$$q_H = \frac{\lambda p_L - p_H}{\lambda(1-\lambda)A} \tag{4}$$

The profit function of the two manufacturers is:

$$\pi_{LM} = (p_L - c_L) q_L \tag{5}$$

$$\pi_{HM} = (p_H - c_H) q_H \tag{6}$$

Proposition 1. By solving the above equations, the optimal price of the products p_L^{DN*}, p_H^{DN*}, the market demand q_L^{DN*}, q_H^{DN*} and the profit of manufactures $\pi_{LM}^{DN*}, \pi_{HM}^{DN*}$ can be obtained, as listed in TABLE II and TABLE III.

Prove:

The utility function of consumers on low-carbon products and ordinary products in the market is:

$$U_L = v - p_L \tag{7}$$

$$U_H = \lambda v - p_H \tag{8}$$

When $U_H < U_L$, consumers will choose to buy low-carbon products, and the demand for low-carbon products in this case will be:

$$q_L = \int_{\frac{p_L - p_H}{(1-\lambda)}}^{A} \frac{1}{A} dv = 1 - \frac{p_L - p_H}{(1-\lambda)A} \tag{9}$$

When $U_H \geq U_L$, price-sensitive buyers in the market will choose to buy ordinary products, and the conditions $U_H \geq U_L$ and $U_H > 0$ should be met. At this time, the demand for ordinary products in the market is:

$$q_H=\int_{\frac{p_H}{\lambda}}^{\frac{p_L-p_H}{(1-\lambda)}}\frac{1}{A}dv=\frac{\lambda p_L-p_H}{\lambda(1-\lambda)A} \tag{10}$$

Therefore, when there is demand for low-carbon and ordinary products in the market, the demand for the two products is:

$$q_L=1-\frac{p_L-p_H}{(1-\lambda)A} \tag{11}$$

$$q_H=\frac{\lambda p_L-p_H}{\lambda(1-\lambda)A} \tag{12}$$

Bring the demand of two products into the profit function of manufacturer to obtain:

$$\pi_{LM}=(p_L-c_L)\left(1-\frac{p_L-p_H}{(1-\lambda)A}\right) \tag{13}$$

$$\pi_{HM}=(p_H-c_H)\left(\frac{\lambda p_L-p_H}{\lambda(1-\lambda)A}\right) \tag{14}$$

The two manufactures will make decisions using Nash game, then it is obtained by calculating partial derivative of manufacturer's profit function:

Let $4-\lambda=\Delta_2, A(1-\lambda)=\Delta_3$.

$$\begin{cases}\dfrac{\partial\pi_{LM}}{\partial p_L}=\Delta_3-2p_L+p_H+c_L=0\\ \dfrac{\partial\pi_{HM}}{\partial p_H}=\lambda p_L-2p_H+c_H=0\end{cases} \tag{15}$$

Simultaneously to solve the equations, get the optimal solution: $p_L^{DN*}=\dfrac{2\Delta_3+2c_L+c_H}{\Delta_2}, p_H^{DN*}=\dfrac{\lambda\Delta_3+\lambda c_L+2c_H}{\Delta_2}$.

Substituting p_L^{DN*}, p_H^{DN*} into $q_L, q_H, \pi_{LM}, \pi_{HM}$, then get:

$$q_L^{DN*}=\frac{2\Delta_3-\Delta_1c_L+c_H}{\Delta_3\Delta_2} \tag{16}$$

$$q_H^{DN*}=\frac{\lambda\Delta_3+\lambda c_L-\Delta_1c_H}{\lambda\Delta_3\Delta_2} \tag{17}$$

$$\pi_{LM}^{DN*}=\frac{(2\Delta_3-\Delta_1c_L+c_H)^2}{\Delta_3\Delta_2^2} \tag{18}$$

$$\pi_{HM}^{DN*}=\frac{(\lambda\Delta_3+\lambda c_L-\Delta_1c_H)^2}{\lambda\Delta_3\Delta_2^2} \tag{19}$$

(2)Only low-carbon product manufactures invest in CSR (hereinafter referred to as "DL Model")

When only low-carbon product manufacturers invest CSR, the utility functions of consumers for ordinary products and low-carbon products are respectively:

$$U_L=v-p_L+\theta d_L \tag{20}$$

$$U_H=\lambda v-p_H \tag{21}$$

Likewise, when there are two products in the market, the demand function and the manufacturer's profit function are:

$$q_L=1-\frac{p_L-p_H-\theta d_L}{(1-\lambda)A} \tag{22}$$

$$q_H=\frac{\lambda p_L-p_H-\lambda\theta d_L}{\lambda(1-\lambda)A} \tag{23}$$

$$\pi_{LM}=(p_L-c_L)q_L-\frac{1}{2}kd_L^2 \tag{24}$$

$$\pi_{HM}=(p_H-c_H)q_H \tag{25}$$

Proposition 2. By solving the above equations, the optimal scale of CSR input d_L^{DL*}, the optimal price of the products p_L^{DL*}, p_H^{DL*}, the market demand q_L^{DL*}, q_H^{DL*} and the profit of manufactures $\pi_{LM}^{DL*}, \pi_{HM}^{DL*}$ can be obtained, see Table II and III.

Prove: refer to Proof 1 in the appendix.

(3)Only ordinary product manufactures invest in CSR (hereinafter referred to as "DH Model")

When only ordinary manufacturers have CSR investment, the utility functions of consumers for ordinary products and low-carbon products are respectively:

$$U_L=v-p_L \tag{26}$$

$$U_H=\lambda v-p_H+\theta d_H \tag{27}$$

Likewise, when there are two products in the market, the demand function and the manufacturer's profit function are:

$$q_L=1-\frac{p_L-p_H+\theta d_H}{(1-\lambda)A} \tag{28}$$

$$q_H=\frac{\lambda p_L-p_H+\theta d_H}{\lambda(1-\lambda)A} \tag{29}$$

$$\pi_{LM}=(p_L-c_L)q_L \tag{30}$$

$$\pi_{HM}=(p_H-c_H)q_H-\frac{1}{2}kd_H^2 \tag{31}$$

Proposition 3. By solving the above equations, the optimal scale of CSR input d_H^{DH*}, the optimal price of the products p_L^{DH*}, p_H^{DH*}, the market demand q_L^{DH*}, q_H^{DH*} and the profit of manufactures $\pi_{LM}^{DH*}, \pi_{HM}^{DH*}$ can be obtained, as listed in Table II and III.

Prove: refer to Proof 2 in the appendix.

(4)Two manufacturers invest in CSR (hereinafter referred to as "DLH Model")

When both manufacturers invest CSR, the utility function of consumers for ordinary products and low-carbon products are respectively:

$$U_L=v-p_L+\theta d_L \tag{32}$$

$$U_H=\lambda v-p_H+\theta d_H \tag{33}$$

Likewise, when there are two products in the market, the demand function and the manufacturer's profit function are:

$$q_L=1-\frac{p_L-p_H+\theta d_H-\theta d_L}{(1-\lambda)A} \tag{34}$$

$$q_H=\frac{\lambda p_L-p_H+\theta d_H-\lambda\theta d_L}{\lambda(1-\lambda)A} \tag{35}$$

$$\pi_{LM}=(p_L-c_L)q_L-\frac{1}{2}kd_L^2 \tag{36}$$

$$\pi_{HM}=(p_H-c_H)q_H-\frac{1}{2}kd_H^2 \tag{37}$$

Proposition 4. By solving the above equations, the optimal scale CSR input d_L^{DLH*}, d_H^{DLH*}, the optimal price of the products p_L^{DLH*}, p_H^{DLH*},the market demand q_L^{DLH*}, q_H^{DLH*} and the profit of manufactures $\pi_{LM}^{DLH*}, \pi_{HM}^{DLH*}$ can be obtained, as listed in Table II and III.

Prove: refer to Proof 3 in the appendix.

Let $2-\lambda=\Delta_1, 4-\lambda=\Delta_2, A(1-\lambda)=\Delta_3,\ Ak(1-\lambda)(4-\lambda)^2=A_1, \theta^2(2-\lambda)(4-\lambda)=A_2,\ Ak(1-\lambda)(4-\lambda)^2-2(2-\lambda)^2\theta^2=B_1, \lambda Ak^2(1-\lambda)(4-\lambda)^2-2(2-\lambda)^2\theta^2=B_2$

TABLE II

CSR INVESTMENT SCALE AND PRODUCT RETAIL PRICE (MODEL D)

CSR Investment Scale / Model	d_L	d_H
DN Model	-	-
DL Model	$\frac{2\Delta_1\theta\left(2\Delta_3-\Delta_1 c_L+c_H\right)}{B_1}$	-
DH Model	-	$\frac{2\Delta_1\theta\left(\lambda\Delta_3-\Delta_1 c_H+\lambda c_L\right)}{B_2}$
DLH Model	$2\Delta_1\theta\frac{2\Delta_3(A_2-\lambda A_1)+(\lambda\Delta_1 A_1-2A_2(1-\lambda))c_L-\lambda A_1 c_H}{4\lambda\Delta_1^2\theta^4-B_1B_2}$	$2\Delta_1\theta\frac{2\lambda\Delta_3 A_2-\Delta_3 A_1-A_1 c_L+(\Delta_1 A_1-2A_2(1-\lambda))c_H}{4\lambda\Delta_1^2\theta^4-B_1B_2}$
Product Price / Model	p_L	p_H
DN Model	$\frac{2\Delta_3+2c_L+c_H}{\Delta_2}$	$\frac{\lambda\Delta_3+\lambda c_L+2c_H}{\Delta_2}$
DL Model	$\frac{2k\Delta_2\Delta_3^2+2(k\Delta_2\Delta_3-\Delta_1^2\theta^2)c_L+k\Delta_2\Delta_3 c_H}{B_1}$	$\frac{k\lambda\Delta_2\Delta_3^2-2\lambda\Delta_1\Delta_3\theta^2+k\lambda\Delta_2\Delta_3 c_L+2(k\Delta_2\Delta_3-\Delta_1\theta^2)c_H}{B_1}$
DH Model	$\frac{2\lambda k\Delta_2\Delta_3^2-2\Delta_1\Delta_3\theta^2+2(\lambda k\Delta_2\Delta_3-\Delta_1\theta^2)c_L+\lambda k\Delta_2\Delta_3 c_H}{B_2}$	$\frac{\lambda^2 k\Delta_2\Delta_3^2+\lambda^2 k\Delta_2\Delta_3 c_L+2(\lambda k\Delta_2\Delta_3-\Delta_1^2\theta^2)c_H}{B_2}$
DLH Model	$\frac{\Delta_3(2\Delta_1\theta^2 A_1-16\Delta_1^2\theta^4-2\lambda A_1^2)+c_L(4\Delta_1^2\theta^4(\lambda^3-9\lambda^2+26\lambda-16)+2\Delta_1\theta^2(\lambda^3-6\lambda^2+7\lambda+4)-2\lambda A_1^2)-(\lambda A_1^2+8\Delta_1^2\theta^4)c_H}{\Delta_2(4\lambda\Delta_1^2\theta^4-B_1B_2)}$	$\frac{\Delta_3(6\lambda^2\Delta_1^2\theta^2 A_1-\lambda^2 A_1^2-3\lambda^2\Delta_1^2\theta^4)-(\lambda^2 A_1^2+3\lambda^2\Delta_2^2\theta^4)c_L+(4A_1A_2-2\lambda kA_1+2\Delta_1^2\theta^4(43\lambda-18\lambda^2+2\lambda^3-32))c_H}{\Delta_2(4\lambda\Delta_1^2\theta^4-B_1B_2)}$

TABLE III

MARKET DEMAND AND MANUFACTURER'S PROFIT (MODEL D)

Market Demand / Model	q_L	q_H
DN Model	$\frac{2\Delta_3-\Delta_1 c_L+c_H}{\Delta_3\Delta_2}$	$\frac{\lambda\Delta_3+\lambda c_L-\Delta_1 c_H}{\lambda\Delta_3\Delta_2}$
DL Model	$\frac{k\Delta_2(2\Delta_3-\Delta_1 c_L+c_H)}{B_1}$	$\frac{\lambda\Delta_3(k\Delta_2\Delta_3-2\Delta_1\theta^2)+\lambda k\Delta_2\Delta_3 c_L+(2\theta^2(1-\lambda)-k\Delta_2\Delta_3)\Delta_1 c_H}{\lambda\Delta_3 B_1}$
DH Model	$\frac{2\lambda k\Delta_2\Delta_3^2-2\Delta_1\Delta_3\theta^2+(2\theta^2(1-\lambda)-\lambda k\Delta_2\Delta_3)\Delta_1 c_L+\lambda k\Delta_2\Delta_3 c_H}{\Delta_3 B_2}$	$\frac{k\Delta_2(\lambda\Delta_3+\lambda c_L-\Delta_1 c_H)}{B_2}$
DLH Model	$\frac{2A_1(\Delta_1\theta^2-\lambda k\Delta_2\Delta_3)-\lambda k\Delta_2 A_1 c_H+(\lambda\Delta_2 A_1-2A_2(1-\lambda))k\Delta_2 c_L}{4\lambda\Delta_1^2\theta^4-B_1B_2}$	$\frac{\lambda^2(2\Delta_1\theta^2 A_1-k^2\Delta_2^3\Delta_3^2)-\lambda^2 k\Delta_2 A_1 c_L+(\Delta_1 A_1-2A_2(1-\lambda))\lambda k\Delta_2 c_H}{\lambda(4\lambda\Delta_1^2\theta^4-B_1B_2)}$
Manufacturer's Profit / Model	π_L	π_H
DN Model	$\frac{(2\Delta_3-\Delta_1 c_L+c_H)^2}{\Delta_3\Delta_2^2}$	$\frac{(\lambda\Delta_3+\lambda c_L-\Delta_1 c_H)^2}{\lambda\Delta_3\Delta_2^2}$
DL Model	$\frac{(2\Delta_3-\Delta_1 c_L+c_H)^2}{B_1}$	$\frac{(\lambda k\Delta_2\Delta_3^2-\lambda\Delta_1\Delta_3\theta^2+\lambda k\Delta_2\Delta_3 c_L+(2(1-\lambda)\theta^2-k\Delta_2\Delta_3)\Delta_1 c_H)^2}{\lambda\Delta_3\Delta_2 B_1^2}$
DH Model	$\frac{(2\lambda k\Delta_2\Delta_3^2-2\Delta_1\Delta_3\theta^2+(2\theta^2(1-\lambda)-\lambda k\Delta_2\Delta_3)\Delta_1 c_L+\lambda k\Delta_2\Delta_3 c_H)^2}{\Delta_3\Delta_2 B_2^2}$	$\frac{(\lambda\Delta_3-\Delta_1 c_H+\lambda c_L)^2}{B_2}$
DLH Model	$\frac{(2\Delta_3-\Delta_1 c_L+c_H+\Delta_1\theta d_L^{DLH*}-\theta d_H^{DLH*})^2}{\Delta_2^2\Delta_3}-\frac{1}{2}\mathrm{k}(d_L^{DLH*})^2$	$\frac{(\lambda\Delta_3+\lambda c_L-\Delta_1 c_H+\Delta_1\theta d_H^{DLH*}-\lambda\theta d_L^{DLH*})^2}{\lambda\Delta_2^2\Delta_3}-\frac{1}{2}\mathrm{k}(d_H^{DLH*})^2$

B. Analysis on the Results of Model D

Conclusion 1. In Model D, when different manufacturers compete to invest in CSR, by comparing the price of low-carbon products, market demand and the profit results of low-carbon manufacturers, we can get:

(1) $p_L^{DL*} > p_L^{DLH*} > p_L^{DH*}, p_L^{DL*} > p_L^{DN*} > p_L^{DH*}$

(2) $q_L^{DL*} > q_L^{DLH*} > q_L^{DH*}, q_L^{DL*} > q_L^{DN*} > q_L^{DH*}$

(3) $\pi_{LM}^{DL*} > \pi_{LM}^{DLH*} > \pi_{LM}^{DH*}, \pi_{LM}^{DL*} > \pi_{LM}^{DN*} > \pi_{LM}^{DH*}$

Prove: refer to Proof 8 in the appendix.

Conclusion 1 shows that in a market with competitive manufacturers, when low-carbon manufacturers invest in CSR, the price of low-carbon products, market demand and profit of low-carbon manufacturers are the highest, while when manufacturers of ordinary products invest in CSR, the price is the lowest. It can be found in the analysis that the profit of low carbon product manufacturer is not optimal when two product manufacturers competitively invest in CSR. Although the price of low-carbon products increases when only low-carbon product manufacturers invest CSR, the increment of market demand for low-carbon products is larger, so only low-carbon product manufacturers invest CSR is higher than the competitive investment of both.

To some extent, this indicates that manufacturers competitive CSR input will result in a reduced market demand for low-carbon products when both manufacturers have CSR inputs. This is because the increase in demand caused by the CSR input of the original low-carbon manufacturer will be allocated between low-carbon and ordinary products due to the CSR input of both parties. In this case, the market demand decreases and profits decrease. From the perspective of profits of manufacturers of low-carbon products, no matter what the behavior of manufacturers of ordinary products is, low-carbon manufacturers should invest in CSR to maximize their own economic interests and avoid loss of profits.

Conclusion 2. In Model D, when different manufacturers compete to invest in CSR, comparing the price of ordinary products, market demand and the profit results of ordinary product manufacturers, we can get：

(1) $p_H^{DH*} > p_H^{DLH*} > p_H^{DL*}, p_H^{DH*} > p_H^{DN*} > p_H^{DL*}$

(2) $q_H^{DH*} > q_H^{DLH*} > q_H^{DL*}, q_H^{DH*} > q_H^{DN*} > q_H^{DL*}$

(3) $\pi_{HM}^{DH*} > \pi_{HM}^{DLH*} > \pi_{HM}^{DL*}, \pi_{HM}^{DH*} > \pi_{HM}^{DN*} > \pi_{HM}^{DL*}$

Prove: refer to Proof 9 in the appendix.

Conclusion 2 is similar to Conclusion 1. The price, market demand and manufacturer profit of ordinary products are the highest when only ordinary product manufacturers invest in CSR, which is higher than that when two manufacturers invest at the same time. Combined with Conclusion 1, manufacturer CSR investments don't improve manufacturer profits in all modes. Only from the perspective of profit of individual manufacturer, it can be obtained that manufacturing enterprises with profit maximization as the goal should make CSR investment to obtain higher market share and enterprise profits regardless of whether the other manufacturers do or not. Conclusion 1 and Conclusion 2 are different from the study of Liu et al. (2021) [18]. In this paper, when a single manufacturer invests in CSR, its own profit is the highest. When two product manufacturers invest in CSR, the profit will decrease. This paper combines consumer utility function with CSR investment. CSR investment is not only affected by consumers' CSR preferences, but also by consumers ' willingness to pay for the two types of products. Under the influence of consumers' dual preferences, there is a competitive efficiency loss when two single-product manufacturers make CSR investment.

C. The Construction and Solution of Model C

In model C, two product manufacturers in the market cooperate with each other to make CSR inputs as a whole. When the supply chain produces and sells products, it can decide whether to carry out CSR for different products to improve the overall profit. Supply chains first decide the retail price of low-carbon and ordinary products, and then decide the scale of CSR investment according to whether they invest in CSR.

(5)Supply chain has no CSR input for either product (hereinafter referred to as "CN Model")

When there is no CSR input by the supply chain, the utility functions of consumers for ordinary products and low carbon products respectively are:

$$U_L = v - p_L \tag{38}$$

$$U_H = \lambda v - p_H \tag{39}$$

When there are two kinds of products in the market, market demand and supply chain profit are:

$$q_L = 1 - \frac{p_L - p_H}{(1-\lambda)A} \tag{40}$$

$$q_H = \frac{\lambda p_L - p_H}{\lambda(1-\lambda)A} \tag{41}$$

$$\pi_{CS} = (p_L - c_L)q_L + (p_H - c_H)q_H \tag{42}$$

Proposition 5. By solving the above equations, the optimal price of the products p_L^{CN*}, p_H^{CN*}, the market demand q_L^{CN*}, q_H^{CN*} and the profit of supply chain π_{CS}^{CN*} can be obtained, as listed in TABLE IV and TABLE V.

Prove: refer to Proof 4 in the appendix.

(6)Supply chain only invests in CSR for low-carbon products (hereinafter referred to as "CL Model")

In the case of CSR input for low-carbon products alone, the utility functions of consumers for ordinary products and low carbon products respectively are:

$$U_L = v - p_L + \theta d_L \tag{43}$$

$$U_H = \lambda v - p_H \tag{44}$$

Market demand and supply chain profit are:

$$q_L = 1 - \frac{p_L - p_H - \theta d_L}{(1-\lambda)A} \tag{45}$$

$$q_H = \frac{\lambda p_L - p_H - \lambda\theta d_L}{\lambda(1-\lambda)A} \tag{46}$$

$$\pi_{CS} = (p_L - c_L)q_L + (p_H - c_H)q_H - \frac{1}{2}kd_L^2 \tag{47}$$

Proposition 6. By solving the above equations, the optimal investment scale of CSR d_L^{CL*}, the optimal price of the products p_L^{CL*}, p_H^{CL*}, the market demand q_L^{CL*}, q_H^{CL*} and the

profit of supply chain π_{CS}^{CL*} can be obtained, as listed in TABLE IV and TABLE V.
Prove: refer to Proof 5 in the appendix.

(7)Supply chain only invests in CSR for ordinary products (hereinafter referred to as "CH Model")

When the supply chain only invests in CSR for ordinary products, the utility functions of consumers for ordinary products and low carbon products respectively are:

$$U_L = v - p_L \tag{48}$$

$$U_H = \lambda v - p_H + \theta d_H \tag{49}$$

Market demand and supply chain profit are:

$$q_L = 1 - \frac{p_L - p_H + \theta d_H}{(1-\lambda)A} \tag{50}$$

$$q_H = \frac{\lambda p_L - p_H + \theta d_H}{\lambda(1-\lambda)A} \tag{51}$$

$$\pi_{CS} = (p_L - c_L)q_L + (p_H - c_H)q_H - \frac{1}{2}kd_H^2 \tag{52}$$

Proposition 7. By solving the above equations, the optimal scale of CSR input d_H^{CH*}, the optimal price of the products p_L^{CH*}, p_H^{CH*}, the market demand q_L^{CH*}, q_H^{CH*} and the profit of supply chain π_{CS}^{CH*} can be obtained, as listed in TABLE IV and TABLE V.
Prove: refer to Proof 6 in the appendix.

(8)Supply chain invests in CSR for two products at the same time (hereinafter referred to as "CLH Model")

When the supply chain invests in CSR for two products at the same time, the utility functions of consumers for ordinary products and low carbon products respectively are:

$$U_L = v - p_L + \theta d_L \tag{53}$$

$$U_H = \lambda v - p_H + \theta d_H \tag{54}$$

Market demand and supply chain profit are:

$$q_L = 1 - \frac{p_L - p_H + \theta d_H - \theta d_L}{(1-\lambda)A} \tag{55}$$

$$q_H = \frac{\lambda p_L - p_H + \theta d_H - \lambda\theta d_L}{\lambda(1-\lambda)A} \tag{56}$$

$$\pi_{CS} = (p_L - c_L)q_L + (p_H - c_H)q_H - \frac{1}{2}kd_H^2 - \frac{1}{2}kd_L^2 \tag{57}$$

Proposition 8. By solving the above equations, the optimal scale of CSR input d_L^{CLH*}, d_H^{CLH*}, the optimal price of the products p_L^{CLH*}, p_H^{CLH*}, the market demand q_L^{CLH*}, q_H^{CLH*} and the profit of supply chain π_{CS}^{CLH*} can be obtained, as listed in TABLE IV and TABLE V.
Prove: refer to Proof 7 in the appendix.

D. Analysis on the Results of C Model

Conclusion 3. In C Model, manufacturers of low-carbon products and manufacturers of ordinary products cooperate to invest in CSR, comparing the price of low-carbon products and market demand, we can get:

(1) $p_L^{CL*} > p_L^{CLH*} > p_L^{CH*} = p_L^{CN*}$

(2) $q_L^{CL*} > q_L^{CLH*} > q_L^{CH*}, q_L^{CL*} > q_L^{CN*} > q_L^{CH*}$

Prove: refer to Proof 10 in the appendix.

Conclusion 3 is the same as Conclusion 1. In the market where two manufacturers cooperate to invest in CSR, the cooperative and competitive sales model only has an impact on the product price and market demand value under CSR investment, and does not change the size relationship between them. Different from Conclusion 1, product price and CSR input scale are determined by the supply chain as a whole in the cooperative mode, compared with manufacturers competing for CSR input. Therefore, whether the supply chain invests in CSR for ordinary products will not affect the price of low-carbon products, while the supply chain invests in CSR for any kind of products will affect the change of market demand, thus affecting the profits of the supply chain.

Conclusion 4. In Model C, manufacturers of low-carbon products and manufacturers of ordinary products cooperate to invest in CSR, comparing the price of ordinary products, market demand, we can get:

(1) $p_H^{CH*} > p_H^{CLH*} > p_H^{CL*} = p_H^{CN*}$

(2) $q_H^{CH*} > q_H^{CLH*} > q_H^{CL*}, q_H^{CH*} > q_H^{CN*} > q_H^{CL*}$

Prove: refer to Proof 11 in the appendix.

In Conclusion 4, the comparison between the price of common products and market demand is the same as that in Conclusion 3. And the price of ordinary products and market demand are affected by the same factors. Unlike Liu et al. (2021) [18], in reference [18], the demand function of products which is homogeneous product has nothing to do with the consumer utility function. At this time, the market demand of products and enterprise profit are the highest when both retailers invest in CSR. There are many differences between this study and literature [18], so the conclusions are different.

Conclusion 5. In model C, manufacturers of low-carbon products and manufacturers of ordinary products cooperate to invest in CSR, comparing manufacturers' profit, we can get:

(1) $\pi_{CS}^{CLH*} > \pi_{CS}^{CL*} > \pi_{CS}^{CN*}$

(2) $\pi_{CS}^{CLH*} > \pi_{CS}^{CH*} > \pi_{CS}^{CN*}$

Prove: refer to Proof 12 in the appendix.

Conclusion 5 shows that the supply chain profit with CSR input is always better than that without CSR input. And the simultaneous investment of two products is the best choice. Therefore, manufacturers should actively cooperate to jointly fulfill their social responsibilities, establish a good corporate image and improve corporate profits.

E. Profit comparison between Model D and Model C

In order to analyze the impact of competition and cooperation on manufacturers' profits, this section will discuss the profit size of model D and model C under different CSR investment modes of manufacturers. In model D, the total profit function of the supply chain is $\pi_{CS}^D = \pi_{LM}^D + \pi_{HM}^D$ .This section is mainly presented in numerical form for consideration of model setup and other issues. This paper assumes that the upper limit of consumers' willingness to pay $A = 100$, and the CSR input cost coefficient of manufacturers $k = 1$.

Let $2-\lambda=\Delta_1, 4-\lambda=\Delta_2, A(1-\lambda)=\Delta_3, 2Ak(1-\lambda)-\theta^2=B_3, 2\lambda Ak(1-\lambda)-\theta^2=B_4$.

TABLE IV

CSR INVESTMENT SCALE AND PRODUCT PRICE (MODEL C)

CSR Investment Scale / Model	d_L	d_H
CN Model	-	-
CL Model	$\frac{\theta(\Delta_3-c_L+c_H)}{B_3}$	-
CH Model	-	$\frac{\theta(\lambda c_L-c_H)}{2B_4}$
CLH Model	$\frac{\theta B_4\Delta_3+\theta(B_4+\lambda\theta^2)c_L-2\lambda k\theta\Delta_3 c_H}{\lambda\theta^4-B_3B_4}$	$\frac{\lambda\theta^3\Delta_3-2\lambda k\theta\Delta_3 c_L+\theta(B_3+\lambda\theta^2)c_H}{\lambda\theta^4-B_3B_4}$
Product Price / Model	p_L	p_H
CN Model	$\frac{A+c_L}{2}$	$\frac{A\lambda+c_H}{2}$
CL Model	$\frac{AB_3+\theta^2\Delta_3+(B_3-\theta^2)c_L+\theta^2 c_H}{2B_3}$	$\frac{A\lambda+c_H}{2}$
CH Model	$\frac{A+c_L}{2}$	$\frac{A\lambda B_4+(B_4-\theta^2)c_H+\lambda\theta^2 c_L}{2B_4}$
CLH Model	$\frac{A\theta^4-2\theta^2\lambda k\Delta_3^2-AB_3B_4+(2\lambda\theta^4-B_3B_4-\theta^2B_4)c_L-2\lambda k\theta^2\Delta_3 c_H}{2(\lambda\theta^4-B_3B_4)}$	$\frac{A\lambda\theta^4-AB_3B_4+(2\lambda\theta^4-B_3B_4-\theta^2B_3)c_H-2\lambda k\theta^2\Delta_3 c_L}{2(\lambda\theta^4-B_3B_4)}$

TABLE V

MARKET DEMAND AND MANUFACTURER'S PROFIT (MODEL D)

Market Demand / Model	q_L	q_H
CN Model	$\frac{\Delta_3-c_L+c_H}{2\Delta_3}$	$\frac{\lambda c_L-c_H}{2\lambda\Delta_3}$
CL Model	$\frac{k(\Delta_3-c_L+c_H)}{B_3}$	$\frac{-\lambda\theta^2\Delta_3+2\lambda k\Delta_3 c_L-(\lambda\theta^2+B_3)c_H}{2\lambda\Delta_3B_3}$
CH Model	$\frac{B_4\Delta_3+2\lambda k\Delta_3 c_H-(\lambda\theta^2+B_4)c_L}{2\Delta_3B_4}$	$\frac{k(\lambda c_L-c_H)}{B_4}$
CLH Model	$\frac{k\Delta_3B_4+k(B_4+\lambda\theta^2)c_L-2\lambda k^2\Delta_3 c_H}{\lambda\theta^4-B_3B_4}$	$\frac{k\lambda^2\theta^2\Delta_3+\lambda k(B_3+\lambda\theta^2)c_H-2\lambda k^2\Delta_3 c_L}{\lambda\theta^4-B_3B_4}$
Supply Chain Profit / Model	π_{CS}	
CN Model	$\frac{\lambda(A-c_L)(\Delta_3-c_L+c_H)+(A\lambda-c_H)(\lambda c_L-c_H)}{4\lambda\Delta_3}$	
CL Model	$\frac{2\lambda\Delta_3k(\Delta_3-c_L+c_H)(A-c_L)+(A\lambda-c_H)(-\lambda\theta^2\Delta_3+2\lambda k\Delta_3c_L-(\lambda\theta^2+B_3)c_H)}{4\lambda\Delta_3B_3}$	
CH Model	$\frac{(A-c_L)(B_4\Delta_3+2\lambda k\Delta_3c_H-(\lambda\theta^2+B_4)c_L)+2\Delta_3k(\lambda c_L-c_H)(A\lambda+c_H)}{4\Delta_3B_4}$	
CLH Model	$(p_L^{CLH*}-c_L)q_L^{CLH*}+(p_H^{CLH*}-c_H)q_H^{CLH*}-\frac{1}{2}k(d_L^{CLH*})^2-\frac{1}{2}k(d_H^{CLH*})^2$	

(9) Models without CSR input

In the case that manufacturers do not have CSR input, the impacts of low carbon, cost difference of common products and change of consumers' willingness to pay on the profits of the whole supply chain are discussed. Specific values are shown in Table VI.

By assigning values to product cost and consumers' willingness to pay, it can be seen that the increase of product cost will reduce the profit of supply chain. When consumers' willingness to pay for ordinary products increases, it helps to improve the profits of the entire supply chain. And the profit of supply chain in cooperative input mode is always higher than that in competitive input mode.

(10)There are models for CSR inputs

It can be seen from Table VII that the supply chain profit increases with the increase of consumer CSR preference under the cooperative input mode. Under the manufacturer's competitive input CSR mode, the relationship between supply chain profit and consumer CSR preference is affected by the cost of the two products and the difference in consumer willingness to pay. Similar to the non-input CSR mode, the profit of the supply chain under the competitive input mode is always smaller than that of the cooperative input mode, and the cooperative input mode is dominant and not affected by the CSR input object.

TABLE VI
SUPPLY CHAIN PROFIT

	λ	$c_L=50$ $c_H=20$	$c_L=80$ $c_H=20$	$c_L=80$ $c_H=70$
$(\pi_{CS}^{DN*},\pi_{CS}^{CN*})$	0.3	Null	(1.02190, 1.19051)	Null
	0.4	Null	(2.03701, 2.50000)	Null
	0.5	(4.61224, 6.50000)	Null	Null
	0.6	(4.38291, 7.29167)	Null	Null
	0.9	Null	Null	(0.22667, 1.11111)

Combining Table VI, Table VII, Conclusion 5 we find an interesting phenomenon. Conclusion 5 shows that CSR investment is beneficial to improve supply chain profits in the cooperative mode. Table VI and Table VII also confirm this conclusion. When competing for CSR investment, CSR investment may not always promote the increase of supply chain profits. CSR inputs have an impact on supply chain profits.

TABLE VII
SUPPLY CHAIN PROFIT

	θ	$c_L=50$ $c_H=20$ $\lambda=0.6$	$c_L=80$ $c_H=20$ $\lambda=0.3$	$c_L=80$ $c_H=70$ $\lambda=0.9$
$(\pi_{CS}^{DL},\pi_{CS}^{CL})$	0.2	(4.38277, 7.29199)	(1.02201, 1.19058)	(0.22649, 1.11111)
	0.5	(4.38195, 7.29362)	(1.02236, 1.19111)	(0.22582, 1.11111)
	0.8	(4.38041, 7.29670)	(1.02302, 1.19211)	(0.22458, 1.11111)
$(\pi_{CS}^{DH},\pi_{CS}^{CH})$	0.2	(4.38294, 7.29253)	(1.02202, 1.19066)	Null
	0.5	(4.38303, 7.29712)	(1.02244, 1.19161)	Null
	0.8	(4.38323, 7.30574)	(1.02322, 1.19342)	Null
$(\pi_{CS}^{DLH},\pi_{CS}^{CLH})$	0.2	(4.38415, 7.29308)	(1.02208, 1.19076)	Null
	0.5	(4.38202, 7.29906)	(1.02194, 1.19266)	Null
	0.8	(4.38054, 7.32049)	(1.02426, 1.19505)	Null

F. Conclusion

Combining Conclusions 1 and 2, it can be concluded that from the perspective of profits of individual manufacturers, CSR input has the characteristic of “who invests, who benefits”. From the perspective of long-term dynamics, manufacturers will invest CSR in a known or unknown state. According to Table VI and Table VII, the profit of supply chain will benefit from CSR input only in the cooperative mode. However, in the competitive mode, CSR input is not beneficial to the improvement of supply chain profits. For example, when $\lambda=0.6,\theta=0.8,c_L=50$, $c_H=20$, the supply chain profit of only the low-carbon manufacturer or both manufacturers investing in CSR is lower than that of no investment.

Therefore, the cooperative mode not only improves the profit of supply chain, but also reduces the efficiency loss of competition. This will help improve the enthusiasm of manufacturers to undertake CSR investment. Therefore, in the competitive CSR market, manufacturers of low-carbon and ordinary products should actively seek cooperation ways to improve the overall benefits of themselves and the supply chain. Therefore, this paper constructs the revenue sharing and two-way cost sharing contract to coordinate the competitive input mode, so that the supply chain profit under the competitive input mode can reach the cooperative input mode, and realize the Pareto optimum.

Therefore, the cooperative mode not only improves the profit of supply chain, but also reduces the efficiency loss of competition. This will help improve the enthusiasm of manufacturers to undertake CSR investment. Therefore, in the competitive CSR market, manufacturers of low-carbon

and ordinary products should actively seek cooperation ways to improve the overall benefits of themselves and the supply chain. Therefore, this paper constructs the revenue sharing and two-way cost sharing contract to coordinate the competitive input mode, so that the supply chain profit under the competitive input mode can reach the cooperative input mode, and realize the Pareto optimum.

IV. Revenue sharing - a two-way cost sharing contract

In the low-carbon supply chain, manufacturers of low-carbon and common products share the revenue from selling products with each other, and share the CSR input costs for each other, that is, the revenue sharing -- two-way cost sharing contract is used.

A. DN Coordination Model (TN Model)

In the TN model, manufacturers of low-carbon and common products share their sales revenue with each other. Assume that the proportion of retained earnings of each manufacturer is t, and the proportion of profits of other manufacturers is $1-t$. At this point, the profit function of the two manufacturers is:

$$\pi_{LM}^{TN}=t(p_L-c_L)q_L+(1-t)(p_H-c_H)q_H \tag{58}$$

$$\pi_{HM}^{TN}=t(p_L-c_L)q_L+(1-t)(p_H-c_H)q_H \tag{59}$$

$$\pi_{CS}^{TN}=(p_L-c_L)q_L+(p_H-c_H)q_H \tag{60}$$

When $p_L^{TN*}=p_L^{CN*},p_H^{TN*}=p_H^{CN*}$ is established, $\pi_{CS}^{TN*}=\pi_{CS}^{CN*}$, the supply chain is Pareto optimal, and the profit of the supply chain reaches the cooperative mode under the competition model. In order to ensure that the two manufacturers have sufficient incentives to perform the contract, the following individual rational constraints should be met: $\pi_{LM}^{DN*}\le\pi_{LM}^{TN*}$, $\pi_{HM}^{DN*}\le\pi_{HM}^{TN*}$. Thus, the coordination coefficient t should satisfy the condition $\underline{t}^{TN}\le t^{TN*}\le\bar{t}^{TN}$. The size of the coordination coefficient depends on the bargaining power of both parties.

Let $\psi_1=(p_L^{CN*}-c_L)q_L^{CN*},\psi_2=(p_H^{CN*}-c_H)q_H^{CN*}$.

When $\psi_1-\psi_2>0$, $\underline{t}^{TN}=\dfrac{\pi_{LM}^{DN*}-\psi_2}{\psi_1-\psi_2},\bar{t}^{TN}=\dfrac{\psi_1-\pi_{HM}^{DN*}}{\psi_1-\psi_2}$;

when $\psi_1-\psi_2<0$, $\underline{t}^{TN}=\dfrac{\pi_{HM}^{DN*}-\psi_1}{\psi_2-\psi_1},\bar{t}^{TN}=\dfrac{\psi_2-\pi_{LM}^{DN*}}{\psi_2-\psi_1}$.

According to the above results, proposition 9 is established.

Proposition 9. When $\underline{t}^{TN}\le t^{TN*}\le\bar{t}^{TN}$ is established, $p_L^{TN*}=p_L^{CN*},p_H^{TN*}=p_H^{CN*},\pi_{CS}^{TN*}=\pi_{CS}^{CN*}$. The supply chain is optimized, and the profits of low-carbon, ordinary product manufacturers in the competition mode reach the cooperation mode. The value of $\underline{t}^{TN},\bar{t}^{TN}$ depends on ψ_1,ψ_2.

B. DL coordination model (TL Model)

In the TL model, manufacturers of low-carbon products will invest in CSR. Therefore, the proportion of low carbon product manufacturers' own profits and CSR input costs is t, the proportion of ordinary product manufacturers' profits is $1-t$, and the proportion of ordinary product manufacturers is the opposite. At this point, the profits of the two manufacturers are:

$$\pi_{LM}^{TL}=t(p_L-c_L)q_L+(1-t)(p_H-c_H)q_H-\frac{t}{2}kd_L^2 \tag{61}$$

$$\pi_{HM}^{TL}=(1-t)(p_L-c_L)q_L+t(p_H-c_H)q_H-\frac{1-t}{2}kd_L^2 \tag{62}$$

$$\pi_{CS}^{TL}=(p_L-c_L)q_L+(p_H-c_H)q_H-\frac{1}{2}kd_L^2 \tag{63}$$

When $p_L^{TL*}=p_L^{CL*},p_H^{TL*}=p_H^{CL*},d_L^{TL*}=d_L^{CL*}$ is established, $\pi_{CS}^{TL*}=\pi_{CS}^{CL*}$. After coordination, the profit of supply chain can reach the profit under the cooperation mode, and the coordination is effective. When the profits of two manufacturers meet $\pi_{LM}^{DL*}\le\pi_{LM}^{TL*},\pi_{HM}^{DL*}\le\pi_{HM}^{TL*}$, the two manufacturers will comply with the coordination contract, and the supply chain can achieve perfect coordination. At this point, the coordination coefficient t satisfies $\underline{t}^{TL}\le t^{TL*}\le\bar{t}^{TL}$. The specific value of the coordination coefficient is determined by the game power of the two manufacturers.

Let $\psi_3=(p_L^{CL*}-c_L)q_L^{CL*}-\dfrac{1}{2}k(d_L^{CL*})^2,\psi_4=(p_H^{CL*}-c_H)q_H^{CL*}$.

When $\psi_3-\psi_4>0,\underline{t}^{TL}=\dfrac{\pi_{LM}^{DL*}-\psi_4}{\psi_3-\psi_4},\bar{t}^{TL}=\dfrac{\psi_3-\pi_{HM}^{DL*}}{\psi_3-\psi_4}$;

when $\psi_3-\psi_4<0$, $\underline{t}^{TL}=\dfrac{\pi_{HM}^{DL*}-\psi_3}{\psi_4-\psi_3},\bar{t}^{TL}=\dfrac{\psi_4-\pi_{LM}^{DL*}}{\psi_4-\psi_3}$.

According to the above results, proposition 10 is established.

Proposition 10. When $\underline{t}^{TL}\le t^{TL*}\le\bar{t}^{TL}$ is established, $p_L^{TL*}=p_L^{CL*},p_H^{TL*}=p_H^{CL*},d_L^{TL*}=d_L^{CL*},\pi_{CS}^{TL*}=\pi_{CS}^{CL*}$. Revenue sharing and two-way cost sharing contracts can effectively coordinate supply chains, and manufacturers of low-carbon and common products have enough incentives to comply with the contract. The value of $\underline{t}^{TL},\bar{t}^{TL}$ depends on ψ_3,ψ_4.

C. DH Coordination Model (TH Model)

Similarly, in TH model, the two product manufacturers retain their own profits and bear their own costs as t, while the profits and CSR costs of other manufacturers are $1-t$. The profit function of manufacturers of low-carbon and common products is as follows:

$$\pi_{LM}^{TH}=t(p_L-c_L)q_L+(1-t)(p_H-c_H)q_H-\frac{1-t}{2}kd_H^2 \tag{64}$$

$$\pi_{HM}^{TH}=(1-t)(p_L-c_L)q_L+t(p_H-c_H)q_H-\frac{t}{2}kd_H^2 \tag{65}$$

$$\pi_{CS}^{TH}=(p_L-c_L)q_L+(p_H-c_H)q_H-\frac{1}{2}kd_H^2 \tag{66}$$

When $p_L^{TH*}=p_L^{CH*},p_H^{TH*}=p_H^{CH*},d_H^{TH*}=d_H^{CH*}$ is established, $\pi_{CS}^{TH*}=\pi_{CS}^{CH*}$. At this time, the profit of supply chain under the competition mode is improved. As a rational constraint condition, when the manufacturer's profit meets

$\pi_{LM}^{DH*} \le \pi_{LM}^{TH*}, \pi_{HM}^{DH*} \le \pi_{HM}^{TH*}$, the manufacturer is motivated to cooperate. At this point, the coordination coefficient t satisfies the condition that $\underline{t}^{TH} \le t^{TH*} \le \bar{t}^{TH}$. The coordination factor depends on the bargaining power of the two manufacturers.

Let $\psi_5 = (p_L^{CH*} - c_L) q_L^{CH*}, \psi_6 = (p_H^{CH*} - c_H) q_H^{CH*} - \frac{1}{2} k (d_H^{CH*})^2$.

When $\psi_5 - \psi_6 > 0$, $\underline{t}^{TH} = \frac{\pi_{LM}^{DH*} - \psi_6}{\psi_5 - \psi_6}, \bar{t}^{TH} = \frac{\psi_5 - \pi_{HM}^{DH*}}{\psi_5 - \psi_6}$;

when $\psi_5 - \psi_6 < 0$, $\underline{t}^{TH} = \frac{\pi_{HM}^{DH*} - \psi_5}{\psi_6 - \psi_5}, \bar{t}^{TH} = \frac{\psi_6 - \pi_{LM}^{DH*}}{\psi_6 - \psi_5}$.

According to the above results, proposition 11 is established.

Proposition 11. When $\underline{t}^{TH} \le t^{TH*} \le \bar{t}^{TH}$ is established, $p_L^{TH*} = p_L^{CH*}, p_H^{TH*} = p_H^{CH*}, d_H^{TH*} = d_H^{CH*}, \pi_{CS}^{TH*} = \pi_{CS}^{CH*}$. Manufacturers of low-carbon and common products can improve their profits without damaging themselves, and the whole supply chain can reach Pareto optimization. The value of $\underline{t}^{TH}, \bar{t}^{TH}$ depends on ψ_5, ψ_6.

D. DLH coordination model (TLH Model)

Similar to C, under the TLH model, the profit function of manufacturers of low-carbon products and ordinary products is:

$$\pi_{LM}^{TLH} = t(p_L - c_L) q_L + (1-t)(p_H - c_H) q_H - \frac{t}{2} k d_L^2 - \frac{1-t}{2} k d_H^2 \quad (67)$$

$$\pi_{HM}^{TLH} = (1-t)(p_L - c_L) q_L + t(p_H - c_H) q_H - \frac{1-t}{2} k d_L^2 - \frac{t}{2} k d_H^2 \quad (68)$$

$$\pi_{CS}^{TLH} = (p_L - c_L) q_L + (p_H - c_H) q_H - \frac{1}{2} k d_L^2 - \frac{1}{2} k d_H^2 \quad (69)$$

When $p_L^{TLH*} = p_L^{CLH*}, p_H^{TLH*} = p_H^{CLH*}, d_L^{TLH*} = d_L^{CLH*}$, $d_H^{TLH*} = d_H^{CLH*}$ is established, $\pi_{CS}^{TLH*} = \pi_{CS}^{CLH*}$. At this point, the supply chain profit under TLH model reaches that under CLH model, and the supply chain realizes Pareto improvement. When the profits of two manufacturers meet the condition $\pi_{LM}^{DLH*} \le \pi_{LM}^{TLH*}, \pi_{HM}^{DLH*} \le \pi_{HM}^{TLH*}$, the two manufacturers can be promoted to comply with the coordination contract. At this point, the size of the coordination coefficient t is $\underline{t}^{TLH} \le t^{TLH*} \le \bar{t}^{TLH}$, and the specific value of the coefficient is determined by the negotiation ability of the two manufacturers.

Let

$\psi_7 = (p_L^{CLH*} - c_L) q_L^{CLH*} - \frac{1}{2} k (d_L^{CLH*})^2, \psi_8 = (p_H^{CLH*} - c_H) q_H^{CLH*} - \frac{1}{2} k (d_H^{CLH*})^2$.

When $\psi_7 - \psi_8 > 0$, $\underline{t}^{TLH} = \frac{\pi_{LM}^{DLH*} - \psi_8}{\psi_7 - \psi_8}$, $\bar{t}^{TLH} = \frac{\psi_7 - \pi_{HM}^{DLH*}}{\psi_7 - \psi_8}$;

when $\psi_7 - \psi_8 < 0$, $\underline{t}^{TLH} = \frac{\pi_{HM}^{DLH*} - \psi_7}{\psi_8 - \psi_7}$, $\bar{t}^{TLH} = \frac{\psi_8 - \pi_{LM}^{DLH*}}{\psi_8 - \psi_7}$.

According to the above results, proposition 12 is established.

Proposition 12. When $\underline{t}^{TLH} \le t^{TLH*} \le \bar{t}^{TLH}$ is established, $p_L^{TLH*} = p_L^{CLH*}, p_H^{TLH*} = p_H^{CLH*}, d_L^{TLH*} = d_L^{CLH*}, d_H^{TLH*} = d_H^{CLH*}$, $\pi_{CS}^{TLH*} = \pi_{CS}^{CLH*}$ Revenue sharing - two-way cost sharing contract coordination is effective. After coordination, supply chain profits reach the cooperative input mode, and manufacturers of low-carbon and common products also have the motivation to actively seek cooperation. The value of $\underline{t}^{TLH}, \bar{t}^{TLH}$ depends on ψ_7, ψ_8.

V. NUMERICAL EXAMPLE

A. Numerical example analysis on Model D

Taking low-carbon products in Model D as an example, this paper further validates Proposition 1. Assuming $\lambda = 0.6, A = 1, c_L = 0.6, c_H = 0.3, \mathrm{k} = 1$, it analyzes the influence of the change of sensitivity coefficient of consumers to manufacturers' social responsibility investment level θ on the price of low-carbon products, market demand and profit of low-carbon manufacturers. In order to ensure the value range of the aforementioned variables, the discussing range of the following values is only limited to $0 < \theta \le 0.5$.

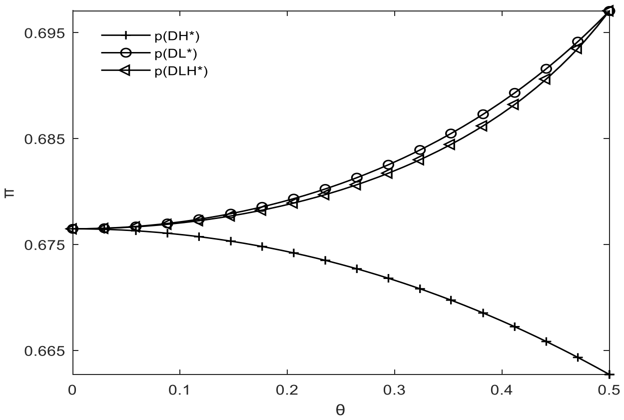

Fig. 1 Price of low-carbon products

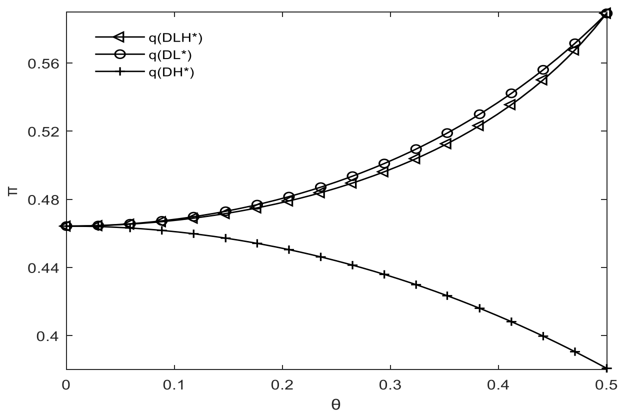

Fig. 2 Market demand of low-carbon products

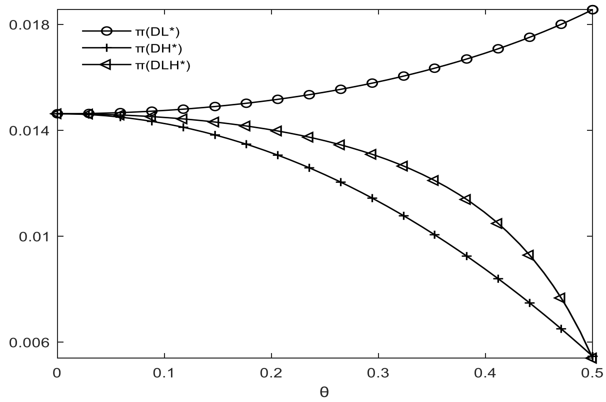

Fig. 3' Profit of low-carbon products manufactures

According to Figure 1, Figure 2 and Figure 3, same with conclusion 1, the price of low-carbon products, market demand and profit of low-carbon manufacturers are the highest when low-carbon product manufacturers invest in CSR. When the product cost and consumers' willingness to pay are constant, in the competitive input mode, it is most beneficial for consumers when manufacturers of low-carbon and ordinary products invest in CSR at the same time. At this point, the price of low-carbon products is the lowest, and the price gradually decreases with the increase of consumers' CSR preference.

VI. Conclusion

Based on the consumer utility function, this paper constructs the market demand function model of low-carbon products and ordinary products, and discusses the impact of competition and cooperative CSR on supply chain. According to the analysis results, CSR investment and improvement of corporate profits are affected by product cost, cooperation and competition, etc., and not all cases can promote the increase of corporate profits. Only under certain conditions, CSR investment is conducive to expanding market demand and improving corporate profits. This is the same as the conclusion of Henri et al. (2013) [2]. When investing in CSR, manufacturers should try to seek cooperation to reduce or avoid competitive efficiency loss and improve the operation of supply chain.

This paper mainly considers the Nash game between two manufacturers and analyzes the influence of consumers' preference for product difference and CSR input scale on enterprises and supply chain. However, in practice, the after-sales service of purchased products, the distance and whether the products can be delivered to the door are all the factors that affect consumers' purchase. Therefore, further research can be done in the future in terms of game types and the factors influencing consumers' purchase.

Appendix

Appendix 1. In Model D, when only low-carbon manufacturers invest in CSR, calculating the partial derivative of the profit function of two manufacturers p_L, p_H, it is obtained:

Let $2-\lambda=\Delta_1, 4-\lambda=\Delta_2, A(1-\lambda)=\Delta_3$.

$$\begin{cases} \dfrac{\partial \pi_{LM}}{\partial p_L} = \Delta_3 - 2p_L + p_H + c_L + \theta d_L = 0 \\ \dfrac{\partial \pi_{HM}}{\partial p_H} = \lambda p_L - 2p_H + c_H - \lambda\theta d_L = 0 \end{cases}$$

Get the expression of p_L, p_H in the form of d_L, then substitute p_L, p_H into π_{LM}, calculate derivative to d_L and obtain

$$d_L^{DL*} = \frac{2\Delta_1\theta(2\Delta_3 - \Delta_1 c_L + c_H)}{k\Delta_2^2\Delta_3 - 2\theta^2\Delta_1^2}$$

In order to maximize the profit of manufacturers, $k\Delta_2^2\Delta_3 - 2(\Delta_1\theta)^2 > 0$.

Substitute d_L^{DL*} into p_L, p_H, get p_L^{DL*}, p_H^{DL*}, then substitute them into $q_L, q_H, \pi_{LM}, \pi_{HM}$.

Appendix 2. In Model D, when only ordinary products manufacturers invest in CSR, the solving process is similar to that in Appendix 1.

Appendix 3. In Model D, when both manufacturers invest in CSR, calculating the partial derivative of the profit function of two manufacturers p_L, p_H, it is obtained:

$$\begin{cases} \dfrac{\partial \pi_{LM}}{\partial p_L} = \Delta_3 - 2p_L + p_H + c_L + \theta d_L - \theta d_H = 0 \\ \dfrac{\partial \pi_{HM}}{\partial p_H} = \lambda p_L - 2p_H + c_H + \theta d_H - \lambda\theta d_L = 0 \end{cases}$$

Get the expression of p_L, p_H in the form of d_L, d_H, substitute p_L, p_H into π_{LM}, π_{HM}, calculate derivative to d_L, d_H, and obtain d_L^{DLH*}, d_H^{DLH*}.

When inequalities $2\Delta_1^2\theta^2 - k\Delta_2^2\Delta_3 < 0, 2\Delta_1^2\theta^2 - \lambda k\Delta_2^2\Delta_3 < 0, 4\lambda\Delta_1^2\theta^4 - (2\Delta_1^2\theta^2 - k\Delta_2^2\Delta_3)(2\Delta_1^2\theta^2 - \lambda k\Delta_2^2\Delta_3) < 0$, the manufacturers' profit can be maximized, and the optimal solution exists. At his time, the cost of low-carbon products satisfies the following conditions:

$$c_L < \frac{2(\lambda k\Delta_2\Delta_3 - \Delta_1\theta^2)\Delta_2\Delta_3}{(\lambda k\Delta_2\Delta_3\Delta_1 - 2\Delta_1\theta^2(1-\lambda))\Delta_2} + \frac{\lambda k\Delta_2\Delta_3 c_H}{\lambda k\Delta_2\Delta_3\Delta_1 - 2\Delta_1\theta^2(1-\lambda)}$$

$$c_L > \frac{2\lambda\Delta_1\Delta_2\Delta_3\theta^2 - \lambda k\Delta_2^2\Delta_3^2}{\lambda k\Delta_2^2\Delta_3} + \frac{(k\Delta_2^2\Delta_3 - 2\Delta_2\theta^2(1-\lambda))\Delta_1 c_H}{\lambda k\Delta_2^2\Delta_3}$$

Substitute d_L^{DLH*}, d_H^{DLH*} into $p_L, p_H, q_L, q_H, \pi_{LM}, \pi_{HM}$.

Appendix 4. In Model C, when the supply chain has no CSR input for two kinds of products, by calculating partial derivative of the profit function of the supply chain p_L, p_H, it is obtained:

$$\begin{cases}\dfrac{\partial \pi_{CS}}{\partial p_L} = \Delta_3 - 2p_L + 2p_H + c_L - c_H = 0 \\ \dfrac{\partial \pi_{CS}}{\partial p_H} = 2\lambda p_L - 2p_H + c_H - \lambda c_L = 0\end{cases}$$

Simultaneously to solve the equations, it is obtained p_L^{CN*}, p_H^{CN*}, then substitute it into q_L, q_H, π_{CS}.

Appendix 5. In Model C, when the supply chain only invests in CSR for low-carbon products, by calculating partial derivative of the profit function of the supply chain p_L, p_H, it is obtained:

$$\begin{cases}\dfrac{\partial \pi_{CS}}{\partial p_L} = \Delta_3 - 2p_L + 2p_H + c_L - c_H + \theta d_L = 0 \\ \dfrac{\partial \pi_{CS}}{\partial p_H} = 2\lambda p_L - 2p_H + c_H - \lambda c_L - \lambda\theta d_L = 0\end{cases}$$

Simultaneously to solve the equations, Substitute p_L, p_H into π_{CS}, and calculate derivative to d_L. When $2k\Delta_3 - \theta^2 > 0$, the supply chain profit is maximized, hence obtain:

$$d_L^{CL*} = \frac{\theta(\Delta_3 - c_L + c_H)}{2k\Delta_3 - \theta^2}$$

Substitute d_L^{CL*} into p_L, p_H, π_{CS}.

Appendix 6. In Model C, when the supply chain only invests in CSR for ordinary products, the solving process is similar to that in Appendix 5.

Appendix 7. In Model C, when the supply chain invests in CSR for two kinds of products, by calculating partial derivative of the profit function of the supply chain p_L, p_H, it is obtained:

$$\begin{cases}\dfrac{\partial \pi_{CS}}{\partial p_L} = \Delta_3 - 2(p_L - p_H) + c_L - c_H + \theta(d_L - d_H) = 0 \\ \dfrac{\partial \pi_{CS}}{\partial p_H} = 2(\lambda p_L - p_H) + c_H - \lambda c_L + \theta(d_H - \lambda d_L) = 0\end{cases}$$

Simultaneously to solve the equations, Substitute p_L, p_H into π_{CS}, and calculate derivative to d_L, d_H,

when $\lambda\theta^4 - (\theta^2 - 2k\Delta_3)(\theta^2 - 2\lambda k\Delta_3) < 0, \theta^2 - 2k\Delta_3 < 0,$ $\theta^2 - 2\lambda k\Delta_3 < 0$, Hessian matrix is negatively determined, and it is obtained:

$$d_L^{CLH*} = \frac{\theta\Delta_3(\theta^2 - 2\lambda k\Delta_3) - 2\theta\lambda k\Delta_3 c_H}{\lambda\theta^4 - (\theta^2 - 2k\Delta_3)(\theta^2 - 2\lambda k\Delta_3)} + \frac{\theta(2\lambda k\Delta_3 - (1-\lambda)\theta^2)c_L}{\lambda\theta^4 - (\theta^2 - 2k\Delta_3)(\theta^2 - 2\lambda k\Delta_3)}$$

$$d_H^{CLH*} = \theta\frac{\lambda\theta^2\Delta_3 - 2\lambda k\Delta_3 c_L}{\lambda\theta^4 - (\theta^2 - 2k\Delta_3)(\theta^2 - 2\lambda k\Delta_3)} + \theta\frac{(2k\Delta_3 - (1-\lambda)\theta^2)c_H}{\lambda\theta^4 - (\theta^2 - 2k\Delta_3)(\theta^2 - 2\lambda k\Delta_3)}$$

In order to ensure $d_L^{CLH*} > 0, d_H^{CLH*} > 0$ the cost range of low-carbon products is obtained as follows:

$$c_L < \frac{(2\lambda k\Delta_3 - \theta^2)\Delta_3}{2\lambda k\Delta_3 - \theta^2 + \lambda\theta^2} + \frac{2\lambda k\Delta_3}{2\lambda k\Delta_3 - \theta^2 + \lambda\theta^2}c_H$$

$$c_L > \frac{\theta^2}{2k} + \frac{2\lambda k\Delta_3 - \theta^2 + \lambda\theta^2}{2\lambda k\Delta_3}c_H$$

Substitute d_L^{CLH*}, d_H^{CLH*} into p_L, p_H, π_{CS}.

Appendix 8. Conclusion 1 is about the price of low-carbon products, market demand and profit of low-carbon manufacturers in Model D.

(1) Price of low-carbon products

Due to $d_L^{DL*} > 0, d_H^{DH*} > 0$,

$$p_L^{DL*} - p_L^{DH*} = \theta\frac{\Delta_1 d_L^{DL*} + d_H^{DH*}}{2\Delta_2} > 0$$

$$p_L^{DLH*} - p_L^{DL*} = \theta\frac{\Delta_1 d_L^{DLH*} - d_H^{DLH*} - \Delta_1 d_L^{DL*}}{2\Delta_2}$$

$$\Delta_1 d_L^{DLH*} - d_H^{DLH*} - \Delta_1 d_L^{DL*} = \frac{2\Delta_1\theta k\Delta_2^3\Delta_3}{(k\Delta_2^2\Delta_3 - 2\theta^2\Delta_1{}^2)} \bullet [\frac{\lambda\Delta_3(k\Delta_2(\Delta_3 + c_L) - 2\Delta_1\theta^2)}{(4\lambda\Delta_1^2\theta^4 - (2\Delta_1^2\theta^2 - k\Delta_2^2\Delta_3)(2\Delta_1^2\theta^2 - \lambda k\Delta_2^2\Delta_3))} + \frac{\Delta_1(2\theta^2(1-\lambda) - k\Delta_2\Delta_3)c_H}{(4\lambda\Delta_1^2\theta^4 - (2\Delta_1^2\theta^2 - k\Delta_2^2\Delta_3)(2\Delta_1^2\theta^2 - \lambda k\Delta_2^2\Delta_3))}]$$

Combing the cost range of low-carbon products given in the certification of Appendix 3, it is obtained:

$$\lambda\Delta_2\Delta_3(k\Delta_2\Delta_3 - 2\Delta_1\theta^2) + \lambda k\Delta_2^2\Delta_3 c_L + \Delta_1\Delta_2(2\theta^2(1-\lambda) - k\Delta_2\Delta_3)c_H > 0$$

$$p_L^{DLH*} - p_L^{DL*} < 0$$

$$p_L^{DLH*} - p_L^{DH*} = \theta\frac{\Delta_1 d_L^{DLH*} - d_H^{DLH*} + d_H^{DH*}}{2\Delta_2} > 0$$

$$\Delta_1 d_L^{DLH*} - d_H^{DLH*} + d_H^{DH*} = \frac{2\Delta_1^2\theta(\lambda k\Delta_2^2\Delta_3 - 2\Delta_1^2\theta^2 + 2\lambda\theta^2)}{(\lambda k\Delta_2^2\Delta_3 - 2(\Delta_1\theta)^2)} \bullet [\frac{2\Delta_2\Delta_3(\Delta_1\theta^2 - \lambda k\Delta_2\Delta_3) - \lambda k\Delta_2^2\Delta_3 c_H}{(4\lambda\Delta_1^2\theta^4 - (2\Delta_1^2\theta^2 - k\Delta_2^2\Delta_3)(2\Delta_1^2\theta^2 - \lambda k\Delta_2^2\Delta_3))} + \frac{(2\lambda\theta^2 + \lambda k\Delta_2^2\Delta_3 - 2\Delta_1^2\theta^2)\Delta_1 c_L}{(4\lambda\Delta_1^2\theta^4 - (2\Delta_1^2\theta^2 - k\Delta_2^2\Delta_3)(2\Delta_1^2\theta^2 - \lambda k\Delta_2^2\Delta_3))}] > 0$$

Then get $p_L^{DL*} > p_L^{DLH*} > p_L^{DH*}$.

$$p_L^{DH*} - p_L^{DN*} < 0, p_L^{DL*} - p_L^{DN*} > 0$$

$$p_L^{DL*} - p_L^{DH*} = \theta\frac{\Delta_1 d_L^{DL*} + d_H^{DH*}}{2\Delta_2} > 0$$

Then get $p_L^{DL*} > p_L^{DN*} > p_L^{DH*}$.

(2) Market demand of low-carbon products

Combing the certification in (1), it is obtained:

$$q_L^{DL*} - q_L^{DH*} = \frac{\Delta_1\theta d_L^{DL*} + \theta d_H^{DH*}}{\Delta_3\Delta_2} > 0$$

$$q_L^{DLH*} - q_L^{DL*} = \theta\frac{\Delta_1 d_L^{DLH*} - d_H^{DLH*} - \Delta_1 d_L^{DL*}}{\Delta_3\Delta_2}$$

$$\Delta_1 d_L^{DLH*} - d_H^{DLH*} - \Delta_1 d_L^{DL*} < 0, q_L^{DLH*} - q_L^{DL*} < 0$$

$$q_L^{DLH*} - q_L^{DH*} = \theta\frac{\Delta_1 d_L^{DLH*} - d_H^{DLH*} + d_H^{DH*}}{\Delta_3\Delta_2} > 0$$

Then get $q_L^{DL*} > q_L^{DLH*} > q_L^{DH*}$.

$$q_L^{DH*} < q_L^{DN*}$$

$$q_L^{DL*} - q_L^{DH*} = \frac{\Delta_1 \theta d_L^{DL*} + \theta d_H^{DH*}}{\Delta_3 \Delta_2} > 0$$

Then get $q_L^{DL*} > q_L^{DN*} > q_L^{DH*}$.

(3) Profit of low-carbon product manufacturers

Using the conclusions (1) and (2) that have been proved, and the conditions that variable exists and is positive, we can get:

$$\pi_{LM}^{DH*} - \pi_{LM}^{DN*} = \frac{(2\Delta_3 - \Delta_1 c_L + c_H - \theta d_H^{DH*})^2}{\Delta_3 \Delta_2^2} - \frac{(2\Delta_3 - \Delta_1 c_L + c_H)^2}{\Delta_3 \Delta_2^2} < 0$$

$$\pi_{LM}^{DL*} - \pi_{LM}^{DN*} = \frac{(2\Delta_3 - \Delta_1 c_L + c_H + \Delta_1 \theta d_L^{DL*})^2}{\Delta_3 \Delta_2^2} - \frac{1}{2}\mathrm{k}(d_L^{DL*})^2 - \frac{(2\Delta_3 - \Delta_1 c_L + c_H)^2}{\Delta_3 \Delta_2^2} > 0$$

$$\pi_{LM}^{DLH*} - \pi_{LM}^{DL*} = \frac{(2\Delta_3 - \Delta_1 c_L + c_H + \Delta_1 \theta d_L^{DLH*} - \theta d_H^{DLH*})^2}{\Delta_3 \Delta_2^2} - \frac{(2\Delta_3 - \Delta_1 c_L + c_H + \Delta_1 \theta d_L^{DL*})^2}{\Delta_3 \Delta_2^2} + \frac{1}{2}\mathrm{k}(d_L^{DL*})^2 - \frac{1}{2}\mathrm{k}(d_L^{DLH*})^2 < 0$$

Therefore,

$$\pi_{LM}^{DL*} > \pi_{LM}^{DLH*} > \pi_{LM}^{DH*},$$
$$\pi_{LM}^{DL*} > \pi_{LM}^{DN*} > \pi_{LM}^{DH*}.$$

Appendix 9. Conclusion 2 is about the price of ordinary products, market demand and profit of ordinary manufacturers in Model D.

The proving process is similar to Appendix 8, so we do not repeat it here.

Appendix 10. Conclusion 3 is about the price of low-carbon products and market demand in Model C.

(1) Price of low-carbon products

$$p_L^{CH*} = p_L^{CN*} = \frac{A + c_L}{2}$$

$$p_L^{CLH*} - p_L^{CL*} = \frac{\theta(d_L^{CLH*} - d_L^{CL*})}{2}$$

$$p_L^{CL*} - p_L^{CH*} > 0$$

$$d_L^{CLH*} - d_L^{CL*} = \theta^4 \frac{-\lambda\theta^2\Delta_3 + 2\lambda k \Delta_3 c_L}{(2k\Delta_3 - \theta^2)(\lambda\theta^4 - B_3 B_4)} + \frac{\theta^4((1-\lambda)\theta^2 - 2k\Delta_3)c_H}{(2k\Delta_3 - \theta^2)(\lambda\theta^4 - B_3 B_4)}$$

Using the cost range of low-carbon products given by Appendix 7 and the negative condition of Hessian matrix, it is obtained:

$$2\lambda k \Delta_3 \theta^3 c_L + ((1-\lambda)\theta^2 - 2k\Delta_3)\theta^3 c_H - \lambda\theta^5\Delta_3 > 0$$

$$p_L^{CLH*} - p_L^{CL*} < 0$$

Then get $p_L^{CL*} > p_L^{CLH*} > p_L^{CH*} = p_L^{CN*}$.

(2) Market demand of low-carbon products

Due to $d_L^{CL*} > 0, d_H^{CH*} > 0$, it is obtained:

$$q_L^{CL*} - q_L^{CH*} = \frac{\theta(d_L^{CL*} + d_H^{CH*})}{2\Delta_3} > 0$$

$$q_L^{CLH*} - q_L^{CL*} = \frac{\theta(d_L^{CLH*} - d_H^{CLH*} - d_L^{CL*})}{2\Delta_3}$$

Using the cost range of low-carbon products given by Appendix 7 and the negative condition of Hessian matrix, it is obtained:

$$d_L^{CLH*} - d_H^{CLH*} - d_L^{CL*} = 2\theta k \Delta_3 \frac{-\lambda\theta^2\Delta_3 + 2k\lambda\Delta_3 c_L}{(\lambda\theta^4 - B_3 B_4)(2\lambda k \Delta_3 - \theta^2)} + 2\theta k \Delta_3 \frac{((1-\lambda)\theta^2 - 2k\Delta_3)c_H}{(\lambda\theta^4 - B_3 B_4)(2\lambda k \Delta_3 - \theta^2)}$$

$$4\lambda k^2 \Delta_3^2 c_L + 2((1-\lambda)\theta^2 - 2k\Delta_3)\Delta_3 c_H - 2\lambda k \theta^2 \Delta_3^2 > 0$$

$$q_L^{CLH*} - q_L^{CL*} < 0$$

$$q_L^{CLH*} - q_L^{CH*} = \frac{\theta(d_L^{CLH*} - d_H^{CLH*} + d_H^{CH*})}{2\Delta_3}$$

$$d_L^{CLH*} - d_H^{CLH*} + d_H^{CH*} = \frac{\theta(2\lambda k\Delta_3 - \theta^2 + \lambda\theta^2)}{(2\lambda k \Delta_3 - \theta^2)} \bullet \left[\frac{\theta^2\Delta_3 - 2\lambda k \Delta_3(\Delta_3 + c_H)}{(\lambda\theta^4 - B_3 B_4)} + \frac{(2\lambda k\Delta_3 - \theta^2(1-\lambda))c_L}{(\lambda\theta^4 - B_3 B_4)}\right]$$

$$(\theta^2 - 2\lambda k \Delta_3)\Delta_3 + (2\lambda k \Delta_3 - \theta^2 + \lambda\theta^2)c_L - 2\lambda k \Delta_3 c_H < 0$$

$$q_L^{CLH*} - q_L^{CH*} > 0$$

Then get: $q_H^{CH*} > q_H^{CLH*} > q_H^{CL*}$.

Due to $d_L^{CL*} > 0, d_H^{CH*} > 0$, it is obtained:

$$q_H^{CH*} > q_H^{CN*}, q_H^{CN*} > q_H^{CL*}$$

Therefore, $q_H^{CH*} > q_H^{CN*} > q_H^{CL*}$.

Appendix 11. Conclusion 4 is about the price of ordinary products and market demand in C Model.

The proving process is similar to Appendix 10, thus we do not need to repeat it.

Appendix 12. Conclusion 5 is about the profit of manufacturers in Model C.

Combing the cost range of low-carbon products given by Appendix 7 and the negative condition of Hessian matrix, and comparing the magnitude of the following equations, it is obtained:

$$\pi_{CS}^{CLH*} > \pi_{CS}^{CL*} > \pi_{CS}^{CN*}, \pi_{CS}^{CLH*} > \pi_{CS}^{CH*} > \pi_{CS}^{CN*}.$$

$$\pi_{CS}^{CL*} - \pi_{CS}^{CN*} = d_L \frac{\theta(\Delta_3 - c_L + c_H) + (\theta^2 - k\Delta_3)d_L}{2\Delta_3} = \frac{\theta^2 k(\Delta_3 - c_L + c_H)^2}{2(2k\Delta_3 - \theta^2)^2} > 0$$

Similarly, obtain:

$$\pi_{CS}^{CH*} - \pi_{CS}^{CN*} > 0, \quad \pi_{CS}^{CLH*} - \pi_{CS}^{CN*} > 0$$

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