Exponential Stability Analysis and Observer-based Control of T-S Fuzzy Networked Systems with Time Delay and Communication Delay

L.F. Zhang, H.J. Yao

Abstract—**The exponentially stable analysis and observer-based dynamic output feedback control of T-S fuzzy networked systems with state delay and communication delay are studied in this paper. The mathematical model of networked systems with state delay and communication delay is established by T-S fuzzy method. Then, the fuzzy observer of the systems is designed by using the systems output. With Lyapunov stability theorem and linear matrix inequality method, the sufficient conditions for the exponential stability of networked systems and error systems are obtained. On this basis, the design strategy of dynamic output feedback control of networked systems is given. The simulation and experimental results show that the proposed control strategy is effective and feasible.**

Index Terms—**Networked systems, fuzzy control, observer, exponential stable, delay**

I. Introduction

Networked control systems refer to the control systems in which the components of the control loop exchange information through the communication network [1–2]. Commands and feedback of the control systems are transmitted in the form of packets in the network. The important feature of networked control systems is that it connects cyberspace and physical space, so it can perform many tasks over a long distance. Moreover, the information of networked control systems is transmitted through the network, which saves unnecessary wiring, reduces the complexity of the systems, and reduces the cost of designing and erecting the systems [3–5]. The important feature of networked control systems is that each controller can efficiently share information, integrate information in large physical space and make decisions. Many advantages and wide applications of networked control systems have attracted the attention and research of many scholars [6–9].

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In recent years, the research on nonlinear networked control systems has gradually attracted extensive attention. As we all know, T-S fuzzy control is an effective method to deal with nonlinear systems. The work of introducing T-S fuzzy control method into nonlinear networked systems is gradually carried out, and fruitful research results have emerged [10–13]. Zare et al. proposed a new control design method for a class of nonlinear systems represented by T-S fuzzy model by using augmented Lyapunov-Krasovskii functional, linear matrix inequality approach and parallel distributed compensation method [14]. The dissipation analysis of T-S fuzzy networked control systems with voluntary defense strategy was studied in [15]. A new delay product relaxation condition was proposed, which fully excavates the time-varying delay information under the given conditions. Using reciprocal convex matrix inequality, true integral inequality and linear convex combination method, a new criterion and corresponding control algorithm were given. The finite-time synchronization problem of sampling control for T-S fuzzy complex dynamic networks with coupling delays was studied in [16]. Based on Lyapunov stability theory, a suitable Lyapunov functional was constructed by using Kronecker product, and a new sufficient condition was established by using the average residence time method. The problem of fault detection for discrete time-delay fuzzy networked control systems with quantization and packet loss was studied in [17]. The input-output method and two term approximation method are used to transform the discrete fuzzy networked systems into the form of interconnection of two subsystems, and two term approximation method is used to approximate the time-varying delay. By eliminating the coupling between Lyapunov matrix and system matrix, the sufficient conditions for the stability of the systems are obtained.

However, the above research results only study the state feedback controller design of nonlinear networked systems, which requires that the systems states are measurable. In reality, the systems states are usually unknown or partially known, which makes the state feedback controller design suffers from strong resistance. In order to overcome the unobservable problem of the systems states, it is necessary to design the state observer of the systems and use the observed states for control design. Therefore, the observer-based fuzzy control design technology of networked control systems urgently needs to be studied [18–22]. Li et al. investigated the defense control problem of T-S fuzzy systems with multiple transmission channels against asynchronous denial of service attacks[23]. A new switching security control method was proposed to tolerate asynchronous denial of service attacks acting independently on each channel. By using the piecewise Lyapunov-Krasovskii functional method, the sufficient conditions for the exponential stability of the newly constructed switching systems were obtained. The problem of observer-based robust fuzzy control for actuator saturated nonlinear networked systems was studied in [24]. T-S fuzzy method was used to establish a fuzzy observer. The saturation fuzzy control law was designed according to the estimated states of the observer. By constructing a new Lyapunov functional, the stability of the systems was analyzed, and the design strategy of the controller was given. Chen et al. studied the observer-based controller design problem for a class of discrete-time nonlinear networked systems[25]. Considering the influence of data packet loss on systems performance in the process of data transmission, the mean square exponential stability condition and H_∞ controller design method are obtained by using Lyapunov functional method.

Although the observer-based feedback control of networked systems has been designed in the above research results, and the asymptotic stability of the systems has been analyzed, the exponential stability of networked systems, especially the exponential stability of networked systems with time delay is not considered. Motivated on the above analysis, the dynamic output feedback fuzzy controller of the nonlinear networked systems is designed to make the systems states and error systems states exponentially stable.

II. Problem Formulation

Consider the following typical networked systems with time delay shown in Figure 1

Rule i:

IF $z_1(t)$ is M_1^i and $z_2(t)$ is M_2^i ,…, and $z_n(t)$ is M_n^i,

THEN

$$\begin{aligned} &\dot{x}(t) = A_i x(t) + A_{di} x(t-d) + B_i u(t), \\ &y(t) = C_i x(t) \qquad i = 1,\ 2,\ \cdots,\ q \qquad (1) \\ &x(t) = \phi(t) \qquad t \in [-d, 0], \end{aligned}$$

where $z(t) = [z_1(t) \ \ z_2(t) \ \ \cdots \ \ z_n(t)]^T$ is the premise variable, $x(t) \in R^n$ is the systems state, q is the number of IF-THEN rule, $M_k^i (i = 1,\ 2,\ \cdots,\ q;\ \ k = 1,\ 2,\ \cdots,\ n)$ are fuzzy sets, $u(t) \in R^m$ is the control input, $y(t) \in R^l$ is the systems output, $A_i, A_{di} \in R^{n\times n}$ are constant matrices, $B_i \in R^{n\times m}$ are input matrices, $C_i \in R^{l\times n}$ are output matrices, d is time delay, $\phi(t) = [\phi_1(t) \ \ \phi_2(t) \ \ \cdots \ \ \phi_n(t)]^T \in R^n$ is the initial condition of the state.

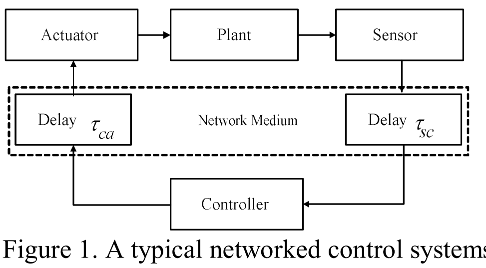

Figure 1. A typical networked control systems

In Figure 1, τ_{sc} and τ_{ca} are the sensor-controller and the controller-actuator delay respectively. The communication delay is given by $\tau = \tau_{sc} + \tau_{ca}$. Using single point fuzzification, product inference engine and central fuzzy elimination method, the global fuzzy model of the systems (1) can be described as

$$\begin{aligned} &\dot{x}(t) = \sum_{i=1}^{q} \mu_i(z(t))[A_i x(t) + A_{di} x(t-d) + B_i u(t-\tau)], \\ &y(t) = \sum_{i=1}^{q} \mu_i(z(t)) C_i x(t), \qquad (2) \\ &x(t) = \phi(t), \qquad t \in [-\bar{d},\ \ 0], \end{aligned}$$

where

$$\bar{d} = \max\{d, \tau\},$$

$$\omega_i(z(t)) = \prod_{k=1}^{n} M_k^i(z_k(t)),$$

$$\mu_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{q} \omega_i(z(t))},$$

where $M_k^i(z_k(t))$ is the membership degree of $z_k(t)$ corresponding to M_k^i, $\omega_i(z(t))$ satisfying

$$\omega_i(z(t)) \geq 0,$$

$$\sum_{i=1}^{q} \omega_i(z(t)) > 0, \quad i = 1,\ 2,\ \cdots,\ q,$$

And we definite

$$\|\phi\|_d = \max_{\substack{t\in[-\bar{d},0] \\ i=1,2,\cdots,n}} \|\phi_i(t)\|.$$

The following fuzzy observer based on T-S model of the systems (2) will be designed as

$$\begin{aligned} &\dot{\hat{x}}(t) = \sum_{i=1}^{q} \mu_i(z(t))\{A_i \hat{x}(t) + A_{di}\hat{x}(t-d) + B_i u(t-\tau) \\ &\qquad + L_i[y(t) - \hat{y}(t)]\}, \\ &\hat{y}(t) = \sum_{i=1}^{q} \mu_i(z(t)) C_i \hat{x}(t), \qquad (3) \\ &\hat{x}(t) = \psi(t), \qquad t \in [-\bar{d} \ \ \ 0], \end{aligned}$$

where L_i are constant matrices.

And then, the fuzzy controller based on the above observer will be designed as

$$u(t) = \sum_{i=1}^{q} \mu_i(z(t)) K_i \hat{x}(t), \qquad (4)$$

The observer error is defined as

$$e(t) = x(t) - \hat{x}(t), \qquad (5)$$

The closed-loop systems can be obtained from equations (2) - (5),

$$\begin{aligned} &\dot{x}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t))\mu_j(z(t))[A_i x(t) + A_{di} x(t-d) \\ &\qquad + B_i K_j x(t-\tau) - B_i K_j e(t-\tau)], \qquad (6) \\ &x(t) = \phi(t), \qquad t \in [-\bar{d},\ \ 0], \end{aligned}$$

The error systems will be obtained,

$$\dot{e}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t))\mu_j(z(t))[(A_i - L_i C_j)e(t) + A_{di} e(t-d)],$$

$$e(t)=\eta(t), \qquad t\in[-\bar{d},\ 0], \qquad (7)$$

where $\eta(t)=\phi(t)-\psi(t)$.

The purpose of this paper is to design a controller in the form of (4) such that the closed-loop systems and the error systems exponentially stable.

Remark 1. The systems studied in this paper is networked systems with both state delay and communication delay. The networked systems is modeled as differential equations with state delay and input delay.

III. Main Results

Definition1 [7] For the systems (2), if there exist constants $\alpha>0$ and $\gamma\geq 1$, such that

$$\|x(t)\|\leq\gamma\sup_{-\bar{d}\leq s\leq 0}\{\|\phi(s)\|\}\mathrm{e}^{-\alpha t},\quad t\geq 0,$$

the systems (2) is exponentially stable.

Lemma1 [8] The linear matrix inequality

$$\begin{bmatrix} Y(x) & W(x)\\ * & R(x)\end{bmatrix}>0$$

is equivalent to

$$R(x)>0,\, Y(x)-W(x)R^{-1}(x)W^T(x)>0,$$

where $Y(x)=Y^T(x), R(x)=R^T(x)$ depend on x.

Theorem1 For the given constants $\alpha>0$ and $i,j=1,\ 2,\ \cdots,\ q$, if there exist matrices $L_i\in R^{n\times l}$ and positive-definite matrices $P,Q\in R^{n\times n}$, such that the following matrix inequality holds,

$$\begin{bmatrix} P(A_i-L_iC_j)+(A_i-L_iC_j)^TP+Q+2\alpha P & PA_{di}\\ * & -e^{-2\alpha d}Q\end{bmatrix}<0, \qquad (8)$$

the error systems (7) is exponentially stable.

Proof. Lyapunov function is selected as

$$V(e(t))=e^T(t)Pe(t)+\int_{t-d}^{t}e^T(s)Q\mathrm{e}^{2\alpha(s-t)}e(s)ds,$$

where $\alpha>0$ is a constant to be determined, $P,Q\in R^{n\times n}$ are positive-definite matrices.

Following the state trajectory of the systems (7), we obtain

$$\begin{aligned}\dot{V}(e(t))&=2e^T(t)P\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))[(A_i-L_iC_j)e(t)\\&\quad+A_{di}e(t-d)]+e^T(t)Qe(t)\\&\quad-e^T(t-d)Q\mathrm{e}^{-2\alpha d}e(t-d)\\&\quad-2\alpha\int_{t-d}^{t}e^T(s)Q\mathrm{e}^{2\alpha(s-t)}e(s)ds.\end{aligned}$$

It can be seen from [6],

$$\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))=1,$$

therefore

$$\begin{aligned}\dot{V}(e(t))&=\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))\begin{bmatrix}e(t)\\e(t-d)\end{bmatrix}^T\\&\quad\times\begin{bmatrix} P(A_i-L_iC_j)+(A_i-L_iC_j)^TP+Q+2\alpha P & PA_{di}\\ * & -\mathrm{e}^{-2\alpha d}Q\end{bmatrix}\\&\quad\times\begin{bmatrix}e(t)\\e(t-d)\end{bmatrix}-2\alpha V(e(t)).\end{aligned}$$

Substituting (8) into the above equality, we obtain

$$\dot{V}(e(t))<-2\alpha V(e(t)),$$

then

$$V(e(t))<V(e(0))\mathrm{e}^{-2\alpha t}\leq[\lambda_{\max}(P)+d\lambda_{\max}(Q)]\|\eta\|_d^2\ \mathrm{e}^{-2\alpha t}. \qquad (9)$$

It is easy to know from the expression of $V(e(t))$

$$V(e(t))\geq\lambda_{\min}(P)\|e(t)\|^2. \qquad (10)$$

Combing (9) and (10), we obtain

$$\|e(t)\|<\sqrt{\frac{\lambda_{\max}(P)+d\lambda_{\max}(Q)}{\lambda_{\min}(P)}}\|\eta\|_d\ \mathrm{e}^{-\alpha t},$$

with Defintion1, we know that the error systems (7) is exponentially stable.

Theorem2 For the given constants $\alpha>0,\beta>0$ and $i,j=1,\ 2,\ \cdots,\ q$, if there exist a constant $\varepsilon>0$, matrices $L_i\in R^{n\times l},K_j\in R^{m\times n}$ and positive-definite matrices $P,Q,P_1,Q_1,Q_2\in \mathrm{R}^{n\times n}$, such that the following matrix inequalities hold,

$$\begin{bmatrix} P(A_i-L_iC_j)+(A_i-L_iC_j)^TP+Q+2\alpha P & PA_{di}\\ * & -\mathrm{e}^{-2\alpha d}Q\end{bmatrix}<0, \qquad (11)$$

$$\begin{bmatrix}\Sigma & P_1A_{di} & P_1B_iK_j\\ * & -\mathrm{e}^{-2\beta d}Q_1 & 0\\ * & * & -\mathrm{e}^{-2\beta\tau}Q_2\end{bmatrix}<0, \qquad (12)$$

where

$$\Sigma=P_1A_i+A_i^TP_1+Q_1+Q_2+2\beta P_1+\varepsilon I,$$

with the controller $u(t)=\sum_{i=1}^{q}\mu_i(z(t))K_i\hat{x}(t)$, the closed-loop systems (6) and the error systems (7) are exponentially stable.

Proof. Lyapunov function is selected as

$$\begin{aligned}V(x(t))&=x^T(t)P_1x(t)+\int_{t-d}^{t}x^T(s)Q_1\mathrm{e}^{2\beta(s-t)}x(s)ds\\&\quad+\int_{t-\tau}^{t}x^T(s)Q_2\mathrm{e}^{2\beta(s-t)}x(s)ds,\end{aligned}$$

where $\beta>0$ is a constant to be determined, $P_1,Q_1,Q_2\in R^{n\times n}$ are positive-definite matrices.

Following the state trajectory of the systems (6), we obtain

$$\begin{aligned}\dot{V}(x(t))&=2x^T(t)P_1\{\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))[A_ix(t)\\&\quad+A_{di}x(t-d)+B_iK_jx(t-\tau)\\&\quad-B_iK_je(t-\tau)]+x^T(t)Q_1x(t)\\&\quad-x^T(t-d)Q_1\mathrm{e}^{-2\beta d}x(t-d)+x^T(t)Q_2x(t)\end{aligned}$$

$$-x^T(t-\tau)Q_2\mathrm{e}^{-2\beta\tau}x(t-\tau)+\beta x^T(t)P_1x(t)$$
$$-\beta x^T(t)P_1x(t)+\varepsilon x^T(t)x(t)-\varepsilon x^T(t)x(t)$$
$$-2\beta\int_{t-d}^{t}x^T(s)Q_1\mathrm{e}^{2\beta(s-t)}x(s)ds$$
$$-2\beta\int_{t-\tau}^{t}x^T(s)Q_2\mathrm{e}^{2\beta(s-t)}x(s)ds$$
$$=\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))\begin{bmatrix}x(t)\\x(t-d)\\x(t-\tau)\end{bmatrix}^T$$
$$\times\begin{bmatrix}\Sigma & P_1A_{di} & P_1B_iK_j\\ * & -\mathrm{e}^{-2\beta d}Q_1 & 0\\ * & * & -\mathrm{e}^{-2\beta\tau}Q_2\end{bmatrix}\begin{bmatrix}x(t)\\x(t-d)\\x(t-\tau)\end{bmatrix}$$
$$-\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))\{2x^T(t)P_1B_iK_je(t-\tau)$$
$$+\varepsilon x^T(t)x(t)\}-2\beta V(x(t)),$$

where $\varepsilon>0$ is a constant, and

$$\Sigma=P_1A_i+A_i^TP_1+Q_1+Q_2+2\beta P_1+\varepsilon I.$$

If the matrix inequality (12) holds, the above equality can be rewritten as

$$\dot V(x(t))\le-\varepsilon x^T(t)x(t)+2\sum_{i=1}^{q}\sum_{j=1}^{q}\|x(t)\|\|P_1B_iK_j\|\|e(t-\tau)\|$$
$$-2\beta V(x(t))$$
$$=-\|x(t)\|(\varepsilon\|x(t)\|-2\sum_{i=1}^{q}\sum_{j=1}^{q}\|P_1B_iK_j\|\|e(t-\tau)\|)$$
$$-2\beta V(x(t)).\tag{13}$$

When $\varepsilon\|x(t)\|-2\sum_{i=1}^{q}\sum_{j=1}^{q}\|P_1B_iK_j\|\|e(t-\tau)\|\le0$, we have

$$\|x(t)\|\le\frac{2\sum_{i=1}^{q}\sum_{j=1}^{q}\|P_1B_iK_j\|\|e(t-\tau)\|}{\varepsilon}.\tag{14}$$

According to Theorem 1, when the matrix inequality (11) is established, the error systems (7) is exponentially stable, that is, the error $e(t)$ is exponentially stable, and the state $x(t)$ of the closed-loop systems is also exponentially stable according to the equation (14), such that the closed-loop systems (6) and the error systems (7) are exponentially stable.

When $\varepsilon\|x(t)\|-2\sum_{i=1}^{q}\sum_{j=1}^{q}\|P_1B_iK_j\|\|e(t-\tau)\|>0$, the equation (13) can be rewritten as

$$\dot V(x(t))<-2\beta V(x(t)),$$

therefore

$$V(x(t))<V(x(0))\mathrm{e}^{-2\beta t}$$
$$\le[\lambda_{\max}(P_1)+d\lambda_{\max}(Q_1)+\tau\lambda_{\max}(Q_2)]\|\phi\|_d^2\,\mathrm{e}^{-2\beta t}.\tag{15}$$

It is easy to know from the expression of V_x

$$V(x(t))\ge\lambda_{\min}(P_1)\|x(t)\|^2.\tag{16}$$

Combing (15) and (16), we obtain

$$\|x(t)\|<\sqrt{\frac{\lambda_{\max}(P_1)+d\lambda_{\max}(Q_1)+\tau\lambda_{\max}(Q_2)}{\lambda_{\min}(P_1)}}\|\phi\|_d\,\mathrm{e}^{-\beta t},$$

the closed-loop systems (6) is exponentially stable. It is also known from Theorem1 that when the matrix inequality (11) holds, the error systems (7) is exponentially stable, so that the closed-loop systems (6) and the error systems (7) are exponentially stable.

In a word, when the matrix inequalities (11) and (12) hold, the closed-loop systems (6) and the error systems (7) are exponentially stable.

Remark 2. In the theorem 2, the sufficient condition (11)-(12) are not linear matrix inequalities, which cannot be solved by the tool of the LMI toolbox in MATLAB. An equivalent sufficient condition in terms of linear matrix inequality will be given in the following theorem.

Theorem3 For the given constants $\alpha>0,\beta>0$ and $i,j=1,\ 2,\ \cdots,\ q$, if there exist constant $\varepsilon>0$, matrices $M_i\in R^{n\times l},\bar K_j\in R^{m\times n}$ and positive-definite matrices $P,Q,X,\bar Q_1,\bar Q_2\in R^{n\times n}$, such that the following matrix inequalities hold,

$$\begin{bmatrix}PA_i-M_iC_j+(PA_i-M_iC_j)^T+Q+2\alpha P & PA_{di}\\ * & -\mathrm{e}^{-2\alpha d}Q\end{bmatrix}<0,\tag{17}$$

$$\begin{bmatrix}\Omega & A_{di}X & B_i\bar K_j & X\\ * & -e^{-2\beta d}\bar Q_1 & 0 & 0\\ * & * & -e^{-2\beta\tau}\bar Q_2 & 0\\ * & * & * & -\frac{1}{\varepsilon}I\end{bmatrix}<0,\tag{18}$$

where

$$\Omega=A_iX+XA_i^T+\bar Q_1+\bar Q_2+2\beta X,$$

with the controller $u(t)=\sum_{i=1}^{q}\mu_i(z(t))\bar K_iX^{-1}\hat x(t)$, the closed-loop systems (6) and the error systems (7) are exponentially stable.

Proof. In the matrix inequality (11), let $M_i=PL_i$, then the matrix inequality (11) can be changed as the equation (17). Multiplying the matrix $diag(P_1^{-1},\ P_1^{-1},\ P_1^{-1})$ on both sides of the matrix inequality (12), and let $X=P_1^{-1}$, we can obtain

$$\begin{bmatrix}\Pi & A_{di}X & B_iK_jX\\ * & -e^{-2\beta d}XQ_1X & 0\\ * & * & -e^{-2\beta\tau}XQ_2X\end{bmatrix}<0,$$

where

$$\Pi=A_iX+XA_i^T+XQ_1X+XQ_2X+2\beta X+\varepsilon XX.$$

Letting $\bar K_j=K_jX$, $\bar Q_1=XQ_1X$, $\bar Q_2=XQ_2X$ and with Lemma1, the above inequality is equivalent to equation (18).

Remark 3. When β is determined, the conditions (17) and (18) are linear in theorem3, which can be easily solved by using the LMI toolbox in MATLAB.

IV. Simulation

Consider networked control systems (2) with delay, where

$$A_1=\begin{bmatrix}-6 & -1 & -2\\ 1 & -2 & 1\\ 1 & 0 & -3\end{bmatrix}, A_{d1}=\begin{bmatrix}0.2 & 1 & 0.1\\ 0.4 & -1 & 0.1\\ -1 & 0.4 & -0.5\end{bmatrix},$$

$$A_2=\begin{bmatrix}-2 & 0.1 & 0.2\\ -2 & -1 & 1.5\\ -1 & 0.2 & -4\end{bmatrix}, A_{d2}=\begin{bmatrix}0.1 & 1.5 & 0\\ 4 & -0.1 & 0.1\\ -0.1 & 0.2 & -0.1\end{bmatrix},$$

$$A_3=\begin{bmatrix}-5 & 1 & -0.1\\ -0.3 & -0.7 & -0.1\\ -2 & -2 & -4\end{bmatrix}, B_1=\begin{bmatrix}2\\ 1\\ 3\end{bmatrix}, B_2=\begin{bmatrix}1\\ -1\\ 0.1\end{bmatrix},$$

$$A_{d3}=\begin{bmatrix}1.5 & 0 & 0.2\\ 0.1 & -2 & 0.2\\ -1.6 & 0.1 & -0.7\end{bmatrix}, B_3=\begin{bmatrix}0.1\\ 2\\ 1.5\end{bmatrix},$$

$$C_1=\begin{bmatrix}0.1 & 0.3 & 1\end{bmatrix}, C_2=\begin{bmatrix}1 & -2 & 0.2\end{bmatrix},$$

$$C_3=\begin{bmatrix}-2 & -0.5 & 1\end{bmatrix}, d=0.2, \tau=0.3,$$

$$\mu_1(z(t))=0.5\sin^2(t), \mu_2(z(t))=0.7\cos(t),$$

$$\mu_3(z(t))=1-0.5\sin^2(t)-0.7\cos(t),$$

$$\bar{d}=\max\{d,\tau\}=0.3.$$

Using the algorithm in [4], the controller can be obtained as

$$u(t)=\begin{bmatrix}-2.4593 & 0.5482 & 1.5392\end{bmatrix}x(t).$$

Using the observer-based fuzzy control approach proposed in this paper to solve the linear matrix inequalities (17)-(18), we obtain the feedback matrices,

$$\bar{K}_1=\begin{bmatrix}1.4726 & -1.7430 & 1.3327\end{bmatrix},$$

$$\bar{K}_2=\begin{bmatrix}1.8594 & -2.4842 & 1.8973\end{bmatrix},$$

$$\bar{K}_3=\begin{bmatrix}-1.3668 & 0.8543 & -1.5899\end{bmatrix},$$

$$X=\begin{bmatrix}2.3970 & 0.9802 & 2.6738\\ 0.9802 & 1.5790 & 1.5987\\ 2.6738 & 1.5987 & 1.6892\end{bmatrix},$$

and the observer gain matrices,

$$L_1=\begin{bmatrix}1.6794 & 0.6455 & -1.4992\end{bmatrix}^T,$$

$$L_2=\begin{bmatrix}-2.4795 & 1.4987 & -2.8654\end{bmatrix}^T,$$

$$L_3=\begin{bmatrix}-0.4632 & -0.3673 & 2.4467\end{bmatrix}^T,$$

The dynamic output feedback fuzzy controller can be designed,

$$u(t)=\big[1.4323\sin^2(t)-0.1347\cos(t)\quad 0.3421\sin^2(t)+2.3513\cos(t)\quad -0.1866\sin^2(t)+1.1286\cos(t)\big]\hat{x}(t).$$

By selecting the initial value condition such as

$$\phi(0)=\psi(0)=\begin{bmatrix}6\\ -3\\ 2\end{bmatrix}.$$

the comparison results of the systems states $x_1(t), x_2(t), x_3(t)$ with two algorithms as Figure 2-4.

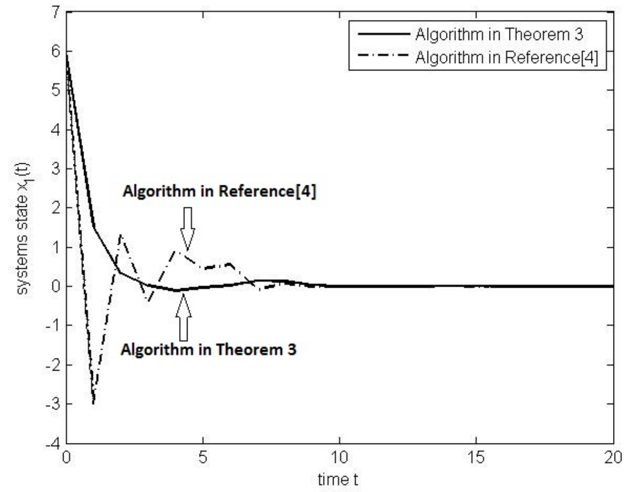

Figure 2. The response curves of the systems state $x_1(t)$

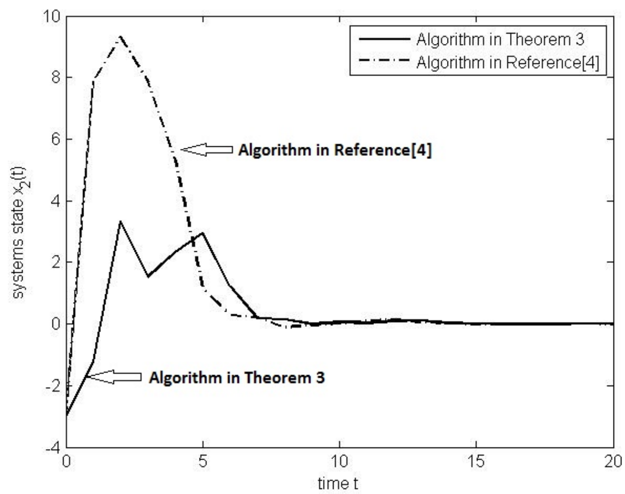

Figure 3. The response curves of the systems state $x_2(t)$

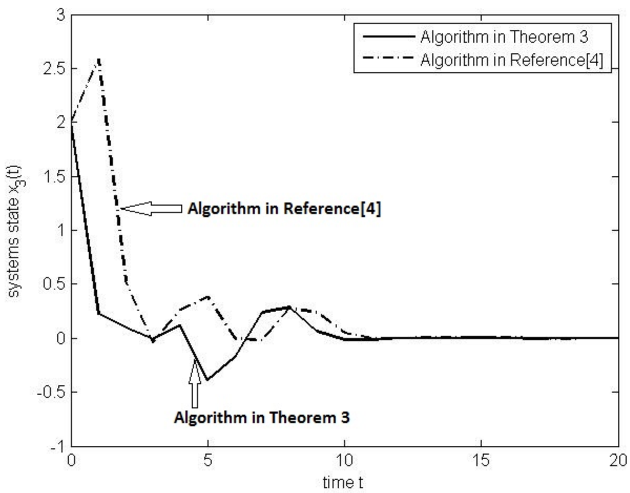

Figure 4. The response curves of the systems state $x_3(t)$

Figure 2 shows the comparison curves of the systems state $x_1(t)$ with two different algorithms. The solid line is the control effect curve obtained by the algorithm of Theorem 3, and the dotted line is the control effect curve obtained by the algorithm in [4]. The solid line can converge to 0 within 5 seconds and the dotted line can converge to 0 within 8 seconds. The smoothness of the solid line is very good, and there is basically no overshoot, while the dotted line has large vibration and overshoot, and the amplitude reaches 2.

Figure 3 shows the comparison curves of the systems state $x_2(t)$ with two different algorithms. The solid line is the control effect curve obtained by the algorithm of Theorem 3, and the dotted line is the control effect curve obtained by the algorithm in [4]. The solid line can converge to 0 within 6 seconds and the dotted line can converge to 0 within 7 seconds. The solid line has good smoothness and small overshoot, and the amplitude reaches 2.5, while the dotted line has large vibration and overshoot, and the amplitude reaches 9.

Figure 4 shows the comparison curves of the systems state $x_3(t)$ with two different algorithms. The solid line is the control effect curve obtained by the algorithm of Theorem3, and the dotted line is the control effect curve obtained by the algorithm in [4].The solid line can converge to 0 within 5 seconds and the dotted line can converge to 0 within 10 seconds. The solid line has good smoothness and small overshoot, and the amplitude reaches 0.4, while the dotted line has large vibration and overshoot, and the amplitude reaches 2.5.

In a word, the algorithm given in Theorem 3 is superior to [4] in convergence speed and smoothness.

V. CONCLUSION

The dynamic output feedback control of T-S fuzzy networked systems with state delay and communication delay is studied. The main work is as follows: (1) Considering the influence of communication delay caused by network on systems performance, the mathematical model of networked systems with state delay and communication delay is established by T-S fuzzy method. (2) The exponentially stable state observer is designed by using Lyapunov stability theory and linear matrix inequality method. (3) Using matrix inequality transformation, the nonlinear exponentially stable condition is transformed into linear matrix inequality, and the design method of dynamic output feedback fuzzy control is obtained. The innovation of this paper is that the influence of communication delay on fuzzy networked systems is considered, and the exponential stability condition and control design process can be easily solved by MATLAB. The design method proposed in this paper can be extended to other types of networked systems, such as distributed delay networked systems, time-varying delay networked systems, random delay networked systems and so on, which will be our next research work.

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