Existence of Positive Steady-state Solution of a Predator-prey Dynamics with Dinosaur Functional Response and Heterogeneous Environment

Wenbin Yang

***Abstract*—It is well known that the effects of the protection zone on the dynamical behavior are significantly different from the non-protection zone. In this paper, we presents a semi-linear ellipic system, which describes the predator-prey relationship with Dinosaur functional response under homogeneous Neumann boundary conditions. This paper concerns the existence of non-constant positive steady-state solutions, in the case (1): Non-degeneracy of $\beta(x)$ with $\beta(x) > 0$ on $\overline{\Omega}$ or case (2): Degeneracy of $\beta(x)$ with $\beta(x) = 0$ in Ω_0, $\beta(x) > 0$ in $\overline{\Omega} \setminus \overline{\Omega}_0$. These results provide theoretical evidence for the complex spatiotemporal dynamics.**

***Index Terms*—reaction-diffusion system, predator-prey relationship, heterogeneous environment, existence.**

I. Introduction

In [1], Wollkind et al. first considered the following predator-prey system:

$$\begin{cases} \dot{u} = ru(1 - \frac{u}{K}) - p(u)v, \\ \dot{v} = v[s(1 - h\frac{v}{u})], \end{cases} \tag{1}$$

where u is the biomass of prey and v is the biomass of predator. To model various different processes of energy transfer in ecology, many kinds of $p(u)$ modes have been developed [2], [3]. They are put forward by different backgrounds and have important dynamic significance in mathematical theory. Incorporated the Dinosaur functional response [5] into (1), we obtain

$$\begin{cases} \dot{u} = ru(1 - \frac{u}{K}) - uve^{-ku}, \\ \dot{v} = v[s(1 - h\frac{v}{u})]. \end{cases} \tag{2}$$

The Dinosaur reaction term is ue^{-ku}, which is an improvement of the Ivlev-type reaction term $h(1 - e^{-ku})$. Given the spatial inhomogeneity in population density, we establish the following diffusive predator-prey model:

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u + ru(1 - \frac{u}{K}) - uve^{-ku} & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = \Delta v + v[s(1 - h\frac{v}{u})] & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0 & x \in \partial\Omega, t > 0, \end{cases} \tag{3}$$

where Ω is a bounded domain in the Euclidean space $\mathbb{R}^N$ ($N \in \{1, 2, 3, \cdots\}$) with smooth boundary, denoted as $\partial\Omega$, n is the unit outer normal vector on $\partial\Omega$. Now we introduce the following non-dimensional quantities:

$$\overline{u} = ru, \overline{v} = rhv, \text{ and } a = r, b = 1/K, \alpha = \frac{1}{rh}, \beta = k/r.$$

By substituting these new variables into (3) and dropping the bars for notational convenience, we obtain

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u + au - bu^2 - \alpha uve^{-\beta u} & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = \Delta v + sv(1 - \frac{v}{u}) & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0 & x \in \partial\Omega, t > 0. \end{cases} \tag{4}$$

The corresponding steady-state system to (4) is

$$\begin{cases} -\Delta u = au - bu^2 - \alpha uve^{-\beta u}, & x \in \Omega, \\ -\Delta v = sv(1 - \frac{v}{u}), & x \in \Omega, \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, & x \in \partial\Omega. \end{cases} \tag{5}$$

In order to discuss the effect of heterogeneous environment on parameters, we obtain

$$\begin{cases} -\Delta u = au - b(x)u^2 - \alpha uve^{-\beta(x)u}, & x \in \Omega, \\ -\Delta v = sv(1 - \frac{v}{u}), & x \in \Omega, \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, & x \in \partial\Omega. \end{cases} \tag{6}$$

where the parameters a, α and s are assumed to be positive constants, and the functions $b(x), \beta(x) \in C^1(\overline{\Omega})$ may vanish in a non-trivial open subdomain Ω_0 of Ω, which imply the degeneracy (if vanish). The dependence of $b(x)$ and $\beta(x)$ on the space variable x represents that the two species interact in a spatially heterogeneous environment. Let $z = z(x)$ be a function in $C^1(\overline{\Omega})$, satisfying

$$z(x) = 0, \ x \in \Omega_0, \quad z(x) > 0, \ x \in \overline{\Omega} \setminus \overline{\Omega}_0. \tag{\textbf{H}}$$

When $\beta \equiv 0$, Du and Hsu [6] considered

$$\begin{cases} -\Delta u = au - bu^2 - \alpha uv, & x \in \Omega, \\ -\Delta v = sv(1 - \frac{v}{u}), & x \in \Omega, \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, & x \in \partial\Omega. \end{cases} \tag{7}$$

They showed that (7) always had a positive solution. When $\alpha = 0$, (6) becomes

$$\begin{cases} -\Delta u = au - b(x)u^2, & x \in \Omega, \\ -\Delta v = sv(1 - \frac{v}{u}), & x \in \Omega, \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, & x \in \partial\Omega, \end{cases} \tag{8}$$

where the first equation is decoupling from the second equation. So, the existence of positive solution of (8) depends directly on the existence of positive solution of the first scalar equation in (8).

In this paper, we always assume that the condition **(H)** holds for $b = b(x)$ in (6), and consider the existence of non-constant positive steady state solutions, in the case (1): Non-degeneracy of $\beta(x)$ with $\beta(x) > 0$ on $\overline{\Omega}$ or case (2): Degeneracy of $\beta(x)$ with $\beta(x) = 0$ in Ω_0 and $\beta(x) > 0$ in $\overline{\Omega} \setminus \overline{\Omega}_0$.

Manuscript received March 24, 2022; revised October 28, 2022. This work was supported by the National Natural Science Foundation of China under Grants 12001425 and 12171296.

Wenbin Yang is an Associate Professor of the School of Science, Xi'an University of Posts and Telecommunications, Xi'an 710121, China. (e-mail: yangwenbin-007@163.com & yangwenbin@xupt.edu.cn).

II. Existence of Positive Steady-state Solution

First, we want to obtain the existence for the positive solutions to (6) by the techniques in [7] in the non-degenerate case: $\beta(x) > 0$ on $\overline{\Omega}$.

Assume that $q(x) \in C(\overline{\Omega})$. Let $\lambda_1^{\Omega}(q)$ or $\lambda_1^{\Omega,N}(q)$ be the first eigenvalue of $-\Delta + q(x)$ in Ω subject to the homogeneous Dirichlet or Neumann boundary condition. By some of the notations in [7], we have $\lambda_1^{\Omega,N}(q)$ or $\lambda_1^{\Omega}(q)$ is increasing in q.

Let $\lambda_1^{\Omega} \doteq \lambda_1^{\Omega}(0)$. Obviously,

$$-\Delta u = au - b(x)u^2,\ x \in \Omega, \quad \frac{\partial u}{\partial n} = 0,\ x \in \partial\Omega \tag{9}$$

has a positive solution if and only if $0 < a < \lambda_1^{\Omega_0}$. In this case (9) has a unique positive solution u_a, $a \to u_a$ is continuous as a map from $(0, \lambda_1^{\Omega_0})$ to $C^{2+\mu}(\Omega)$, and $\|u_a\|_\infty \to \infty$ as $a \to \lambda_1^{\Omega_0} - 0$.

Lemma 1. (Maximum principle, [8, Proposition 2.2]) *Assume that $g \in C(\overline{O} \times \mathbb{R})$.*

(i) *If $w \in C^2(O) \cap C^1(\overline{O})$ satisfies*

$$\Delta w(x) + g(x, w(x)) \geq 0 \text{ in } O,\ \partial_\nu w \leq 0 \text{ on } \partial O,$$

and $w(x_0) = \max_{\overline{O}} w$, then $g(x_0, w(x_0)) \geq 0$.

(ii) *If $w \in C^2(O) \cap C^1(\overline{O})$ satisfies*

$$\Delta w(x) + g(x, w(x)) \leq 0 \text{ in } O,\ \partial_\nu w \geq 0 \text{ on } \partial O,$$

and $w(x_0) = \min_{\overline{O}} w$, then $g(x_0, w(x_0)) \leq 0$.

Lemma 2. (Harnack inequality, [9, Lemma 4.3]) *Let $w \in C^2(\Omega) \cap C^1(\overline{\Omega})$ be a positive solution to $\Delta w(x) + c(x)w(x) = 0$, where $c \in C(\overline{\Omega})$, satisfying the homogeneous Neumann boundary condition. Then, there exists a positive constant C which depends only on B where $\|c\|_\infty \leq B$ such that*

$$\max_{\overline{\Omega}} w \leq C \min_{\overline{\Omega}} w.$$

Theorem 3. *Let $\beta(x) > 0$ on $\overline{\Omega}$. Given any small $\epsilon > 0$ and large $M > 0$, there exists a positive constant C, which depends only on ϵ and Ω, such that any positive solution (u, v) to (6) satisfies $\|u\|_\infty + \|v\|_\infty \leq C$ if either (i) $0 < a < \lambda_1^{\Omega_0} - \epsilon$ or (ii) $\lambda_1^{\Omega_0} + \epsilon < a < M$.*

Proof: Let (u, v) be a positive solution to (6).

(i) From the first equation in (6), we have $-\Delta u \leq au - \alpha(x)u^2$. By the upper and lower solution method for elliptic equation, existence and uniqueness of positive solution to (6), $u \leq u_a$ on Ω. Let $v(x_0) = \max_{\overline{\Omega}} v$ for some $x_0 \in \overline{\Omega}$. By Lemma 1, it follows from the second equation in (6) that $v(x_0) \leq u(x_0)$, and so $v \leq u_a$. Hence, Lemma 1 holds for case (i).

(ii) By an indirect argument, supposing the conclusion is not true, we can find some $\epsilon, M > 0$ and one sequence $a_i \in (\lambda_1^{\Omega_0} + \epsilon, M)$ such that the corresponding positive solution (u_i, v_i) to (6) with $a = a_i$, i.e.,

$$\begin{cases} -\Delta u_i = a_i u_i - b(x)u_i^2 - \alpha u_i v_i e^{-\beta u_i}, & x \in \Omega, \\ -\Delta v_i = s v_i (1 - \frac{v_i}{u_i}), & x \in \Omega, \\ \frac{\partial u_i}{\partial n} = \frac{\partial v_i}{\partial n} = 0, & x \in \partial\Omega \end{cases} \tag{10}$$

satisfies $\|u_i\|_\infty + \|v_i\|_\infty \to \infty$ as $i \to \infty$. Without loss of generality, we assume that $a_i \to a \in [\lambda_1^{\Omega_0} + \epsilon, M]$. By Lemma 1 again, it follows from the second equation in (10) that $v_i \leq u_i$. Thus, for the sequence $\{u_i\}$, it is necessary that $\|u_i\|_\infty \to \infty$ as $i \to \infty$.

Let $\widetilde{u}_i = \frac{u_i}{\|u_i\|_\infty}$. From the first equation in (10), $-\Delta u_i \leq a_i u_i$. Then we obtain

$$\int_\Omega |\nabla \widetilde{u}_i|^2 + |\widetilde{u}_i|^2 \ dx \leq (a_i + 1) \int_\Omega |\widetilde{u}_i|^2 \ \leq (M+1)|\Omega| dx.$$

By the similar proof statements of Proposition 2.1 in [11], we obtain that $\widetilde{u}_i \to \widetilde{u}$ weakly in $H^1(\Omega)$ and strongly in $L^2(\Omega)$, and $\widetilde{u} \not\equiv 0$ in $L^p(\Omega)$.

Let $c_i = v_i e^{-\beta u_i}$. By virtue of $v_i \leq u_i$, we have $c_i \leq u_i e^{-\beta u_i}$. Since $\beta(x) > 0$ on $\overline{\Omega}$, there must exist a $\beta_0 > 0$ such that $\beta \geq \beta_0 > 0$ in $\overline{\Omega}$, and then $\|c_i\|_\infty < C_0$ dependent of β for some $C_0 > 0$. By choosing a subsequence if necessary, we assume that $c_i \to c$ weakly in $L^2(\Omega)$ and $c \in L^\infty(\Omega)$. From the first equation of (10), we obtain

$$\begin{cases} -\Delta u_i = a_i u_i - b(x)u_i^2 - \alpha u_i c_i, & x \in \Omega, \\ \frac{\partial u_i}{\partial n} = 0, & x \in \partial\Omega, \end{cases} \tag{11}$$

Let $\varphi \in C_0^1(\Omega_0)$ and $\varphi \equiv 0 \in C(\overline{\Omega} \setminus \overline{\Omega}_0)$. Since $b(x) = 0$ on $\overline{\Omega}_0$, integrating over Ω by parts, it follows from (11) that

$$\int_{\Omega_0} \nabla \widetilde{u}_i \cdot \nabla \varphi = a_i \int_{\Omega_0} \widetilde{u}_i \varphi \ dx - \alpha \int_{\Omega_0} \widetilde{u}_i c_i dx.$$

Let $i \to \infty$. Then $\widetilde{u}$ satisfies

$$\int_{\Omega_0} \nabla \widetilde{u} \cdot \nabla \varphi = a_i \int_{\Omega_0} \widetilde{u} \varphi \ dx - \alpha \int_{\Omega_0} \widetilde{u} c dx.$$

Hence, $\widetilde{u}$ is a weak solution of the problem:

$$-\Delta \widetilde{u} = a\widetilde{u} - \alpha \widetilde{u} c,\ x \in \Omega, \quad \frac{\partial \widetilde{u}}{\partial n} = 0,\ x \in \partial\Omega. \tag{12}$$

By the similar proof statements of Proposition 2.1 in [11], the smoothness of the boundary $\partial\Omega_0$ yields $\widetilde{u} \in H_0^1(\Omega_0)$.

Recall that $\widetilde{u} \not\equiv 0$ in $L^p(\Omega)$. Then, $\widetilde{u} \geq, \not\equiv 0$ is a solution of (12). Since $a - \alpha c \in L^\infty(\Omega)$, it follows from Lemma 2 that $\widetilde{u} > 0$ in Ω_0. Moreover, the standard theory guarantees $\widetilde{u} \in C^1(\overline{\Omega}_0)$. Note that $\widetilde{u}_i = \frac{u_i}{\|u_i\|_\infty}$ and $v_i \leq u_i$. Since $\|u_i\|_\infty \to \infty$ as $i \to \infty$,

$$c_i = v_i e^{-\beta u_i} \leq u_i e^{-\beta u_i} = \frac{\widetilde{u}_i \|u_i\|_\infty}{e^{\beta \widetilde{u}_i \|u_i\|_\infty}} \to 0$$

a.e. in Ω_0 as $i \to \infty$. This shows that $c = 0$ a.e in Ω_0. Hence, $a = \lambda_1^{\Omega_0}$ from (12), which contradicts $a > \lambda^{\Omega_0} + \epsilon$. The proof is complete. ■

Remark 4. *Theorem 3 (i) still holds for the degenerate case of $\beta(x)$, which could be seen in the proof process of Theorem 6.*

Remark 5. *Considering s as the bifurcation parameter, we want to consider the bifurcation solutions in the semi-trivial (non-degenerate) solution curve $\Gamma_u = \{(s; u_a, 0) : d \in (0, \infty)\}$. By employing the Crandall and Rabinowitz bifurcation theorem [10], we find that there will be no bifurcations around $(u_a, 0)$, except the bifurcation parameter $s = 0$, which is invalid since the parameter s is a positive constant. That is completely different from Theorems 2.2 and 2.3 in [7], so we will consider the degenerate case in the next moment.*

Now, we will give the result for the existence of positive solutions to (6) in the degenerate case: $\beta(x) = 0$ in Ω_0 and $\beta(x) > 0$ in $\overline{\Omega} \setminus \overline{\Omega}_0$.

Theorem 6. *If $a \in (0, \lambda_1^{\Omega_0})$, then (6) has at least one positive solution.*

Proof: Let (u, v) be a positive solution to

$$\begin{cases} -\Delta u = au - b(x)u^2 - t\alpha uve^{-\beta(x)u}, & x \in \Omega, \\ -\Delta v = sv(1 - \frac{v}{u}), & x \in \Omega, \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, & x \in \partial\Omega, \end{cases} \tag{13}$$

with $t \in [0, 1]$ and

$$f(t, u, v) \doteq au - b(x)u^2 - t\alpha uve^{-\beta(x)u},$$
$$g(u, v) \doteq sv(1 - \frac{v}{u}).$$

Using the same proof process as Theorem 3(i), we obtain that

$$0 \le v \le u \le u_a \le \|u_a\|_\infty, \quad \min_{\overline{\Omega}} v \ge \min_{\overline{\Omega}} u \tag{14}$$

by Lemma 1, where u_a is the unique positive solution to (9).

Note that $b, \beta \in C^1(\overline{\Omega})$. Then, the solution (u, v) to (6) belongs to $[C^2(\overline{\Omega})]^2$ by elliptic regularity. Let $u(x_0) = \min_{\overline{\Omega}} u$ for some $x_0 \in \overline{\Omega}$. By Lemma 1 again, we have

$$a - b(x_0)u(x_0) - t\alpha v(x_0)e^{-\beta(x_0)u(x_0)} \le 0,$$

which yields that $a \le \|b\|_\infty \min_{\overline{\Omega}} u + \alpha\|v\|_\infty$, and then $a \le \|b\|_\infty \min_{\overline{\Omega}} u + \alpha\|u\|_\infty$.

Let $c(x) = a - b(x)u - t\alpha ve^{-\beta(x)u}$. Then the equation of u becomes

$$-\Delta u = c(x)u, \ x \in \Omega, \quad \frac{\partial u}{\partial n} = 0, \ x \in \partial\Omega.$$

Since $\|c\|_\infty \le a + \|b\|_\infty\|u\|_\infty + \alpha\|v\|_\infty \le a + (\|b\|_\infty + \alpha)\|u_a\|_\infty$, by Lemma 2, there exists a positive constant C dependent of a such that $\max_{\overline{\Omega}} u \le C \min_{\overline{\Omega}} u$. Then

$$\begin{aligned} a \le \|b\|_\infty \min_{\overline{\Omega}} u + \alpha\|u\|_\infty &\le \|b\|_\infty \min_{\overline{\Omega}} u + \alpha\|u\|_\infty \\ &\le (\|b\|_\infty + \alpha C) \min_{\overline{\Omega}} u, \end{aligned}$$

which implies that $\min_{\overline{\Omega}} u \ge a(\|b\|_\infty + \alpha C)^{-1}$.

Define a space E by

$$E = \{(u, v) \in C(\Omega) \times C(\Omega) : m < u, v < M\}$$

with $m = \frac{a}{2}(\|b\|_\infty + \alpha C)^{-1}$ and $M = 2\|u_a\|_\infty$. By the above discussion, for all $t \in [0, 1]$, (13) has no solution on ∂E.

Let $L(t; u, v) = ((-\Delta + I)^{-1} f(t, u, v), (-\Delta + I)^{-1}) g(u, v)$. Then, $L : [0, 1] \times \overline{E} \to C(\overline{\Omega}) \times C(\overline{\Omega})$ is compact. For $(u, v) \in \overline{E}$, (u, v) is a solution of (13) if and only if (u, v) is a fixed point of $L(t : \cdot)$, i.e. $(u, v) = L(t; u, v)$. It is obvious that $(u, v) \ne L(t; u, v)$ for all $t \in [0, 1]$ and $(u, v) \in \partial E$. Hence, the degree $deg(I - L(t; \cdot), E, 0)$ is well defined and independent of $t \in [0, 1]$.

Setting $t = 0$, (13) becomes

$$\begin{cases} -\Delta u = au - b(x)u^2, & x \in \Omega, \\ -\Delta v = sv(1 - \frac{v}{u}), & x \in \Omega, \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, & x \in \partial\Omega. \end{cases} \tag{15}$$

One can see that (15) has a unique positive solution $(u, v) = (u_a, v_a)$, where $u = u_a$ is the unique positive solution of (9) and $v = v_a$ is the unique positive solution of

$$-\Delta v = sv(1 - \frac{v}{u_a}), \ x \in \Omega, \quad \frac{\partial v}{\partial n} = 0, \ x \in \partial\Omega.$$

And therefore, $deg(I - L(0; \cdot), E, 0) = index(I - L(0; \cdot), (u_a, v_a))$. Now we prove that (u_a, v_a) as a solution of (15) is non-degenerate and linearized stable. In fact, the linearized eigenvalue problem of (15) at (u_a, v_a) is

$$\begin{cases} -\Delta\phi = a\phi - 2b(x)u_a\phi + \mu\phi, & x \in \Omega, \\ -\Delta\psi = s(1 - \frac{2v_a}{u_a})\psi + s\frac{v_a^2}{u_a^2}\phi + \mu\psi, & x \in \Omega, \\ \frac{\partial\phi}{\partial n} = \frac{\partial\psi}{\partial n} = 0, & x \in \partial\Omega, \end{cases} \tag{16}$$

where μ denotes the eigenvalue and (ϕ, ψ) denotes the corresponding eigenfunction. It follows from the first equation of (15) that $\lambda_1^{\Omega,N}(b(x)u_a - a) = 0$. If $\phi \not\equiv 0$, by the first equation of (16), we obtain that

$$\mu \ge \lambda_1^{\Omega,N}(2b(x)u_a - a) > \lambda_1^{\Omega,N}(b(x)u_a - a) = 0.$$

If $\phi \equiv 0$, then $\psi \not\equiv 0$. It follows from the second equation of (15) that $\lambda_1^{\Omega,N}(s(\frac{v_a}{u_a} - 1)) = 0$. By the second equation of (16), we obtain that

$$\mu \ge \lambda_1^{\Omega,N}(s(\frac{2v_a}{u_a} - 1)) > \lambda_1^{\Omega,N}(s(\frac{v_a}{u_a} - 1)) = 0.$$

In conclusion, we always have $\mu > 0$ and therefore $index(I - L(0; \cdot), (u_a, v_a)) = 1$. So we have that $deg(I - L(1; \cdot), E, 0) = 1$, and then $L(1; \cdot)$ has at least one fixed point in E. In other words, (6) has at least one positive solution. ■

Remark 7. (Numerical example) *We consider the effect of degenerate on the positive solution to (6): fixed $b(x)$, but variable $\beta(x)$. Let $\Omega = (0, 5\pi)$, $b(x) = \begin{cases} 1 & \pi \le x \le 4\pi \\ 0 & otherwise \end{cases}$ and $\beta(x)$ presents the following three cases:*

case 1. $\beta(x) = \begin{cases} 1 & 0.5\pi \le x \le 4.5\pi \\ 0 & otherwise \end{cases}$ $(Figure\ 1)$,

case 2. $\beta(x) = \begin{cases} 1 & \pi \le x \le 4\pi \\ 0 & otherwise \end{cases}$ $(Figure\ 2)$,

case 3. $\beta(x) = \begin{cases} 1 & 2\pi \le x \le 3\pi \\ 0 & otherwise \end{cases}$ $(Figure\ 4)$.

For better comparison, we also give the parameter homogeneous case: $b(x) \equiv 1$ and $\beta(x) \equiv 1$ (Figure 5). In fact, if $a = 0.7, \alpha = 1, s = 1.5, b = 1$ and $\beta = 1$, the kinetic system corresponding to (4) has a unique positive equilibrium point (Figure 3), which is linearly stable (Locally asymptotically stable).

REFERENCES

[1] J.D. Wollkind, J.A. Logan, "Temperature-dependent predator-prey mite ecosystem on apple tree foliage", Journal of Mathematical Biology, vol. 6, issue 3, pp. 265–283, 1978.

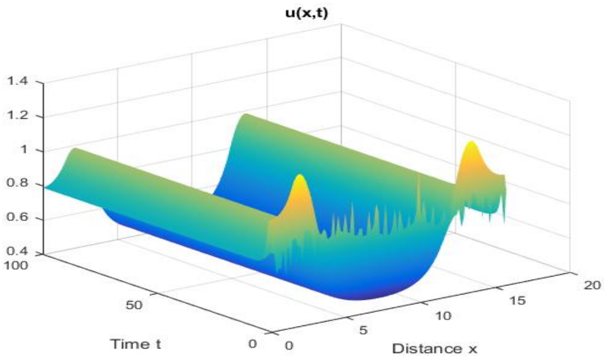
[2] C.S. Holling, "The functional response of predator to prey density and its role in mimicry and population regulation", Memoirs of the Entomological Society of Canada, vol. 91, issue 45, pp. 385-398, 1959.

[3] J.R. Beddington, "Mutual interference between parasites or predators and its effect on searching efficiency", Journal of Animal Ecology, vol. 44, issue 1, pp. 331–340, 1975.

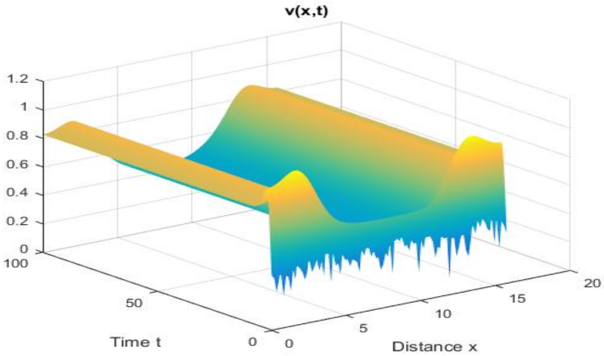
[4] S.B. Hsu, T.W. Huang, "Global stability for a class of predator-prey systems", SIAM Journal on Applied Mathematics, vol. 55, issue 3, pp. 763–783, 1995.

[5] X. Feng, K. Shi, J. Tian, T. Zang, "Existence, multiplicity, and stability of positive Solutions of a predator-prey model with Dinosaur functional response", Mathematical Problems in Engineering, vol. 2017, issue 2, pp. 1-10, 2017.

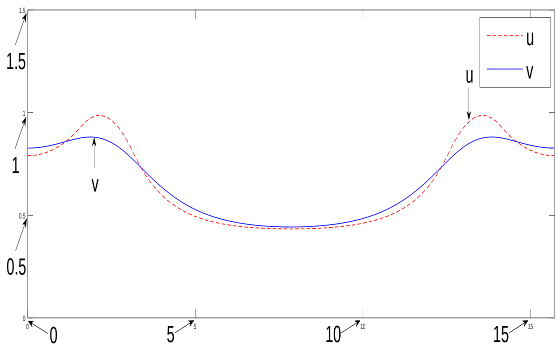
[6] Y. Du, S.B. Hsu, "A diffusive predator-prey model in heterogeneous environment", Journal of Differential Equations, vol. 203, issue 2, pp. 331–364, 2004.



(a)

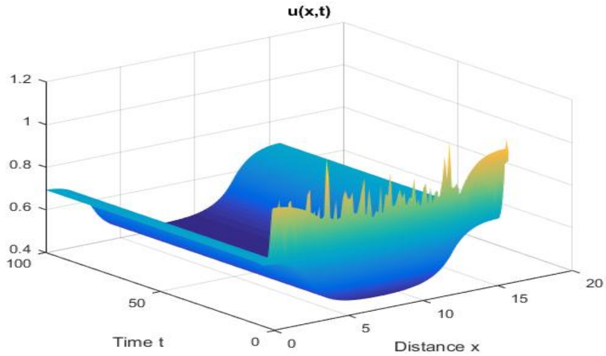


(b)

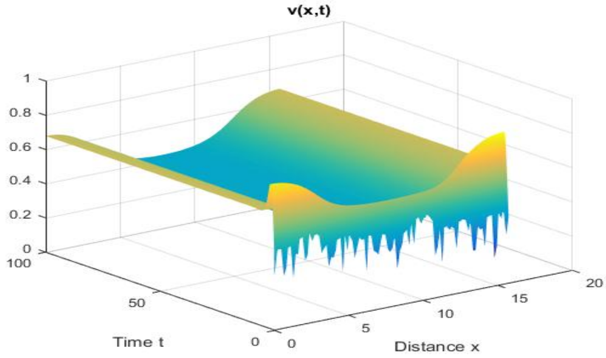


(c)

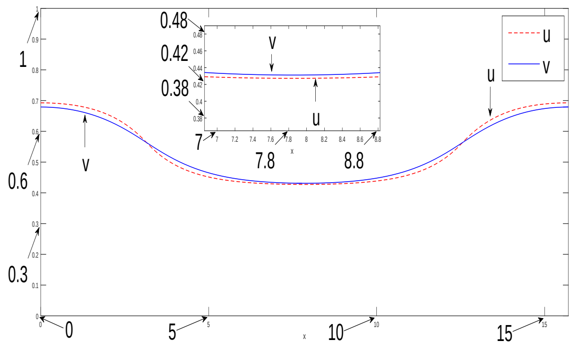
Fig. 1. Numerical simulation of the spatio-temporal positive solution to (4) with $a = 0.7, \alpha = 1$ and $s = 1.5$: case 1.



(a)



(b)



(c)

Fig. 2. Numerical simulation of the spatio-temporal positive solution to (4) with $a = 0.7, \alpha = 1$ and $s = 1.5$: case 2.

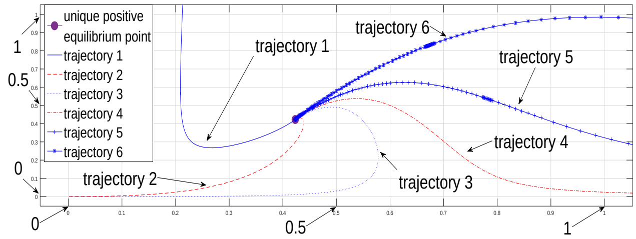


Fig. 3. Stability of the kinetic system corresponding to (4) with $a = 0.7, \alpha = 1, s = 1.5, b = 1$ and $\beta = 1$.

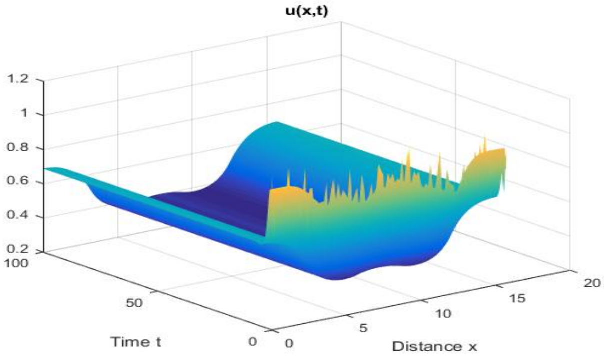
[7] R. Peng, M. Wang, M. Yang, "Positive solutions of a diffusive prey-predator model in a heterogeneous environment", Mathematical & Computer Modelling, vol. 11-12, issue 46, pp. 1410–1418, 2007.

[8] Y. Lou, W.M. Ni, "Diffusion, self-diffusion and cross-diffusion", Journal of Differential Equations, vol. 131, issue 1, pp. 79–131, 1996.

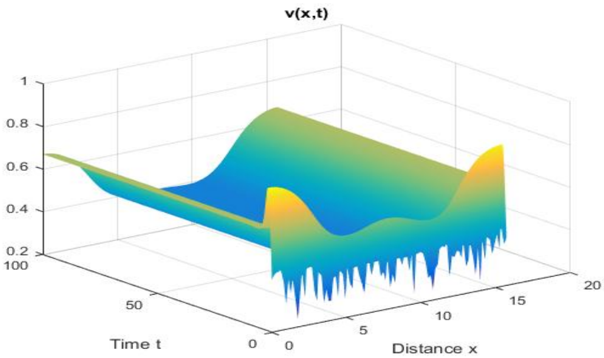
[9] C.S. Lin, W.M. Ni, I. Takagi, "Large amplitude stationary solutions to a chemotaxis system", Journal of Differential Equations, vol. 72, issue 1, pp. 1–27, 1988.

[10] M.G. Crandall, P.H. Rabinowitz, "Bifurcation from simple eigenvalues", Journal of Functional Analysis, vol. 8, issue 2, pp. 321–340, 1971.

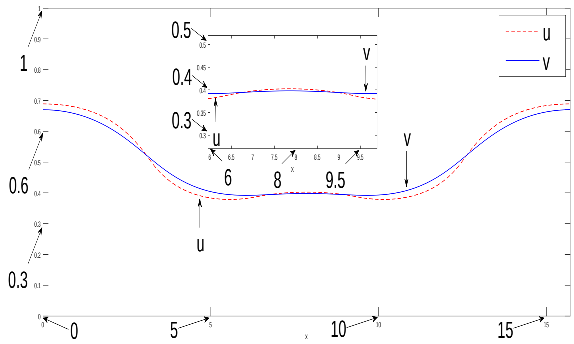
[11] R. Peng, M. Wang, M. Yang, "Positive solutions of a diffusive prey-predator model in a heterogeneous environment", Mathematical and Computer Modelling, vol. 46, issue 11-12, pp. 1410–1418.



(a)

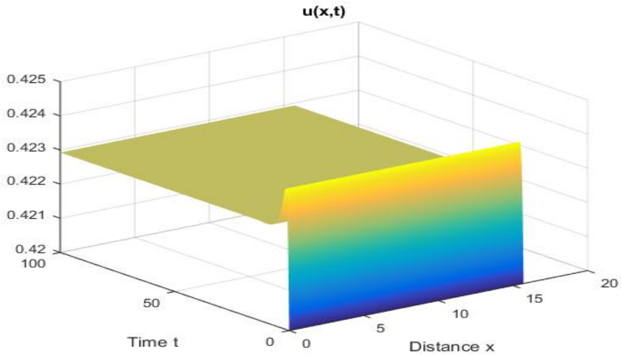


(b)

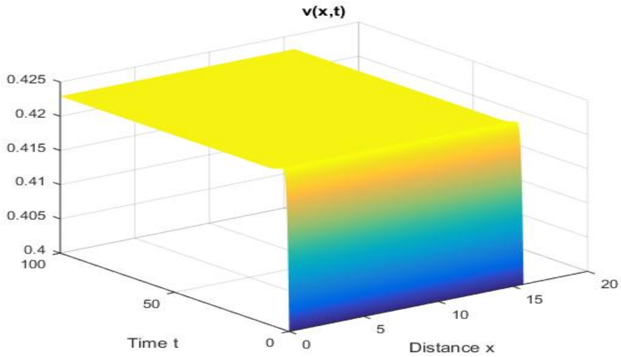


(c)

Fig. 4. Numerical simulation of the spatio-temporal positive solution to (4) with $a = 0.7, \alpha = 1$ and $s = 1.5$: case 3.



(a)



(b)

Fig. 5. Numerical simulation of the spatio-temporal positive solutions to (4) with $a = 0.7, \alpha = 1$ and $s = 1.5$: $b(x) \equiv 1$ and $\beta(x) \equiv 1$.