A Relative Closeness Coefficient Model Based on the Distance of Virtual DMUs Cross-Efficiency Method for Ranking Thai Economic Development

Narong Wichapa, and Suphan Sodsoon

Abstract—**Economic development is a process that seeks to improve economic well-being to enhance the quality of life of a community. So, sustainable economic development is an important goal for all countries. In this study, the cross-efficiency scores of each decision-making unit (DMU) were calculated using the virtual DMUs cross-efficiency approach, which is based on the distance between ideal and anti-ideal DMUs. The interval cross-efficiency matrix was generated using the viewpoints of minimization and maximization of the objective function (Virtual DMUs distance) of the virtual DMUs cross-efficiency method. After that, we formulated a novel optimization model based on the perspectives of TOPSIS, called the relative closeness coefficient model (RCC model), to convert interval cross-efficiency scores into crisp scores for ranking the DMUs. The proposed method was validated using four numerical examples involving six nursing homes, fourteen bank branches, fourteen international passenger airlines, and the economic development of twenty Thai provinces. The ranking results were compared with other cross-efficiency method rankings. The results show that the ranking results derived from the proposed method were easy to use but effective for solving DEA ranking problems.**

Index Terms— **cross-efficiency method, relative closeness coefficient, TOPSIS, data envelopment analysis, economic development, virtual DMUs**

I. Introduction

ECONOMIC development is related to increases in consumption, savings, investments and rising incomes of the population. Undoubtedly, it is a principal goal of all countries. Thailand has worked ceaselessly over the past four decades to build its economy by moving from a low-income country to a middle-income country. Gross Domestic Product, also known as GDP, is the market worth of all finished goods and services produced inside Thailand's borders over time. Economic growth can be assessed from the perspective of GDP. Since economic development is a process that seeks to improve economic wellbeing to enhance the quality of life of a community, the Thai government has formulated a national economic and social development plan and other policies to continually encourage budget allocation, investments, and infrastructure development [1]. The Gross Provincial Product (GPP) is provincial national income data that can describe a province's overall economy, based on the same concept as the GDP; the GPP can assess the well-being of the people of a province. Therefore, allocating government budgets and other resources for producing all finished goods and services can improve the people's livelihoods in the province. However, each province's budget allocation and other resources should be measured using these inputs to improve GPP. Therefore, measuring efficiency and ranking each province using these input factors are essential to set up appropriate government policies for the economic development of each province.

Data envelopment analysis (DEA) model, pioneered by Charn et al. [2], is a nonparametric technique for estimating the relative efficiency of a group of homogenous decision-making units (DMUs) with various inputs and outputs [3]. Over the past four decades, DEA approaches have garnered increasing interest from academics and researchers in a variety of domains such as economics [4], engineering [5], logistics [6], agriculture [7] et cetera. If the relative efficiency score of a DMU is determined to be one, then the DMU is considered efficient. Otherwise, it is said that the DMU is inefficient. However, due to self-assessment, the DMU can measure its own performance using the most beneficial weight. Consequently, it is impossible to distinguish between efficient DMUs [8]. The data envelopment analysis cross-efficiency method (DEA-CE method), offered by Sexton [9], can be used to discriminate DMUs and make weight choices more acceptable to overcome the disadvantages of the traditional DEA models. Unlike traditional DEA models, the DEA-CE method adopts the traditional DEA models with peer-assessment and self-assessment to assess and rank homogenous DMUs. This method allows each DMU to calculate its relative efficiency score from its favorable DEA weights and n-1 peer-assessment relative efficiency scores generated by the favorable DEA weights of other DMUs. Then, each DMU's

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average cross-efficiency score (ACE score) must be calculated first. Each DMU can be ranked based on its ACE value; if the DMU has a higher ACE score, it will be ranked healthier [10]. This principle has been extended to other cross-efficiency methods for improving the performance of DMU rankings [11], especially the second goal method. Due to its strong discrimination ability, the DEA-CE principles have been widely applied to DEA ranking problems in various fields [12].

Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is one of the multi-attribute decision analysis methods (MADM methods) that was originally offered in 1981by Hwang and Yoon [13]. TOPSIS operates on the premise that the selected alternative must have the shortest Euclidean distance from the positive ideal solution (PIS) and the greatest Euclidean distance from the negative ideal solution (NIS). Based on TOPSIS principles, the distances to both PIS and NIS must be assessed concurrently, and a preference order is ordered according to its relative closeness coefficient. According to Kim et al. [14], Shih et al. [15] and our observations, the principal benefits of the TOPSIS technique are (i) the rationale of human choice as represented by sound logic; (ii) relative closeness coefficient that concurrently considers both the best and worst alternatives and (iii) a simple calculation process. These principal benefits make TOPSIS a primary MADM method, and it has been widely used [16],[17],[18],[19],[20],[21].

According to the literature review, the virtual DMUs cross-efficiency method, proposed by Wang et al. [22], is one of the powerful DEA cross-efficiency methods for discrimination among DMUs. It is an attractive method because the concepts of TOPSIS were taken into the traditional cross-efficiency model, especially Model III [22], in which the distances between the ideal decision-making unit (IDMU), the anti-ideal decision-making unit (ADMU) and the evaluated decision-making unit (DMU_d), called virtual DMUs distances, are considered simultaneously. Unfortunately, they consider only the maximization perspective of the virtual DMUs distance (Max Z). In fact, the minimization perspective of the virtual DMUs distance (Min Z) should not be overlooked. The cross-efficiency ranking that considers both maximization and minimization perspectives of the virtual DMUs distance is more comprehensive and reliable than individual perspectives. In this paper, both perspectives (Min Z and Max Z) of the cross-efficiency model obtained from model III [22] are considered simultaneously, to produce the interval cross-efficiency matrix (ICE matrix). Subsequently, we introduce a new optimization model based on perspectives of TOPSIS, called the relative closeness coefficient model (RCC model), to convert interval cross-efficiency scores into crisp scores (relative closeness coefficients) to rank all DMUs. The principal advantages of the proposed method are that it can be applied to tackle large-size problems with interval data, and it can be employed to incorporate various DEA cross-efficiency methods for discrimination among DMUs. Moreover, the proposed model is simple to calculate using any optimization solver. Using the proposed method, decision-makers can make more trustworthy choices than with the individual cross-efficiency methods.

The remainder of this study is structured as follows. Section 2 is the literature review. Section 3 presents the traditional cross-efficiency approach. Section 4 presents the cross-efficiency approach based on virtual DMUs. Section 5 introduces the proposed RCC model for ranking DMUs. Section 6 presents two numerical examples of the proposed method. Section 7 is the Conclusion.

II. Literature Review

The DEA cross-efficiency approach is one of the most effective approaches to rank the DMUs for DEA problems. Despite the widespread use of cross-efficiency approaches the non-uniqueness of optimal weights has become one of the most significant problems [23]. To address this problem, Sexton et al. [9] developed the DEA cross-efficiency models based on the secondary goal method. Doyle and Green [23] presented the most well-known secondary goal-based cross-efficiency models, the aggressive and benevolent cross-efficiency models. Based on this concept, Liang et al. [24] developed secondary goal-based cross-efficiency models for this work [23]. Wang and Chin [25] further developed alternative models for the cross-efficiency method to expand the number of DEA methodological options. Wu et al. [26] extended the DEA cross-efficiency model based on the secondary goal method for weight selection, considering both desirable and undesirable targets of all the DMUs. Wang et al. [27], [28] proposed another approach for the secondary goal model, the DEA neutral model. Liu et al. [29] presented a revised neutral model based on the secondary goal. Besides, the game cross-efficiency method was offered for the cross-efficiency evolution process. Liang et al. [30] proposed an effective model based on the game cross-efficiency model. The concept of this method has been extended to various game cross-efficiency models [31], [32]. Besides, Wang et al. [33] offered the DEA cross-efficiency models based on virtual DMUs for ranking all DMUs. The proposed models are effective and attractive models because the ideas of TOPSIS were taken into the traditional cross-efficiency model. In Model III [22], the virtual DMUs distance has been maximized for DMU rankings. Unfortunately, they offer only maximization of the objective function (virtual DMUs distance) of the proposed DEA cross-efficiency model. The information contained in the different perspectives of DEA cross-efficiency models should be considered together for increased reliability in the decision-making process. Undoubtedly, ranking results considering both perspectives provide more comprehensive and reliable perspectives.

Combining the DEA cross-efficiency score an additional significant method. The arithmetic average technique is the most prevalent. However, it cannot be utilized to analyze the correlation between weights and efficiency scores [33]. Additionally, many researchers [34],[35],[36],[37],[38] have proposed combination cross-efficiency methods: the perspective of entropy weight, CRITIC weight, standard deviation weight and others. Although the entropy weight is widely used in the cross-efficiency method, its application to interval data of cross-efficiency scores has been offered recently. Wang et al. [39] proposed a Shannon's entropy method to convert the interval cross-efficiency scores into crisp scores to rank all DMUs. Lu and Liu [40] proposed a novel Gibbs entropy optimization model to evaluate the optimal Gibbs entropy values to rank all DMUs. This model is exciting because it can be utilized to convert interval cross-efficiency scores into crisp scores to rank DMUs. It is simple to apply optimization solvers in computing. Recently, Wichapa et al. [41] presented a novel Gibbs entropy model for combining interval cross-efficiency scores, which are derived from aggressive and benevolent cross-efficiency models for ranking DMUs. The main advantages of this method are that it can be applied to solve large-size ranking problems with uncertainty, and it can be used to combine other cross-efficiency models to rank DMUs. Besides, it is simple but powerful to use in computing by using optimization solvers. Inspired by the above ideas of the combination of the cross-efficiency score, we present an alternative ranking method for interval cross-efficiency score derived from both maximization and minimization perspectives of the objective function (Virtual DMUs distance) of the proposed cross-efficiency model (M-III [22]) to rank DMUs. To be precise, the proposed method should be able to solve large-size problems with interval data, and it should be able to combine other cross-efficiency models for ranking DMUs. Furthermore, the proposed model should be simple to calculate with any optimization solver. Certainly, by using the proposed method, decision-makers can achieve more reliable decisions than individual perspectives.

In this paper, we consider the combination of maximization and minimization perspectives of virtual DMUs distance of model III [22] for ranking DMUs. Our method uses the virtual DMUs cross-efficiency models to make full use of the information contained in each model (each perspective). In addition, the novel RCC model, based on the concepts of TOPSIS, is developed to transform interval cross-efficiency scores, derived from both maximization and minimization perspectives of model III [18], into crisp values for ranking all DMUs.

III. Cross-Efficiency Method

A. CCR Model

The CCR model, which was developed by Charnes et al. [2], is a powerful mathematical model for assessing the performance of a group of DMUs with multiple outputs and multiple inputs. Assume that each DMU$_j$ (j = 1,2,…, n) with the inputs (x_{ij}, i = 1,.2,…,m) produces outputs of DMU$_j$ (y_{rj}, r = 1,2,..., s). Let u_{rk} and v_{ik} be the weights of outputs and inputs respectively. For a set of DMU$_d$ ($1 \le d \le n$), the cross-efficiency score (θ_{dd}) can be obtained by the CCR model as in model (1).

$$\begin{aligned} &\text{Max}\, Z = \theta_{dd} = \sum_{r=1}^{s} u_{rd} y_{rd} \\ &\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} \le 0, \forall j, j = 1,2,3,...,n \\ &\sum_{i=1}^{m} v_{id} x_{id} = 1, \forall j \\ &v_{id} \ge 0, \forall i, i = 1,2,3,...,m \\ &u_{rd} \ge 0, \forall r, r = 1,2,3,...,s \end{aligned} \tag{1}$$

B. The Concept of the Cross-Efficiency Method

After solving Equation (1), let u_{rd}^* and v_{id}^* be the optimal weights of outputs and inputs respectively for a given DMU$_d$, then the cross-efficiency scores of each DMU$_j$ (j=1,2,3,…n, $j \ne d$) peer-evaluated by DMU$_d$ are given by

$$\theta_{dj} = \frac{\sum_{r=1}^{s} u_{rd}^* y_{rj}}{\sum_{i=1}^{m} v_{id}^* x_{ij}}, d, j = 1,2,3,\ldots,n \tag{2}$$

As a result, the average cross-efficiency score of DMU$_j$ ($\bar{\theta}_j$), defined by Sexton [9], is as follows.

$$\bar{\theta}_j = \frac{1}{n} \sum_{d=1}^{n} \theta_{dj}, d, j = 1,2,3,...,n \tag{3}$$

IV. Cross-Efficiency Method Based on Ideal and Anti-Ideal Decision-Making Units

This section presents the well-known cross-efficiency method based on virtual DMUs for calculating interval cross-efficiency scores.

The following are definitions of an Ideal Decision-Making Unit (IDMU) and an Anti-Ideal Decision-Making Unit (ADMU) [22]:

Definition 1. A virtual DMU is an IDMU if it requires the fewest inputs to generate the most outputs. If a virtual DMU consumes the greatest number of inputs but produces the smallest number of outputs, it can be categorized as an ADMU.

Based on Definition 1, the inputs, and outputs of an IDMU can be described as follows:

$$\begin{aligned} &x_i^p = \min\{x_{ij}\},\ i = 1,2,3,...,n \\ &y_r^p = \max\{y_{rj}\}\ r = 1,2,3,...,s \end{aligned}$$

The inputs and outputs of the ADMU are defined as

$$\begin{aligned} &x_i^n = \max\{x_{ij}\},\ i = 1,2,3,...,n \\ &y_r^n = \min\{y_{rj}\}\ r = 1,2,3,...,s \end{aligned}$$

Definition 2. The distances of IDMU, ADMU and between IDMU and ADMU are defined as

$$\begin{aligned} &D_{\text{d}}^{\text{IDMU}} = \sum_{i=1}^{m} v_{id}(x_{id} - x_i^p) + \sum_{r=1}^{s} u_{rd}(y_i^p - y_{rd}), d = 1,2,...,n \\ &D_{\text{d}}^{\text{ADMU}} = \sum_{i=1}^{m} v_{id}(x_i^n - x_{id}) + \sum_{r=1}^{s} u_{rd}(y_{rd} - y_i^n), d = 1,2,...,n \\ &D_{\text{ADMU}}^{\text{IDMU}} = \sum_{i=1}^{m} v_{id}(x_i^n - x_i^p) + \sum_{r=1}^{s} u_{rd}(y_i^p - y_i^n), d = 1,2,...,n \end{aligned}$$

In Wang's Model-III, the maximization of the virtual DMUs distance is given by

$$
\begin{aligned}
& Max\ D_{\mathrm{ADMU}}^{\mathrm{IDMU}} = \sum_{i=1}^{m} v_{id}(x_i^n - x_i^p) + \sum_{r=1}^{s} u_{rd}(y_i^p - y_i^n) \\
& \sum_{i=1}^{m} v_{id} \cdot x_{id} = 1, \\
& \sum_{r=1}^{s} u_{rd} \cdot y_{rd} = \theta_{dd}^{*}, \qquad (4)\\
& \sum_{r=1}^{s} u_{rd} \cdot y_{rj} - \sum_{i=1}^{m} v_{id} \cdot x_{ij} \le 0,\ \forall j,\ j = 1,2,\dots,n, \\
& v_{id} \ge 0,\ i = 1,2,\dots,m \\
& u_{rd} \ge 0\ \ r = 1,2,\dots,s
\end{aligned}
$$

Details of the other virtual DMUs cross-efficiency models are shown in Wang et al. [22].

V. PROPOSED METHOD

This section presents a new method for ranking a set of DMUs based on the virtual DMUs distance.Fig.1 depicts the proposed framework in this paper.

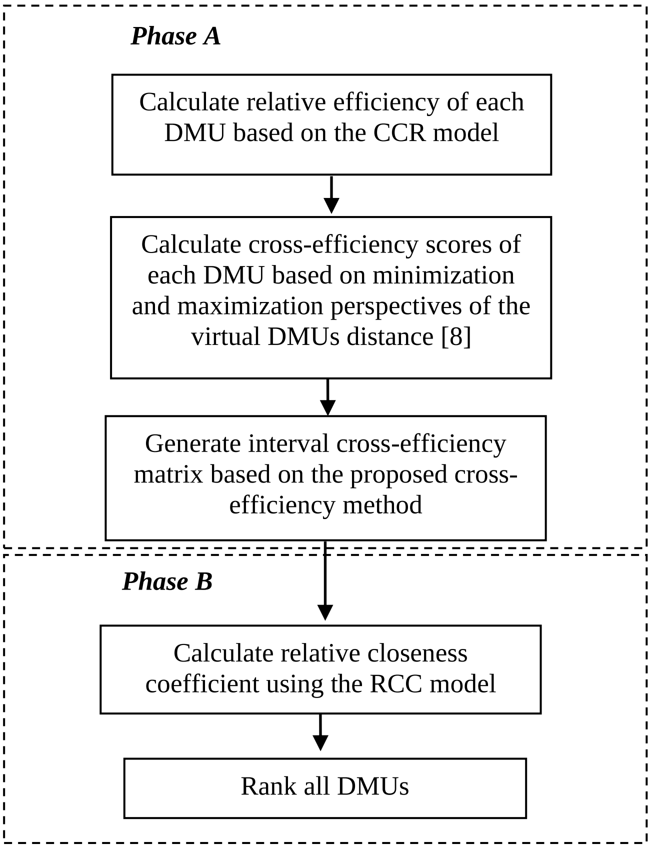

Fig.1. The framework for this paper.

A. Generating the ICE matrix

This section presents a combination of both perspectives of min Z and max Z of model (4) to enhance the effectiveness and reliability of the assessment. Details of calculation steps are as follows.

(1) Calculate the relative efficiency scores of each DMU based on the CCR model, namely the CCR score.

(2) Calculate the cross-efficiency scores of each DMU based on min Z and max Z of model (4).

(3) Generate ICE matrix based on the cross-efficiency scores of both perspectives of model (4).

Based on model (4), both min Z and max Z perspectives must be calculated first. As a result, the ICE matrix can be generated using the minimization perspective ($X^{\mathrm{min}Z}$) and the maximization perspective ($X^{\mathrm{max}Z}$). Details are shown in Eq.(5) and Eq.(6).

$$
X^{\mathrm{min}Z} = \begin{bmatrix} x_{11}^{\mathrm{min}Z} & x_{12}^{\mathrm{min}Z} & x_{13}^{\mathrm{min}Z} & \cdots & x_{1n}^{\mathrm{min}Z} \\ x_{21}^{\mathrm{min}Z} & x_{22}^{\mathrm{min}Z} & x_{23}^{\mathrm{min}Z} & \cdots & x_{2n}^{\mathrm{min}Z} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{m1}^{\mathrm{min}Z} & x_{m2}^{\mathrm{min}Z} & x_{m3}^{\mathrm{min}Z} & \cdots & x_{mn}^{\mathrm{min}Z} \end{bmatrix}
$$

$$
X^{\mathrm{max}Z} = \begin{bmatrix} x_{11}^{\mathrm{max}Z} & x_{12}^{\mathrm{max}Z} & x_{13}^{\mathrm{max}Z} & \cdots & x_{1n}^{\mathrm{max}Z} \\ x_{21}^{\mathrm{max}Z} & x_{22}^{\mathrm{max}Z} & x_{23}^{\mathrm{max}Z} & \cdots & x_{2n}^{\mathrm{max}Z} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{m1}^{\mathrm{max}Z} & x_{m2}^{\mathrm{max}Z} & x_{m3}^{\mathrm{max}Z} & \cdots & x_{mn}^{\mathrm{max}Z} \end{bmatrix} \qquad (5)
$$

and

where indices i and j are DMU and target DMU, respectively. The $x_{ij}^{\min Z}$ and $x_{ij}^{\max Z}$ are cross-efficiency scores of the minimization and maximization perspectives, respectively. The ICE matrix based on values of ($X^{\mathrm{min}Z}$) and ($X^{\mathrm{max}Z}$) can be generated as

$$
X = \begin{bmatrix} [x_{11}^l, x_{11}^u] & [x_{12}^l, x_{12}^u] & & \cdots & [x_{1n}^l, x_{1n}^u] \\ [x_{21}^l, x_{21}^u] & [x_{22}^l, x_{22}^u] & & \cdots & [x_{2n}^l, x_{2n}^u] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ [x_{m1}^l, x_{m1}^u] & [x_{m2}^l, x_{m2}^u] & & \cdots & [x_{mn}^l, x_{mn}^u] \end{bmatrix} \qquad (6)
$$

where $X = \left[X^{\min Z}, X^{\max Z} \right]$, $x_{ij}^l = \min\left\{ x_{ij}^{\min Z}, x_{ij}^{\max Z} \right\}$ and $x_{ij}^u = \max\left\{ x_{ij}^{\min Z}, x_{ij}^{\max Z} \right\}$, $\forall i = 1,2,\dots,m,\ \forall j = 1,2,\dots,n$.

B. Formulating the Relative Closeness Coefficient Model

This section offers a novel optimization model based upon the ideas of TOPSIS for ranking DMUs, called the RCC model. The distance between IDMU, ADMU and DMU_i will be considered. The proposed RCC model is described as follows. Let RCC_i (i=1,2,3,…, n) be the relative closeness coefficient of DMU_i, η and ρ are the maximum weights of the summation of distance from negative ideal point to DMU_i and the summation of distances from positive ideal point to DMU_i respectively. Let x^n and x^p be the negative ideal and positive ideal values of all elements in ICE matrix, $x^n = \min\left\{ x_{ij}^l, x_{ij}^u \right\}$ and $x^p = \max\left\{ x_{ij}^l, x_{ij}^u \right\}$, $i, j = 1,2,\dots,n$, then the RCC model is

$$
\mathrm{RCC_i} = \frac{\sum_{j=1}^{n} \eta((x_{ij}^l - x^n) + (x_{ij}^u - x^n))}{\sum_{j=1}^{n} \eta((x_{ij}^l - x^n) + (x_{ij}^u - x^n)) + \sum_{j=1}^{n} \rho((x^p - x_{ij}^l) + (x^p - x_{ij}^u))},
$$

$$
\forall i, j = 1,2,\dots,n
$$

$$
\eta, \rho \ge 0
$$

To achieve the optimal solution of RCC_i (maximum relative closeness efficient), the maximum value of RCC_i (the η and ρ must be maximum values) can be formulated as

$$\text{Max RCC}_i = \frac{\sum_{j=1}^{n} \eta((x_{ij}^{l} - x^{n}) + (x_{ij}^{u} - x^{n}))}{\sum_{j=1}^{n} \eta((x_{ij}^{l} - x^{n}) + (x_{ij}^{u} - x^{n})) + \sum_{j=1}^{n} \rho((x^{p} - x_{ij}^{l}) + (x^{p} - x_{ij}^{u}))},$$

$$\forall i, j = 1,2,3,..,n$$

$$\eta, \rho \geq 0, \eta = \rho$$

The above model is a nonlinear fractional programming model. Based on the concept of Charnes and Cooper [42], the above model can be converted to model (7).

$$\begin{aligned} &\text{Max RCC}_i = \sum_{j=1}^{n} \eta((x_{ij}^{l} - x^{n}) + (x_{ij}^{u} - x^{n})) \\ &\sum_{j=1}^{n} \eta((x_{ij}^{l} - x^{n}) + (x_{ij}^{u} - x^{n})) + \\ &\sum_{j=1}^{n} \rho((x^{p} - x_{ij}^{l}) + (x^{p} - x_{ij}^{u})) = 1, \\ &\forall i, j = 1,2,3,..,n, \\ &\eta, \rho \geq 0, \eta = \rho; \end{aligned} \tag{7}$$

The objective function of the proposed model is to be maximized; the RCC_i, η and ρ can be achieved using model (7). After obtaining the RCC_i, a set of DMUs can be ranked by preference in descending order of the RCC_i value. The larger the RCC_i value, the better the DMU's ranking.

VI. NUMERICAL EXAMPLES

Four numerical examples, including six nursing homes, fourteen bank branches, fourteen international passenger airlines, and the economic development of twenty provinces in Thailand, are provided in this section to illustrate the potential applications of the proposed method and its effectiveness in ranking DMUs. Details are provided below.

A. Six Nursing Homes

The six nursing homes proposed by Sexton et al. [9] consist of two inputs (X_1 and X_2) and two outputs (Y_1 and Y_2). Let X_1 and X_2 be staff hours per day and supplies per day respectively. Let Y_1 and Y_2 be total Medicare-plus-Medicaid reimbursed patient days and total privately paid patient days respectively. The data set for the six nursing homes is shown in TABLE I.

TABLE I
DATA SET FOR SIX NURSING HOMES

DMUs	X_1	X_2	Y_1	Y_1	CCR
1	1.50	0.20	1.40	0.35	1.0000
2	4.00	0.70	1.40	2.10	1.0000
3	3.20	1.20	4.20	1.05	1.0000
4	5.20	2.00	2.80	4.20	1.0000
5	3.50	1.20	1.90	2.50	0.9775
6	3.20	0.70	1.40	1.50	0.8675

The CCR scores of DMUs must be calculated first using Eq. (1). Next, both min Z and max Z perspectives of Eq. (4), were solved using LINGO software. As a result, the cross-efficiency matrices of each perspective were generated as listed in TABLE II and TABLE III, respectively.

TABLE II
THE CROSS-EFFICIENCY MATRIX OF MAXIMIZATION PERSPECTIVE

DMUs/Target DMUs	1	2	3	4	5	6
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	0.8640	1.0000	1.0000	1.0000
3	0.5000	0.5000	1.0000	0.8295	0.8295	0.8295
4	0.7000	0.7000	1.0000	1.0000	1.0000	1.0000
5	0.7083	0.7083	0.9676	0.9775	0.9775	0.9775
6	0.7551	0.7551	0.8046	0.8675	0.8675	0.8675

TABLE III
THE CROSS-EFFICIENCY MATRIX OF MINIMIZATION PERSPECTIVE

DMUs/Target DMUs	1	2	3	4	5	6
1	1.0000	0.4977	0.7111	0.2889	1.0000	1.0000
2	0.3505	1.0000	0.2667	0.6500	1.0000	1.0000
3	1.0000	0.4129	1.0000	0.4063	0.8295	0.8295
4	0.4056	1.0000	0.4103	1.0000	1.0000	1.0000
5	0.4301	0.9506	0.4136	0.8844	0.9775	0.9775
6	0.4099	0.8027	0.3333	0.5804	0.8675	0.8675

After obtaining the cross-efficiency matrices of both perspectives, the ICE matrix was generated using Eq. (6). Details of the ICE matrix of the six nursing homes problem are shown in TABLE IV.

TABLE IV
THE INTERVAL CROSS-EFFICIENCY MATRIX OF SIX NURSING HOMES

DMUs	1	2	3	4	5	6
1	[1.0000, 1.0000]	[0.4977, 1.0000]	[0.7111, 1.0000]	[0.2889, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]
2	[0.3505, 1.0000]	[1.0000, 1.0000]	[0.2667, 0.8640]	[0.6500, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]
3	[0.5000, 1.0000]	[0.4129, 0.5000]	[1.0000, 1.0000]	[0.4063, 0.8295]	[0.8295, 0.8295]	[0.8295, 0.8295]
4	[0.4056, 0.7000]	[0.7000, 1.0000]	[0.4103, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]
5	[0.4301, 0.7083]	[0.7083, 0.9506]	[0.4136, 0.9676]	[0.8844, 0.9775]	[0.9775, 0.9775]	[0.9775, 0.9775]
6	[0.4099, 0.7551]	[0.7551, 0.8027]	[0.3333, 0.8046]	[0.5804, 0.8675]	[0.8675, 0.8675]	[0.8675, 0.8675]

After obtaining the ICE matrix, the values of RCC_i were calculated. For example, to obtain the value of RCC_1 value, the relevant data listed in TABLE IV were entered into Eq. (7):

$$\begin{aligned} x^{n} &= \min\{[1.0000, 1.0000], [0.4977, 1.0000], ..., [0.8675, 0.8675]\} \\ &= 0.2667 \\ x^{p} &= \max\{[1.0000, 1.0000], [0.4977, 1.0000], ..., [0.8675, 0.8675]\} \\ &= 1.0000 \end{aligned}$$

As a result, the mathematical model for finding the optimal value of RCC_1 is

$$
\begin{aligned}
\text{Max } RCC_1 = {} & \\
& \eta[(1.0000-0.2667)+(1.0000-0.2667)+ \\
& (0.4977-0.2667)+(1.0000-0.2667)+ \\
& (0.7111-0.2667)+(1.0000-0.2667)+ \\
& (0.2889-0.2667)+(1.0000-0.2667)+ \\
& (1.0000-0.2667)+(1.0000-0.2667)+ \\
& (1.0000-0.2667)+(1.0000-0.2667)];
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
& \eta[(1.0000-0.2667)+(1.0000-0.2667)+ \\
& (0.4977-0.2667)+(1.0000-0.2667)+ \\
& (0.7111-0.2667)+(1.0000-0.2667)+ \\
& (0.2889-0.2667)+(1.0000-0.2667)+ \\
& (1.0000-0.2667)+(1.0000-0.2667)+ \\
& (1.0000-0.2667)+(1.0000-0.2667)]+ \\
& \rho[(1.0000-1.0000)+(1.0000-1.0000)+ \\
& (1.0000-0.4977)+(1.0000-1.0000)+ \\
& (1.0000-0.7111)+(1.0000-1.0000)+ \\
& (1.0000-0.2889)+(1.0000-1.0000)+ \\
& (1.0000-1.0000)+(1.0000-1.0000)+ \\
& (1.0000-1.0000)+(1.0000-1.0000)]=1; \\
& \eta \geq 0; \rho \geq 0; \eta = \rho;
\end{aligned}
$$

In this paper, the mathematical model of RCC_1 was solved using the LINGO software. As a result, RCC_1 is 0.8293. TABLE V provides information on each RCC_i value.

TABLE V
THE RELATIVE CLOSENESS COEFFICIENT OF EACH DMU

DMUs	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6
RCC_i	0.8293	0.7876	0.6553	0.7973	0.7671	0.6339
Rank	1	3	5	2	4	6

Finally, the ranking comparisons for the proposed method, benevolent, aggressive and Wang's models [22], namely Model-III (M-III) and Model-IV (M-IV), are provided in TABLE VI.

TABLE VI
THE RANKING COMPARISONS FOR SIX NURSING HOMES

DMUs	Ben.	Rank	Agg.	Rank	M-III	Rank
1	1.0000	1	0.7639	1	1.0000	1
2	0.9773	3	0.7004	3	0.9773	2
3	0.8580	5	0.6428	5	0.7481	6
4	1.0000	1	0.7184	2	0.9000	3
5	0.9758	4	0.6956	4	0.8861	4
6	0.8570	6	0.6081	6	0.8195	5

TABLE VI
(CONTINUED)

DMUs	M-IV	Rank	Proposed	Rank
1	0.7639	1	0.8293	1
2	0.7004	3	0.7876	3
3	0.6428	5	0.6553	5
4	0.7184	2	0.7973	2
5	0.6956	4	0.7671	4
6	0.6081	6	0.6339	6

As seen in TABLE VI, the rankings of each cross-efficiency model were obtained. The aggressive, M-IV and proposed models assess that DMU_1> DMU_4 > DMU_2 > DMU_4 > DMU_3>DMU_6, but the Model-III is different. Besides, the benevolent model cannot distinguish between DMU_1 and DMU_4.

Fig. 2 shows the results of the proposed method with M-III model. From this figure, it can be seen that the proposed method tends to be inconsistent with the M-III model. Only the DMU_3 is the same ranking. However, based on TABLE VI, the proposed method tends to be consistent with the other methods.

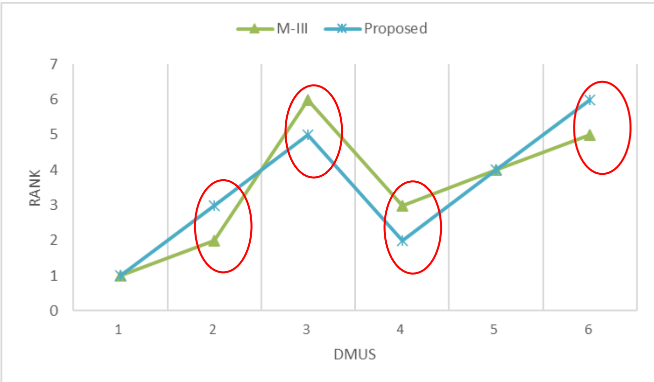

Fig.2. The ranking comparisons for the six nursing homes problem

Spearman's rank correlation test was used to evaluate the correlation coefficients (r_s) of each method. The details of the r_s values of each method are shown in TABLE VII.

TABLE VII
THE CORRELATION TEST FOR SIX NURSING HOMES

	Benevolent	Aggressive	M-III	M-IV	Proposed
Benevolent	-	0.986	0.841	0.986	0.986
Aggressive	0.986	-	0.886	1.000	1.000
M-III	0.841	0.886	-	0.886	0.886
M-IV	0.986	1.000	0.886	-	1.000
Proposed	0.986	1.000	0.886	1.000	-

As seen in TABLE VII, the r_s values for the proposed method and benevolent, aggressive, Model-III and Model-IV, are obtained as r_s = 0.986, 1.000, 0.886 and 1.000, respectively. The proposed method is highly correlated with the well-known cross-efficiency models, including benevolent, aggressive and Model-IV. By comparison with well-known models, the proposed method provides higher r_s while Model-III provides lower r_s. This means that the proposed method is more consistent with other methods than the M-III model. The comparison for the proposed method and traditional M-III is shown in Fig.3.

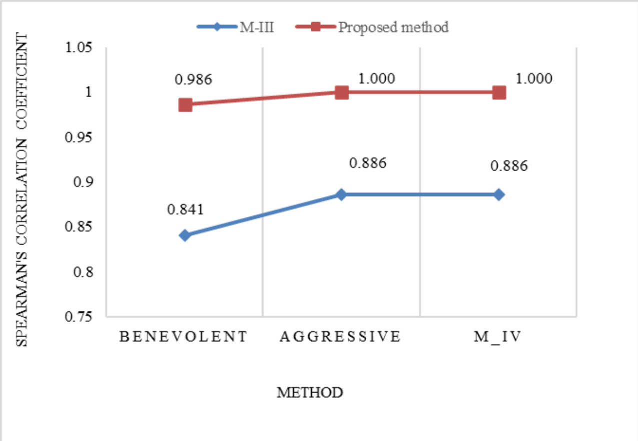

Fig.3. Comparison of the proposed method and traditional M-III

As seen in Fig.3, Spearman's rank correlation test was used to evaluate the r_s values of each method. As a result, the r_s for the proposed method and the benevolent, aggressive and M-IV were computed as r_s = 0.986, 1.000 and 1.000 respectively. The r_s for the traditional model (M-III) and the benevolent, aggressive and M-IV were computed as r_s = 0.841, 0.886 and 0.886 respectively. The results show that the proposed method is more reliable than the traditional M-III model [22].

B. *Sherman and Gold's Dataset for 14 Bank Branches*

Sherman and Gold [43] presented an actual dataset of 14 bank branches with three inputs and four outputs. The rent (thousands of dollars), fulltime equivalent personnel, and supplies (thousands of dollars) are input 1, input 2 and input 3, respectively. Details for Sherman and Gold's dataset for 14 bank branches are shown in TABLE VIII.

TABLE VIII
DATA SET OF 14 BANK BRANCHES

DMUs	X_1	X_2	X_3	Y_1	Y_2	Y_3	Y_4
1	140000	42900	87500	484000	4139100	59860	2951430
2	48800	17400	37900	384000	1685500	139780	3336860
3	36600	14200	29800	209000	1058900	65720	3570050
4	47100	9300	26800	157000	879400	27340	2081350
5	32600	4600	19600	46000	370900	18920	1069100
6	50800	8300	18900	272000	667400	34750	2660040
7	40800	7500	20400	53000	465700	20240	1800250
8	31900	9200	21400	250000	642700	43280	2296740
9	36400	76000	21000	407000	647700	32360	1981930
10	25700	7900	19000	72000	402500	19930	2284910
11	44500	8700	21700	105000	482400	49320	2245160
12	42300	8900	25800	94000	511000	26950	2303000
13	40600	5500	19400	84000	287400	34940	1141750
14	76100	11900	32800	199000	694600	67160	3338390

Using the same calculation procedure of Section A, the CCR scores of DMUs were calculated using Eq. (1). As a result, the CCR scores for DMU_1, DMU_2, DMU_3, DMU_4, DMU_5, DMU_6, DMU_7, DMU_8, DMU_9, DMU_{10}, DMU_{11}, DMU_{12}, DMU_{13} and DMU_{14} were obtained as: 1.0000, 1.0000, 1.0000, 1.0000, 0.9041, 1.0000, 0.7821, 1.0000, 1.0000, 1.0000, 0.9668, 0.8522, 0.9049, and 1.0000 respectively. As seen in the CCR scores, the efficient DMUs are DMU_1, DMU_2, DMU_3, DMU_4, DMU_6, DMU_8, DMU_9, DMU_{10}, and DMU_{14}, and they cannot differentiate between these DMUs. Using the same calculation method as Section *A* of six nursing homes, the ICE matrix for cross-efficiency score of 14 bank branches is shown in Table IX.

TABLE IX
THE INTERVAL CROSS-EFFICIENCY MATRIX OF 14 BANK BRANCHES

DMU	1	2	3	4	5	6	7
1	[1.000, 1.000]	[0.276, 0.996]	[0.270, 0.722]	[0.839, 0.916]	[0.839, 0.839]	[0.337, 0.870]	[0.722, 0.722]
2	[1.000, 1.000]	[0.770, 1.000]	[0.730, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[0.840, 1.000]	[1.000, 1.000]
3	[0.772, 0.852]	[0.553, 0.946]	[1.000, 1.000]	[0.837, 0.902]	[0.879, 0.879]	[0.743, 1.000]	[1.000, 1.000]
4	[0.751, 0.957]	[0.425, 0.976]	[0.610, 0.944]	[1.000, 1.000]	[1.000, 1.000]	[0.609, 0.921]	[0.944, 0.944]
5	[0.444, 0.785]	[0.830, 0.832]	[0.433, 0.808]	[0.880, 0.887]	[0.904, 0.904]	[0.398, 0.734]	[0.808, 0.808]
6	[0.828, 0.869]	[0.603, 1.000]	[1.000, 1.000]	[0.935, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]
7	[0.559, 0.631]	[0.721, 0.641]	[0.674, 0.782]	[0.717, 0.768]	[0.762, 0.762]	[0.558, 0.579]	[0.782, 0.782]
8	[0.718, 0.726]	[0.088, 0.854]	[0.869, 0.924]	[0.794, 0.855]	[0.841, 0.841]	[0.854, 0.966]	[0.924, 0.924]
9	[0.093, 0.729]	[0.526, 0.716]	[0.122, 0.743]	[0.094, 0.102]	[0.097, 0.097]	[0.128, 1.000]	[0.122, 0.122]
10	[0.523, 0.560]	[0.572, 0.777]	[0.902, 0.989]	[0.649, 0.759]	[0.728, 0.728]	[0.497, 0.904]	[0.902, 0.902]
11	[0.563, 0.567]	[0.593, 0.871]	[0.786, 0.791]	[0.669, 0.742]	[0.740, 0.740]	[0.566, 0.708]	[0.791, 0.791]
12	[0.499, 0.581]	[0.539, 0.616]	[0.710, 0.817]	[0.687, 0.760]	[0.747, 0.747]	[0.563, 0.650]	[0.817, 0.817]
13	[0.364, 0.519]	[0.603, 0.570]	[0.445, 0.615]	[0.607, 0.641]	[0.661, 0.661]	[0.431, 0.579]	[0.615, 0.615]
14	[0.540, 0.593]	[0.000, 0.829]	[0.751, 0.802]	[0.710, 0.782]	[0.788, 0.788]	[0.643, 0.688]	[0.802, 0.802]

TABLE IX
(CONTINUED)

DMU	8	9	10	11	12	13	14
1	[0.359, 0.660]	[0.459, 0.934]	[0.268, 0.485]	[0.256, 0.256]	[0.268, 0.268]	[0.246, 0.246]	[0.256, 0.299]
2	[0.878, 1.000]	[0.626, 1.000]	[0.732, 0.872]	[1.000, 1.000]	[0.929, 0.929]	[1.000, 1.000]	[1.000, 1.000]
3	[0.716, 1.000]	[0.484, 0.772]	[1.000, 1.000]	[0.914, 0.914]	[1.000, 1.000]	[0.864, 0.864]	[0.882, 0.914]
4	[0.610, 0.716]	[0.202, 0.698]	[0.623, 0.841]	[0.709, 0.709]	[0.736, 0.736]	[0.679, 0.679]	[0.709, 0.724]
5	[0.389, 0.443]	[0.951, 0.377]	[0.452, 0.743]	[0.785, 0.785]	[0.727, 0.727]	[0.786, 0.786]	[0.785, 0.812]
6	[1.000, 1.000]	[0.205, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[0.971, 0.971]	[1.000, 1.000]
7	[0.361, 0.551]	[0.795, 0.449]	[0.685, 0.782]	[0.727, 0.727]	[0.760, 0.760]	[0.694, 0.694]	[0.721, 0.727]
8	[1.000, 1.000]	[1.000, 1.000]	[0.875, 0.927]	[0.904, 0.904]	[0.948, 0.948]	[0.866, 0.866]	[0.880, 0.904]
9	[0.228, 1.000]	[0.334, 1.000]	[0.123, 0.472]	[0.094, 0.094]	[0.116, 0.116]	[0.085, 0.085]	[0.088, 0.094]
10	[0.438, 0.876]	[0.376, 0.451]	[1.000, 1.000]	[0.853, 0.853]	[1.000, 1.000]	[0.781, 0.781]	[0.804, 0.853]
11	[0.473, 0.685]	[0.308, 0.494]	[0.794, 0.826]	[0.967, 0.967]	[0.926, 0.926]	[0.956, 0.956]	[0.953, 0.967]
12	[0.452, 0.632]	[0.341, 0.424]	[0.726, 0.850]	[0.799, 0.799]	[0.852, 0.852]	[0.757, 0.757]	[0.779, 0.799]
13	[0.428, 0.473]	[0.453, 0.357]	[0.459, 0.608]	[0.869, 0.869]	[0.723, 0.723]	[0.905, 0.905]	[0.869, 0.892]
14	[0.552, 0.670]	[0.000, 0.507]	[0.762, 0.833]	[1.000, 1.000]	[0.928, 0.928]	[1.000, 1.000]	[1.000, 1.000]

After obtaining the ICE matrix for cross-efficiency score of 14 bank branches, the optimal RCC_i values were obtained using model (7). Finally, the ranking comparisons for all DMUs are provided in TABLE X.

TABLE X
RANKING COMPARISONS FOR 14 BANK BRANCHES

DMUs	Ben.	Rank	Agg.	Rank	M-III	Rank
DMU_1	0.6011	12	0.4391	13	0.6580	12
DMU_2	0.9758	2	0.8826	1	0.9858	1
DMU_3	0.9503	3	0.7886	4	0.8678	3
DMU_4	0.7746	8	0.6754	6	0.8462	5
DMU_5	0.6327	11	0.6011	10	0.7446	7
DMU_6	0.9886	1	0.8731	2	0.9654	2
DMU_7	0.6826	10	0.5525	12	0.6691	11
DMU_8	0.8813	4	0.8114	3	0.8640	4
DMU_9	0.4523	14	0.2269	14	0.1770	14
DMU_{10}	0.8094	5	0.6581	8	0.7181	9
DMU_{11}	0.7767	7	0.6658	7	0.7351	8
DMU_{12}	0.7029	9	0.6064	9	0.6941	10
DMU_{13}	0.5887	13	0.5811	11	0.6402	13
DMU_{14}	0.7794	6	0.6979	5	0.7686	6

TABLE X
(CONTINUED)

DMU	M-IV	Rank	Proposed	Rank
DMU_1	0.4182	13	0.5160	13
DMU_2	0.8724	2	0.9534	2
DMU_3	0.8499	3	0.8846	3
DMU_4	0.6914	8	0.7604	5
DMU_5	0.6308	10	0.6426	10
DMU_6	0.8798	1	0.9748	1
DMU_7	0.6192	11	0.6268	11
DMU_8	0.8100	4	0.8683	4
DMU_9	0.2578	14	0.2521	14
DMU_{10}	0.7554	5	0.7411	7
DMU_{11}	0.7234	7	0.7330	8
DMU_{12}	0.6663	9	0.6638	9
DMU_{13}	0.5875	12	0.5815	12
DMU_{14}	0.7382	6	0.7561	6

As seen in TABLE X, the rankings of each cross-efficiency model were obtained. Fig. 4 shows the results of the proposed method with M-III model. From this figure, it can be seen that the proposed method tends to be consistent with the M-III model.

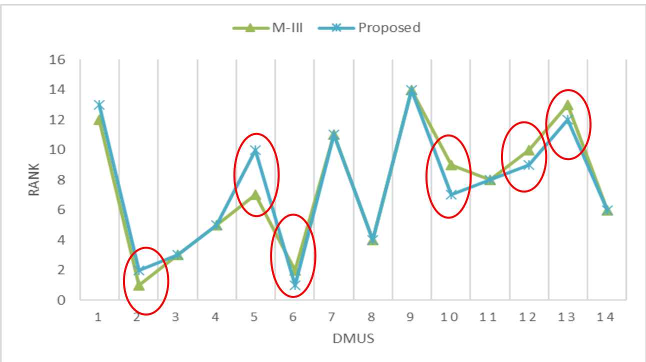

Fig.4. The ranking comparisons for the 14 bank branches problem

The r_s values of each method were tested using Spearman's rank correlation test. The details of the r_s values of each method are shown in TABLE XI.

TABLE XI
THE CORRELATION TEST FOR 14 BANK BRANCHES

	Benevolent	Aggressive	M-III	M-IV	Proposed
Benevolent	-	0.938	0.991	0.960	1.000
Aggressive	0.938	-	0.956	0.978	0.938
M-III	0.991	0.956	-	0.969	0.991
M-IV	0.960	0.978	0.969	-	0.960
Proposed	1.000	0.938	0.991	0.960	-

As seen in TABLE XI, the Spearman correlation coefficients (r_s) for the proposed method and benevolent, aggressive, Model-III and Model-IV, are obtained as r_s = 1.000, 0.938, 0.991 and 0.960, respectively. This means that the proposed method is more consistent with other methods than the M-III model. The comparison results for the proposed method and traditional M-III are shown in Fig.5.

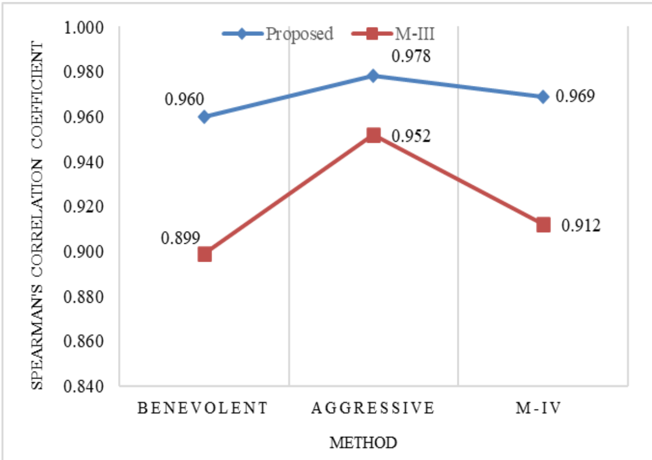

Fig.5. Comparison of results for the proposed method and M-III

As seen in Fig.5, the comparison results of the proposed method and M-III show that the proposed method has a higher correlation coefficient (r_s) than the traditional M-III model [22].

C. *Fourteen International Passenger Airlines Problem*

Tofallis [44] presented an actual dataset of fourteen international passenger airlines with three inputs and two outputs. Let X_1, X_2 and X_3 be aircraft capacity, operating cost, and non-flight assets respectively. Let Y_1 and Y_2 be passenger kilometers and non-passenger revenue respectively. Details for the dataset for fourteen international passenger airlines are shown in TABLE XII.

TABLE XII
DATA SET OF FOURTEEN INTERNATIONAL PASSENGER AIRLINES

DMU	X_1	X_2	X_3	Y_1	Y_2	CCR
DMU_1	5723	3239	2003	26677	697	0.8684
DMU_2	5895	4225	4557	3081	539	0.3379
DMU_3	24099	9560	6267	124055	1266	0.9475
DMU_4	13565	7499	3213	64734	1563	0.9581
DMU_5	5183	1880	783	23604	513	1.0000
DMU_6	19080	8032	3272	95011	572	0.9766
DMU_7	4603	3457	2360	22112	969	1.0000
DMU_8	12097	6779	6474	52363	2001	0.8588
DMU_9	6587	3341	3581	26504	1297	0.9477
DMU_{10}	5654	1878	1916	19277	972	1.0000
DMU_{11}	12559	8098	3310	41925	3398	1.0000
DMU_{12}	5728	2481	2254	27754	982	1.0000
DMU_{13}	4715	1792	2485	31332	543	1.0000
DMU_{14}	22793	9874	4145	122528	1404	1.0000

Based on the same calculation procedure of Section *A* and Section *B*, the ICE matrix for cross-efficiency score of fourteen international passenger airlines is shown in TABLE XIII.

TABLE XIII
THE INTERVAL CROSS-EFFICIENCY MATRIX OF FOURTEEN INTERNATIONAL PASSENGER AIRLINES

DMU	1	2	3	4	5	6	7
1	[0.868, 0.868]	[0.450, 0.450]	[0.623, 0.623]	[0.868, 0.868]	[0.849, 0.487]	[0.473, 0.473]	[0.788, 0.791]
2	[0.172, 0.172]	[0.338, 0.338]	[0.047, 0.047]	[0.172, 0.172]	[0.174, 0.102]	[0.025, 0.025]	[0.272, 0.258]
3	[0.883, 0.883]	[0.194, 0.194]	[0.948, 0.948]	[0.883, 0.883]	[0.884, 0.481]	[0.690, 0.690]	[0.683, 0.695]
4	[0.958, 0.958]	[0.426, 0.426]	[0.703, 0.703]	[0.958, 0.958]	[0.941, 0.706]	[0.697, 0.697]	[0.785, 0.789]
5	[0.965, 0.965]	[0.366, 0.366]	[1.000, 1.000]	[0.965, 0.965]	[1.000, 1.000]	[1.000, 1.000]	[0.736, 0.729]
6	[0.882, 0.882]	[0.111, 0.111]	[0.956, 0.956]	[0.882, 0.882]	[0.878, 0.611]	[0.977, 0.977]	[0.608, 0.627]
7	[0.921, 0.921]	[0.778, 0.778]	[0.477, 0.477]	[0.921, 0.921]	[0.880, 0.471]	[0.338, 0.338]	[1.000, 1.000]
8	[0.781, 0.781]	[0.611, 0.611]	[0.516, 0.516]	[0.781, 0.781]	[0.770, 0.371]	[0.292, 0.292]	[0.859, 0.848]
9	[0.785, 0.785]	[0.728, 0.728]	[0.508, 0.508]	[0.785, 0.785]	[0.789, 0.401]	[0.268, 0.268]	[0.907, 0.882]
10	[0.782, 0.782]	[0.635, 0.635]	[0.652, 0.652]	[0.782, 0.782]	[0.825, 0.557]	[0.356, 0.356]	[0.794, 0.759]
11	[1.000, 1.000]	[1.000, 1.000]	[0.429, 0.429]	[1.000, 1.000]	[1.000, 1.000]	[0.442, 0.442]	[1.000, 0.966]
12	[0.946, 0.946]	[0.634, 0.634]	[0.750, 0.750]	[0.946, 0.946]	[0.960, 0.538]	[0.440, 0.440]	[0.940, 0.920]
13	[1.000, 1.000]	[0.426, 0.426]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 0.375]	[0.456, 0.456]	[1.000, 1.000]
14	[1.000, 1.000]	[0.228, 0.228]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 0.746]	[1.000, 1.000]	[0.728, 0.740]

TABLE XIII
(CONTINUED)

DMU	8	9	10	11	12	13	14
1	[0.788, 0.788]	[0.703, 0.703]	[0.475, 0.559]	[0.475, 0.868]	[0.751, 0.704]	[0.623, 0.559]	[0.623, 0.473]
2	[0.272, 0.272]	[0.281, 0.281]	[0.242, 0.181]	[0.242, 0.172]	[0.206, 0.279]	[0.047, 0.181]	[0.047, 0.025]
3	[0.683, 0.683]	[0.623, 0.623]	[0.288, 0.630]	[0.288, 0.883]	[0.785, 0.626]	[0.948, 0.630]	[0.948, 0.690]
4	[0.785, 0.785]	[0.699, 0.699]	[0.493, 0.565]	[0.493, 0.958]	[0.811, 0.702]	[0.703, 0.565]	[0.703, 0.697]
5	[0.736, 0.736]	[0.778, 0.778]	[0.648, 0.785]	[0.648, 0.965]	[1.000, 0.782]	[1.000, 0.785]	[1.000, 1.000]
6	[0.608, 0.608]	[0.510, 0.510]	[0.170, 0.514]	[0.170, 0.882]	[0.718, 0.514]	[0.956, 0.514]	[0.956, 0.977]
7	[1.000, 1.000]	[0.839, 0.839]	[0.605, 0.572]	[0.605, 0.921]	[0.781, 0.838]	[0.477, 0.572]	[0.477, 0.338]
8	[0.859, 0.859]	[0.821, 0.821]	[0.582, 0.638]	[0.582, 0.781]	[0.753, 0.819]	[0.516, 0.638]	[0.516, 0.292]
9	[0.907, 0.907]	[0.948, 0.948]	[0.738, 0.759]	[0.738, 0.785]	[0.837, 0.945]	[0.508, 0.759]	[0.508, 0.268]
10	[0.794, 0.794]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 0.782]	[1.000, 1.000]	[0.652, 1.000]	[0.652, 0.356]
11	[1.000, 1.000]	[1.000, 1.000]	[1.000, 0.697]	[1.000, 1.000]	[1.000, 1.000]	[0.429, 0.697]	[0.429, 0.442]
12	[0.940, 0.940]	[1.000, 1.000]	[0.792, 0.885]	[0.792, 0.946]	[1.000, 1.000]	[0.750, 0.885]	[0.750, 0.440]
13	[1.000, 1.000]	[1.000, 1.000]	[0.526, 1.000]	[0.526, 1.000]	[0.984, 1.000]	[1.000, 1.000]	[1.000, 0.456]
14	[0.728, 0.728]	[0.648, 0.648]	[0.337, 0.621]	[0.337, 1.000]	[0.857, 0.652]	[1.000, 0.621]	[1.000, 1.000]

After obtaining the ICE matrix, the optimal RCC_i values were obtained using Eq.(7). Finally, the ranking comparisons for all DMUs are provided in TABLE XIV.

TABLE XIV
RANKING COMPARISONS FOR FOURTEEN INTERNATIONAL PASSENGER AIRLINES

DMUs	Ben.	Rank	Agg.	Rank	M-III	Rank
DMU_1	0.7543	12	0.5990	12	0.6582	12
DMU_2	0.1894	14	0.1652	14	0.1790	14
DMU_3	0.7678	9	0.6226	11	0.6811	11
DMU_4	0.8222	6	0.6734	7	0.7292	7
DMU_5	0.8912	3	0.7983	1	0.8469	1
DMU_6	0.7554	11	0.6385	9	0.6831	10
DMU_7	0.8214	7	0.6478	8	0.7133	8
DMU_8	0.7242	13	0.5855	13	0.6465	13
DMU_9	0.7590	10	0.6309	10	0.6948	9
DMU_{10}	0.7803	8	0.6813	6	0.7469	6
DMU_{11}	0.9193	1	0.7742	2	0.8337	3
DMU_{12}	0.8850	4	0.7314	5	0.8049	4
DMU_{13}	0.9190	2	0.7503	3	0.8366	2
DMU_{14}	0.8659	5	0.7316	4	0.7846	5

TABLE XIV
(CONTINUED)

DMU	M-IV	Rank	Proposed	Rank
DMU_1	0.6069	13	0.6547	12
DMU_2	0.1811	14	0.1593	14
DMU_3	0.6224	11	0.6800	10
DMU_4	0.6699	8	0.7205	7
DMU_5	0.7789	1	0.8425	1
DMU_6	0.6342	10	0.6684	11
DMU_7	0.6748	7	0.7102	8
DMU_8	0.6077	12	0.6445	13
DMU_9	0.6576	9	0.6954	9
DMU_{10}	0.6914	6	0.7577	6
DMU_{11}	0.7742	2	0.8315	3
DMU_{12}	0.7464	4	0.8135	4
DMU_{13}	0.7655	3	0.8399	2
DMU_{14}	0.7243	5	0.7747	5

Fig. 6 shows the results of the proposed method with the M-III model. From this figure, it can be seen that the proposed method tends to be highly consistent with the M-III model. Only the DMU_3 and DMU_6 were ranked differently by the two methods.

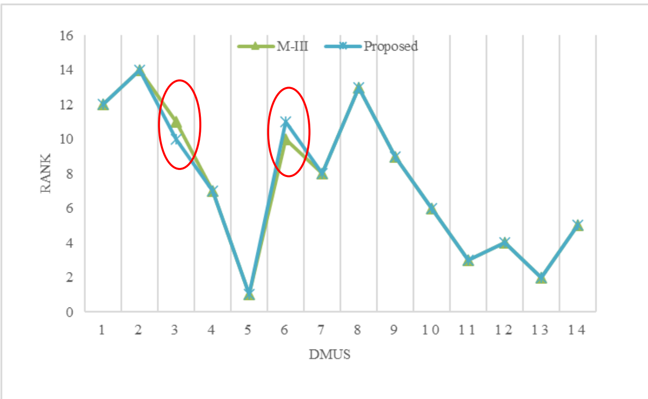

Fig.6. Ranking comparisons for fourteen international passenger airlines

The details of the r_s values of each method are shown in TABLE XV.

TABLE XV
CORRELATION TEST FOR THE FOURTEEN INTERNATIONAL PASSENGER AIRLINES PROBLEM

	Benevolent	Aggressive	M-III	M-IV	Proposed
Benevolent	-	0.952	0.956	0.952	0.965
Aggressive	0.952	-	0.987	0.982	0.978
M-III	0.956	0.987	-	0.987	0.996
M-IV	0.952	0.982	0.987	-	0.982
Proposed	0.965	0.978	0.996	0.982	-

As seen in TABLE XV, the r_s values for the proposed method and benevolent, aggressive, Model-III and Model-IV, are computed as r_s = 0.965, 0.978, 0.996 and 0.982, respectively. This means that the proposed method is strongly consistent with the other methods. The comparison results for the proposed method and traditional M-III are shown in Fig.7.

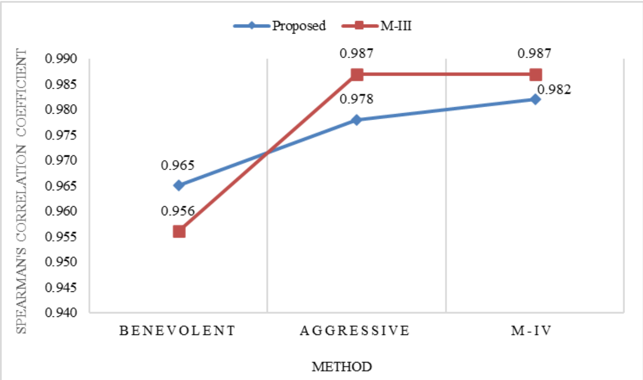

Fig.7. Comparison results for the proposed method and M-III.

As seen in Fig.7, the proposed method is more consistent with benevolent than M-III while M-III is more consistent with aggressive and M-IV.

D. Economic Development of Twenty Thai Provinces

The northeastern region of Thailand covers an area of 160,000 square kilometers, including 20 provinces, with a total population of approximately 22 million. Fig.8 shows a map of Northeast of Thailand.

Fig.8. A map of the Northeast of Thailand.

The region is poorer than the rest of the country, due to the land being difficult to farm, and the fact that the region does not benefit from industries like central and eastern Thailand. Economic development, therefore, depends on the agricultural sector and partly on the industrial sector. The economic crops of the northeastern region are rice, corn, sugar cane, cassava and rubber. Measuring the efficiency of the use of inputs is one way to find ways to develop the region's economy. The northeastern provinces have twenty DMUs with seven inputs (X_1, X_2, X_3, X_4, X_5, X_6 and X_7) and one output (Y_1). The X_1, X_2, X_3, X_4, X_5, X_6, X_7 and Y_1 are electrical energy consumption (Ktoe), fuel consumption (Ktoe), agricultural land (km^2), non-agricultural land (km^2) annual budget (million baht), labor, total distance of rural roads (km) and Gross Provincial Product (baht) respectively. The data set for twenty Thai provinces, together with the CCR scores of each DMU, is provided in TABLE XVI.

TABLE XVI
DATA SET OF TWENTY THAI PROVINCES

DMUs	Name	X_1	X_2	X_3	X_4
DMU_1	Bueng Kan	26.41	54.79	2669	1637
DMU_2	Loei	43.38	136.22	4342	7083
DMU_3	Nong Khai	40.91	112.92	1813	1214
DMU_4	Nong Bua Lamphu	26.75	69.32	2700	1159
DMU_5	Udon Thani	131.03	682.32	6191	5539
DMU_6	Nakhon Phanom	39.96	115.13	3042	2471
DMU_7	Mukdahan	26.84	76.20	2062	2278
DMU_8	Sakon Nakhon	64.80	207.88	4957.0	4649
DMU_9	Kalasin	66.87	150.67	4532.0	2415
DMU_{10}	Khon Kaen	213.03	616.67	6750	4136
DMU_{11}	Maha Sarakham	68.52	176.24	4509	783
DMU_{12}	Roi Et	82.14	228.37	5946	2353
DMU_{13}	Chaiyaphum	75.08	202.96	5323	7455
DMU_{14}	Nakhon Ratchasima	526.62	1101.04	13416	7078
DMU_{15}	Buriram	102.76	358.66	7021	3302
DMU_{16}	Surin	79.34	245.16	6732	1392
DMU_{17}	Yasothon	31.58	88.36	2748	1414
DMU_{18}	Sisaket	74.33	215.59	6509	2331
DMU_{19}	Amnat Charoen	20.04	61.58	2325	836
DMU_{20}	Ubon Ratchathani	137.96	359.42	8582	7162

TABLE XVI
(CONTINUED)

DMUs	X_5	X_6	X_7	Y_1	CCR
DMU_1	198.6	176738	5708.35	24711	1.0000
DMU_2	225.6	298001	9382.58	54985	1.0000
DMU_3	213.3	213263	4003.22	41515	1.0000
DMU_4	210.9	216339	5760.05	30003	1.0000
DMU_5	313.3	634742	17824.82	113887	0.8288
DMU_6	283.7	273489	10268.96	45053	1.0000
DMU_7	207.5	199818	5120.59	27316	0.9058
DMU_8	283.9	405249	14968.04	60737	0.8376
DMU_9	311.0	423177	12818.81	58617	0.9716
DMU_{10}	350.7	912976	17641.00	214018	1.0000
DMU_{11}	270.7	406719	9691.40	59208	1.0000
DMU_{12}	285.4	541263	16681.66	78134	0.9221
DMU_{13}	259.6	497756	13795.04	65698	0.8429
DMU_{14}	414.3	1164344	29734.61	303996	1.0000
DMU_{15}	339.5	648254	17772.54	89356	0.8319
DMU_{16}	297.2	529759	15883.96	81007	1.0000
DMU_{17}	231.6	292488	8345.61	28747	0.8207
DMU_{18}	310.2	554953	17414.62	72752	0.9274
DMU_{19}	218.4	145848	4297.49	20267	0.9173
DMU_{20}	346.8	907108	23075.98	126088	0.9581

As seen in TABLE XVI, the efficient DMUs are DMU_1, DMU_3, DMU_4, DMU_6, DMU_{10}, DMU_{11}, DMU_{14} and DMU_{16}, and they cannot discriminate among DMUs.

Using the same calculation procedure as Section A, the ICE matrix for cross-efficiency score of twenty Thai provinces is shown in Table XVII.

TABLE XVII

THE INTERVAL CROSS-EFFICIENCY MATRIX OF TWENTY THAI PROVINCES

DMUs	1	2	3	4	5
DMU_{1}	[1.000, 1.000]	[0.738, 0.927]	[0.897, 1.000]	[0.858, 1.000]	[0.817, 0.817]
DMU_{2}	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[0.948, 1.000]	[1.000, 1.000]
DMU_{3}	[0.991, 1.000]	[0.801, 0.968]	[1.000, 1.000]	[0.994, 1.000]	[0.952, 0.952]
DMU_{4}	[0.984, 1.000]	[0.885, 0.905]	[0.993, 1.000]	[1.000, 1.000]	[1.000, 1.000]
DMU_{5}	[0.503, 0.525]	[0.551, 0.686]	[0.516, 0.603]	[0.517, 0.679]	[0.829, 0.829]
DMU_{6}	[0.975, 0.987]	[0.889, 0.935]	[0.959, 1.000]	[0.940, 0.970]	[0.990, 0.990]
DMU_{7}	[0.857, 0.885]	[0.803, 0.818]	[0.865, 0.902]	[0.847, 0.882]	[0.861, 0.861]
DMU_{8}	[0.765, 0.771]	[0.739, 0.760]	[0.759, 0.790]	[0.760, 0.773]	[0.838, 0.838]
DMU_{9}	[0.920, 0.944]	[0.692, 0.858]	[0.887, 0.919]	[0.825, 0.931]	[0.820, 0.820]
DMU_{10}	[0.994, 1.000]	[0.793, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]
DMU_{11}	[0.859, 0.859]	[0.682, 0.813]	[0.844, 0.863]	[0.843, 0.877]	[0.845, 0.845]
DMU_{12}	[0.862, 0.868]	[0.750, 0.818]	[0.857, 0.877]	[0.854, 0.873]	[0.913, 0.913]
DMU_{13}	[0.786, 0.808]	[0.690, 0.762]	[0.790, 0.796]	[0.764, 0.766]	[0.762, 0.762]
DMU_{14}	[0.813, 0.852]	[0.455, 0.882]	[0.678, 0.840]	[0.639, 0.844]	[0.586, 0.586]
DMU_{15}	[0.673, 0.681]	[0.668, 0.686]	[0.677, 0.727]	[0.683, 0.754]	[0.832, 0.832]
DMU_{16}	[0.848, 0.862]	[0.806, 0.823]	[0.855, 0.881]	[0.876, 0.903]	[0.992, 0.992]
DMU_{17}	[0.723, 0.762]	[0.659, 0.718]	[0.728, 0.806]	[0.738, 0.768]	[0.815, 0.815]
DMU_{18}	[0.827, 0.833]	[0.772, 0.777]	[0.822, 0.858]	[0.840, 0.846]	[0.927, 0.927]
DMU_{19}	[0.802, 0.827]	[0.787, 0.798]	[0.819, 0.824]	[0.833, 0.869]	[0.873, 0.873]
DMU_{20}	[0.858, 0.884]	[0.721, 0.815]	[0.864, 0.878]	[0.843, 0.859]	[0.858, 0.858]

TABLE XVII

(CONTINUED)

DMUs	6	7	8	9	10
DMU_{1}	[0.818, 0.920]	[0.952, 0.952]	[0.817, 0.817]	[0.985, 0.985]	[0.238, 0.294]
DMU_{2}	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[0.402, 0.457]
DMU_{3}	[0.951, 1.000]	[1.000, 1.000]	[0.952, 0.952]	[0.937, 0.937]	[0.365, 0.724]
DMU_{4}	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[0.273, 0.353]
DMU_{5}	[0.578, 0.802]	[0.516, 0.516]	[0.829, 0.829]	[0.485, 0.485]	[0.582, 0.632]
DMU_{6}	[1.000, 1.000]	[0.995, 0.995]	[0.990, 0.990]	[0.941, 0.941]	[0.303, 0.470]
DMU_{7}	[0.880, 0.897]	[0.906, 0.906]	[0.861, 0.861]	[0.837, 0.837]	[0.253, 0.420]
DMU_{8}	[0.787, 0.822]	[0.767, 0.767]	[0.838, 0.838]	[0.768, 0.768]	[0.389, 0.396]
DMU_{9}	[0.804, 0.901]	[0.932, 0.932]	[0.820, 0.820]	[0.972, 0.972]	[0.351, 0.410]
DMU_{10}	[0.961, 1.000]	[0.988, 0.988]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]
DMU_{11}	[0.816, 0.849]	[0.846, 0.846]	[0.845, 0.845]	[0.889, 0.889]	[0.402, 0.416]
DMU_{12}	[0.877, 0.877]	[0.868, 0.868]	[0.913, 0.913]	[0.922, 0.922]	[0.417, 0.497]
DMU_{13}	[0.748, 0.798]	[0.817, 0.817]	[0.762, 0.762]	[0.842, 0.842]	[0.392, 0.459]
DMU_{14}	[0.558, 0.708]	[0.737, 0.737]	[0.586, 0.586]	[0.811, 0.811]	[0.713, 1.000]
DMU_{15}	[0.716, 0.801]	[0.681, 0.681]	[0.832, 0.832]	[0.691, 0.691]	[0.404, 0.475]
DMU_{16}	[0.876, 0.946]	[0.842, 0.842]	[0.992, 0.992]	[0.899, 0.899]	[0.382, 0.498]
DMU_{17}	[0.795, 0.817]	[0.799, 0.799]	[0.815, 0.815]	[0.777, 0.777]	[0.238, 0.332]
DMU_{18}	[0.854, 0.891]	[0.837, 0.837]	[0.927, 0.927]	[0.894, 0.894]	[0.355, 0.433]
DMU_{19}	[0.817, 0.895]	[0.783, 0.783]	[0.873, 0.873]	[0.735, 0.735]	[0.180, 0.277]
DMU_{20}	[0.826, 0.879]	[0.895, 0.895]	[0.858, 0.858]	[0.958, 0.958]	[0.466, 0.636]

TABLE XVII

(CONTINUED)

DMUs	11	12	13	14	15
DMU_{1}	[0.353, 0.816]	[0.985, 0.985]	[0.891, 0.891]	[0.170, 0.443]	[0.817, 0.817]
DMU_{2}	[0.207, 0.575]	[1.000, 1.000]	[1.000, 1.000]	[0.326, 0.332]	[1.000, 1.000]
DMU_{3}	[0.710, 0.963]	[0.937, 0.937]	[0.884, 0.884]	[0.265, 0.765]	[0.952, 0.952]
DMU_{4}	[0.528, 1.000]	[1.000, 1.000]	[0.910, 0.910]	[0.194, 0.558]	[1.000, 1.000]
DMU_{5}	[0.475, 0.500]	[0.485, 0.485]	[0.489, 0.489]	[0.495, 0.585]	[0.829, 0.829]
DMU_{6}	[0.425, 0.843]	[0.941, 0.941]	[0.878, 0.878]	[0.216, 0.528]	[0.990, 0.990]
DMU_{7}	[0.292, 0.674]	[0.837, 0.837]	[0.781, 0.781]	[0.179, 0.388]	[0.861, 0.861]
DMU_{8}	[0.318, 0.627]	[0.768, 0.768]	[0.752, 0.752]	[0.292, 0.423]	[0.838, 0.838]
DMU_{9}	[0.505, 0.891]	[0.972, 0.972]	[0.916, 0.916]	[0.257, 0.544]	[0.820, 0.820]
DMU_{10}	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[0.832, 1.000]	[1.000, 1.000]
DMU_{11}	[1.000, 1.000]	[0.889, 0.889]	[0.845, 0.845]	[0.298, 0.768]	[0.845, 0.845]
DMU_{12}	[0.633, 0.906]	[0.922, 0.922]	[0.894, 0.894]	[0.373, 0.624]	[0.913, 0.913]
DMU_{13}	[0.223, 0.550]	[0.842, 0.842]	[0.843, 0.843]	[0.319, 0.345]	[0.762, 0.762]
DMU_{14}	[0.755, 0.909]	[0.811, 0.811]	[0.819, 0.819]	[1.000, 1.000]	[0.586, 0.586]
DMU_{15}	[0.543, 0.697]	[0.691, 0.691]	[0.679, 0.679]	[0.359, 0.565]	[0.832, 0.832]
DMU_{16}	[0.902, 0.985]	[0.899, 0.899]	[0.868, 0.868]	[0.371, 0.758]	[0.992, 0.992]
DMU_{17}	[0.402, 0.770]	[0.777, 0.777]	[0.717, 0.717]	[0.169, 0.410]	[0.815, 0.815]
DMU_{18}	[0.587, 0.889]	[0.894, 0.894]	[0.860, 0.860]	[0.320, 0.573]	[0.927, 0.927]
DMU_{19}	[0.505, 0.826]	[0.735, 0.735]	[0.656, 0.656]	[0.126, 0.545]	[0.873, 0.873]
DMU_{20}	[0.398, 0.766]	[0.958, 0.958]	[0.954, 0.954]	[0.475, 0.495]	[0.858, 0.858]

TABLE XVII

(CONTINUED)

DMUs	16	17	18	19	20
DMU_{1}	[0.725, 0.801]	[0.810, 0.810]	[0.817, 0.817]	[0.832, 0.832]	[0.985, 0.985]
DMU_{2}	[0.502, 0.893]	[0.951, 0.951]	[1.000, 1.000]	[0.977, 0.977]	[1.000, 1.000]
DMU_{3}	[0.935, 0.958]	[0.979, 0.979]	[0.952, 0.952]	[0.975, 0.975]	[0.937, 0.937]
DMU_{4}	[0.939, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]
DMU_{5}	[0.524, 0.810]	[0.819, 0.819]	[0.829, 0.829]	[0.824, 0.824]	[0.485, 0.485]
DMU_{6}	[0.785, 0.970]	[1.000, 1.000]	[0.990, 0.990]	[1.000, 1.000]	[0.941, 0.941]
DMU_{7}	[0.611, 0.829]	[0.871, 0.871]	[0.861, 0.861]	[0.864, 0.864]	[0.837, 0.837]
DMU_{8}	[0.587, 0.802]	[0.818, 0.818]	[0.838, 0.838]	[0.828, 0.828]	[0.768, 0.768]
DMU_{9}	[0.815, 0.828]	[0.817, 0.817]	[0.820, 0.820]	[0.816, 0.816]	[0.972, 0.972]
DMU_{10}	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]	[1.000, 1.000]
DMU_{11}	[0.860, 1.000]	[0.841, 0.841]	[0.845, 0.845]	[0.843, 0.843]	[0.889, 0.889]
DMU_{12}	[0.882, 0.908]	[0.893, 0.893]	[0.913, 0.913]	[0.892, 0.892]	[0.922, 0.922]
DMU_{13}	[0.489, 0.709]	[0.737, 0.737]	[0.762, 0.762]	[0.738, 0.738]	[0.842, 0.842]
DMU_{14}	[0.588, 0.724]	[0.585, 0.585]	[0.586, 0.586]	[0.606, 0.606]	[0.811, 0.811]
DMU_{15}	[0.699, 0.825]	[0.815, 0.815]	[0.832, 0.832]	[0.815, 0.815]	[0.691, 0.691]
DMU_{16}	[1.000, 1.000]	[0.964, 0.964]	[0.992, 0.992]	[0.972, 0.972]	[0.899, 0.899]
DMU_{17}	[0.730, 0.813]	[0.821, 0.821]	[0.815, 0.815]	[0.793, 0.793]	[0.777, 0.777]
DMU_{18}	[0.868, 0.922]	[0.901, 0.901]	[0.927, 0.927]	[0.899, 0.899]	[0.894, 0.894]
DMU_{19}	[0.799, 0.886]	[0.891, 0.891]	[0.873, 0.873]	[0.917, 0.917]	[0.735 ,0.735]
DMU_{20}	[0.711, 0.831]	[0.835, 0.835]	[0.858, 0.858]	[0.827, 0.827]	[0.958, 0.958]

Finally, the optimal RCC_i values were obtained using the same calculation steps used for the numerical example of six nursing homes. TABLE XVIII shows the ranking comparisons for each DMU.

TABLE XVIII
THE RANKING COMPARISONS FOR TWENTY THAI PROVINCES

DMUs	Ben.	Rank	Agg.	Rank	M-III	Rank
DMU_1	0.8971	7	0.7571	12	0.8187	12
DMU_2	0.9656	3	0.8261	6	0.9062	5
DMU_3	0.9577	4	0.8483	4	0.9309	3
DMU_4	0.9803	2	0.8627	3	0.9353	2
DMU_5	0.6337	20	0.5961	20	0.6584	20
DMU_6	0.9567	5	0.8201	7	0.9140	4
DMU_7	0.8368	13	0.7209	13	0.8019	13
DMU_8	0.7818	15	0.6735	18	0.7476	15
DMU_9	0.8813	9	0.7641	11	0.8244	11
DMU_{10}	0.9994	1	0.9415	1	0.9868	1
DMU_{11}	0.8587	12	0.8264	5	0.8257	9
DMU_{12}	0.8884	8	0.8012	8	0.8523	7
DMU_{13}	0.7701	16	0.6627	19	0.7242	18
DMU_{14}	0.7346	19	0.6839	15	0.6862	19
DMU_{15}	0.7368	18	0.6754	17	0.7261	17
DMU_{16}	0.9157	6	0.8684	2	0.8910	6
DMU_{17}	0.7660	17	0.6832	16	0.7459	16
DMU_{18}	0.8733	10	0.7890	9	0.8415	8
DMU_{19}	0.8120	14	0.7143	14	0.7830	14
DMU_{20}	0.8690	11	0.7659	10	0.8254	10

TABLE XVIII
(CONTINUED)

DMU	M-IV	Rank	Proposed	Rank
DMU_1	0.7965	11	0.7829	12
DMU_2	0.9028	3	0.8712	5
DMU_3	0.8803	6	0.8942	3
DMU_4	0.9130	2	0.8978	2
DMU_5	0.6488	20	0.5821	20
DMU_6	0.8868	4	0.8713	4
DMU_7	0.7814	13	0.7433	13
DMU_8	0.7321	15	0.6909	15
DMU_9	0.7936	12	0.7892	11
DMU_{10}	0.9468	1	0.9873	1
DMU_{11}	0.8149	9	0.7967	9
DMU_{12}	0.8322	7	0.8192	7
DMU_{13}	0.7130	17	0.6741	18
DMU_{14}	0.6550	19	0.6854	16
DMU_{15}	0.7125	18	0.6684	19
DMU_{16}	0.8833	5	0.8625	6
DMU_{17}	0.7299	16	0.6800	17
DMU_{18}	0.8264	8	0.8010	8
DMU_{19}	0.7653	14	0.7225	14
DMU_{20}	0.8087	10	0.7951	10

As seen in TABLE XVIII, all models assess that DMU_1 is the best, and DMU_5 is the worst.

Fig. 9 shows the results of the proposed method with the M-III model. From this figure, it can be seen that the proposed method tends to be highly consistent with the M-III model. Only the DMU_{14}, DMU_{15} and DMU_{17} were ranked differently by the two methods.

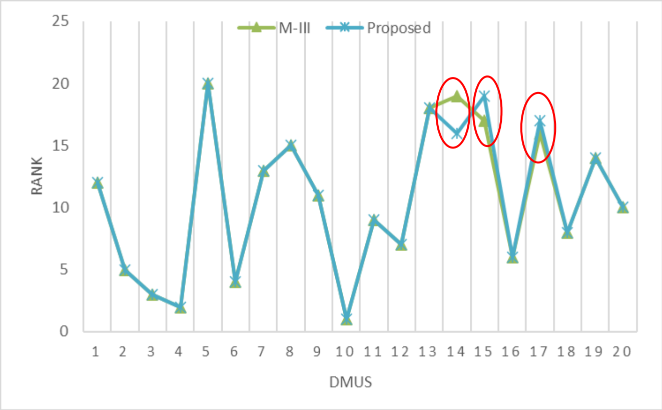

Fig.9. The ranking comparisons for the economic development of twenty provinces of Thailand.

In TABLE XIX, the r_s values for the proposed method and benevolent, aggressive and Model-III and Model-IV are evaluated as r_s = 0.952, 0.953, 0.989 and 0.979, respectively. This ensures that the proposed method has a strong correlation with other cross-efficiency models.

TABLE XIX
THE CORRELATION TEST FOR TWENTY THAI PROVINCES

	Benevolent	Aggressive	M-III	M-IV	Proposed
Benevolent	-	0.890	0.958	0.964	0.952
Aggressive	0.890	-	0.946	0.938	0.953
M-III	0.958	0.946	-	0.986	0.989
M-IV	0.964	0.938	0.986	-	0.979
Proposed	0.952	0.953	0.989	0.979	-

The comparison results for the proposed method and traditional M-III are shown in Fig.10.

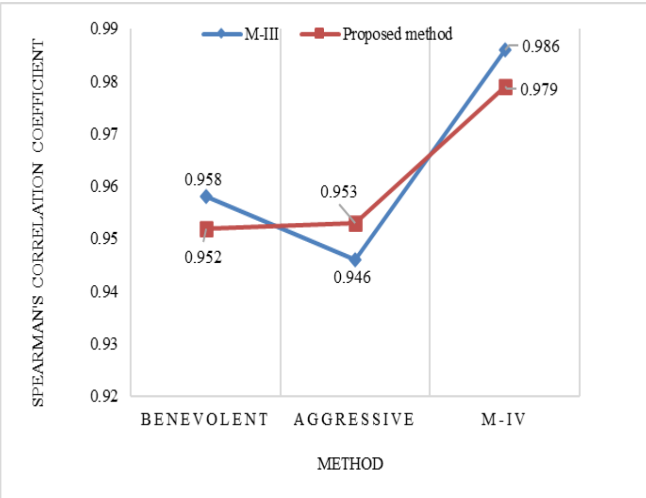

Fig.10. The comparison results for the proposed method and M-III.

As shown in Fig.10, the proposed method and the conventional M-III model have a high correlation with the other models.

Since the original model, Wang's model-III [22], is only considered with a maximization perspective, some ranking problems in DEA may be less reliable. Hence, the proposed method was developed to consider both minimization and maximization perspectives of the virtual DMUs distance in model-III [22], and then the RCC model was developed to combine both perspectives for ranking DMUs with interval cross-efficiency score. The main advantages of the proposed method are that it can be used to address DEA ranking problems with interval data, and it is straightforward but effective. In addition, the RCC model, a new optimization model based on TOPSIS concepts, is simple to solve with any optimization solver.

VII. Conclusion

There are various mathematical models based on the concepts of the cross-efficiency method that have been offered to tackle ranking problems in DEA. One of the most effective and well-known cross-efficiency methods is the secondary goal method. However, the results of each model's ranking for similar problems may differ. Therefore, it is wise to try alternative methods that provide more reliable results and are easier to use but are powerful in solving the DEA's ranking problems. In this paper, both min Z and max Z perspectives of the virtual DMUs cross-efficiency model, namely Model-III [22], were utilized to generate the interval cross-efficiency matrix. Differently from previous work, this research presents a new ranking model based on TOPSIS perspectives, called the RCC model, to tackle interval cross-efficiency scores of interval cross-efficiency matrix for ranking all DMUs. Based on four numerical examples of six nursing homes, Sherman and Gold's dataset for 14 bank branches, fourteen international passenger airlines and twenty Thai provinces, these have illustrated the main advantages, potential and applications of the proposed ranking method. In addition, the RCC model is simple but powerful, and is a flexible model to solve DEA ranking problems. The results from the proposed approach show that it gives more reliable results than individual perspectives for some DEA ranking problems.

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