Finite Time Adaptive Control for Nonlinear Systems with Input Delay

Guangyu Li, Xinyu Ouyang*, Nannan Zhao, Feng Zhang, and Mengzhou Tang

***Abstract*—The main task of this paper is to control a class of nonlinear systems with input delay in finite time. The Pade approximation approach is used to reduce the impact of input delay. In addition, unknown functions are approximated by fuzzy logic systems (FLSs). Then the adaptive controller is intended by backstepping technique, which make sure the the stability of the system in finite time and also the close-loop signals are bounded in finite time. Finally, a simulation shows that the proposed method is effective.**

***Index Terms*—finite time, input delay, nonlinear systems, adaptive controller, backstepping method, fuzzy logic systems.**

I. INTRODUCTION

As we all know, nonlinear systems exist in various domains. So far, scholars worldwide have done much research on nonlinear system control and put forward various control schemes, for instance, adaptive control [1–4], fuzzy control [5–7] and neural network control [8, 9]. However, because of the nonlinear system's complexness, some challenging issues still require to be additionally studied.

Time delays occur in network transport and biological, physical, and chemical changes. In the work of networked management systems, mechanical transmission systems and other systems, input delay can invariably occur. Usually, the existence of input delay can affect the steadiness of the system and cut back the system's performance. Therefore, the way to eliminate the adverse impact of input delay on the system has become a research hotspot within the system control field. In [10–12], the authors designed controllers and analyzed the stability of linear systems with input delays. Compared with the control of input delay in linear systems, it is more complicated in nonlinear systems. In [13], Pezeshki et al. designed the new Lyapunov-Krasovskii functions and combined free weighted matrices and average residence time techniques to propose new stability conditions for nonlinear systems with input delays. In [14–16], the Pade approximation methodology was used to contend with the influence of input delay. In addition, [17] on the basis of Pade approximation methodology, the compensation systems were added to further reduce the impact of input delay on nonlinear systems.

Furthermore, in engineering, we need the system to be stable within the shortest time. Therefore, many scholars are not satisfied with the analysis results of the asymptotic stability, and gradually begin to study the finite time stability analysis methods. In recent decades, with the study of finite time stability, finite time control of nonlinear systems has achieved rich results [18–23]. In [22], Liu et al. designed a finite time controller combining event-triggered and prescribed performance control. And then in [23], under the condition of finite time stability, the author considered the case of unknown disturbance and actuator failure of the nonlinear systems.

If there are time delays, the finite time stability of the system is a great challenge. At present, the finite time control of linear time-delay systems has been gradually improved [24], but it will be more difficult to study in nonlinear systems. Now, finite time control still has limitations in nonlinear time delay systems [25–27]. Therefore, it's necessary to more study the finite time control problem of nonlinear time-delay systems.

The research content is to cut back the influence of input delay links in nonlinear systems and design controllers to fulfill the finite time control conditions. The main work is as follows:

(1) Introducing Pade approximate technology to reduce the effect of time delay. In addition, fuzzy systems are used to eliminate the influence of uncertain functions.

(2) The created controller can guarantee the system's stability in a finite time, and all signals are bounded.

The remaining structure of this paper is as follows. Section II provides the problem description and main lemma. Section III introduces the controller design theme. The simulation results and conclusion are provided in Section IV and Section V respectively.

II. SYSTEM DESCRIPTIONS AND PRELIMINARIES

A. Problem Formulation

Consider a class of nonlinear system with input delay as follows

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + d_i(\bar{x}_i, t), \\ \dot{x}_n = u(t-\tau) + f_n(\bar{x}_n) + d_n(\bar{x}_n, t), \\ y = x_1, \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}} \in R^i (i = 1, 2, \cdots, n)$ represent the system state vectors; $y \in R$ is the system output; $u(t-\tau)$ indicates the control input with time delay,

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Guangyu Li is postgraduate student of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (e-mail: 1047645570@qq.com)

Xinyu Ouyang is professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China.(Corresponding author, e-mail: 13392862@qq.com)

Nannan Zhao is professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (Corresponding author, e-mail: 723306003@qq.com)

Feng Zhang is doctoral candidate of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (e-mail: 15941237106@163.com)

Mengzhou Tang is postgraduate student of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (e-mail: 445700237@qq.com)

τ represents the known input delay; $f_i(\cdot)(i = 1, 2, \cdots n)$ are the indeterminate functions; $d_i(\cdot)(i = 1, 2, \cdots, n)$ are the unknown interferences.

The aim of the study is to construct a controller that will stabilize the system in finite time. At the same time, the system output signal y can track the reference signal y_r in finite time.

For achieve the control effect, the following assumptions are proposed.

Assumption 1: The unknown disturbances $d_i(\cdot)(i = 1, \cdots, n)$ existence limit. Inequality $|d_i(\cdot)| \leq D_i$ holds, where $D_i(i = 1, \cdots, n)$ are positive constants.

Assumption 2: The reference signal y_r and $y_r{}^{(i)}(i = 1, 2, \cdots, n)$ are known and bounded, where $y_r{}^{(i)}$ represents the ith derivative of y_r.

Use the Pade approximation approach proposed in [28, 29] to cut back the influence of input delay within the system (1). From [29], one has

$$\begin{aligned} &\ell\{u(t-\tau)\} = e^{-\tau s}\ell\{u(t)\} = \frac{e^{-\frac{\tau s}{2}}}{e^{\frac{\tau s}{2}}}\ell\{u(t)\} \\ &\approx \frac{(1-\tau s/2)}{(1+\tau s/2)}\ell\{u(t)\}, \end{aligned} \tag{2}$$

among them $\ell\{u(t)\}$ represents the Laplace transform of $u(t)$, s is Laplace variable. Next, we introduce λ_n and let it satisfy the following equation

$$\frac{(1-\tau s/2)}{(1+\tau s/2)}\ell\{u(t)\} = \ell\{\lambda_n\} - \ell\{u(t)\}. \tag{3}$$

Therefore, according to the above formula, we can get

$$\dot{\lambda}_n = -\eta\lambda_n + 2\eta u, \tag{4}$$

where $\eta = 2/\tau$.

Thus

$$u(t-\tau) = \lambda_n - u(t). \tag{5}$$

Substituting (5) into (1), the system (1) can be written as follows

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + d_i(\bar{x}_i, t), \\ \dot{x}_n = f_n(\bar{x}_n) + \lambda_n - u + d_n(\bar{x}_n, t), \\ y = x_1. \end{cases} \tag{6}$$

After introducing λ_n, similar to [29], we consider the problem of later system controller design and establish the following compensation system

$$\begin{cases} \dot{\lambda}_i = \lambda_{i+1} - q_i\lambda_i, \\ \dot{\lambda}_{n-1} = -\frac{1}{\eta}\lambda_n - q_{n-1}\lambda_i, \\ \dot{\lambda}_n = -\eta\lambda_n + 2\eta u, \end{cases} \tag{7}$$

where the design parameters $q_1 > \frac{1}{2}$, $q_i > 1$, $i = 2, 3, \cdots, n-1$.

B. Fuzzy Logic Systems (FLSs)

In this study, fuzzy logic systems are introduced to deal with uncertain functions. Use the following IF-THEN rules:

IF x_1 is N_1^ι, x_2 is N_2^ι, ..., x_m is N_m^ι,

THEN y is M^ι,

where $x = [x_1, x_2, \cdots, x_m]^{\mathrm{T}} \in R^m$ indicates the FLSs input; y is the FLSs output; $N_i{}^\iota$ and M^ι are fuzzy sets; whereas $\mu_{N_i^\iota}(x_i)$ and $\mu_{M^\iota}(y)$ are Membership functions; the number of fuzzy rules is represented by $\iota(\iota = 1, 2, \cdots, m)$.

The FLSs can be described as

$$y(x) = \frac{\sum_{j=1}^{\iota}\tilde{y}_j\prod_{i=1}^{m}\mu_{N_i^\iota}(x_i)}{\sum_{j=1}^{\iota}\left[\prod_{i=1}^{m}\mu_{N_i^\iota}(x_i)\right]}, \tag{8}$$

where $\tilde{y}_j = \max\{\mu_{M^\iota}(y)|y \in R\}$.

The membership function is shown below

$$\phi_j(x) = \frac{\prod_{i=1}^{m}\mu_{N_i^\iota}(x_i)}{\sum_{j=1}^{\iota}\left[\prod_{i=1}^{m}\mu_{N_i^\iota}(x_i)\right]},$$

where $\phi(x) = [\phi_1(x), \phi_2(x), \cdots, \phi_\iota(x)]$ and let $W = [\tilde{y}_1, \tilde{y}_2, \cdots, \tilde{y}_\iota]^{\mathrm{T}} = [W_1, W_2, \cdots, W_\iota]^{\mathrm{T}}$. Then the FLSs (8) can be written as

$$y(x) = W^{\mathrm{T}}\phi(x). \tag{9}$$

Lemma 1: [5] For any smooth function $f(x)$ on the set Λ, there is a scalar quantity $\varepsilon > 0$ to establish the following inequality

$$\sup_{x\in\Lambda}|f(x) - W^{\mathrm{T}}\phi(x)| \leq \varepsilon, \tag{10}$$

where ε is estimation error.

C. Finite Time Stability

Definition 1: (see[19]) For nonlinear system $\dot{\varsigma} = f(\varsigma)$, if there is a constant $\varepsilon > 0$ and $0 < T(\varepsilon, \varsigma_0) < \infty$, so that

$$\|\varsigma(t)\| < \varepsilon, t \geqslant t_0 + T,$$

where $\varsigma(t_0) = \varsigma_0$. As a result, the system is semi-global practical finite time stable(SGPFS).

To achieve the control objective of the system, the following lemmas need to be introduced.

Lemma 2: For $z_i \in R$, $i = 1, 2, \cdots, n$, $0 < l \leq 1$, the following formula holds

$$(\sum_{i=1}^{n}|z_i|)^l \leq \sum_{i=1}^{n}|z_i|^l \leq n^{1-l}(\sum_{i=1}^{n}|z_i|)^l. \tag{11}$$

Lemma 3: [30] When δ and ς are arbitrary values, and ρ, α and μ are arbitrary positive constants, the following inequality is true

$$|\delta|^\rho|\varsigma|^\alpha \leq \frac{\rho}{\rho+\alpha}\mu|\delta|^{\rho+\alpha} + \frac{\alpha}{\rho+\alpha}\mu^{\frac{-\rho}{\alpha}}|\varsigma|^{\rho+\alpha}. \tag{12}$$

Lemma 4: [19] In terms of the system $\dot{\delta} = g(\delta)$. $V(\delta)$ is a positive definite smooth function, and there are $c > 0$, $0 < \beta < 1$, and $h > 0$, one has

$$\dot{V}(\delta) \leq -cV^\beta(\delta) + H(t \geq 0), \tag{13}$$

then the system $\dot{\delta} = g(\delta)$ is SGPFS.

III. Adaptive Controller Design

This section introduces the adaptive fuzzy controller design method and analyzes the stability of the system (1).

A. Controller Design

Before designing the controller, a set of state coordinate transformation is introduced.

$$\begin{cases} e_1 = y - y_r - \lambda_1, \\ e_i = x_i - \alpha_{i-1} - \lambda_i, \\ e_n = x_n - \alpha_{n-1} + \frac{1}{\eta}\lambda_n, \end{cases} \tag{14}$$

where $\alpha_i(i = 1, 2, \cdots n-1)$ are virtual controllers.

Step 1: Choose a following Lyapunov function

$$V_1 = \frac{1}{2}e_1^2 + \frac{\gamma_1}{2}\tilde{\theta}_1^2, \tag{15}$$

where $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$, $\hat{\theta}_1$ is an estimate of θ_1, and $\gamma_1 > 0$ is the design parameter.

By (6), (7) and (14), we can get

$$\begin{aligned} \dot{e}_1 =& \dot{x}_1 - \dot{y}_r - \dot{\lambda}_1 \\ =& e_2 + \alpha_1 + f_1 + d_1 - \dot{y}_r + q_1\lambda_1. \end{aligned}$$

Then, one has

$$\begin{aligned} \dot{V}_1 =& e_1\dot{e}_1 - \gamma_1\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ =& e_1(e_2 + \alpha_1 + f_1 + d_1 - \dot{y}_r + q_1\lambda_1) - \gamma_1\tilde{\theta}_1\dot{\hat{\theta}}_1. \end{aligned} \tag{16}$$

By Young's inequality, one has

$$e_1 d_1 \leq \frac{e_1^2 D_1^2}{2a_1^2} + \frac{a_1^2}{2}. \tag{17}$$

Substituting (17) for (16), the following inequality holds

$$\begin{aligned} \dot{V}_1 \leq & e_1(e_2 + \alpha_1 + f_1 + \frac{e_1 D_1^2}{2a_1^2} - \dot{y}_r + q_1\lambda_1) \\ & + \frac{a_1^2}{2} - \gamma_1\tilde{\theta}_1\dot{\hat{\theta}}_1. \end{aligned} \tag{18}$$

Then, defined $F_1(X_1) = f_1 + e_1 D_1^2/2a_1^2 + q_1\lambda_1$, $X_1 = [x_1, y_r, \lambda_1]^{\mathrm{T}}$. According to Lemma 1, $F_1(X_1)$ can be approximated by FLS $W_1^{\mathrm{T}}\phi_1(X_1)$. By giving a scalar quantity $\varepsilon_1 > 0$, and an approximate error $\delta_1(X_1)$, we can get

$$F_1(X_1) = W_1^{\mathrm{T}}\phi_1(X_1) + \delta_1(X_1), |\delta_1(X_1)| \leq \varepsilon_1. \tag{19}$$

Using Young's inequality, we can get

$$\begin{aligned} e_1 F_1(X_1) =& e_1[W_1^{\mathrm{T}}\phi_1(X_1) + \delta_1(X_1)] \\ \leq & \frac{e_1^2\|W_1\|^2\phi_1(X_1)^{\mathrm{T}}\phi_1(X_1)}{2\rho_1^2} + \frac{\rho_1^2}{2} + \frac{e_1^2}{2} + \frac{\varepsilon_1^2}{2} \\ \leq & \frac{e_1^2\theta_1\phi_1(X_1)^{\mathrm{T}}\phi_1(X_1)}{2\rho_1^2} + \frac{\rho_1^2}{2} + \frac{e_1^2}{2} + \frac{\varepsilon_1^2}{2}, \end{aligned} \tag{20}$$

where $\theta_1 = \|W_1\|^2$ and there is positive constant ρ_1. Then, combine (20) with (18), we can get

$$V_1 \leq e_1(e_2 + \frac{e_1\theta_1\phi_1^{\mathrm{T}}\phi_1}{2\rho_1^2} + \frac{e_1}{2} + \alpha_1 - \dot{y}_r) + h_1 - \gamma_1\tilde{\theta}_1\dot{\hat{\theta}}_1, \tag{21}$$

where $h_1 = \frac{a_1^2}{2} + \frac{\rho_1^2}{2} + \frac{\varepsilon_1^2}{2}$.

Based on the preceding information, the virtual controller is selected as follows

$$\alpha_1 = -\frac{1}{2}e_1 - c_1 e_1^{2\beta-1} - \frac{e_1\hat{\theta}_1\phi_1^{\mathrm{T}}\phi_1}{2\rho_1^2} + \dot{y}_r, \tag{22}$$

where $c_1 > 0$ is an optional parameter. Now, we can get

$$\dot{V}_1 \leq -c_1 e_1^{2\beta} + \frac{e_1^2\tilde{\theta}_1\phi_1^{\mathrm{T}}\phi_1}{2\rho_1^2} + h_1 - \gamma_1\tilde{\theta}_1\dot{\hat{\theta}}_1 + e_1 e_2, \tag{23}$$

and choose the following adaptive law

$$\dot{\hat{\theta}}_1 = \frac{e_1^2\phi_1^{\mathrm{T}}\phi_1}{2\gamma_1\rho_1^2} - k_1\hat{\theta}_1, \tag{24}$$

where k_1 is an optional positive constant. Then

$$\dot{V}_1 \leq -c_1 e_1^{2\beta} + k_1\gamma_1\tilde{\theta}_1\hat{\theta}_1 + h_1 + e_1 e_2. \tag{25}$$

Step 2: Choose the Lyapunov function as follows

$$V_2 = V_1 + \frac{1}{2}e_2^2 + \frac{\gamma_2}{2}\tilde{\theta}_2^2, \tag{26}$$

where $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$, $\hat{\theta}_2$ is an estimate of θ_2, and $\gamma_2 > 0$ is the design parameter.

By (6), (7) and (14), we can get

$$\begin{aligned} \dot{e}_2 =& \dot{x}_2 - \dot{\alpha}_1 - \dot{\lambda}_2 \\ =& e_3 + \alpha_2 + f_2 + d_2 - \dot{\alpha}_1 + q_2\lambda_2. \end{aligned} \tag{27}$$

Then, derivation of V_2 as follows

$$\begin{aligned} \dot{V}_2 \leq & -c_1 e_1^{2\beta} + k_1\gamma_1\tilde{\theta}_1\hat{\theta}_1 + h_1 \\ & + e_2(e_1 + e_3 + \alpha_2 + f_2 + d_2 - \dot{\alpha}_1 \\ & + q_2\lambda_2) - \gamma_2\tilde{\theta}_2\dot{\hat{\theta}}_2. \end{aligned} \tag{28}$$

By Young's inequality, the following inequality is true

$$e_2 d_2 \leq \frac{e_2^2 D_2^2}{2a_2^2} + \frac{a_2^2}{2}. \tag{29}$$

Substituting (29) for (28), the following inequality holds

$$\begin{aligned} \dot{V}_2 \leq & -c_1 e_1^{2\beta} + k_1\gamma_1\tilde{\theta}_1\hat{\theta}_1 + h_1 \\ & + e_2(e_1 + e_3 + \alpha_2 + f_2 + \frac{e_2 D_2^2}{2a_2^2} - \dot{\alpha}_1 + q_2\lambda_2) \\ & + \frac{a_2^2}{2} - \gamma_2\tilde{\theta}_2\dot{\hat{\theta}}_2. \end{aligned} \tag{30}$$

Now, defined $F_2(X_2) = e_1 + f_2 + e_2 D_2^2/2a_2^2 - \dot{\alpha}_1 + q_2\lambda_2$, $X_2 = [x_1, x_2, y_r, \dot{y}_r, \lambda_1, \lambda_2, \hat{\theta}_1]^{\mathrm{T}}$. According to lemma 1, give a scalar quantity $\varepsilon_2 > 0$, by using FLS $W_2^{\mathrm{T}}\phi_2(X_2)$ to approximate $F_2(X_2)$, there has

$$F_2(X_2) = W_2^{\mathrm{T}}\phi_2(X_2) + \delta_2(X_2), |\delta_2(X_2)| \leq \varepsilon_2,$$

where $\delta_2(X_2)$ is approximate error.

By Young's inequality, the following inequality is true

$$\begin{aligned} e_2 F_2(X_2) =& e_2[W_2^{\mathrm{T}}\phi_2(X_2) + \delta_2(X_2)] \\ \leq & \frac{e_2^2\|W_2\|^2\phi_2(X_2)^{\mathrm{T}}\phi_2(X_2)}{2\rho_2^2} + \frac{\rho_2^2}{2} + \frac{e_2^2}{2} + \frac{\varepsilon_2^2}{2} \\ \leq & \frac{e_2^2\theta_2\phi_2(X_2)^{\mathrm{T}}\phi_2(X_2)}{2\rho_2^2} + \frac{\rho_2^2}{2} + \frac{e_2^2}{2} + \frac{\varepsilon_2^2}{2}, \end{aligned} \tag{31}$$

where $\theta_2 = \|W_2\|^2$ and there is positive constants ρ_2. Then, combine (31) with (30) ,we can get

$$\begin{aligned} \dot{V}_2 \leq & -c_1 e_1^{2\beta} + k_1\gamma_1\tilde{\theta}_1\hat{\theta}_1 + h_2 \\ & + e_2(e_3 + \alpha_2 + \frac{e_2\theta_2\phi_2^{\mathrm{T}}\phi_2}{2\rho_2^2} + \frac{e_2}{2}) \\ & - \gamma_2\tilde{\theta}_2\dot{\hat{\theta}}_2, \end{aligned} \tag{32}$$

where $h_2 = h_1 + \frac{a_2^2}{2} + \frac{\rho_2^2}{2} + \frac{\varepsilon_2^2}{2}$.

Construct the following virtual controller and adaptive law to ensure the stability of the system

$$\alpha_2 = -\frac{e_2}{2} - c_2 e_2^{2\beta-1} - \frac{e_2\hat{\theta}_2\phi_2^{\mathrm{T}}\phi_2}{2\rho_2^2}, \tag{33}$$

$$\dot{\hat{\theta}}_2 = \frac{e_2^2\phi_2^{\mathrm{T}}\phi_2}{2\gamma_2\rho_2^2} - k_2\hat{\theta}_2, \tag{34}$$

where c_2 and k_2 are positive design parameters. Consequently, $\dot{V}_2$ is rewritten as follows

$$\dot{V}_2 \leq -\sum_{j=1}^{2} c_j e_j^{2\beta} + \sum_{j=1}^{2} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j + h_2 + e_2e_3. \tag{35}$$

Step i ($3 \leq i \leq n-1$): The Lyapunov function is designed as follows:

$$V_i = V_{i-1} + \frac{1}{2}e_i^2 + \frac{\gamma_i}{2}\tilde{\theta}_i^2, \tag{36}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, $\hat{\theta}_i$ is an estimate of θ_i, and $\gamma_i > 0$ is the design parameter.

By (6), (7) and (14), we can get

$$\begin{aligned}\dot{e}_i =& \dot{x}_i - \dot{\alpha}_{i-1} - \dot{\lambda}_i \\ =& e_{i+1} + \alpha_i + f_i + d_i - \dot{\alpha}_{i-1} + q_i\lambda_i.\end{aligned} \tag{37}$$

Then, we have

$$\begin{aligned}\dot{V}_i \leq& -\sum_{j=1}^{i-1} c_j e_j^{2\beta} + \sum_{j=1}^{i-1} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j + h_{i-1} + e_{i-1}e_i \\ &+ e_i(e_{i+1} + \alpha_i + \lambda_{i+1} + f_i + d_i - \dot{\alpha}_{i-1} - \lambda_{i+1} \\ &+ q_i\lambda_i) - \gamma_i\tilde{\theta}_i\dot{\hat{\theta}}_i.\end{aligned} \tag{38}$$

Similarly step 1, the following holds

$$e_id_i \leq \frac{e_i^2D_i^2}{2a_i^2} + \frac{a_i^2}{2}. \tag{39}$$

Substitute the above formula into (38) and the following inequality is ture

$$\begin{aligned}\dot{V}_i \leq& -\sum_{j=1}^{i-1} c_j e_j^{2\beta} + \sum_{j=1}^{i-1} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j + h_{i-1} \\ &+ e_i(e_{i+1} + \alpha_i + f_i + \frac{e_iD_i^2}{2a_i^2} + q_i\lambda_i - \dot{\alpha}_{i-1} + e_{i-1}) \\ &+ \frac{a_i^2}{2} - \gamma_i\tilde{\theta}_i\dot{\hat{\theta}}_i.\end{aligned} \tag{40}$$

Defined $F_i(X_i) = f_i + e_iD_i^2/2a_i^2 + q_i\lambda_i - \dot{\alpha}_{i-1} + e_{i-1}$, where $X_i = [\bar{x}_i, \bar{y}_r^{(i-1)}, \bar{\lambda}_i, \bar{\hat{\theta}}_{i-1}]^{\mathrm{T}}$ with $\bar{y}_r^{(i-1)} = [y_r, y_r^{(1)}, \cdots, y_r^{(i-1)}]^{\mathrm{T}}$, $\bar{\lambda}_i = [\lambda_1, \lambda_2, \cdots, \lambda_i]^{\mathrm{T}}$ and $\bar{\hat{\theta}}_{i-1} = [\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_{i-1}]^{\mathrm{T}}$. From Lemma 1, $F_i(X_i)$ can be approximated by FLS $W_i^{\mathrm{T}}\phi_i(X_i)$. By giving a scalar quantity $\varepsilon_i > 0$ and an approximate error $\delta_i(X_i)$, the $F_i(X_i)$ is written as follows

$$F_i(X_i) = W_i^{\mathrm{T}}\phi_i(X_i) + \delta_i(X_i), |\delta_i(X_i)| \leq \varepsilon_i.$$

Using Young's inequality, we can get

$$\begin{aligned}e_iF_i(X_i) =& e_i[W_i^{\mathrm{T}}\phi_i(X_i) + \delta_i(X_i)] \\ \leq& \frac{e_i^2\|W_i\|^2\phi_i(X_i)^{\mathrm{T}}\phi_i(X_i)}{2\rho_i^2} + \frac{\rho_i^2}{2} + \frac{e_i^2}{2} + \frac{\varepsilon_i^2}{2} \\ \leq& \frac{e_i^2\theta_i\phi_i(X_i)^{\mathrm{T}}\phi_i(X_i)}{2\rho_i^2} + \frac{\rho_i^2}{2} + \frac{e_i^2}{2} + \frac{\varepsilon_i^2}{2},\end{aligned} \tag{41}$$

where $\theta_i = \|W_i\|^2$ and there are positive constants ρ_i. Then, combine (41) with (40), we have

$$\begin{aligned}\dot{V}_i \leq& -\sum_{j=1}^{i-1} c_j e_j^{2\beta} + \sum_{j=1}^{i-1} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j + h_i \\ &+ e_i(e_{i+1} + \alpha_i + \frac{e_i\theta_i\phi_i^{\mathrm{T}}\phi_i}{2\rho_i^2} + \frac{e_i}{2}) - \gamma_i\tilde{\theta}_i\dot{\hat{\theta}}_i,\end{aligned} \tag{42}$$

where $h_i = h_{i-1} + \frac{a_i^2+\rho_i^2+\varepsilon_i^2}{2}$.

Similar to the previous steps, α_i and $\dot{\hat{\theta}}_i$ are composed as follows

$$\alpha_i = -\frac{e_i}{2} - c_i{e_i}^{2\beta-1} - \frac{e_i\hat{\theta}_i\phi_i^{\mathrm{T}}\phi_i}{2\rho_i^2}, \tag{43}$$

$$\dot{\hat{\theta}}_i = \frac{e_i^2{\phi_i}^{\mathrm{T}}\phi_i}{2\gamma_i\rho_i^2} - k_i\hat{\theta}_i, \tag{44}$$

where c_i and k_i are positive design parameters. Consequently, $\dot{V}_i$ is written as

$$\dot{V}_i \leq -\sum_{j=1}^{i} c_j e_j^{2\beta} + \sum_{j=1}^{i} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j + h_i + e_ie_{i+1}. \tag{45}$$

Step n: The following Lyapunov functions will be considered

$$V_n = V_{n-1} + \frac{1}{2}e_n^2 + \frac{\gamma_n}{2}\tilde{\theta}_n^2, \tag{46}$$

where $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$, $\hat{\theta}_n$ is an estimate of θ_n, and $\gamma_n > 0$ is the design parameter.

By (6), (7) and (14), we get the derivative of e_n as follows

$$\begin{aligned}\dot{e}_n =& \dot{x}_n - \dot{\alpha}_{n-1} + \frac{1}{\eta}\dot{\lambda}_n \\ =& f_n + \lambda_n - u + d_n - \dot{\alpha}_{n-1} \\ &+ \frac{1}{\eta}(-\eta\lambda_n + 2\eta u) \\ =& f_n + u + d_n - \dot{\alpha}_{n-1}.\end{aligned} \tag{47}$$

Then, get the derivative of V_n

$$\begin{aligned}\dot{V}_n \leq& -\sum_{j=1}^{n-1} c_j e_j^{2\beta} + \sum_{j=1}^{n-1} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j \\ &+ h_{n-1} + e_n(e_{n-1} + f_n \\ &+ u + d_n - \dot{\alpha}_{n-1}) - \gamma_n\tilde{\theta}_n\dot{\hat{\theta}}_n,\end{aligned} \tag{48}$$

where

$$e_nd_n \leq \frac{e_n^2D_n^2}{2a_n^2} + \frac{a_n^2}{2}. \tag{49}$$

Substituting the above formula into (48), we get

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n-1} c_j e_j^{2\beta} + \sum_{j=1}^{n-1} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j + h_{n-1} \\ & + e_n(e_{n-1} + f_n + u + \frac{e_n D_n^2}{2a_n^2} - \dot{\alpha}_{n-1}) \\ & + \frac{a_n^2}{2} - \gamma_n\tilde{\theta}_n\dot{\hat{\theta}}_n.\end{aligned} \tag{50}$$

Similar to step i, defined $F_n(X_n) = e_{n-1} + f_n + \frac{e_n D_n^2}{2a_n^2} - \dot{\alpha}_{n-1}$, where $X_n = [\bar{x}_n, \bar{y}_r^{(n-1)}, \bar{\lambda}_n, \bar{\hat{\theta}}_{n-1}]$. From Lemma 1, we can get

$$F_n(X_n) = W_n^{\mathrm{T}}\phi_n(X_n) + \delta_n(X_n), |\delta_n(X_n)| \leq \varepsilon_n,$$

where $\varepsilon_n > 0$ is a scalar quantity, and $\delta_n(X_n)$ is an approximate error.

By Young's inequality, the following inequality is true

$$\begin{aligned}e_n F_n(X_n) = & e_n[W_n^{\mathrm{T}}\phi_n(X_n) + \delta_n(X_n)] \\ \leq & \frac{e_n^2\|W_n\|^2\phi_n(X_n)^{\mathrm{T}}\phi_n(X_n)}{2\rho_n^2} + \frac{\rho_n^2}{2} + \frac{e_n^2}{2} + \frac{\varepsilon_n^2}{2} \\ \leq & \frac{e_n^2\theta_n\phi_n(X_n)^{\mathrm{T}}\phi_n(X_n)}{2\rho_n^2} + \frac{\rho_n^2}{2} + \frac{e_n^2}{2} + \frac{\varepsilon_n^2}{2},\end{aligned} \tag{51}$$

where $\theta_n = \|W_n\|^2$ and there is a positive constant ρ_n. Then (50) can be rewritten as

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n-1} c_j e_j^{2\beta} + \sum_{j=1}^{n-1} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j + h_n \\ & + e_n(u + \frac{e_n\theta_n\phi_n^{\mathrm{T}}\phi_n}{2\rho_n^2} + \frac{e_n}{2}) - \gamma_n\tilde{\theta}_n\dot{\hat{\theta}}_n,\end{aligned} \tag{52}$$

where $h_n = h_{n-1} + \frac{a_n^2}{2} + \frac{\rho_n^2}{2} + \frac{\varepsilon_n^2}{2}$.

To ensure the system's stability, the input u and adaptive law are selected as follows

$$u = -\frac{e_n}{2} - c_n e_n^{2\beta-1} - \frac{e_n\hat{\theta}_n\phi_n^{\mathrm{T}}\phi_n}{2\rho_n^2}, \tag{53}$$

$$\dot{\hat{\theta}}_n = \frac{e_n^2\phi_n^T\phi_n}{2\gamma_n\rho_n^2} - k_n\hat{\theta}_n, \tag{54}$$

where c_n and k_n are positive design parameter. Consequently, $\dot{V}_n$ is rewritten as follows:

$$\dot{V}_n \leq -\sum_{j=1}^{n} c_j e_j^{2\beta} + \sum_{j=1}^{n} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j + h_n. \tag{55}$$

B. Stability Analysis

Theorem 1: It is considered that the nonlinear system (1), (6), adaptive law (24), (34), (44), (54), controller (53), and all system signals are SGPFS. The output signal can effectively track the preset signal within finite time under the condition that the Assumption 1 and the Assumption 2 holds.

Proof 1: Let $V{=}V_n$, the inequality from (55) is the following

$$\dot{V} \leq -\sum_{j=1}^{n} c_j e_j^{2\beta} + \sum_{j=1}^{n} k_j\gamma_j\tilde{\theta}_j\hat{\theta}_j + h_n. \tag{55}$$

According to the interpretation of $\tilde{\theta}_i$ and Yang's inequality, the following formula holds

$$\tilde{\theta}_i\hat{\theta}_i \leq -\frac{1}{2}\tilde{\theta}_i^2 + \frac{1}{2}\theta_i^2. \tag{55}$$

Then, (1) becomes

$$\dot{V} \leq -\sum_{j=1}^{n} c_j e_j^{2\beta} - \frac{1}{2}\sum_{j=1}^{n} k_j\gamma_j\tilde{\theta}_j^2 + \frac{1}{2}\sum_{j=1}^{n} k_j\gamma_j\theta_j^2 + h_n, \tag{55}$$

where define $c = \min\{c_j, k_j, j = 1, 2, \cdots n\}$. Apply Lemma 2 then (1) is written as follows

$$\begin{aligned}\dot{V} \leq & -2^\beta c(\sum_{j=1}^{n}\frac{e_j^2}{2})^\beta - c(\sum_{j=1}^{n}\frac{\gamma_j}{2}\tilde{\theta}_j^2)^\beta \\ & + c(\sum_{j=1}^{n}\frac{\gamma_j}{2}\tilde{\theta}_j^2)^\beta - c\sum_{j=1}^{n}\frac{\gamma_j}{2}\theta_j^2 \\ & + \frac{1}{2}\sum_{j=1}^{n} k_j\gamma_j\theta_j^2 + h_n.\end{aligned}$$

Apply Lemma 3 to the formula $c(\sum_{j=1}^{n}\frac{\gamma_j}{2}\tilde{\theta}_j^2)^\beta$ with $\delta = 1$, $\varsigma = \sum_{j=1}^{n}\frac{\gamma_j}{2}\tilde{\theta}_j^2$, and $\rho = 1-\beta$, $\alpha = \beta$ and $\mu = \beta^{\frac{\beta}{1-\beta}}$ to get that

$$c(\sum_{j=1}^{n}\frac{\gamma_j}{2}\tilde{\theta}_j^2)^\beta \leq c(1-\beta)\mu + c\sum_{j=1}^{n}\frac{\gamma_j}{2}\theta_j^2. \tag{55}$$

Then, (1) becomes

$$\begin{aligned}\dot{V} \leq & -2^\beta c(\sum_{j=1}^{n}\frac{e_j^2}{2})^\beta - c(\sum_{j=1}^{n}\frac{\gamma_j}{2}\tilde{\theta}_j^2)^\beta \\ & + c(1-\beta)\mu + \frac{1}{2}\sum_{j=1}^{n} k_j\gamma_j\theta_j^2 + h_n.\end{aligned}$$

Using Lemma 2, there are

$$\dot{V} \leq -\tilde{c}V^\beta + H, \tag{55}$$

where

$$\begin{aligned}\tilde{c} &= \min\{2^\beta c, c\}, \\ H &= c(1-\beta)\mu + \frac{1}{2}\sum_{j=1}^{n} k_j\gamma_j\theta_j^2 + h_n.\end{aligned}$$

According to lemma 4(refer to lemma3 of Wu), we let $T^*{=}\frac{1}{(1-\beta)\sigma\tilde{c}}[V^{1-\beta}(e(0),\theta(0)) - (\frac{h}{(1-\sigma)\tilde{c}})^{(1-\beta)/\beta}]$ with $0 < \sigma < 1$, $e(0) = [e_1(0), e_2(0), \cdots, e_n(0)]^{\mathrm{T}}$ and $\theta(0) = [\theta_1(0), \theta_2(0), \cdots, \theta_n(0)]^{\mathrm{T}}$. Thus, for any $t \geq T^*$, $V^\beta(e,\theta) \leq \frac{h}{(1-\sigma)\tilde{c}}$. That means V_n is SGPFS. Therefore, it can be seen that e_i and $\tilde{\theta}_i$ are bounded. From (53), we can know that u is bounded. Thus, define a constant $b_1 > 0$, it makes $|u| < b_1$.

Now, we need to prove whether λ_i is bounded. Consider the following Lyapunov function

$$V_\lambda = \frac{1}{2}\sum_{j=1}^{n}\lambda_j^2, \tag{55}$$

then, by deriving, one has

$$\begin{aligned}\dot{V}_\lambda =& \sum_{j=1}^{n-2} \lambda_j(\lambda_{j+1} - q_j\lambda_j) + \lambda_{n-1}(-\frac{1}{\eta}\lambda_n - q_{n-1}\lambda_{n-1}) \\ &+ \lambda_n(-\eta\lambda_n + 2\eta u) \\ &\leq -\sum_{j=1}^{n} \bar{q}_j\lambda_j^2 + \eta b_1^2 \leq -c_\lambda \sum_{j=1}^{n} \lambda_j^2 + \eta b_1^2,\end{aligned}$$

where $\bar{q}_1 = q_1 - \frac{1}{2}$, $\bar{q}_i = q_i - 1, i = 2, 3, \cdots n-2$, $\bar{q}_{n-1} = q_{n-1} + \frac{1}{2\eta} - \frac{1}{2}$, $\bar{q}_n = \frac{1}{2\eta}$ and $c_\lambda = \min\{\bar{q}_i\}, (i = 1, \cdots, n)$.

By Lemma 3, we get

$$\left(\sum_{j=1}^{n} \lambda_j^2\right)^\beta \leq \sum_{j=1}^{n} \lambda_j^2 + (1-\beta)\beta^{\frac{\beta}{1-\beta}}. \tag{55}$$

Substituting (1) into (1), one has

$$\begin{aligned}\dot{V}_\lambda \leq& -c_\lambda 2^\beta \left(\sum_{j=1}^{n} \frac{\lambda_j^2}{2}\right)^\beta + \eta b_1^2 + (1-\beta)\beta^{\frac{\beta}{1-\beta}} \\ \leq& -\bar{c}_\lambda V_\lambda + H_\lambda,\end{aligned}$$

where $\bar{c}_\lambda = c_\lambda 2^\beta$ and $H_\lambda = \eta b_1^2 + (1-\beta)\beta^{\frac{\beta}{1-\beta}}$.According to Lemma 4, it can be proved that λ_i is SGPFS. By (14), it can be deduced that x_i is SGPFS.

Besides this, from the definition of V, it can be seen that for $\forall t \geq T^*$, the following inequality holds

$$|y - y_r| \leq 2(\frac{h}{(1-\sigma)\bar{c}})^{\frac{1}{2\beta}}. \tag{55}$$

Therefore, the Theorem1 can be proved.

IV. Simulation Examples

The efficacy of the developed controller is verified in this section using a simulated example. First, consider the following nonlinear system with input delay and outside disturbance

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + d_1(x_1, t), \\ \dot{x}_2 = u(t-\tau) + f_2(x_1, x_2) + d_2(x_1, x_2, t), \\ y = x_1, \end{cases} \tag{56}$$

where x_1 and x_2 are the system state vectors, y indicates the system output. $\tau = 0.01$ represents the input delay; the external perturbations are $d_1(x_1, t) = 0.01\cos(t)\sin(x_1)$ and $d_2(x_1, x_2, t) = 0.01\cos(t)\sin(x_1x_2)$; the nonlinear functions $f_1(x_1) = 0.1\sin(x_1)$, $f_2(x_1, x_2) = 0.1\sin(x_1)\cos(x_2)$. The target function selected for tracing is $y_r = \sin(t)$.

The compensation system introduced is as follows:

$$\begin{cases} \dot{\lambda}_1 = -\frac{1}{\eta}\lambda_2 - q_1\lambda_1, \\ \dot{\lambda}_2 = -\eta\lambda_2 + 2\eta u. \end{cases}$$

The parameters in the simulation are designed as $\gamma_1 = 4$, $\gamma_2 = 5$, $k_1 = 5$, $k_2 = 10$, $c_1 = 3$, $c_2 = 5$, $q_1 = 1.1$, $\rho_1 = 4$, $\rho_2 = 4$, $\beta = 99/100$. The selected initial system conditions are $[x_1(0), x_2(0)]^{\mathrm{T}} = [1.5, -0.3]^{\mathrm{T}}$, $[\theta_1(0), \theta_2(0)]^{\mathrm{T}} = [0.5, 0.5]^{\mathrm{T}}$ and $[\lambda_1(0), \lambda_2(0)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$. Figs. 1-4 shows the simulation results. Fig. 1 represents system output $y(t)$ and reference signals $y_r(t)$. Fig. 2 represents the actual controller u of the system and Fig. 3 is adaptive laws $\hat{\theta}_1$ and $\hat{\theta}_2$. Fig. 4 is tracking error.

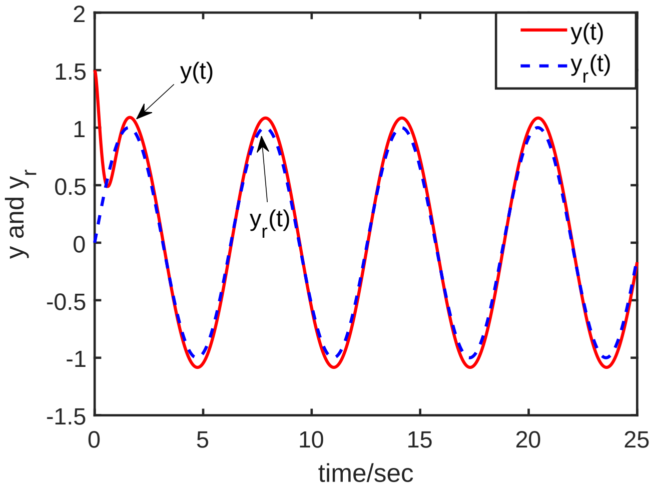

Fig. 1. Reference signal $y_r(t)$ and system actual out $y(t)$.

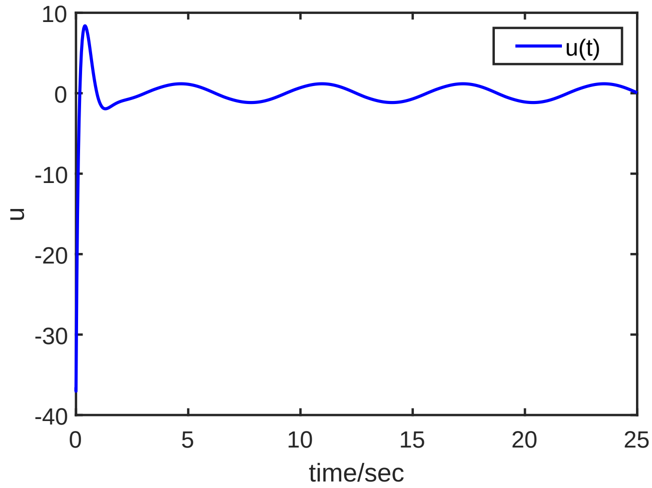

Fig. 2. The system actual control signal u.

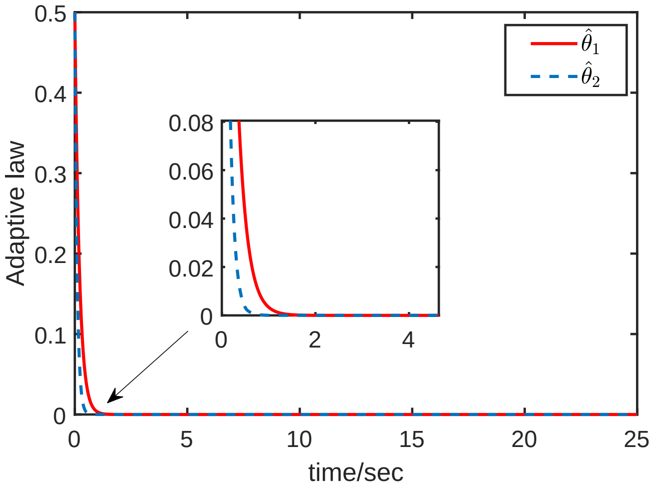

Fig. 3. Adaptive law $\hat{\theta}_1$ and $\hat{\theta}_2$.

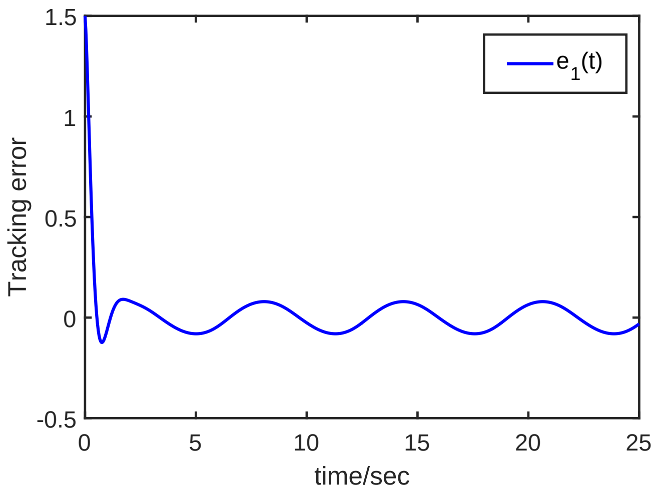

Fig. 4. The tracking error e_1.

V. Conclusion

In the research, the finite time control method is applied to the nonlinear system with input delay. Introducing Pade approximate technology to reduce the effect of time delay. In addition, fuzzy systems are used to eliminate the influence of uncertain functions. The controller is designed by Lyapunov functions to ensure the stability of the system.

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