Fractional-order SS1IR Model for Predicting Public Opinion Dissemination in Social Networks

Linna Li, Yuze Li and Jianke Zhang

Abstract—**With over one billion Internet users in China, a large amount of public opinion is spreading rapidly in breadth and depth at an unprecedented speed in social networks. The establishment of an accurate public opinion dissemination (POD) model is vital for predicting and maintaining the construction of online civilization. On the basis of the traditional SIR model, a fractional-order differential is combined to solve the problem of "anomalous dissemination" in the process of actual POD. The influence of web hypers on POD in social networks is also discovered, and the S1 super-spreading node is introduced. In this paper, we propose a fractional-order SS1IR POD prediction model. The experimental results show that the model fits real data well and has small errors. Therefore, our model can play an aggressive and significant role for predicting POD in social networks.**

Index Terms—**Public opinion dissemination; SIR model; Fractional-order SS1IR model; Super-spreading node**

I. Introduction

In recent years, under the background of rapid development of social networks, interactive social networks represented by Facebook, Twitter and Weibo platforms have gradually replaced traditional media as the main platform for POD. According to the 50th Statistical Report on the Development Status of China's Internet [1] released by China Internet Network Information Center (CNNIC), it is pointed out. By June 2022, the number of Chinese Internet users had reached 1.051 billion, and the Internet penetration rate had reached 74.4%. This has also triggered many potential crises inherent in social networks, as people's demand for diversified, real-time and personalized information has increased dramatically, and massive amounts of information are spreading rapidly in social networks at an unprecedented speed in both breadth and depth [2]. Different dissemination subjects will have diverse views on the same information, and the competition between different views and the large malicious dissemination of public opinion information by web hyper "Self-media marketing number" will all contribute to the dissemination of information. If undesirable information is spread maliciously, which leads to large-scale mass incidents, it will not only affect the construction of network civilization, but also affect the order and stability of society.

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Over the past decade, the issue of POD in social networks has received a lot of scholarly attention, and which has achieved a lot of results. Many researchers have studied and proposed various models for predicting the POD, mainly in a macro to micro modeling approach, that is, focusing on the interactions between groups in social networks or changes in the views of groups to examine the behavior of individuals. The main models include epidemic models, reaction-diffusion equation models [3], hydrodynamic models [4], [5] and other models, such as opinion models [6], grey models [7] and emotional models [8]. It has been found that the process of POD is extremely similar to that of epidemic viruses. Therefore, many researchers, based on the traditional infectious disease model (SIR model), have searched for correlation points to research the opinion dissemination model, and after continuous optimization, have applied it to the research of interactive social network opinion dissemination.

As early as 1927, Kermack and McKendrick [9] were the first to propose the SIR epidemic model (Susceptible-Infected-Recovered) considering immune mechanisms when they researched the epidemic pattern of the plague in London and Bombay. In 1932, Kermack and McKendrick [10] again proposed the SIS infectious disease model (Susceptible-Infected-Susceptible) for certain types of diseases that can be re-infected (encephalitis, gonorrhea, etc). Anderson and Kephart [11] proposed a new SEIR infectious disease model by adding the latent state E (Exposed) to the SIR model. Due to the similarity between the POD in social networks and the spread of viruses on biological networks, that is, initial outbreak-mid-term surge-late stabilization, which is quite similar to the development process of POD in social networks, the infectious disease model was widely applied to the problem of POD in social networks. In 1964, Daley and Kendall [12] proposed the first classical POD model (DK model) on the basis of the SIR contagion model, and many subsequent scholars have expanded on it and studied the POD model in depth based on the common SIR, SIS, SEIR and other contagion models.

Recently, a growing amount of research has been conducted both in China and abroad on the POD in social networks. Again, due to the sudden outbreak of COVID-19, the hotness of infectious disease model and social network POD has increased sharply again. In [13], Ha established a top-down research model of POD by refining the propagation of SIR model. Chen [14] proposed the $\mathrm{SIR_1R_2}$ POD model on the basis of the SIR model, and found that the factors of emotional changes of the Internet users would have an important impact on the POD. In [15], Wei proposed a friendship-based altruistic incentive public opinion diffusion model (SIH model) and used theoretical analysis to determine the criteria for the persistence of infected and susceptible individuals. In

[16], emotional factors and user relationship intimacy were introduced into public opinion diffusion, focusing on the interaction of emotional information among friends and the influence on diffusion. Yanan [17] considered the mobility of users and built a model of information spreading in social networks, demonstrating that user mobility increases the connection between users and further expands the spread of public opinion. In [18], Qiu proposed a new rumor propagation model SIR-im with an influence mechanism. Orman [19] implements the identification of overlapping communities in social networks.

Most of the existing epidemic models are integer-order models, that is, first-order systems of differential equations models. The integer-order models are incompetent in describing "anomalous dissemination" and are not suitable for describing the dynamic characteristics of system development. In the social network POD, there are anomalous dissemination and evolution behaviors of explosive and accelerated information dissemination in the early stage and slow and slow dissemination in the later stage. The fractional-order model can adjust the differential order in different time intervals and combine the characteristics of "anomalous dissemination" of actual events to predict the complete dynamic process of POD more accurately. In view of this, we establish a new fractional-order POD model. We also find the correlation points with the POD by combining the super-spreaders in COVID-19 and propose the "S1 super-spreading node" on the basis of the SIR model. Finally, a fractional-order SS1IR POD model is established. The paper is structured as follows. In Section 2, we propose and elaborate the fractional-order SS1IR POD model. In Section 3, we apply the fractional-order SS1IR model to the events of "Yi Yangqianxi is suspected of high school entrance examination irregularities in enrollment", compare the predicted values with the real data, and judge the validity of the model by calculating the errors (MAE, RMSE) to determine the effectiveness of the model. Finally, in Section 4, we give a conclusion.

II. Fractional-order POD SS1IR model building

In this section, the definition of conventional integer-order POD SIR model and the most details of the fractional-order POD SS1IR model and the definition of conformable derivative are introduced systematically.

A. Traditional Integer-Order SIR Model

In this segment, the state transfer rules of traditional integer-order SIR model and differential equation system for the model are introduced.

1) Model assumption:

Assumption 1. The total amount of dissemination subjects N remains unchanged, the status of subjects is split into three groups: S (Susceptible), which refers to those individuals not infected; I (Infective), which refers to those who are infected and can infect the unknown; R (Removal), for those who are exempt to the infection to no longer infected.

Assumption 2. The infection rate (exposure per patient per day) is β, the immunization rate (cured per day as a percentage of total patients) is γ.

2) Model state transfer rule:

The densities of susceptible, infected and removed individuals are denoted by $S(t)$, $I(t)$, and $R(t)$ at time t, respectively. The traditional integer-order SIR model is illustrated in Fig. 1.

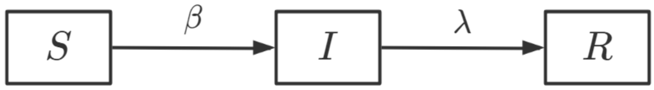

Fig. 1. Traditional integer-order SIR model

The particular features about each node of S, I, and R are described as shown below:

a) For S (Susceptible) node:

S-node refers to not yet infected, but has the possibility of infection. According to the assumption, S-node is transformed into I-node with infection rate β after being infected by the infectious disease. Then S-node transforms $\beta S(t)$ I-node in unit time at moment t, and the number of sick people is $NI(t)$. Therefore, the number of all sick people infected per unit time at moment t is $\beta S(t)I(t)$.

b) For I (Infective) node:

I-node refers to infected and has the ability to spread the virus. According to the assumption that I-node is infected and then transformed into R-node at an immunity rate γ due to treatment or autoimmunity to recover their health, I-node is transformed into $\gamma I(t)$ I-node in unit time at moment t.

c) For R (Removal) node:

R-node refers to a permanent immunity to this infectious disease and not to be re-infected.

3) Model building:

On the basis of the analysis on the model state transfer rule, the differential equation system of traditional integer-order SIR model could be deduced as follows:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta S(t)I(t) \\ \dfrac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t) \\ \dfrac{dR(t)}{dt} = \gamma I(t) \\ S(t) + I(t) + R(t) = N \end{cases} \tag{1}$$

Where $S(0) = S_0$, $I(0) = I_0$, $R(0) = R_0$, $\beta > 0$, $\gamma > 0$.

The traditional integer-order SIR model has good stability and anti-interference performance. However, the integer-order model cannot depict the "anomalous dissemination" of POD and is not memorable, which is not suitable for describing the dynamic characteristics of the system development. Consequently, fractional-order model is used to predict the POD on hot events. Moreover, it is considered that in the actual social network POD, web hyper (Self-media marketing number) can influence the trend of POD. Therefore, the state refinement is carried out on the basis of the SIR model, and the "S1 super-spreading node" is proposed. With these considerations, a fractional-order SS1IR POD model is established in this paper.

B. Fractional-order SS1IR Model

In this segment, the new fractional-order derivatives used in this model and the state transfer rules and structural details of the model are described in detail.

1) Conformable fractional derivative (CFD):

Khalil et al [20] proposed a new definition of fractional derivative, called CFD. It is the most natural and productive definition. For $\alpha = 1$, the definition corresponds with the classical definitions of the first-order derivatives. For $0 \geq \alpha \geq 1$, the definition corresponds with the classical definitions on polynomials (up to a constant). The definition of CFD is shown below.

Definition 2.1([20]). For all $t > 0$, $\alpha \in (0, 1)$. Given a function $f : [0, \infty) \to R$. The CFD of the function f with order α is defined as:

$$D^{\alpha} f(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon} \tag{2}$$

Where, the value at $t = 0$ is $(D^{\alpha} f)(0) = \lim_{t \to 0}(D^{\alpha} f)(t)$.

If f is α-differentiable in $(0, a), a > 0$, and $\lim_{t \to 0^+} f^{\alpha}(t)$ exist, then it is possible to define

$$f^{\alpha}(0) = \lim_{t \to 0^+} f^{\alpha}(t) \tag{3}$$

Occasionally, $f^{\alpha}(t)$ is used to denote the notation of CFDs off of order α for $D^{\alpha} f(t)$. Furthermore, we may state that α is α-differentiable if its CFD of order α exists. It is important to note that

$$D^{\alpha}(t^{p}) = pt^{p-\alpha}$$

Additionally, the definition is consistent with earlier definitions of polynomials made by R-L and Caputo (up to a constant multiple).

The aforementioned definition leads to the relevant theorem that follows.

Theorem 2.1([20]). Let $\alpha \in (0, 1]$, and f be α-differentiable at a point $t > 0$.
If f is differentiable. Then,

$$D^{\alpha} f(t) = t^{1-\alpha} \frac{df}{dt}(t) \tag{4}$$

From Theorem 2.1, we know that CFD is the first order derivative when the order $\alpha = 1$ in CFD.

2) Model assumption:

The traditional integer-order SIR model has not taken into account the situation of web hypers in social networks and ignores the promotion and influence of "super spreaders" on the POD process. Web hypers [21] are people who plan, implement, and promote specific objects with the help of online media to make them influential and well-known. The dissemination influence of these web hypers is mainly manifested as the diffusion effect on information, that is, the phenomenon of POD that attracts the attention and positive response of the general audience. In some emergencies in recent years, the web hypers conveyed negative emotions and magnified social contradictions, and the "social flow waterfall" type emotional infection affected people's cognitive ability and level, and POD appeared irrational, which might even trigger new instability factors.

Assumption 1. The total amount of dissemination subjects N is a constant quantity, and the status of dissemination subjects is split into three categories: S (Susceptible), which refers to individuals not in receipt of the information; S1 (Super-spreading), which refers to people who know the information and have super spread ability; I (Infective), which refers to people who know the information and make ordinary spread; and R (Removal), for those who are immune to the information and no longer spread.

Assumption 2. Let the percentage of change in the quantity of susceptible persons over time due to the influence of information dissemination be proportional to the product of the quantity of susceptible persons at that time and the quantity of infected persons (super spreaders, ordinary spreaders) at that time, and the proportionality coefficient (spread rate) is β. Where the super spread rate is set to $\beta_{S \to S1}$, the ordinary spread rate is set to $\beta_{S \to I}$, and the spread rate that is first super-spread and then gradually immunized with the passage of time to ordinary spread is set to $\beta_{S1 \to I}$.

Assumption 3. Let the rate of transition from infected to removed persons be proportional to the number of infected persons (super-spreaders, ordinary spreaders) at that time, and the removal rate be γ. Where, the removal rate is set to $\gamma_{S1 \to R}$ after super-spreaders and $\gamma_{I \to R}$ after ordinary spreaders. This model additionally takes into account that individual susceptible persons may be indifferent to the information and become removed directly, and the removal rate is set to $\gamma_{S \to R}$.

Assumption 4. Recovered in this model are not permanently immune, and it is possible that with the passage of time, some of them have forgotten the information and transform again into susceptible individuals with a conversion rate of η.

3) Model state transfer rule:

The densities of susceptible, infected, super-spreading and removed individuals are denoted by $S(t)$, $I(t)$, $S_1(t)$ and $R(t)$ at time t, respectively. In addition, β is the spread rate, γ is the removal rate, and η is the conversion rate of reinfection after recovery. The fractional-order SS1IR model is shown in Fig. 2.

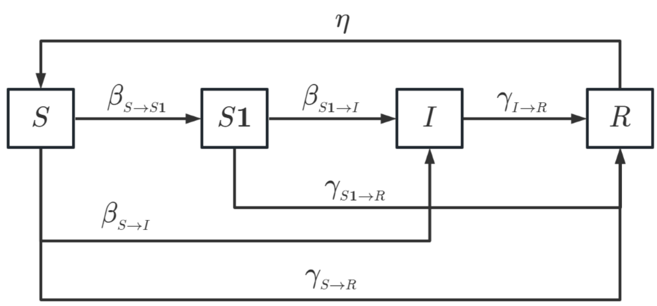

Fig. 2. Fractional-order SS1IR Model

The specific details about each node of $S, S1, I$, and R are described as shown below:

a) For S (Susceptible) node:

S-node refers to a node that has not yet received the information but has the possibility of receiving it. Susceptible persons of S-node will be divided into three states of dynamic changes in the dynamic process of POD: ① Part of S-nodes, after receiving the information, exist as web hypers, disseminate the information at super spread rate $\beta_{S \to S1}$ and transform into $S1$ state; ② Part of S-nodes disseminate the information at ordinary spread rate $\beta_{S\to I}$ and transform into I state; ③ Individual S-nodes may be immune and indifferent to such information and will be directly transformed into R state with the removal rate $\gamma_{S\to R}$.

b) For $S1$ (Super-spreading) node:

S1-node refers to a node that has received the information and has the ability to super spread the information. Super-spreaders of S1 node will be divided into two states of dynamic changes in the dynamic process of POD: ① Due to the generation of hotter public opinion events in the same period, lowering the spread rate of this event will be transformed into I state with the spread rate $\beta_{S1\to I}$; ② Gradually immunize and forget about the information due to the passage of time, and eventually this part will be transformed into R state with the removal rate $\gamma_{S1\to R}$.

c) For I (Infective) node:

I-node refers to a node that has received the message and has the ability to disseminate it. As time passes, the spreader will find the message uninteresting and indifferent to it, then it will transform to R state with recovery rate $\gamma_{I\to R}$.

d) For R (Removal) node:

R-node refers to a node that immunity to the information and no longer spreads it. This model rehabilitators are not permanently immune, and it is possible that with the passage of time, some of them have forgotten about the information, transformed to the S state again with a conversion rate of η, and restarted into a round of POD.

4) Model building:

On the basis of the analysis on the model state transfer rule, the fractional differential equation of the fractional-order SS1IR model could be deduced as follows:

$$\begin{cases} D^{\alpha}S(t) = -\beta_{S\to S1}S(t)I(t) - \beta_{S\to I}S(t) \\ \qquad\qquad - \gamma_{S\to R}S(t) + \eta R(t) \\ D^{\alpha}S_1(t) = \beta_{S\to S1}S(t)I(t) - \beta_{S1\to I}S_1(t) \\ \qquad\qquad - \gamma_{S1\to R}S_1(t) \\ D^{\alpha}I(t) = \beta_{S\to I}S(t) + \beta_{S1\to I}S_1(t) - \gamma_{I\to R}I(t) \\ D^{\alpha}R(t) = \gamma_{S\to R}S(t) + \gamma_{S1\to R}S_1(t) + \gamma_{I\to R}I(t) \\ \qquad\qquad - \eta R(t) \\ S(t) + S_1(t) + I(t) + R(t) = N \end{cases} \tag{5}$$

According to the definition of (Eq. 4), we can know that:

$$D^{\alpha}S(t) = t^{1-\alpha}\frac{dS}{dt}(t)$$

Where α is the order of the fractional derivative.

Similarly, we rewrite the dynamic equation (Eq. 5) for the fractional-order SS1IR model above. As follows (Eq. 6).

Where $S(0) = S_0$, $S_1(0) = S_{1_0}$, $I(0) = I_0$, $R(0) = R_0$, the values of $\beta_{S\to S1}$, $\beta_{S\to I}$, $\beta_{S1\to I}$, $\gamma_{S\to R}$, $\gamma_{S1\to R}$, $\gamma_{I\to R}$, and η are all greater than 0.

$$\begin{cases} t^{1-\alpha}\dfrac{dS(t)}{dt} = -\beta_{S\to S1}S(t)I(t) - \beta_{S\to I}S(t) \\ \qquad\qquad - \gamma_{S\to R}S(t) + \eta R(t) \\ t^{1-\alpha}\dfrac{dS_1(t)}{dt} = -\beta_{S1\to I}S_1(t) - \gamma_{S1\to R}S_1(t) \\ \qquad\qquad + \beta_{S\to S1}S(t)I(t) \\ t^{1-\alpha}\dfrac{dI(t)}{dt} = \beta_{S\to I}S(t) + \beta_{S1\to I}S_1(t) \\ \qquad\qquad - \gamma_{I\to R}I(t) \\ t^{1-\alpha}\dfrac{dR(t)}{dt} = \gamma_{S\to R}S(t) + \gamma_{S1\to R}S_1(t) \\ \qquad\qquad + \gamma_{I\to R}I(t) - \eta R(t) \\ S(t) + S_1(t) + I(t) + R(t) = N \end{cases} \tag{6}$$

III. EXPERIMENTAL RESULTS AND ANALYSIS

In this section, in order to verify the stability and reliability of the fractional-order SS1IR model for predicting the POD. This paper uses Matlab as the simulation platform to simulate the dissemination of popular public opinion events and compare the prediction results of traditional SIR model and integer-order SS1IR model to compare and verify the feasibility of the model through graphs. Finally, we compare the actual retweeting data of POD in social networks with the prediction data of the model and perform error analysis.

The data of this experiment are the retweets in social networks from 08:00 on April 2nd to 24:00 on April 5th, 2022 about the "Yi Yangqianxi is suspected of high school entrance examination irregularities in enrollment". Because this event involves the keywords of celebrity, education and examination, which are easy to be noticed by the society, the controversy and attention are large, so this case is chosen for this verification.

A. Effect of Differential Order α on the model

Based on the obtained actual data of POD, the total number of models is assumed to be 14260, and the initial values are $S(0) = 14254, S_1(0) = 2, I(0) = 4, R(0) = 0$. To obtain the best fit, the least squares method was used to fit the data in this paper. On the basis of the real data fitting curves, the values of spread rate, recovery rate and conversion rate of each node can be calculated. As follows: $\beta_{S\to S1} = 2.9 \times 10^{-5}$, $\beta_{S\to I} = 1.9 \times 10^{-3}$, $\beta_{S1\to I} = 0.5$, $\gamma_{S\to R} = 0.2 \times 10^{-2}$, $\gamma_{S1\to R} = 0.5 \times 10^{-1}$, $\gamma_{I\to R} = 0.1 \times 10^{-1}$, and $\eta = 0.1 \times 10^{-2}$.

In Fig. 3, these three figures show the effects of different differential order α on the fractional-order SS1IR POD model. Taking $\alpha = 0.8$, $\alpha = 1$, and $\alpha = 1.2$, respectively, the dynamic change graphs of the fractional-order SS1IR model are derived. We can see that when the order α is set smaller, the curve of the fractional-order SS1IR POD model converges more slowly. Conversely, the order α is set larger, the curve of the fractional-order SS1IR POD model converges faster.

B. Comparison Between the Integer-order Model and the Fractional-order SS1IR Model

According to the obtained actual data and the obtained values of each parameter, the order of SS1IR model is not changed, and the integer-order SS1IR POD prediction model is used, that is, the differential order $\alpha = 1$ is set to remain constant. The simulation diagram of the model results is shown in Fig. 4.

From the experimental simulation of the integer-order model in Fig. 4, it can be found that when $t \in (0, 15]$, the predicted POD graph of the model is basically similar to the actual data. This is the early stage of the POD, and information is spreading at an ever-increasing rate. When $t \in (15, 50]$, the prediction result graph of the integer-order model has a large error with the real data.

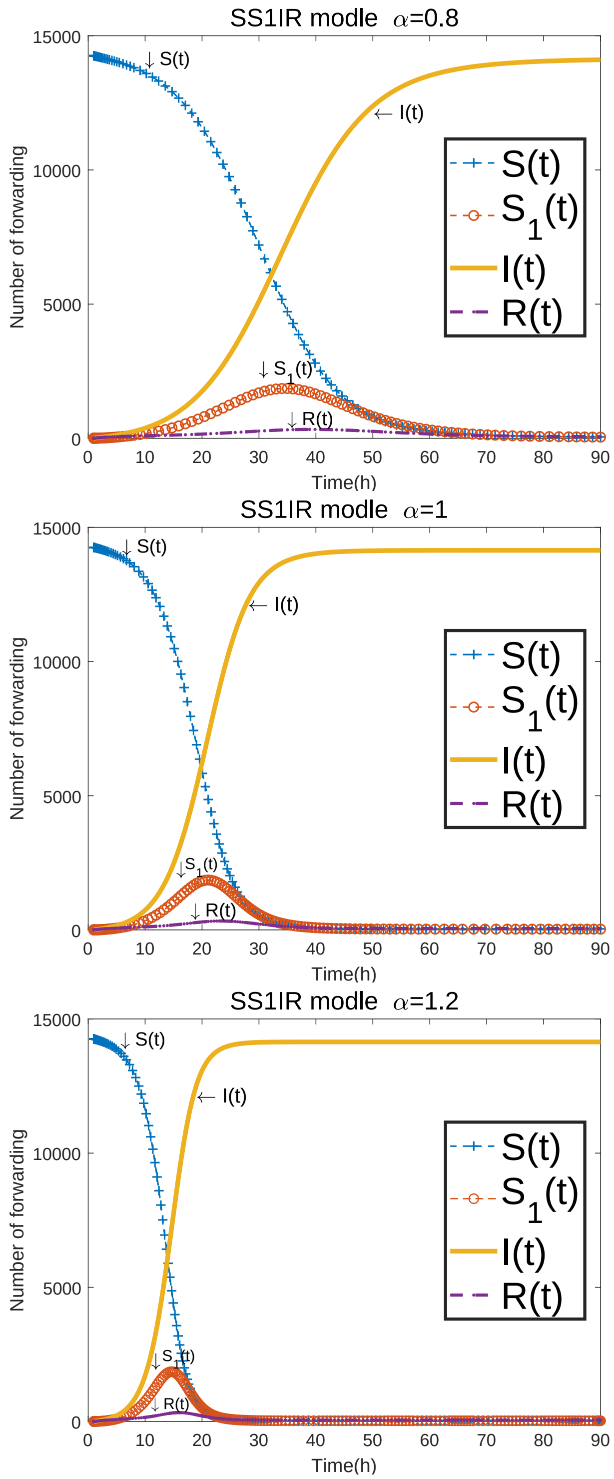

Fig. 3. The effect differential order α on fractional-order SS1IR model

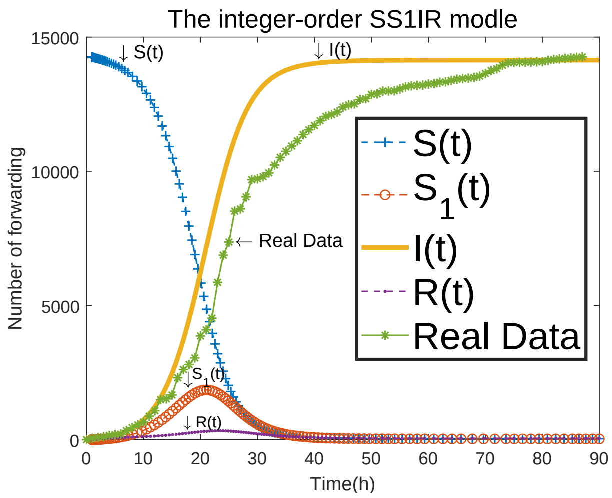

Fig. 4. Simulation diagram of the integer-order SS1IR POD model

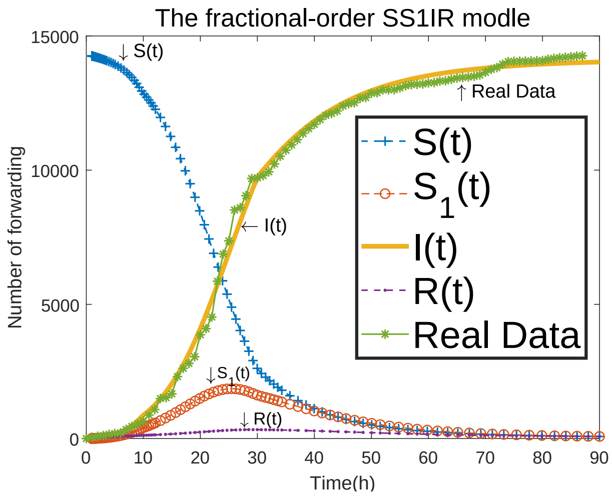

Fig. 5. The fractional-order SS1IR POD model

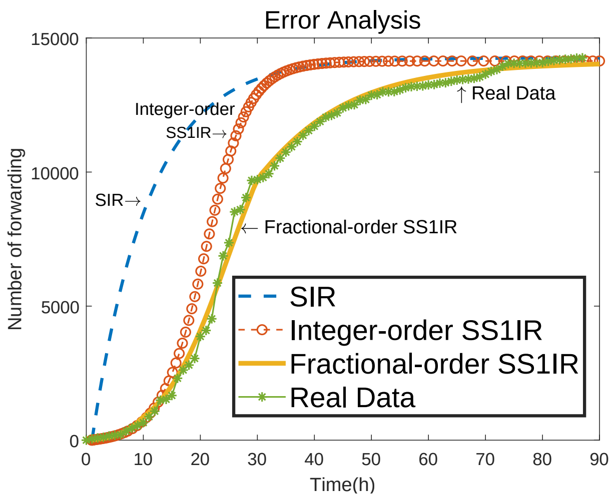

Fig. 6. Error Analysis

As the differential order of the integer-order model is fixed at $\alpha = 1$, but at this time it is in the middle stage of POD, the rate of spread decreases and is not constant. Integer-order model failed to reflect the "anomalous dissemination" characteristic of public opinion, and the entire dissemination process proceeds at a constant rate. This leads to errors between the predicted and actual spread curves, and the fit of the experimental results is low. When $t \in (50, 87]$, it is in the late stage of POD and should be in a slow spread state. However, at this point the predicted data from the integer-order model has stopped spreading and tends to level off. The prediction results are again in error with the real data.

Due to the fact that the actual propagation process of public opinion information in social networks is usually "anomalous dissemination", however, the integer-order model cannot predict the "anomalous dissemination". In this regard, the proposed fractional-order SS1IR model can segment the prediction time, that is, to express the dissemination rate at different stages in the whole stages of POD with different differential orders, which can make the forecast curve closer to real data and the empirical results have a better fit.

According to the characteristics of POD at each stage, using the dichotomous search method, different values of α are set at different time intervals. When $t \in [0, 10]$, set $\alpha = 1.05$ to accommodate the feature that information is spreading at an ever-increasing speed in the early stages of POD; when $t \in (10, 30]$, set $\alpha = 0.86$ to accommodate the decreasing rate of information spread in the middle stage of POD; When $t \in (30, 87]$, set $\alpha = 0.68$ to accommodate the feature that information spreads more slowly and tends to be smoother in the later stages of POD. The simulation graphs of the fractional-order SS1IR model outcomes are shown in Fig. 5.

Comparing Fig. 4 and 5, we can find that the fractional-order model has better predictions than the integer-order model. The prediction curves can approximate the actual data curves when using different fractional-order differentials α at different stages of POD, and the experimental results have a better fit and smaller errors.

C. Error Analysis

To further investigate and confirm the precision and validity of the model, this paper uses the mean absolute error (MAE) and root mean square error (RMSE) to analyse the predicted and real data of the traditional SIR model, the integer-order SS1IR model and the fractional-order SS1IR model respectively. The specific simulation results are shown in Fig. 6

MAE refers to the average of the distances between the model predictions and the actual data. The formula is shown below.

$$MAE = \frac{1}{m}\sum_{i=1}^{m} |F - F^{'}| \tag{7}$$

RMSE refers to the deviation of the predicted value of the model from the actual data. The formula is shown below.

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(F - F^{\prime})^2} \tag{8}$$

Where F represents the actual data, $F^{'}$ represents the predicted data in the different models and m denotes the quantity of data. The smaller the MAE and RMSE, the better the curve fit and the better the forecast of the model.

A total of 87 data were collected for calculation in this simulation experiment and the MAE and RMSE values were compared between the traditional SIR model, the integer-order SS1IR model and the fractional-order SS1IR model to verify the prediction of the model. The results are shown in Tab. I.

TABLE I
ERROR RESULTS FOR DIFFERENT MODELS

Model	SIR model	Integer-order SS1IR model	Fractional-order SS1IR model
MAE	3734.9	1176.1	158.9
RMSE	4538.6	1734.7	237.4

The results indicate that the fractional-order SS1IR model has the smallest MAE and RMSE values. It can be shown that the fractional-order SS1IR model fits the curve better, with higher accuracy and better fit, and can more accurately depict the actual trend of POD in social networks.

IV. CONCLUSIONS

In this paper, we discuss a fractional-order SS1IR model for predicting the POD in social networks. On the basis of the traditional SIR model, a fractional-order differential equation is applied to predict the POD, taking into account the "anomalous dissemination" of information to achieve better objectives. It also discovers the influence of web hypers on the POD in social networks, introduces the S1 super-spreading node, and establishes the fractional-order SS1IR POD model. The experimental results show that the model can better predict the POD with less error than the real data, which plays an active and effective role in the prediction of POD in social networks.

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