Research on Energy Saving Optimization of Random Traction Strategy for Urban Rail Transit

Dongzhi Li, Xuelei Meng, Zheng Han, Shichao Xu, Bo Zhang, Lihui An, and Ruidong Wang

***Abstract*—Urban rail transit usually adopts the traditional four-stage method of traction - uniform speed - coasting - braking as the control sequence, and simply adopts the maximum acceleration to traction and braking in the operation process, ignoring the adaptability of the control strategy and the actual operation ramp. Considering the random interval length and the control strategy of random operating conditions, and running at a certain adaptive constant acceleration under traction and braking conditions, the integral formula of energy consumption calculation based on constant acceleration was further deduced on the basis of the existing mechanical formula, and a new type of single mass point random operating traction calculation model was constructed. The improved quantum genetic algorithm (IQGA) was designed to solve the model, and the algorithm and model were compared with the traditional QGA and the four-stage manipulation method respectively. The results show that compared with the traditional quantum genetic algorithm, the improved quantum genetic algorithm can effectively prevent the algorithm from falling into premature and improve the possibility of obtaining better solutions by introducing the niche strategy and combining the divergent search strategy of quantum mutation and quantum catastrophe. Compared with the traditional four-stage method, the model can further realize the energy saving operation of the train according to the specific situation of the ramp, and the energy saving is 4.06% while the time deviation from the train diagram is only 0.1601s, which proves the feasibility of the model. This method can provide a more practical energy-saving control scheme for urban rail transit.**

***Index Terms*—Urban rail transit; Optimal train operation; Energy saving; Constant acceleration**

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I. Introduction

Urban rail transit has the characteristics of fast, safe and punctual. With the increase of rail transit operating mileage, the proportion of traction energy consumption in the total energy consumption of rail transit also increases. How to reduce traction energy consumption has gradually become the focus of urban rail transit train optimization.

Currently, the existing research results are mainly based on the four-stage control strategy proposed by [1-4], which means that the control is carried out in the way of "full traction - uniform speed - coasting - full braking" without considering the slope changes. On this basis, [5-8] makes train operation closer to the actual situation by constructing multi-particle optimization model and considering regenerative braking. [7-11] construct a single objective model to solve the problem with the goal of minimizing energy consumption. [12] constructed a multi-objective model aiming at minimum energy consumption and minimum tracking interval, and took Beijing Yizhuang Line as an example for calculation. [13] designed particle swarm optimization algorithm to solve the optimal train control strategy with the goal of minimizing energy consumption, running time and docking accuracy. A double-layer programming model for energy saving when trains run on time is designed in [14], and the solution of minimum energy consumption is obtained on a just-in-time basis. A two-stage optimization method for energy saving operation of subway trains under fluctuating ramps and timing constraints is proposed in [15] and solved by genetic algorithm.

On the basis of these research, a multi-objective model of train energy saving traction is constructed by using random traction conditions and operating with constant acceleration under traction and braking conditions, and the energy saving traction strategy of urban rail transit is explored.

II. Problem Description

In urban rail transit, because the ramp’s situation are different, if trains operate between stations according to the four phases of a single method of operation, it will no doubt to ignore some details, such as downhill gravitational potential energy use. If the downhill ramp is located in the four stage method of uniform interval, the train is bound to maintain uniform part and consumes more energy to maintain the operation of the four stage method. Through the ramp situation into account, a simulation model of energy saving traction of single point train is constructed to provide a more energy saving operation suggestion.

III. RANDOM MANIPULATION MODEL

A. Model Assumptions

In order to make the train more considerate of the line situation, the model is set as the train in the process of operation does not run as four stage method, but a random traction mode. Each operating condition can be converted to each other in the appropriate position.

Due to the randomness of the operating line condition, the corresponding segment length and the assigned acceleration, this problem is actually a combinatorial optimization problem. For this problem, the idealized hypothesis is proposed as follows:

1) Conversion time of each working condition is 0.
2) Treat the train as a mass point regardless of its length.
3) The interval speed limit is the general value, namely $80\,km/h$.
4) The maximum allowable constant acceleration is used to draw the brake reverse calculation curve.

According to this model assumption, the operating condition of the train can be changed as shown in the Fig.1, and the train running curve is laid out as shown in Fig. 2.

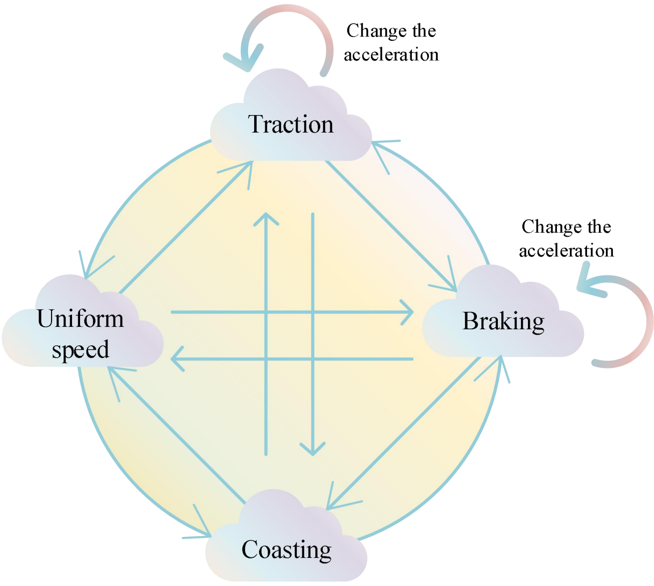

Fig. 1. Schematic diagram of train operating condition conversion

As the fig. 1 shows, the operating conditions can be converted between two pairs. In particular, traction and braking conditions can be internally transformed by changing the acceleration.

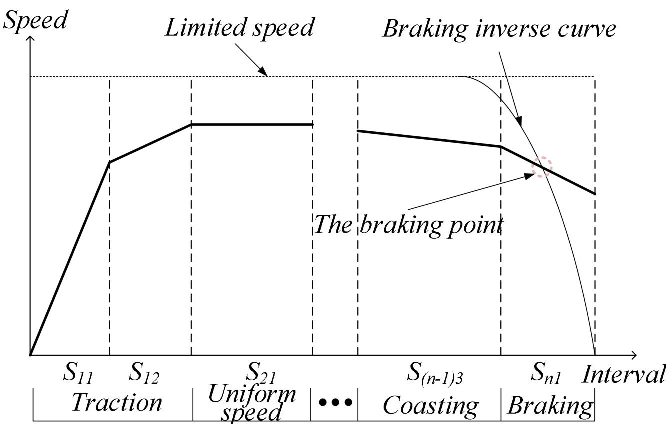

Fig. 2. The train operation curve drawing sketch

And as shown in fig. 2, the station spacing will be divided into n segments, and each segment is assigned with working conditions randomly. Among them, the first section must be the traction condition. At the same time, since the corresponding length of the working condition spans two ramps in this section, it is divided into two parts, and the acceleration is allocated respectively. And the intervals are named S_{11} and S_{12}, so that the corresponding slope and curve radius of each interval are equal, and the required parameters, such as energy consumption and time, can be directly obtained by using the formula. In this figure, the running curve intersects with the braking curve at the n section, at which point maximum braking must be applied to enable the train to stop at the designated station. At this point, the part of the curve behind the intersection is replaced by the braking inverse curve to obtain the complete train operation curve.

B. Symbol Description

Literature [16-18] provides relevant mechanical calculation formulas for traction calculation. On this basis, related formulas for traction and braking with constant acceleration are further derived. The related parameters are shown in Table I.

TABLE I
PARAMETER DESCRIPTION

Symbol	Meaning	Unit
n	Total number of segments segmented between stations	-
m_i	The number of segments subdivided under the i-th segment according to the different ramps	-
E_{ij}	The energy consumed by running in segments	$kW \cdot h$
E_{OD}	Total energy consumption between stations	$kW \cdot h$
S_{ij}	The length of the section	m
S	Station spacing	m
t_{ij}	The time a train runs in a section	s
T	The operation time specified on the operation chart	s
Gk_i	The working condition of the i-th section	-
a_{ij}	Acceleration running on the segment	m/s^2
W	Ramp resistance	N
w_r^{ij}	Additional curve resistance on section	N/kN
A	Empirical coefficient of basic resistance calculation	-
B	Empirical coefficient of basic resistance calculation	-
C	Empirical coefficient of basic resistance calculation	-
m	Weight of train	t
g	Acceleration of gravity	m/s^2
v	The speed of the train	m/s
$v_{\lim it}$	Interval running speed limit	m/s
v_0	The initial velocity into the segment	m/s
i_{ij}	The slope of the section	‰
R_{ij}	The curve radius of the segment	m
w_i^{ij}	Additional ramp resistance on section	N/kN

C. Objective function

1) Objective function 1: traction energy consumption minimization

The first priority of this problem is to save energy by considering the situation of the slope, so as to minimize the sum of the energy consumption of the train in each slope section.

$$\min E_{OD} = \sum_{i=1}^{n}\sum_{j=1}^{m_i} E_{ij} \tag{1}$$

$$W = [A + B(3.6v) + C(3.6v)^2 + w_i^{ij} + w_r^{ij}]\text{mg} \tag{2}$$

Wherein, (1) is the goal of energy minimization, and the ramp resistance can be calculated according to (2).

Then, when $Gk_i = 1$, the energy of each slope section can be calculated as:

$$\begin{aligned} E_{\text{ij}} &= \int_0^{s_{ij}} [W + 1000ma_{ij}]ds \\ &= mg\{[(A + i_{ij} + \frac{600}{R_{ij}} + \frac{1000a_{ij}}{g})v_0 + 3.6Bv_0^2 \\ &+12.96Cv_0^3]t_{ij} \\ &+[\frac{1}{2}a_{ij}(A + i_{ij} + \frac{600}{R_{ij}} + \frac{1000a}{g}) + 3.6Ba_{ij}v_0 \\ &+19.44Ca_{ij}v_0^2]t_{ij}^2 \\ &+(1.2Ba_{ij}^2 + 12.96Ca_{ij}^2 v_0)t_{ij}^3 + \frac{1}{4}Ca_{ij}^3 t_{ij}^4\} \end{aligned} \tag{3}$$

When $Gk_i = 2$, the energy of each slope section can be calculated as:

$$E_{\text{ij}} = mgv_0 t_{ij}\left|A + 3.6Bv_0 + 12.96Cv_0^2 + i + \frac{600}{R_{ij}}\right| \tag{4}$$

When $Gk_i = 3$, the trains run in idle mode, with no energy consumption, so the energy of each slope section can be calculated as:

$$E_{\text{ij}} = 0 \tag{5}$$

When $Gk_i = 4$, the energy of each slope section can be calculated as:

$$\begin{aligned} E_{\text{ij}} &= -\int_0^{s_{ij}} [W + 1000ma_{ij}]ds \\ &= -mg\{[(A + i_{ij} + \frac{600}{R_{ij}} + \frac{1000a_{ij}}{g})v_0 + 3.6Bv_0^2 \\ &+12.96Cv_0^3]t_{ij} \\ &+[\frac{1}{2}a_{ij}(A + i_{ij} + \frac{600}{R_{ij}} + \frac{1000a}{g}) + 3.6Ba_{ij}v_0 \\ &+19.44Ca_{ij}v_0^2]t_{ij}^2 \\ &+(1.2Ba_{ij}^2 + 12.96Ca_{ij}^2 v_0)t_{ij}^3 + \frac{1}{4}Ca_{ij}^3 t_{ij}^4\} \end{aligned} \tag{6}$$

2) Objective function 2: minimize time deviation

If energy saving operation is only considered, it can generally be achieved by appropriately extending the interstation running time. However, the delayed train propagation caused by this will undoubtedly affect the execution of the operation chart. Therefore, the deviation between the total running time of the train in each slope section and the fixed time of the train diagram should be minimized.

$$\min\left|\sum_{i=1}^{n}\sum_{j=1}^{m_i} t_{ij} - T\right| \tag{7}$$

D. Constraints

To limit the value range of each model parameter, the following constraints are set:

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i} S_{ij} = S \tag{8}$$

$$Gk_i = \begin{cases} 1,\ \text{Traction} \\ 2,\ \text{Uniform speed} \\ 3,\ \text{Coasting} \\ 4,\ \text{Braking} \end{cases},\quad i \in [1, n] \tag{9}$$

$$v \in \begin{cases} \{0\},\ \mathrm{S}_{ij} = \mathrm{S}_{11} \text{ or } \mathrm{S}_{\mathrm{nm}_n} \\ (0,\ v_{\lim it}),\ \text{others} \end{cases} \tag{10}$$

$$a_{ij} \in \begin{cases} (0, F_q^v) & , Gk_i = 1 \\ \{0\} & , Gk_i = 2 \\ \{0\} & , Gk_i = 3 \\ (0, F_z^v) & , Gk_i = 4 \end{cases} \quad i \in [1, n],\ \text{j} \in [1, m_i] \tag{11}$$

Among them, (8) ensures the accuracy of train stopping. (9) constrains the value of working condition. (10) constrains the value of speed, and (11) limits the range of acceleration.

IV. Solving Algorithm

Since this model needs to determine interval segmentation and allocate operating conditions and acceleration, coupled with the model has a large solution space, the improved quantum genetic algorithm with good search performance and difficulty in falling into local optimal is used to solve the model.

A. Quantum genetic algorithm

Quantum genetic algorithm (QGA) is different from traditional genetic algorithm (GA) in that its chromosome adopts quantum bit encoding based on qubit and superposition state, which generates new solutions through observation and has better search performance.

According to [19], before observation, each qubit is in a superposition of 0 and 1, which can be expressed by Dirac notation:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{12}$$

In the formula, α and β are probability amplitude of $|0\rangle$ and $|1\rangle$ respectively, and normalization conditions should be met as:

$$|\alpha|^2 + |\beta|^2 = 1 \tag{13}$$

Quantum chromosomes with length N constituted by them can be expressed as follows:

$$\begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \beta_1 & \beta_2 & \cdots & \beta_n \end{bmatrix} \tag{14}$$

In this algorithm, the quantum revolving gate is usually used to make the superposition states of each qubit interact with each other, so as to change the probability amplitude and realize population updating. The updating process always meets the normalization conditions, which can be expressed as:

$$|\psi'\rangle = R(\theta)\times|\psi\rangle = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} \tag{15}$$

Where, $R(\theta)$ is the quantum revolving door matrix, and θ is the rotation Angle, which can be checked according to the table in [20].

B. Improved quantum genetic algorithm

The traditional quantum genetic algorithm is prone to prematurity and local optimization, so the algorithm is improved as follows.

1) Niche

During population initialization, the niche evolutionary strategy was introduced in [20], different from the traditional homogenization, so that the initial population was more evenly dispersed in the solution space. The j-th qubit initialization method of the i-th individual of the population was as follows:

$$\begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix} = \begin{bmatrix} \sqrt{i/N} \\ \sqrt{1-i/N} \end{bmatrix} \tag{16}$$

2) Quantum variation and quantum catastrophe

Quantum variation refers to the complete reversal of the evolutionary direction of an individual by exchanging A and B with a small probability, thus enhancing the genetic diversity of the population and reducing the possibility of falling into local optimum.

If the non-inferior solution remains unchanged for several generations in the iterative process, it indicates that it may fall into local optimum and cannot jump out. In this case, quantum catastrophe is introduced to initialize the qubit coding, which can immediately jump out of the current possible local optimum [21].

C. Algorithm design for this model

According to the characteristics of the model, the solution steps are designed as follows.

Step 1: (16) is used to initialize the qubit coding and observe it to obtain the length of each interval and corresponding working conditions. Heuristic definition of the first working condition must be traction.

Step 2: According to individual current working conditions, each segment acceleration is adaptively allocated, and individual fitness is calculated accordingly. According to the formula, non-inferior solutions are screened, non-inferior solutions and current individuals are recorded, and the particle with the smallest time deviation is taken as the current global optimal particle.

Step 3: The qubit code is observed once, and then the acceleration is allocated adaptively and the fitness of the solution is calculated. If the current solution dominates the original solution, the individual is updated; if they are not dominated by each other, it is determined randomly.

Step 4: The current individual is combined with the previous non-inferior solution, and the new non-inferior solution is screened and recorded.

Step 5: Determine whether catastrophe conditions are met. If yes, carry out quantum catastrophe.

Step 6: According to (15), the globally optimal particle qubit code is updated through the quantum revolving gate, and then quantum mutation is carried out with a small probability.

Step 7: Judge whether the preset number of iterations has been reached. If yes, go to Step 8; if no, go to Step 3.

Step 8: The optimal solution in the final Pareto solution is selected according to (17), and the speed-curve diagram of train energy saving operation is drawn accordingly.

The algorithm flow chart is shown in Fig. 3.

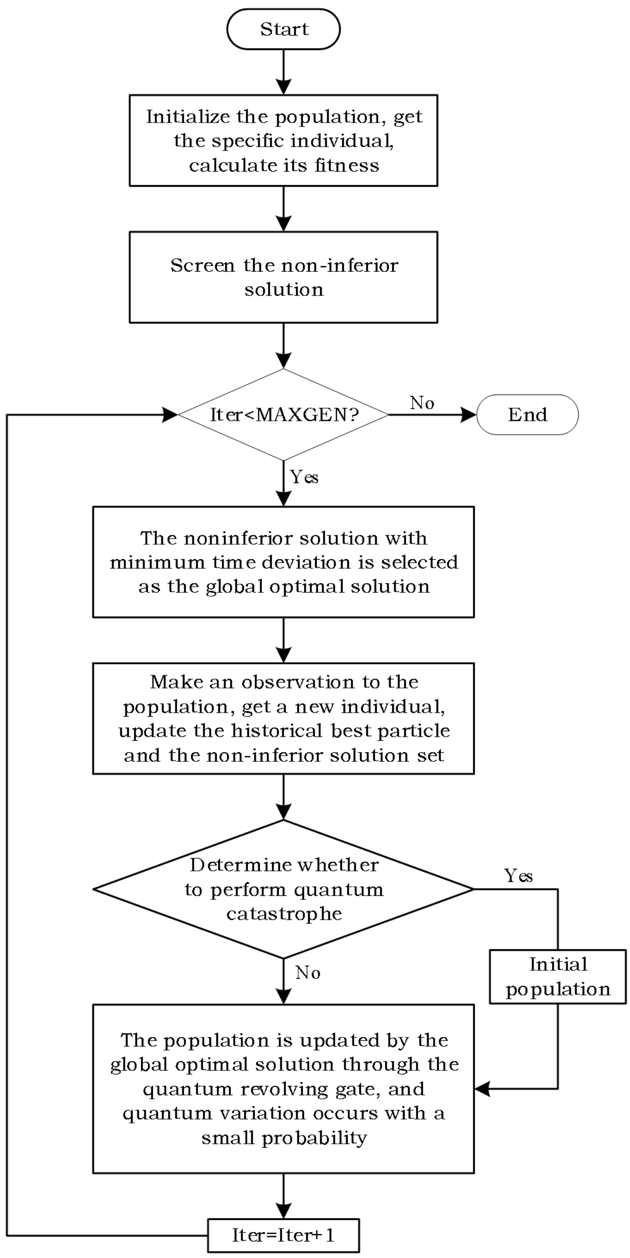

Fig. 3. Algorithm flow chart

V. Case Design and Simulation Analysis

A. Basic Data

The design line data is shown in Table II. On this line, type A car is used for traction, and the train characteristic data is shown in Table III. In addition to these, the total mass of the train is 172 t.

TABLE II
LINES DATA

Index	The ramp starting point(m)	Slope(‰)	Radius of curve(m)	Speed limit(m/s)
1	0	20	-	80
2	550	-13.8	-	80
3	724	-16.8	500	80
4	780	-23	500	80
5	947	-23.5	500	80
6	1050	-3	-	80
7	1290	10	-	80
8	1588	13	400	80
9	1650	15	400	80
10	1755	23	-	80
11	1781	18	400	80
12	1900	10	400	80
13	1953	4	400	80
14	2100	3.4	400	80
15	2263	13.4	-	80
16	2300	3	-	80
17	2580	3.5	-	80
18	2950	5.75	-	80
19	3199	-5.75	350	80
20	3400	-	-	-

TABLE III
DATA OF TRAIN CHARACTERISTICS

Operating speed (km/h)	Traction force (kN)	Braking force (kN)
0	400	389
10	400	389
20	400	389
30	400	389
40	400	389
50	320	389
60	244	389
70	180	389
80	138	321
90	122	254
100	88	206

According to the calculation, the interval running time obtained by using the strategy of full acceleration - cruising at speed limit - full deceleration needs at least 166.9584 *s*. Here, the fixed time T=200*s* is set to compare and explore the traditional four-stage method and the random traction strategy.

The improved quantum genetic algorithm (IQGA) was used to solve the traditional four-stage method, and the best solution was obtained: the position of the working condition transition point of the optimal scheme was 89 *m*, 2306 *m* and 33333.7 *m*, and the energy consumption E_{ct} was 32.8967 $kW \cdot h$.

B. Model solution and analysis

When solving the model constructed above, because the final Pareto solution is relatively dense, it is difficult to intuitively select the optimal solution. Therefore, the energy gain gained per unit time deviation is used as the benefit evaluation function of the solution, and the formula is as follows:

$$\max Z = \begin{cases} \dfrac{E_{ct} - E_{OD}}{\sum_{i=1}^{n}\sum_{j=1}^{m_i} t_{ij}}, & E_{ct} > E_{OD} \\ 0, & E_{ct} \le E_{OD} \end{cases} \quad (17)$$

Take $n = 5$ temporarily, and set the number of iterations as 200. The iteration effect of the algorithm before and after improvement is shown in Fig. 4. It can be seen that although the traditional quantum genetic algorithm occasionally gets a better solution in the early stage, it falls into local optimum prematurely and cannot jump out, while the improved quantum genetic algorithm can jump out steadily at intervals.It can be seen that the improved quantum genetic algorithm has a better simulation effect, and the Pareto surface obtained from this solution is shown in Fig. 5. The optimal benefit solution is that energy consumption is 31.3173 $kW \cdot h$ and time deviation is 0.1601 s, which means that for every unit of time deviation, it can gain 9.8651 $kW \cdot h$ of energy. The comparison with the scheme obtained by the traditional four-stage method is shown in Fig. 6.

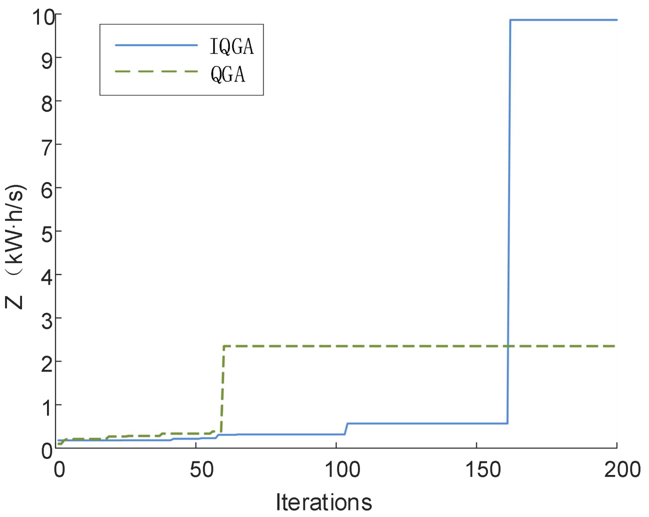

Fig. 4. Algorithm iteration graph

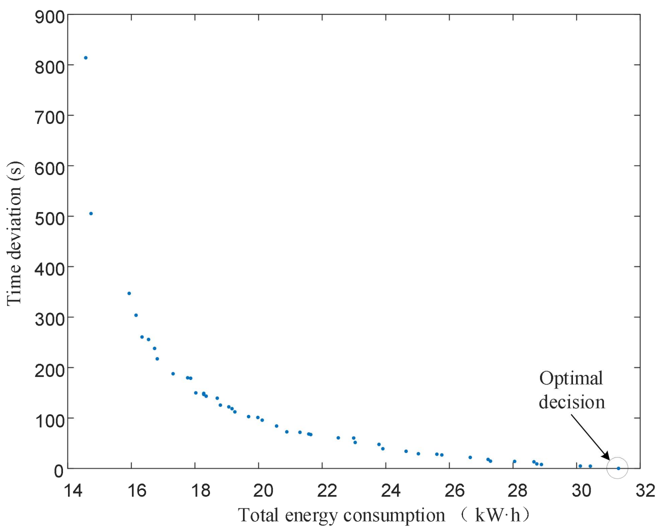

Fig. 5. Pareto solution of optimal control

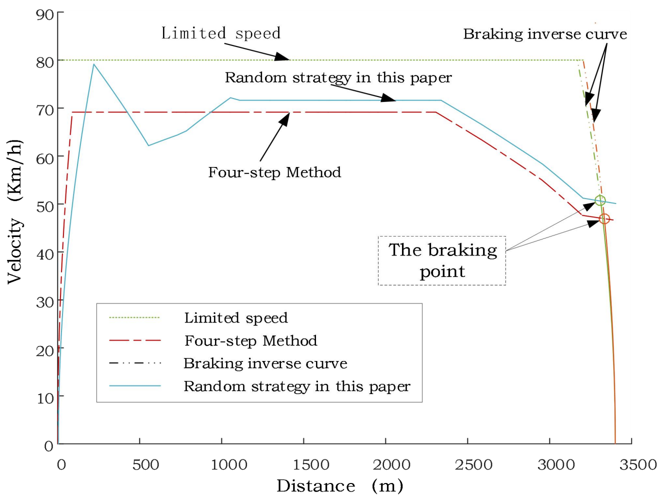

Fig. 6. Comparison chart of scheme

The optimal scheme obtained is that the train is pulled from 0 to 216 m, and then, it coast to 1108 m, and after that, it coast again after cruising at a constant speed to 2334 m. The braking point is located at 3310.9 m. It can be seen that the train makes use of the downhill between 550 m and 1108 m to coast. The speed of the train can be increased even without energy consumption, indicating that the model can achieve the effect of energy saving by considering the situation of the ramp.

C. *Sensitivity analysis*

1) *Sensitivity analysis for the number of segments*

In this simulation model, the change of segment number of interval segmentation has a great influence on the result. On the one hand, if the number of segments is too small, there is no good solution can be obtained. On the other hand, if the number of segments is too large, the solving time of the program will greatly increase. Therefore, the sensitivity analysis of this variable is carried out. As the evaluation function increases, the benefit changes as shown in Fig. 7, and the corresponding optimal operation sequence is shown in Table IV. There is no solution when n = 1, and no energy-saving solution when n = 2 and 3, so it will not be described here.

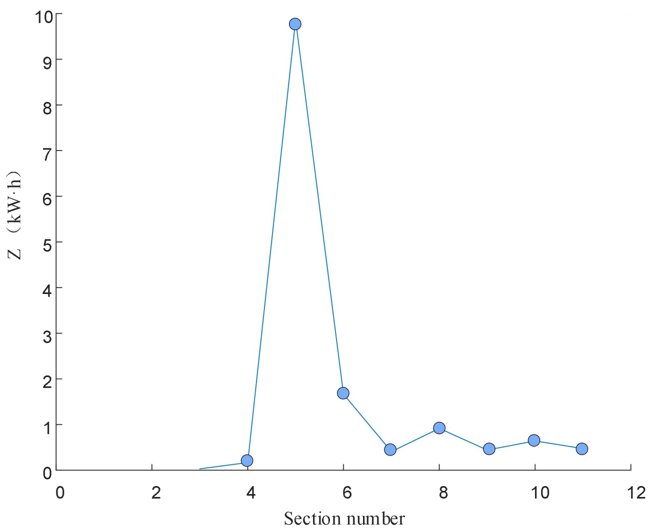

Fig. 7. Benefit trend diagram

TABLE IV
THE OPTIMAL CONTROL SEQUENCE

n	Sequence of manipulation (1: Traction; 2: Uniform speed; 3: Coasting; 4: Braking)									
4	1	2	3	4	-	-	-	-	-	-
5	1	3	2	3	4	-	-	-	-	-
6	1	3	2	2	3	4	-	-	-	-
7	1	3	2	3	3	3	4	-	-	-
8	1	3	1	2	3	3	3	4	-	-
9	1	3	3	1	3	3	3	3	4	-
10	1	1	2	3	2	2	3	3	2	4

As can be seen from Fig. 7, when n = 4, the optimal control is the four-stage method, while when n = 5, 6 and 7, the optimal solution changes into the traction - coasting - uniform speed - coasting - braking control strategy. In other words, according to the situation of the ramp, a section of coasting control is added in the early stage, which significantly increases the benefit. When n = 5, the better solution is already obtained. Following with the increase of n, due to the variable dimension enlargement, search algorithm gets worse in the solution. Only expanding populations and iteration steps can solve this problem. But it will undoubtedly reduce the algorithm search efficiency. Therefore, based on the idea of the algorithm efficiency, it is desirable to set *n* equal to 5 on this line.

2) Sensitivity analysis for the quantum mutation probability

After research, it is found that the setting of quantum mutation probability also has a significant influence on the quality of solution. In the solution above, the probability is set as 0.0005. Therefore, it is divided into four level from 0 to 0.0015 and brought into the solution. And the algorithm iterative results are showed as Fig. 8.

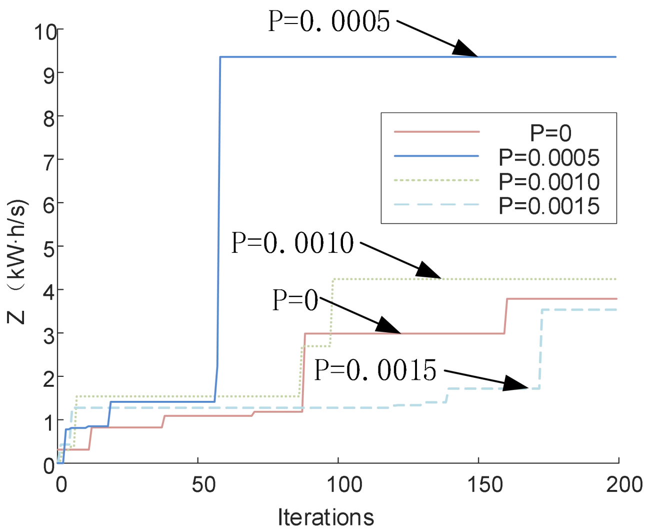

Fig. 8. Algorithm iteration graph

As shown in the figure, when the mutation probability was zero, it was the experimental control group.

When the probability rises to 0.0005, it can be seen that the solution value rises rapidly and reaches the optimal value around 50 generations. In particular, the simulation was re-run here, so the results are slightly different from the previous ones.

As the probability increases to 0.001, the optimal solution decreases. When p=0.0015, it can be seen that it is already lower than that when p=0. After analysis, it is found that the efficiency of the algorithm decreases due to too many mutation operations, and then the desired effect can not be achieved within the specified number of iterations。

VI. CONCLUSION

1) On the basis of the current subway traction calculation method, a new single-point traction calculation model is constructed based on the train traction mode with constant acceleration and the traction strategy with random working conditions, and the simulation accuracy of energy consumption's calculation is improved through integral calculation.

2) For the algorithm of model solving, the niche strategy is introduced for the generation of initial solution to improve the algorithm, and quantum mutation and quantum catastrophe are used as the divergence strategy. The example shows that the designed algorithm is more efficient than the traditional quantum genetic algorithm.

3) According to the calculation example, compared with the traditional four-stage method, the energy consumption of the optimal solution in this paper decreases by 4.06%, indicating that the model can better fit the actual situation in practical operation and give a more energy-saving traction strategy according to the specific ramp.

4) Through the sensitivity analysis of the number of segments and mutation rate, it is known that in different examples, the parameter settings have a great influence on the results. The low mutation probability setting can better promote the efficiency of the algorithm. With the increase of mutation probability setting, too many mutation operations will reduce the efficiency of the algorithm. In calculations on a scale comparable to the examples used in the paper, setting n = 5 and p = 0.0005 is appropriate.

5) In the process of simulation, the current solution is improved compared with the traditional four-stage method, but there is still room for improvement. How to improve the efficiency of the algorithm or reduce the solution space without losing its optimal solution is a problem worth studying in the future. In addition, considering the length of the train, the construction of multi-particle simulation model, or based on this to explore the minimum tracking interval of the train and energy saving operation of regenerative braking, is also the key direction of subsequent research.

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