Network Public Opinion Prediction Based on Conformable Fractional Non-homogeneous Discrete Grey Model

Qiujuan Tong, Luwen Wang, Linna Li, Jianke Zhang

***Abstract*—The traditional grey model assumes that the original data series conforms to the homogeneous exponential trend rather than the non-homogeneous exponential trend. However, compared with the integer-order gray model, the fractional-order gray model is more efficient and flexible in time series forecasting. Hence, in this paper, a conformable fractional order calculus is introduced to extend the integer order gray model into a fractional order gray model. A conformable fractional non-homogeneous exponential discrete grey model (abbreviated as CFNDGM) is proposed, and the particle swarm algorithm is further developed to optimize its order. Specifically, we first use the Baidu index generated by "Xi'an Epidemic" and "MU5735" to build a model. Then use the least squares method to solve the model parameters, obtain the predicted simulation value through the response expression, and finally obtain the data prediction result. The simulation validates that the prediction accuracy of the fractional order non-homogeneous grey model is higher than that of the integer order non-homogeneous grey model.**

***Index Terms*—internet opinion forecasting, CFNDGM model, fractional calculus, fractional accumulation, non-homogeneous index**

I. Introduction

IN the network environment of ample data information, social network platforms represented by "Sina Weibo, Today's Toutiao, Zhihu" has developed rapidly and become an indispensable part of people's lives. These social networking platforms not only greatly meet people's needs to access and share real-time news, but also express opinions with personal emotions and positions anytime, anywhere. However, it potentially leads to a public sentiment with the fermentation of events. Public opinion and the impact of various uncertain factors will bring up multiple unrealistic online rumors [1], eventually developing into uncontrollable social public events.

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There are various methods of network public opinion prediction, such as the infectious epidemic model [2]-[3] (SEIR), time series model [4]-[5] (ARIMA), grey model [6]-[7] (GM), and so on. The grey model is established basis on the grey system theory. Specifically, in the early 1980s, Julong Deng first proposed the new technical term "gray system" and applied the depth of color to characterize the information. For example, black represents unknown information, white represents clear information, and fuzzy information is replaced by gray. Grey models are highly effective in small sample time series forecasting and are currently widely used in various application fields, such as energy marketing, energy economics [8], environmental issues [9], agriculture [10], etc.

Although the gray model has been widely used in various fields and achieved good results, its prediction results still need further improvement. In the GM (1,1) model, discrete equations are used for parameter estimation, and continuous equations are employed during the simulation and prediction. Different representations of discrete and continuous equations cannot be exactly equal. So jumping from discrete to continuous equations creates problems with simulation and prediction errors, even if they are purely exponential based sequences also have this problem. In 2009, Nai-Ming Xie *et al*. proposed the discrete grey model [11] (abbreviated as DGM) to address this problem. In the accumulation process of the traditional gray model, the generated accumulation operator violates the new information priority principle of gray system theory. In 2013, Lifeng Wu *et al*. used the fractional grey model to solve this problem, and experiments proved that this model has better prediction performance than the traditional grey model [12]. Whether the GM (1,1) model, the DGM model, or the fractional gray model are models based on the assumption that the original data series conform to a homogeneous exponential trend, which few systems meet. Most systems align with the assumption that the original data is a non-homogeneous exponential trend. Accordingly, in 2013, Nai-Ming Xie *et al*. proposed a non-homogeneous exponential discrete grey model [13] (abbreviated as NDGM).

Since fractional calculus has better memory function and genetic function and is easier to reveal the internal laws of system objects, it is introduced into the gray system [14]. There are two commonly used fractional gray models, one is a discrete gray prediction model based on fractional accumulation, and the other is a continuous gray prediction model based on Caputo fractional order [15]. In 2014, Khalil *et al.* [16] proposed and proved a new definition of fractional derivative, called the conformable fractional derivative. This definition of derivatives is much simpler than the "old" definition of fractional derivatives, and conformable fractional derivatives have nice properties. Many problems that were difficult or impossible to solve using the "old" definition can often be solved. For example, Riemann-Liouville and Caputo derivatives are two well-known derivatives, and both types of derivative do not obey Leibniz's law. In recent years, conformable fractional derivatives have been widely used. Hammad and Khalil [17] proposed a fractional Fourier series (FFS) based on conformable fractional derivatives and proved that FFS is very effective for solving partial differential equations. Combining the above analysis, improved on the non-homogeneous exponential discrete gray model, this paper proposes a new gray model, the conformable fractional non-homogeneous exponential discrete grey model (abbreviated as CFNDGM).

This paper uses Python to capture the data generated by the Baidu index of popular online events "Xi'an epidemic" and "MU5735". A conformable fractional differential non-homogeneous exponential gray model is established, and the model is implemented by Matlab. The model is used to predict the event data. The prediction data results were compared with the prediction accuracy of models such as the conformable fractional order discrete grey model (CFDGM) and NDGM *et al.*

II. NDGM Model

Suppose the original sequence of the sequence is $x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$, The sequence obtained by first-order accumulation of $x^{(0)}$ is $x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)$,In $x^{(1)}(i) = \sum_{k=0}^{i} x^{(0)}(k)$, $i = 1,2,\dots n$. Equation (1) is called the non-homogeneous exponential discrete grey model [13]:

$$\begin{cases} \hat{x}^{(1)}(k+1) = \beta_1 \hat{x}^{(1)}(k) + \beta_2 \cdot k + \beta_3 \\ \hat{x}^{(1)}(1) = x^{(1)}(1) + \beta_3 \end{cases} \quad (1)$$

In the above formula, $\hat{x}^{(1)}(k)$ is the simulated value of $x^{(1)}(k)$, which is also the iterative value of the NDGM model. β_1 , β_2, β_3 and β_4 are the parameters of the NDGM model, which are solved by the least squares method using the numerical matrix B and the numerical vector Y:

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = (B^T B)^{-1} B^T Y \quad (2)$$

where:

$$Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{bmatrix} \quad (3)$$

$$B = \begin{bmatrix} x^{(1)}(1) & 1 & 1 \\ x^{(1)}(2) & 2 & 1 \\ \vdots & \vdots & \vdots \\ x^{(1)}(n-1) & k-1 & 1 \end{bmatrix} \quad (4)$$

The prediction model of $\hat{x}^{(0)}$ obtained by first-order cumulative reduction is:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k),\ k = 1,2,\dots,n-1 \quad (5)$$

The recursion for $\hat{x}^{(1)}(k+1)$ in equation (1) is shown below:

$$\begin{aligned} \hat{x}^{(1)}(k+1) &= \beta_1 \hat{x}^{(1)}(k) + \beta_2 \cdot k + \beta_3 \\ &= \beta_1\left(\beta_1 \hat{x}^{(1)}(k-1) + \beta_2 \cdot (k-1) + \beta_3\right) + \beta_2 \cdot k + \beta_3 \\ &= \beta_1^2 \hat{x}^{(1)}(k-1) + \beta_2 \cdot (\beta_1(k-1) + k) + \beta_3 \cdot (1+\beta_1) \\ &= \beta_1^3 \hat{x}^{(1)}(k-2) + \beta_2 \cdot (\beta_1^2(k-2) + \beta_1(k-1) + k) + \beta_3 \cdot (1+\beta_1+\beta_1^2) \\ &= \dots = \beta_1^k \hat{x}^{(1)}(1) + \beta_2 \sum_{j=1}^{k} j \beta_1^{k-j} + \frac{1-\beta_1^k}{1-\beta_1} \cdot \beta_3 \end{aligned} \quad (6)$$

The value of parameter β_4 is calculated by a method similar to the method of least squares. Then construct an unconstrained optimization model and minimize the error values of $x^{(1)}(k)$ and $\hat{x}^{(1)}(k)$. The optimization formula is as follows:

$$\min_{\beta_4} \sum_{k=1}^{n} \left[\hat{x}^{(1)}(k) - x^{(1)}(k)\right]^2 \quad (7)$$

then:

$$\beta_4 = \left(\sum_{k=1}^{n-1} [x^{(1)}(k+1) - \beta_1^k x^{(1)}(1) - \beta_2 \sum_{j=1}^{k} j \beta_1^{k-j} - \frac{1-\beta_1^k}{1-\beta_1} \cdot \beta_3] \cdot \beta_1^k\right) / \left(1 + \sum_{k=1}^{n-1} (\beta_1^k)^2\right) \quad (8)$$

III. Consistent Fractional Accumulation

In this section, we first introduce the definition of the consistent fractional derivative and its properties and then introduce the consistent cumulative sum and consistent difference. Extend the definition of fractional derivative from $r \in (0,1)$ to $r \in (n, n+1), n \in N$, and through the recursive formula, deduce the accumulation formula when the order r is greater than 1.

A. *Calculus definition and properties of consistent fractions*

The advantage of using fractional-order models over integer-order models is that integer-order differentials are local, while fractional-order differentials are global. The fractional order model can clearly show the abnormal nodes in the propagation process. The optimal effect can be obtained by adjusting a small number of parameters and accurately predicting the complete dynamic process of public opinion propagation. Therefore, the fractional derivative proposed by Khalil [16] *et al.* is introduced, which is called the conformable fractional derivative. Then, the conformable fractional differential is defined as follows:

Definition 1 [18]. For $0 < r \le 1$, the left conformable fractional derivative (CFD) is defined as

$$D_{s|x}^{r} f(x) = \lim_{\epsilon \to \infty} \frac{f\left(x + \epsilon (x-s)^{1-r}\right) - f(x)}{\epsilon} \quad (9)$$

and the right CFD is defined as

$$D_{x|s'}^{r} f(x) = -\lim_{\epsilon \to \infty} \frac{f\left(x + \epsilon (s'-x)^{1-r}\right) - f(x)}{\epsilon} \quad (10)$$

Definition 2 [18]. For $0 < r \le 1$, the left conformable fractional integral (CFI) is defined as

$$I_{s|x}^{r} f(x) = \int_{s}^{x} (\xi - s)^{r-1} f(\xi) d\xi \quad (11)$$

and the right CFI is defined as

$$I^r_{x|s'} f(x) = \int_x^{s'} (s' - \xi)^{r-1} f(\xi) d\xi \quad (12)$$

Theorem 1 [18]. The CFD and CFI obey the following relations:

$$D^r_{s|x} I^r_{s|x} f(x) = f(x) \quad (13)$$

$$D^r_{x|s'} I^r_{x|s'} f(x) = f(x) - f(a) \quad (14)$$

Theorem 2 [18]. The CFD satisfies the following properties:

$$D^r_\chi\big(af(x) + bg(x)\big) = aD^r_\chi f(x) + bD^r_\chi g(x) \quad (15)$$

$$D^r_\chi\big(f(x)g(x)\big) = \big[D^r_\chi f(x)\big]g(x) + f(x)D^r_\chi g(x) \quad (16)$$

$$D^r_\chi f\big(g(x)\big) = \big[D^r_{g(x)} f\big(g(x)\big)\big]\big[D^r_\chi g(x)\big]g(x)^{r-1} \quad (17)$$

Without loss of generality, it is also possible to define relations between higher order derivatives and derivatives similar to those usually defined. Definition 1 actually takes the following form:

$$D^r_{s|x} f(x) = \lim_{\epsilon\to\infty} \frac{f\big(x + \epsilon(x-s)^{[r]-r}\big) - f(x)}{\epsilon} = (x-s)^{[r]-r}\frac{df(x)}{dx} \quad (18)$$

where $[\cdot]$ is the ceil function, denotes rounding upwards, i.e. [r] is the smallest integer greater than or equal to r. Therefore, a similar formula can be used to define higher-order uniform derivatives.

Definition 3 [16]. The r-order $(r \in (n, n+1))$ consistent fractional order is defined as follows.

$$D^r_{s|x} f(x) = \lim_{\epsilon\to\infty} \frac{f^{(n)}\big(x+\epsilon(x-s)^{[r]-r}\big) - f^{(n)}(x)}{\epsilon} \quad (19)$$

For all $x > 0, r \in (n, n+1), n \in N+, f$ is differentiable on $n + 1$, and, setting $h = \epsilon(x-s)^{1-r}$, it follows that $\epsilon = h(x-s)^{r-1}$, so that

$$D^r_{s|x} f(x) = (x-s)^{[r]-r} \lim_{\epsilon\to\infty} \frac{f^{(n)}(x+h) - f^{(n)}(x)}{\epsilon} = (x-s)^{[r]-r}\frac{d^n f(x)}{dx^n} \quad (20)$$

When $r = n + 1$, we have$[r] - r$, so we have:

$$D^{n+1}_{s|x} f(x) = \lim_{\epsilon\to\infty} \frac{f^{(n)}(x+h) - f^{(n)}(x)}{\epsilon} = \frac{d^{n+1}f(x)}{dx^{n+1}} \quad (21)$$

B. Conformable fractional accumulation and conformable fractional difference definitions

The classical grey prediction model is evolved from Newton's Leibniz formula, to set up the function $y(t)\epsilon(a, b)$ with order $r\epsilon(0,1)$, the r-order consistent cumulative sum and r-order consistent difference [19] of $y(t)$ in the interval (a, b) are denoted as:

$$\nabla^r_{CF} y^{(r)}(k) = \sum_{j=1}^k \frac{\Gamma(k-j+[r])}{\Gamma(k-j+1)\Gamma([r])} \cdot \frac{y^{(0)}(j)}{j^{[r]-r}} \quad (22)$$

$$\Delta^r_{CF} y^{(0)}(k) = k^{[r]-r}\sum_{j=1}^k \frac{(-1)^{k-j}\Gamma(1+[r])}{\Gamma(k-j+1)\Gamma([r]-k+j+1)} \cdot \hat{y}^{(r)}(k) \quad (23)$$

By analogy with the definition of high-order consistent fractional derivative in Definition 3, it can be obtained that the high-order consistent fractional cumulative sum is defined as follows:

Definition 4 [20]. When the conformable fractional accumulation (CFA) order $r \in (n, n+1), n \in N+$,is defined as follows:

$$\nabla^r f(k) = \nabla^n \left(\frac{f(k)}{k^{[r]-r}}\right) \quad (24)$$

When r = n + 1, the CFA produces an (n+1)-order accumulation ∇^{n+1}, and we can introduce the recursive equation by definition:

$$\nabla^r f(k) = \nabla\left(\nabla^{n-1}\left(\frac{f(k)}{k^{[r]-r}}\right)\right) = \sum_{j=1}^k \left(\nabla^{r-1} f(j)\right), r \geq 1 \quad (25)$$

Definition 4 [20]. The conformable fractional difference (CFD) at order $r \in (n, n+1)$, $n \in N$, is defined as follows:

$$\Delta^r f(k) = k^{[r]-r}\Delta^n f(k) \quad (26)$$

IV. CFNDGM MODEL

Suppose the sequence original sequence is $x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$, which is cumulated by equation (27) of order r to obtain $x^{(r)} = (x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n))$,

where:

$$x^{(r)}(k) = \nabla^r x^{(0)}(k) = \begin{cases} \sum_{j=1}^k \frac{\Gamma(k-j+[r])}{\Gamma(k-j+1)\Gamma([r])} \cdot \frac{x^{(0)}(j)}{j^{[r]-r}}, & 0 < r \leq 1, \\ \sum_{j=1}^k x^{(r-1)}(j), & r > 1 \end{cases} \quad (27)$$

The conformable fractional non-homogeneous exponential discrete gray model (CFNDGM) is expressed as:

$$\begin{cases} \hat{x}^{(1)}(k+1) = \beta_1 \hat{x}^{(1)}(k) + \beta_2 \cdot k + \beta_3 \\ \hat{x}^{(1)}(1) = x^{(1)}(1) + \beta_3 \end{cases} \quad (28)$$

In the above equation $\hat{x}^{(1)}(k)$ is the simulated value of $x^{(1)}(k)$. The parameters β_1, β_2, $\beta_3$and $\beta_4$of the CFNDGM are solved for by least squares as follows:

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = (B^T B)^{-1} B^T Y \quad (29)$$

where:

$$Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{bmatrix} \quad (30)$$

$$B = \begin{bmatrix} x^{(1)}(1) & 1 & 1 \\ x^{(1)}(2) & 2 & 1 \\ \vdots & \vdots & \vdots \\ x^{(1)}(n-1) & k-1 & 1 \end{bmatrix} \quad (31)$$

The prediction model for $\hat{x}^{(0)}$ is obtained by differential reduction as:

When the order of CFNDGM is $r \in (0,1]$:

$$x^{(0)}(k) = \Delta^r x^{(0)}(k) = k^{[r]-r}\sum_{j=1}^k \frac{(-1)^{k-j}\Gamma(1+[r])}{\Gamma(k-j+1)\Gamma([r]-k+j+1)} \cdot \hat{x}^{(r)}(k) \quad (32)$$

When order $r \in (1,2)$:

$$\hat{x}^{(r-1)}(k+1) = \hat{x}^{(r-1)}(k+1) - \hat{x}^{(r-1)}(k),$$
$$k = 1,2,\dots,n-1. \quad (33)$$

V. EXAMPLE APPLICATION

A. Baidu index of "Xi'an Epidemic"

This paper takes the Baidu index of the "Xi'an epidemic" in early 2022 as an example to establish a fractional non-homogeneous index gray prediction model CFNDGM, the original data of the event $X(0)$. Using the particle swarm optimization algorithm [21] (pso), comparing the average absolute percentage error of equation (34), the order r=0.055427 with the smallest error is obtained. Therefore, r=0.055427 is the best order of the fractional order heterogeneous exponential grey prediction model, and the CFNDGM is established as follows:

Baidu index raw data for "Epidemic Xi'an" is $X(0)$:

$$X(0) = (138430,111993,93714,109570, 105209,136514,76575,67917,60583,44783, 37466,38747,29536,28963,36742,25799)$$

Conformable fractional accumulation of $X(0)$ using equation (27) yields $X^{(r)}(k)$:

$$X^{(r)}(k) = (138430.00,196619.69,229818.97, 259399.23,282404.37,307532.28,319717.42, 329244.15,336847.40,341935.32,345825.46, 349531.18,352150.27,354544.92,357391.08,359271.37)$$

By $\beta = (B^TB)^{-1}B^TY$, and equations (30) and (31) to calculate $\beta = (\beta_1, \beta_2, \beta_3)^T$ in the parameter $\beta_1, \beta_2, \beta_3$ and using equation (8) to calculate the parameter β_4, the final value of the parameter is:

$$\beta_1 = 0.74458, \quad \beta_2 = 261.93,$$
$$\beta_3 = 89327.39, \quad \beta_4 = -158.235$$

Using the response expression of equation (28), the simulated value of $X^{(r)}$ is found $\hat{X}^{(r)}$:

$$\hat{X}^{(r)} = (138430.00, 104357.32,114702.05,113041.33, 105114.39,94397.39,82948.70,71931.47, 61948.19,53257.13,45912.15,39852.08, 34957.12,31083.46,28083.79,25818.73)$$

Using equation (32) and equation (33) for conformable fractional difference, the prediction sequence is solved to obtain the prediction sequence $\hat{X}^{(0)}$:

$$\hat{X}^{(r)} = (138271.53, 192679.37, 233505.46, 264205.44,287355.74, 304877.23, 318202.08, 71993.24,62021.54, 53331.60, 45978.77, 39903.82, 34988.88, 31091.84, 28066.77)$$

B. Baidu Index of "MU5735"

The data generated from the Baidu index of the "MU5735" accident was selected as the original data $X(0)$ to build the fractional order non-flush exponential grey prediction model CFNDGM. Using the particle swarm optimization algorithm [21] (pso), the average absolute percentage error of equation (34) is solved and compared, and the order with the smallest error r=1.034642 is obtained. Therefore, r=1.034642 is the best order of the fractional order heterogeneous exponential grey prediction model, and the CFNDGM is established as follows:

Baidu index raw data for "MU5735" is $X(0)$:

$$X(0) = (3698,2260,1528,1242,924, 1195,839,592,363,304,277,274)$$

Conformable fractional accumulation of $X(0)$ by equation (27) gives $X^{(r)}(k)$:

$$X^{(r)}(k) = (3698,8553.46,13938.02,19648.34, 25554.07,31671.71,37917.57,44242.96, 50611.87,57013.71,63442.91,69896.99)$$

By $\beta = (B^TB)^{-1}B^TY$, and equations (30) and (31) to calculate $\beta = (\beta_1, \beta_2, \beta_3)^T$ in the parameter $\beta_1, \beta_2, \beta_3$ and using equation (8) to calculate the parameter β_4, the final value of the parameter is:

$$\beta_1 = 0.69609, \ \beta_2 = 1974.7,$$
$$\beta_3 = 4013.5, \beta_4 = -158.472$$

Using the response expression of equation (28), the simulated value of $X^{(r)}$ is found $\hat{X}^{(r)}$:

$$\hat{X}^{(r)} = (3698.13, 4864.32, 5360.72, 5706.26, 5946.79,6114.22, 6230.77, 6311.89, 6368.36,6407.67, 6435.04, 6454.08)$$

Using equations (32) and (33) for conformable fractional difference, the prediction sequence is solved to obtain the prediction sequence $\hat{X}^{(0)}$:

$$\hat{X}^{(0)} = (3698, 2277.04, 1433.60, 1317.36, 1137.43, 944.12,762.64, 603.90, 470.99, 362.95,277.0, 209.71)$$

VI. PREDICTION RESULTS PRECISION ANALYSIS

In this section, the error values and the fitting degree of prediction curves of the above two cases will be analyzed. Establish integer-order models: non-homogeneous exponential grey model NDGM, discrete grey model DGM and grey model GM (1,1). The fractional order model order is determined through MAPE, and the fractional discrete grey model CFDGM and the fractional grey model CFGM are established. The data prediction of the above two cases is made through these five models, and the model prediction results are shown in Table. I and Table. II. The predicted results were compared with those of CFNDGM. Finally, the established grey model is used to forecast future data.

$$MAPE = \sum_{j=1}^{N} \left| \frac{\hat{x}^{0}(j) - x^{(0)}(j)}{x^{(0)}(j)} \right| \times \frac{100\%}{n} \quad (34)$$

A. MAPE error value analysis

Through two cases in this paper, the errors between the prediction results of six different models and the original data are analyzed respectively. The values of mean absolute percentage error (MAPE), mean square error (MSE), and mean absolute error (MAE) are obtained respectively. The calculation results are shown in Table. III and Table. IV below:

The minimum values of the two cases under different error calculation formulas can be obtained from the data in Table. III and Table. IV. The minimum error of "Xi'an Epidemic situation" was: MAPE 10.8404%, MSE 16. 281× 10^7 and MAE 7519.032. The minimum error values of "MU5735" are respectively: MAPE 11.7340%, MSE 12384.72, and MAE 80.8833. The minimum error values are obtained under the CFNDGM model. By comparing MAPE, it can be concluded that the CFNDGM model has the smallest error between the predicted value and the real value among the six models. The error values of CFNDGM in the two cases were 2.421% ~ 5.1376% and 1.3659% ~ 1.6756% smaller than the prediction errors of the remaining five models, respectively. As an improved model of NDGM, the CFNDGM model has significantly improved the accuracy of its prediction results. The prediction accuracy of the two cases increased by 5.1376% and 1.6756% respectively. Comparing the MAPE values of CFNDGM and CFDGM, which are both fractional order models, it can be concluded that the prediction accuracy of CFNDGM is higher, and the prediction accuracy is increased by 2.417% and 1.3659% respectively.

TABLE I
ACTUAL VALUE AND MODEL PREDICTIONS FOR THE "XI'AN EPIDEMIC"

time				
3.10	Actual value	138430.00	DGM	138430.00
	CFNDGM	138430.00	CFGM	138430.00
	NDGM	138430.00	GM (1,1)	138430.00
	CFDGM	138430.00		
3.11	Actual value	111993.00	DGM	128110.82
	CFNDGM	104357.32	CFGM	104573.38
	NDGM	123029.09	GM (1,1)	127159.06
	CFDGM	105740.53		
3.12	Actual value	93714.00	DGM	115156.91
	CFNDGM	114702.05	CFGM	112446.50
	NDGM	113469.50	GM (1,1)	114489.19
	CFDGM	112797.74		
3.13	Actual value	109570.00	DGM	115156.91
	CFNDGM	113041.33	CFGM	112446.50
	NDGM	104314.36	GM (1,1)	114489.19
	CFDGM	110685.75		
3.14	Actual value	105209.00	DGM	93046.15
	CFNDGM	105114.39	CFGM	104185.97
	NDGM	95546.56	GM (1,1)	92810.86
	CFDGM	103755.31		
3.15	Actual value	136514.00	DGM	83637.80
	CFNDGM	94397.39	CFGM	95019.88
	NDGM	87149.71	GM (1,1)	83563.37
	CFDGM	94484.18		
3.16	Actual value	76575.00	DGM	75180.78
	CFNDGM	82948.70	CFGM	84877.12
	NDGM	791081.11	GM (1,1)	75237.28
	CFDGM	84339.99		
3.17	Actual value	67917.00	DGM	67578.88
	CFNDGM	71931.47	CFGM	74672.07
	NDGM	71406.74	GM (1,1)	67740.79
	CFDGM	74192.31		
3.18	Actual value	60583.00	DGM	60745.66
	CFNDGM	61948.19	CFGM	64934.90
	NDGM	64031.20	GM (1,1)	60991.23
	CFDGM	64540.90		
3.19	Actual value	44783.00	DGM	54603.37
	CFNDGM	53257.13	CFGM	55952.14
	NDGM	56967.70	GM (1,1)	54914.19
	CFDGM	55652.67		
3.20	Actual value	37466.00	DGM	49082.15
	CFNDGM	45912.15	CFGM	47855.73
	NDGM	50203.05	GM (1,1)	49442.65
	CFDGM	47647.74		
3.21	Actual value	38747.00	DGM	44119.22
	CFNDGM	39852.08	CFGM	40680.98
	NDGM	43724.60	GM (1,1)	44516.28
	CFDGM	40554.89		
3.22	Actual value	29536.00	DGM	39658.11
	CFNDGM	34957.12	CFGM	34404.50
	NDGM	37520.23	GM (1,1)	40080.77
	CFDGM	34347.76		
3.23	Actual value	28963.00	DGM	35648.09
	CFNDGM	31083.46	CFGM	28969.23
	NDGM	31578.37	GM (1,1)	36087.20
	CFDGM	28968.51		
3.24	Actual value	36742.00	DGM	32043.54
	CFNDGM	28083.79	CFGM	24300.73
	NDGM	25887.89	GM (1,1)	32491.54
	CFDGM	24343.26		
3.25	Actual value	25799.00	DGM	28803.46
	CFNDGM	25818.73	CFGM	20317.67
	NDGM	20438.16	GM (1,1)	29254.15
	CFDGM	20392.01		

CFNDGM has order r = 0.055427; CFDGM has order r = 0.290993; CFGM has order r = 0.262.

TABLE II
ACTUAL VALUE AND MODEL PREDICTIONS FOR THE " MU5735"

time				
3.22	Actual value	3698.00	DGM	3698.00
	CFNDGM	3698.00	CFGM	3698.00
	NDGM	3698.00	GM (1,1)	3698.00
	CFDGM	3698.00		
3.23	Actual value	2260.00	DGM	2108.13
	CFNDGM	2277.04	CFGM	2063.02
	NDGM	2127.66	GM (1,1)	2094.23
	CFDGM	2083.85		
3.24	Actual value	1528.00	DGM	1695.78
	CFNDGM	1433.60	CFGM	1711.55
	NDGM	1695.13	GM (1,1)	1688.23
	CFDGM	1715.22		
3.25	Actual value	1242.00	DGM	1364.08
	CFNDGM	1317.36	CFGM	1392.15
	NDGM	1353.41	GM (1,1)	1360.94
	CFDGM	1389.33		
3.26	Actual value	924.00	DGM	1097.27
	CFNDGM	1137.43	CFGM	1120.13
	NDGM	1083.42	GM (1,1)	1097.10
	CFDGM	1115.50		
3.27	Actual value	1195.00	DGM	882.64
	CFNDGM	944.12	CFGM	895.11
	NDGM	870.12	GM (1,1)	884.41
	CFDGM	890.68		
3.28	Actual value	839.00	DGM	709.99
	CFNDGM	762.64	CFGM	711.91
	NDGM	701.59	GM (1,1)	712.95
	CFDGM	708.44		
3.29	Actual value	592.00	DGM	571.12
	CFNDGM	603.90	CFGM	564.25
	NDGM	568.44	GM (1,1)	574.74
	CFDGM	561.91		
3.30	Actual value	363.00	DGM	459.41
	CFNDGM	470.99	CFGM	446.04
	NDGM	463.24	GM (1,1)	463.32
	CFDGM	444.73		
3.31	Actual value	304.00	DGM	369.55
	CFNDGM	362.95	CFGM	351.86

	NDGM	380.14	GM (1,1)	373.49
	CFDGM	351.39		
4.01	Actual value	277.00	DGM	297.26
	CFNDGM	277.00	CFGM	277.09
	NDGM	314.47	GM (1,1)	301.49
	CFDGM	277.26		
4.02	Actual value	274.00	DGM	239.12
	CFNDGM	209.71	CFGM	217.91
	NDGM	262.59	GM (1,1)	242.72
	CFDGM	218.53		

CFNDGM has order r = 1.034642; CFDGM has order r = 0.863741; CFGM has order r = 0.832.

TABLE III
"Xi'an Epidemic" Error Values for Six Models

	MAPE (%)	MSE (10^7)	MSE (10^7)
CFNDGM	10.8404	16.2281	5719.03
NDGM	15.9780	22.8312	10078.68
CFDGM	13.2610	16.9981	8338.50
DGM	15.2666	16.9304	8477.79
CFGM	13.5246	16.9303	8477.79
GM (1,1)	15.6015	25.8406	10184.51

TABLE IV
"MU5735" Error Values for Six Models

	MAPE (%)	MSE	MSE (10^7)
CFNDGM	11.7340	12384.72	80.8833
NDGM	13.4096	18803.20	106.7850
CFDGM	13.0999	20585.26	112.6683
DGM	13.2908	18833.98	107.8625
CFGM	13.1748	21057.67	114.0517
GM (1,1)	13.3891	18864.01	108.0933

B. Prediction result fit analysis

According to the data in Table. I and Table. II, draw the data curves of the prediction results of the two cases under different models. At the same time, it is compared with the original data curve and the curve of CFNDGM forecast results. The horizontal axis represents time, and the vertical axis represents data values, as shown in the following figure.

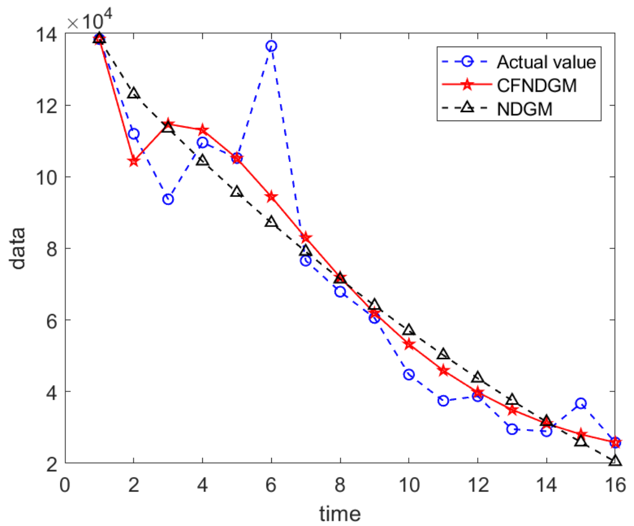

Fig. 1 Fitting effect of CFNDGM and NDGM for the "Xi'an Epidemic "

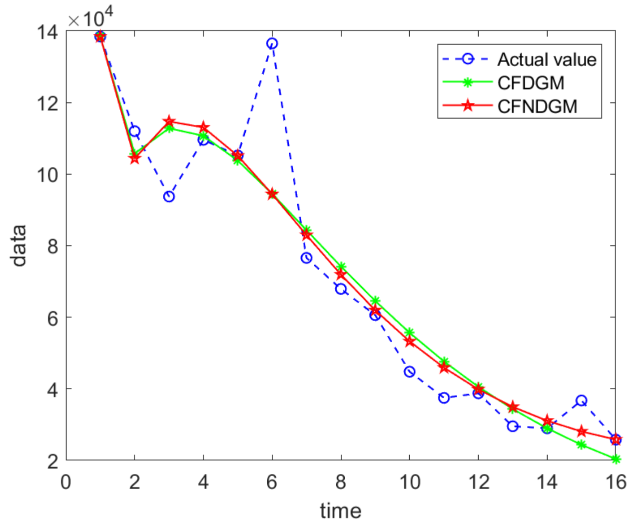

Fig. 2 Fitting effect of CFDGM and CFNDGM for the "Xi'an Epidemic "

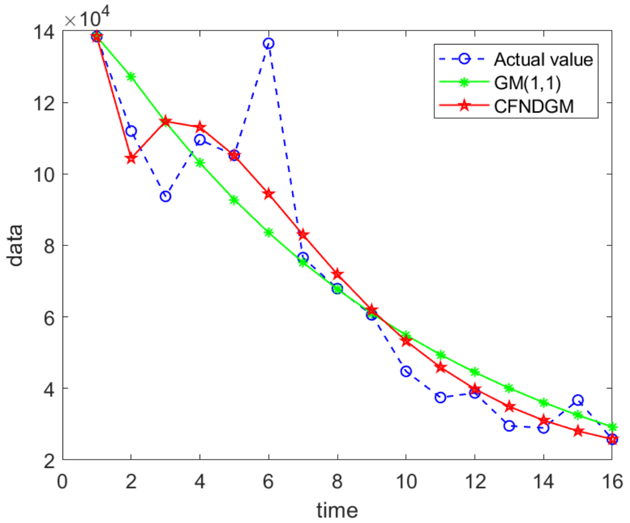

Fig. 3 Fitting effect of CFNDGM and GM (1,1) for the "Xi'an Epidemic "

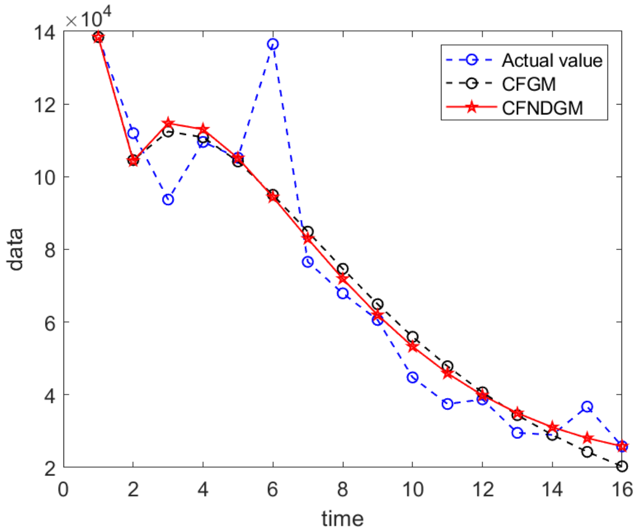

Fig. 4 Fitting effect of CFNDGM and GM (1,1) for the "Xi'an Epidemic "

Calculate the relative error percentage between the original data in Table. I and Table. II and the predicted values of different models, and draw Table. V and Table. VI, as shown below:

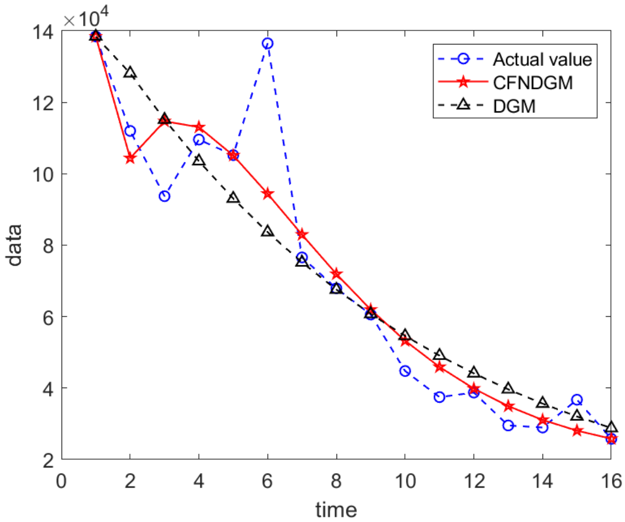

Fig. 5 Fitting effect of CFNDGM and DGM for the "Xi'an Epidemic "

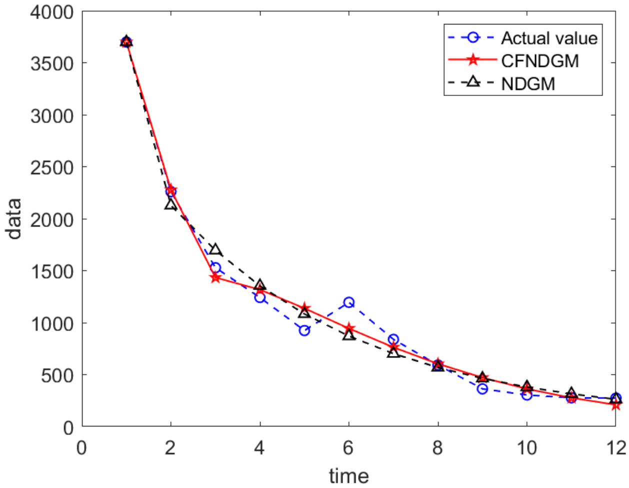

Fig. 6 Fitting effect of CFNDGM and NDGM for the "MU5735 "

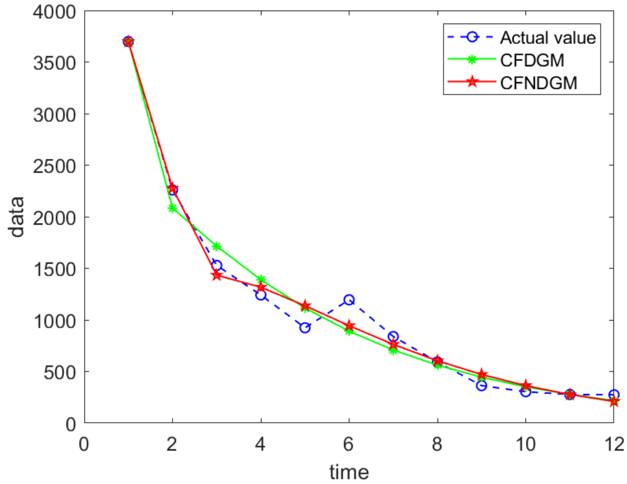

Fig. 7 Fitting effect of CFNDGM and CFDGM for the "MU5735"

Combined with the fitting graph of "Xi 'an Epidemic" under different prediction models and the data in Table. V, the prediction results were analyzed. Compare the relative error values of CFNDGM and NDGM in Fig. 1 and Table V. It can be concluded that the relative error of CFNDGM is smaller than the relative error value of NDGM in 12 out of 16 predicted values. Therefore, the prediction result of the CFNDGM model is better than that of the NDGM model. When the models are all fractional order models, the relative error value of the CFNDGM model is greater than the relative error value of the CFDGM model, and such points account for less than one-third of the whole. Therefore, it can be concluded that the prediction accuracy of the nonhomogeneous model is higher than that of the homogeneous model.

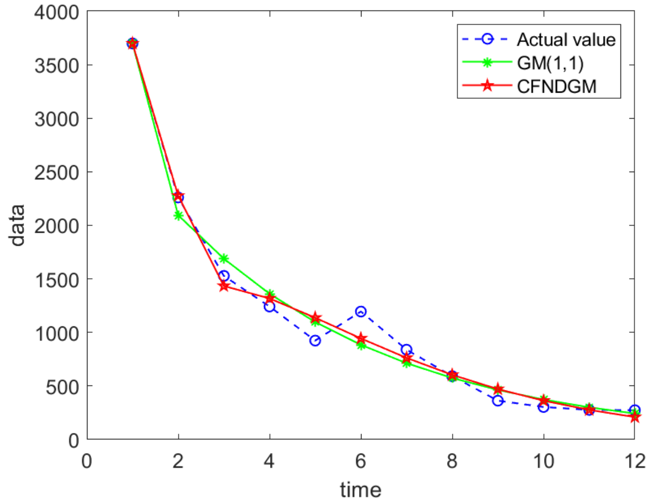

Fig. 8 Fitting effect of CFNDGM and GM (1,1) for the "MU5735 "

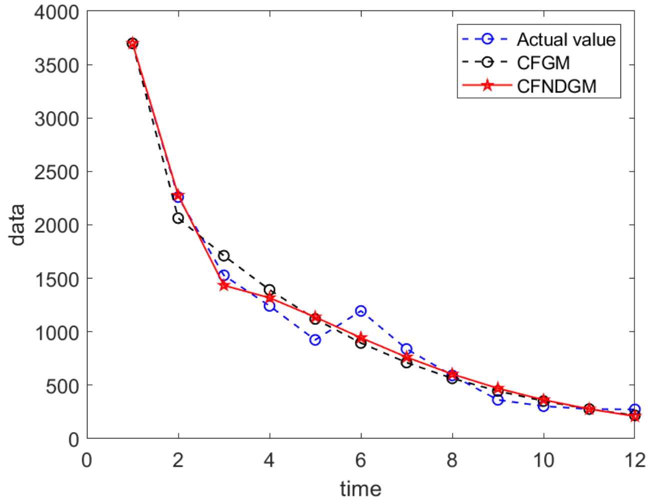

Fig. 9 Fitting effect of CFNDGM and CFGM for the "MU5735 "

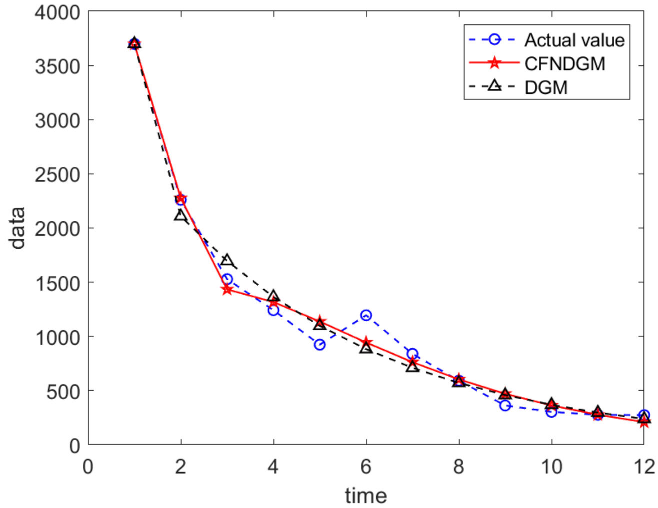

Fig. 10 Fitting effect of CFNDGM and DGM for the "MU5735 "

Combining the fitting diagrams of prediction results of different models with Table. VI, the prediction effect of

"MU5735" is analyzed. The data curve of "MU5735" is smoother than that of the "Xi'an epidemic". Comparing Table 6 and Fig. 6, it can be seen that only at the fifth and twelfth points, the prediction of NDGM is better than that of CFNDGM. In addition to these two, 8 of the remaining points. The CFNDGM model predicts better. Therefore, on the whole, the prediction results of CFNDGM are more accurate, and the average relative error percentage is 1.68% smaller than that of NDGM. It can be seen from Fig. 7 - Fig. 10 that, except for the third prediction point, there is a very obvious gap between the prediction results of CFNDGM and those of other models. For the rest of the prediction points, the fitting curve of the CFNDGM model and the fitting curve of the remaining five models are not much different. However, Table. VI shows that the prediction accuracy of CFNDGM is higher than that of other models, and the average relative error percentage is about 1.5% higher. Therefore, the CFNDGM model is the model with the highest prediction accuracy among the six prediction models.

TABLE V
THE RELATIVE PERCENTAGE ERROR OF THE PERDICTDE VALUE OF THE "Xi'an Epidemic"

time				
3.10	CFNDGM	0.00%	DGM	0.00%
	NDGM	0.00%	CFGM	0.00%
	CFDGM	0.00%	GM (1,1)	0.00%
3.11	CFNDGM	6.82%	DGM	14.39%
	NDGM	9.85%	CFGM	6.63%
	CFDGM	5.58%	GM (1,1)	13.54%
3.12	CFNDGM	22.40%	DGM	22.88%
	NDGM	21.08%	CFGM	19.99%
	CFDGM	20.36%	GM (1,1)	22.17%
3.13	CFNDGM	3.17%	DGM	5.53%
	NDGM	4.80%	CFGM	1.16%
	CFDGM	1.02%	GM (1,1)	5.92%
3.14	CFNDGM	0.09%	DGM	11.56%
	NDGM	9.18%	CFGM	0.97%
	CFDGM	1.38%	GM (1,1)	11.78%
3.15	CFNDGM	30.85%	DGM	38.73%
	NDGM	36.16%	CFGM	30.40%
	CFDGM	30.79%	GM (1,1)	38.79%
3.16	CFNDGM	8.32%	DGM	1.82%
	NDGM	3.31%	CFGM	10.84%
	CFDGM	10.14%	GM (1,1)	1.75%
3.17	CFNDGM	5.91%	DGM	0.50%
	NDGM	5.14%	CFGM	9.95%
	CFDGM	9.24%	GM (1,1)	0.26%
3.18	CFNDGM	2.25%	DGM	0.27%
	NDGM	5.69%	CFGM	7.18%
	CFDGM	6.53%	GM (1,1)	0.67%
3.19	CFNDGM	18.92%	DGM	21.93%
	NDGM	27.21%	CFGM	24.94%
	CFDGM	24.27%	GM (1,1)	22.62%
03.20	CFNDGM	22.54%	DGM	31.00%
	NDGM	34.00%	CFGM	27.73%
	CFDGM	27.18%	GM (1,1)	31.97%
3.21	CFNDGM	2.85%	DGM	13.86%
	NDGM	12.85%	CFGM	4.99%
	CFDGM	4.67%	GM (1,1)	14.89%
3.22	CFNDGM	18.35%	DGM	34.27%
	NDGM	27.03%	CFGM	16.48%
	CFDGM	16.29%	GM (1,1)	35.70%
3.23	CFNDGM	7.32%	DGM	23.08%
	NDGM	9.03%	CFGM	0.02%
	CFDGM	0.02%	GM (1,1)	24.60%
3.24	CFNDGM	23.56%	DGM	12.79%
	NDGM	29.54%	CFGM	33.86%
	CFDGM	33.75%	GM (1,1)	11.57%
3.25	CFNDGM	0.08%	DGM	11.65%
	NDGM	20.78%	CFGM	21.25%
	CFDGM	20.96%	GM (1,1)	13.39%
avg	CFNDGM	10.84%	DGM	15.27%
	NDGM	15.95%	CFGM	13.52%
	CFDGM	13.26%	GM (1,1)	15.60%

CFNDGM has order r = 0.055427; CFDGM has order r = 0.290993; CFGM has order r = 0.262.

TABLE VI
THE RELATIVE PERCENTAGE ERROR OF THE PERDICTDE VALUE OF THE "MU5735"

time				
3.22	CFNDGM	0.00%	DGM	0.00%
	NDGM	0.00%	CFGM	0.00%
	CFDGM	0.00%	GM (1,1)	0.00%
3.23	CFNDGM	0.75%	DGM	6.72%
	NDGM	5.86%	CFGM	8.72%
	CFDGM	7.79%	GM (1,1)	7.33%
3.24	CFNDGM	6.18%	DGM	10.98%
	NDGM	10.94%	CFGM	12.01%
	CFDGM	12.25%	GM (1,1)	10.49%
3.25	CFNDGM	6.07%	DGM	9.83%
	NDGM	8.97%	CFGM	12.09%
	CFDGM	11.86%	GM (1,1)	9.58%
3.26	CFNDGM	23.10%	DGM	18.75%
	NDGM	17.25%	CFGM	21.23%
	CFDGM	20.73%	GM (1,1)	18.73%
3.27	CFNDGM	20.99%	DGM	26.14%
	NDGM	27.19%	CFGM	25.10%
	CFDGM	25.47%	GM (1,1)	25.99%
3.28	CFNDGM	9.10%	DGM	15.38%
	NDGM	16.38%	CFGM	15.15%
	CFDGM	15.56%	GM (1,1)	15.02%
3.29	CFNDGM	2.01%	DGM	3.53%
	NDGM	3.98%	CFGM	4.69%
	CFDGM	5.08%	GM (1,1)	2.92%
3.30	CFNDGM	29.75%	DGM	26.56%
	NDGM	27.61%	CFGM	22.88%
	CFDGM	22.52%	GM (1,1)	27.64%
3.31	CFNDGM	19.39%	DGM	21.56%
	NDGM	25.05%	CFGM	15.74%
	CFDGM	15.59%	GM (1,1)	22.86%
4.01	CFNDGM	0.00%	DGM	0.03%
	NDGM	13.53%	CFGM	7.31%
	CFDGM	0.09%	GM (1,1)	8.84%
4.02	CFNDGM	23.46%	DGM	12.73%
	NDGM	4.16%	CFGM	20.47%
	CFDGM	20.24%	GM (1,1)	11.42%
avg	CFNDGM	11.73%	DGM	13.29%
	NDGM	13.41%	CFGM	13.18%
	CFDGM	13.10%	GM (1,1)	13.40%

CFNDGM has order r = 1.034642; CFDGM has order r = 0.863741; CFGM has order r = 0.832.

The CFNDGM model proposed in this paper is a grey prediction model derived from the improvement of the integer order inhomogeneous discrete grey model. Compare the fitting curves of the "Xi'an epidemic situation" under different models. It can be seen that when there is a sudden increase of data in the original data, the integer order prediction model can not predict the outliers in the data, but presents an overall downward trend. Therefore, the fractional order model can better reflect the real curve of data. In the case of the same fractional order model, although the overall trend of the prediction curve of the CFNDGM model is similar to that of CFDGM and CFGM, the average relative error of the prediction results is smaller than that of these two models. The accuracy was improved by 2.42% and 2.68%, respectively. To sum up, the CFNDGM model has the best prediction effect in the two cases.

C. Numerical prediction

In addition to modeling and predicting a small amount of data, the gray model can also predict the next stage of data through the established model. Three models, CFNDGM, CFDGM, and NDGM were selected to predict the Baidu Index of "Xi'an Epidemic" and "MU5735", as shown in Tables. VII and Table. VIII below.

TABLE VII
PREDICTED DATA OF "XI'AN EPIDEMIC"

	CFNDGM	NDGM	CFDGM
03-26	24162.63	15219.00	17034.75
03-27	23005.70	10220.66	14195.30
03-28	22254.25	5433.78	11803.24

TABLE VIII
PREDICTED DATA OF "MU5735"

	CFNDGM	NDGM	CFDGM
04-03	157.70	221.61	172.07
04-04	117.91	189.22	135.38
04-05	87.73	163.64	106.44

VII. CONCLUSION

Establishing a suitable model is very important for preventing and controlling the spread of public opinion on the Internet and maintaining social stability. Thus, it is necessary to improve the accuracy of the model prediction results by improving the model. Accordingly, in this paper, the non-homogeneous exponential discrete gray model is improved by the conformable fractional accumulation and the conformable fractional difference, and the fractional non-homogeneous gray model CFNDGM is proposed. Furthermore, we use the particle swarm algorithm to optimize the order r to obtain the order with the smallest mean absolute percentage error. The validity and accuracy of the model are verified by using two popular network events as examples. From the above example, we can conclude that after the expansion from the integer order to the fractional-order model, the error caused by the jump from the differential to the difference is eliminated, so that the prediction accuracy of the fractional non-homogeneous gray model is more accurate than that of the integer-order non-homogeneous gray model. Comparing the CFDGM model with the CFNDGM model, it can be concluded that in the case of fractional order, the assumption of the inhomogeneous exponential sequence is more in line with the original sequence, and the prediction result is more accurate. In summary, the prediction accuracy and fit of the CFNDGM model are higher than those of other models mentioned in this paper, so it can more effectively promote the progress of network public opinion prediction.

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