Optimization of the NSGA-III Algorithm Using Adaptive Scheduling

Xijun Zhang, Yunfang Zhong, Baoqi Zhang, Shengyuan Nie

***Abstract*—To address the NSGA-III algorithm's problems of low population diversity, high computational complexity, and slow convergence rate, in this paper an enhanced NSGA-III algorithm is proposed based on adaptive scheduling. The proposed algorithm increases the diversity of Pareto solutions by utilizing a crossover-mutation operator to adapt to dynamic changes of the population number during the population evolution stage. In addition, during the reference point generation process, the number of reference points is adjusted dynamically to avoid increasing the computational complexity under the conditions of numerous reference points and the screening function loss of population individuals caused by fewer reference points. The proposed algorithm is compared with the NSGA-III algorithm and the improved NSGA-III algorithm on the platEMO platform. Experimental results demonstrate that compared to the other algorithms, the proposed algorithm reduces the number of redundant reference points to a certain extent, increases the population diversity, and improves the convergence and distribution of the Pareto solution sets.**

***Index Terms*—Adaptive scheduling, crossover mutation, multi-objective optimization, NSGA-III algorithm, reference point generation.**

I. Introduction

MULTI-OBJECTIVE optimization problems can be found in many fields, but their requirements can differ. For instance, logistics and distribution problems [1] require cost minimization and population service maximization; power system optimization problems [2] pursue the lowest fuel cost and the least active network loss; portfolio optimization problems [3] aim at the lowest investment risk and the highest investment returns. With the emergence of numerous multi-objective problems, the corresponding optimization algorithms have been continuously improving. Deb *et al.* [4] proposed the NSGA-II algorithm based on the Pareto optimal solution discussion, where the elite strategy was introduced to maintain the diversity of solutions, thus avoiding population degradation, reducing computational complexity, and dramatically improving algorithm performance. Qingfu Zhang *et al.* [5] proposed the MOEA/D algorithm based on the idea of decomposition, which combined a mathematical programming method and an evolutionary algorithm to transform multi-objective optimization problems into single-objective optimization through an aggregation function, thus obtaining high-quality solutions and reducing computational complexity. Deb *et al.* [6] proposed the NSGA-III algorithm, which used a set of uniform reference points to segment the target space and guide the evolution of the population, maintain the diversity of the solution, and solve the problem of poor performance of the NSGA-II algorithm in high-dimensional target spaces. In recent years, various improvement strategies based on the NSGA-III algorithm have been proposed to further optimize this algorithm's performance.

Amin *et al.* [7] proposed an improved NSGA-III algorithm called the Elite NSGA-III to improve the diversity and accuracy of NSGA-III solutions by establishing elite group profiles. Bi *et al.* [8] proposed an NSGA-III algorithm based on an improved elimination operator. First, the reference points were identified by the maximum niche count, and then, the individuals related to the reference points were sorted according to the cross-boundary distance of the penalty point method to improve algorithm performance. Jie Qian *et al.* [9] proposed the INSGA-III algorithm, which adopted preliminary competitive solutions optimized using the traditional NSGA-III method as the initial population and integrated an innovative adaptive dominant strategy. Finally, more evenly distributed preferable Pareto fronts were obtained. To solve multi-objective optimal power flow problems, Gang Guo *et al.* [10] proposed the novel NSGA-FA algorithm, which combined the NSGA-III and MFA algorithms, whereby a preferable Pareto front with superior diversity and fast convergence was achieved. Huantong Geng *et al.* [11] proposed an improved NSGA-III algorithm based on reference point selection strategy. In their approach, useless reference points were eliminated based on an evaluation of their importance, which accelerated the convergence and optimization efficiency of the algorithm. However, the scale of reference points was not specified. Shantian Pang *et al.* [12] proposed an improved NSGA-III algorithm based on the reference point crowding degree, which removed some of the individuals in a population by calculating their crowding degree. In this way, the convergence speed and distribution of Pareto solution sets were be improved. However, this algorithm generates too many repeated populations in the parent recombination operation, resulting in an exponential increase in the non-dominated solution of a population.

In order to improve the NSGA-III algorithm and its improved version, in this paper the reference point selection mechanism and population initialization of NSGA-III are improved. The main contributions are as follows:

1) Aiming at the problem of reference point generation, a method for controlling the generated reference point number

Manuscript received December 19, 2022; revised February 8, 2023. This work was supported in part by the National Natural Science Foundation of China (Grant Nos. 62162040 and 61966023), the Gansu Province Higher Education Innovation Fund Project (Grant No. 2021A-028), Gansu Provincial Science and Technology Plan Funded Natural Science Foundation Key Project(Grant No. 22JR5RA226) and the Gansu Provincial Science and Technology Program Funding Project(Grant No. 21ZD4GA028)

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is proposed. This method solves the problems of too many or too few reference points and improves the convergence speed and distribution of Pareto solution sets.

2) For population crossover mutations, an adaptive method of parameter and probability value is proposed to make a population more universal and increase population diversity.

3) The proposed algorithm is verified through comparative experiments, and the results indicate that its performance indices are better than those of other algorithms. Also, the proposed algorithm can effectively avoid falling into a local optimum and has improved robustness, convergence, and distribution compared to existing algorithms.

II. NSGA-III Algorithm and Its Improvements

A. NSGA-III Algorithm

The NSGA-III algorithm retains the original elite strategy of the NSGA-II algorithm. However, it uses the reference point mechanism instead of the crowding degree to sort the population individuals in the algorithm selection stage in a non-dominated manner, making the algorithm perform well in high-dimensional spaces and maintaining population diversity.

The specific steps of the NSGA-III algorithm are shown in TABLE I:

TABLE I
MAIN OF NSGA-III ALGORITHM

Begin
Input: Construct a reference point H [13] generate the initial population P_t randomly
Output: P_{t+1}
Step 1: Q_t = Recombination + Mutation(P_t)
Step 2: $R_t = P_t \cup Q_t$
Step 3: F = nondominated_ sort(R_t)
$P_{t+1} = \emptyset$
i = 1
while$(P_{t+1} + F_i \leq N)$
{
$P_{t+1} = P_t \cup F_i$
$i = i + 1$
}
Step 4: Standardize the target space
Step 5: Perform the association operation between the individual and the reference line
Step 6: The remaining k populations are selected according to the number of niches at the reference point [14] to enter the next iteration
$k = N - P_{t+1}$
Step7: $P_{t+1} = N$
End

The main shortcomings of the NSGA-III algorithm are as follows:

1) As the target spatial dimensions increase, the number of reference points becomes much larger than the population size, resulting in a sharp increase in computational complexity, weakening the Pareto selection pressure pressure, and reducing the convergence speed of the algorithm.

2) The number of reference points in NSGA-III dictates the algorithm's running time, and too many or too few reference points can affect the screening function of the population. For instance, when the number of reference points is equals to one, all individuals must be associated with the reference point. In contrast, when the number of reference points tends to infinity, the distance between individuals and the nearest reference line is zero.

3) In the population evolution stage, the artificial crossover-mutation operator is not universal, resulting in uneven probability and reduced population diversity.

B. AR-NSGA-III Algorithm

Compared to the NSGA-III algorithm, the AR-NSGA-III algorithm introduces the following improvements to reference point selection:

1) Using the quartile information of the population in the decision space, the evolution stage of the population is determined through the quantification of the difference in entropy.

2) The reference points are associated according to the population distribution in the target space, and the importance of each reference point is evaluated to determine suitable ones.

The AR-NSGA-III algorithm can reduce the interference of weak reference points on an individual selection in populations effectively and has improved effectiveness and performance over the original algorithm. However, there is no specific rule on the judgment of the evolutionary stage of the algorithm, and no reasonable threshold has been adopted to make the division of the evolutionary stage more accurate.

III. Adaptive Scheduling-based NSGA-III Algorithm

A. AS-NSGA-III Algorithm

Considering the deficiencies of the NSGA-III algorithm in terms of population diversity, algorithm convergence, and computational complexity, in this paper an algorithm that can improve the reference point generation and the crossover mutation of the population in the initial stage is proposed. In the proposed reference point generation approach, a method is proposed to adapt the number of reference points H to the population size N. According to (1), when the target spatial dimension is constant, the number of reference points is directly defined by the selection of partition p. Therefore, the number of reference points can be controlled through the dynamic adjustment of p, so the number of reference points is always approximately equal to the population size, which reduces the computational complexity and maintains the individual screening function of the population. In the crossover mutation operator, parameters t_1, t_2 and probability pc, pm are selected dynamically according to the population size N to avoid probability inequality and population diversity reduction caused by the artificial crossover-mutation operator in the process of evolution. The flowchart of the AS-NSGA-III algorithm is presented in Fig. 1.

B. Reference Point Design in AS-NSGA-III Algorithm

The specific steps of the reference point generation process are as follows:

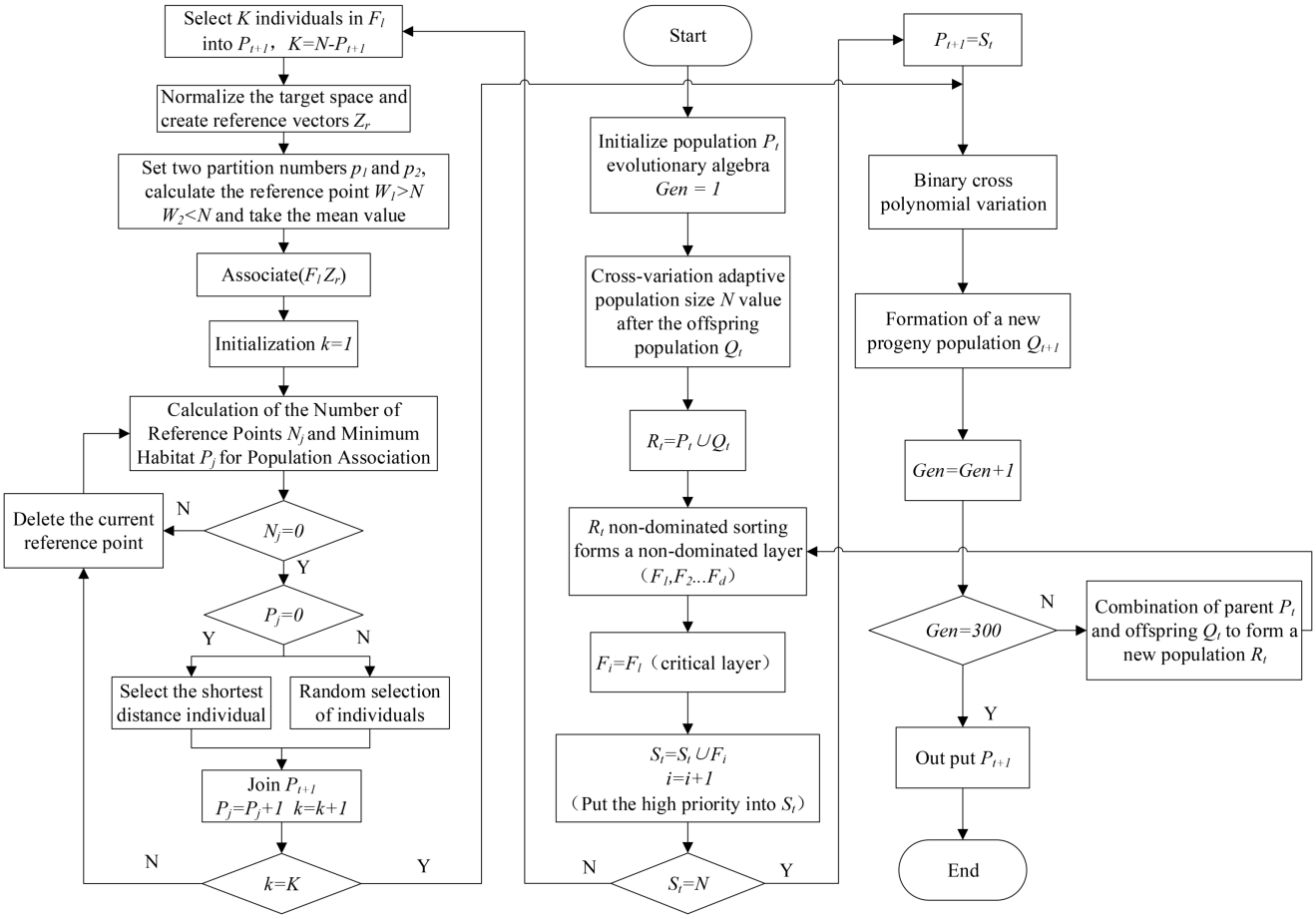

Fig. 1. The flowchart of the AS-NSGA-III algorithm

1) Determine the reference points on a hyperplane: The NSGA-III uses a set of predefined reference points to ensure solution diversity. The reference points are on an M-dimensional hyperplane, where M represents the number dimensionof the target space. If each target is split into p parts, $H(M,p)$ reference points will be generated. For a hyperplane dimension of M = 3 and a number of division copies p = 4, the distribution of reference points on the hyperplane is shown in Fig. 2.

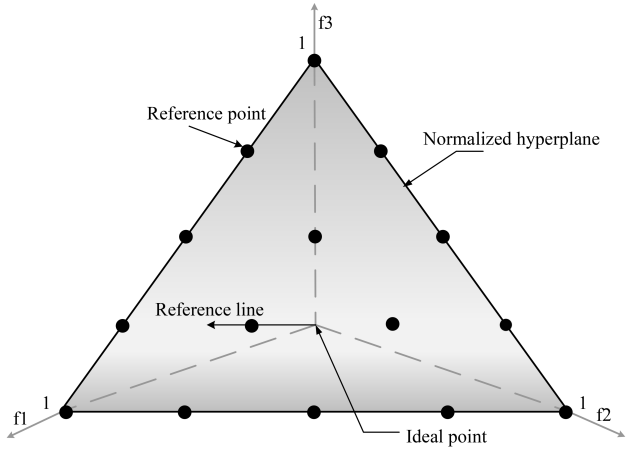

Fig. 2. Distribution of reference points on a hyperplane

When the number of hyperplane dimensions is constant, according to (1), there is a slight increase in the number of shares p, causing the number of reference points to soar. If M = 10 and p = 10, then 92,378 reference points will be generated, which will increase the number of useless reference points and algorithm complexity but will decrease population diversity.

Therefore, in this study, the number of reference points adapts to the population size N, and the number of reference points in each dimension is divided into p_1, p_2, a set of reference points W_1 are obtained by (2), and the set of reference points W_2 is solved using (3). For $W_1 > N$ and $W_2 < N$, it holds that:

$$W_1 = C_{M-1+\mathrm{p}_1}^{\mathrm{p}_1} \tag{1}$$

where W_1 denotes a set of uniformly distributed reference points generated via the permutation and combination of M and p in the hyperplane space.

Further, it holds that:

$$W_2 = C_{M-1+\mathrm{p}_2}^{\mathrm{p}_2} - C_{M-1+\mathrm{p}_1}^{\mathrm{p}_1} \tag{2}$$

where W_2 denotes the difference between the reference point sets generated based on partition copies p_2 and p_1.

Moreover, the following condition holds:

$$W = [W_1, W_2] \tag{3}$$

Assume that $M = 3$, $p_1 = 11$, and $p_2 = 10$; then, if the target number is $M = 3$, the population size is $N = 100$, and according to (1)–(3), $W_1 = 105$, $W_2 = 91$, and $W = 98$, so the initial number of reference points is 98. In this process, W_1 is generated using Das and Dennis's method [13], and

W_2 is obtained based on the difference between W_2 and W_1. In this way, the disadvantages caused by repetition leading to too many or too few reference points are avoided, and the time cost of reference point selection is reduced effectively.

2) Associated population reference points: Assume that K individuals in the population are associated with the corresponding reference lines. Then, the reference lines are define, which represent the lines between the central point and reference points in the target space. Next, the perpendicular distance between each individual in the population and its nearest reference line is calculated. Finally, each population individual is associated with its nearest reference line.

3) Reference point niche number calculation: After the association operation is completed, the three situations explained in the following may occur.

The number of associated populations at each reference point is calculated and the reference point with the least number of associated populations is selected to enter P_{t+1};

1) If the numbers of associated populations of multiple reference points are the same and they all are the minimum, one reference point is selected to enter P_{t+1};

2) If a reference point is associated with one or more individuals, the population with the smallest vertical distance from the individual to the reference line is selected to enter P_{t+1};

3) If no individual is associated with the reference point, the reference point is deleted directly to reduce the computational complexity.

The reference point generation steps of the AS-NSGA-III algorithm are shown in TABLE II.

TABLE II
MAIN STEPS OF THE REFERENCE POINT GENERATION IN THE AS-NSGA-III ALGORITHM

Begin
Input: Uniform point(N, M)
Output: Pareto optimal solution PF
the number of reference points Z
Step 1: H_1, H_2 are set to one
Step 2: While comb $(H_1 + M - 1, M - 1) = N$
H_1 increases by one
Step 3: Assignment upon the completion of conditions
$(H_2 = H_1, H_1 = H_1 - 1)$
Step 4: W_1 array is assigned to H_1
Step 5: W_2 is randomly assigned and based on the comb
$(H_2 - H_1)$ spread of combinations by a column
Step 6: Merge W_1 and W_2 and assign value to W
Step 7: W Standardized processing
Step 8: Z_f is assigned to $W.shape[0]$ (the first dimension length
of the matrix)
Step 9: return PF, Z
End

C. Adaptive Crossover Operator Design in AS-NSGA-III Algorithm

Artificially set crossover-mutation parameters are not universal, which can lead to problems, such as probability inequality and reduced population diversity, while the Pareto solution can fall into local optima. Therefore, in the stage of population evolution, the genetic operator uses simulated binary crossover and polynomial mutation to increase population diversity. The crossover parameter t_1, the mutation parameter t_2, the crossover probability pc, and the mutation probability pm all adapt to the population size N to prevent the decline in population diversity caused by the unevenly set probability. The cross-mutation operator parameter settings are given in TABLE III.

TABLE III
CROSS-MUTATION OPERATOR PARAMETER SETTINGS

N_GENERATIONS = 300	Iteration number
N = 100	Population size
name = "DTLZ"	Test function selection
t_1 = random.randint(1,N)	Cross parameters
t_2 = random.randint(1,N)	Mutation parameters
pc = random.randint(1,1/N)	Crossover probability
pm = random.randint(1,1/N)	Mutation probability

IV. Simulation Experiments

To verify the performance of the proposed AS-NSGA-III algorithm in a highly-dimensional multi-objective space, in this paper used the multi-objective optimization platform platEMO [15] in MATLAB. The simulation parameters were set as follows. population size was $N = 100$, the maximum number of iterations was $Gmax = 300$, and the maximum number of evaluations was $maxFE = N * Gmax = 30000$. Each algorithm was executed 30 times independently for each test function, and the average result was used to ensure the fairness of the experiment.

A. Test Functions

To test the effectiveness of the proposed algorithm, a series of DTLZ [16] test functions proposed by Deb, Thiele *et al.* were adopted. Here, DTLZ1-DTLZ4 were used to test and compare different algorithms, where the number of decision variables D was equal algorithms, wherein the number of decision variables $M + k - 1$ and k was set to 5.

B. Algorithm Performance Evaluation Indexes

To verify the effectiveness of the algorithm, three common evaluation indexes were used:

1) Generation distance (GD) index : GD[17] was used to measure the distance between the individuals in the non-dominated solution set and the individuals in the real Pareto front. Smaller GD values correspond to a better convergence of the front end. The GD was calculated using:

$$GD\left(P, P^*\right) = \frac{\sqrt{\sum_{y \in P} min_{x \in P^*} dis\left(x, y\right)^2}}{|P|} \tag{4}$$

where P^* is a solution set obtained by the multi-objective evolutionary algorithm; P is a set of uniformly distributed reference points sampled on the real Pareto front; $|P|$ represents the number of individuals in the point set P; $dis(x, y)$ is the minimum Euclidean distance between point y in the solution set P^* and point x in the reference set P;

2) Spacing (SP) index: This index [18] was used for spatial evaluation because it can measure the distribution of individuals in the target space of the Pareto front approximation solution. Smaller SP values are obtained when the solution set is more evenly distributed. The mathematical equation of the SP index is:

$$SP(P) = \sqrt{\frac{1}{|P|-1}\sum_{i=1}^{|P|}\left(\bar{d}-d_i\right)^2} \tag{5}$$

where P is a set of uniformly distributed reference points sampled on the real Pareto front; $|P|$ represents the number of solutions in P; d_i is the Euclidean distance between two continuous vectors on the non-dominated boundary in the solution set; $\bar{d}$ is the average value of d_i;

3) Inverse generation distance (IGD) index: The ant generation distance [19], which represents the minimum Euclidean distance from the entire Pareto surface to the final solution set, was also used as an evaluation index. Smaller values correspond to better convergence and diversity of the algorithm. The IGD index is defined as follows:

$$IGD\,(P,P^*) = \frac{\sum_{x\in P^*} min_{y\in P} dis\,(x,y)}{|P^*|} \tag{6}$$

where P^* is the solution set obtained by the multi-objective evolutionary algorithm; P is a set of uniformly distributed reference points sampled on the real Pareto front; $|P|$ represents the number of individuals in the point set P; $dis(x,y)$ represents the minimum Euclidean distance between point y in the solution set P^* and point x in the reference set P.

C. Experimental Results Analysis

The proposed algorithm was compared with the traditional NSGA-II and NSGA-III algorithms and the AR-NSGA-III [11] to verify its effectiveness. The performance of the algorithms was compared using the three evaluation indexes, and the results were as follows:

1) GD index results: The GD results of the four algorithms on the DTLZ test functions are presented in Table IV, where M denotes the target space dimension, D is the decision space, and the bold font indicates the best result.

As shown in TABLE IV, with the increase in the objective function M, the number of reference points increased, which increased the GD index value of each algorithm but decreased the convergence speed of the solution set. However, under a certain dimension condition, the proposed algorithm could constrain the number of reference points by controlling the reference point partition p, thus accelerating the convergence speed of Pareto solution sets obtained by the algorithm. Among the 16 test functions, the proposed algorithm achieved the best IGD index result on the 15, indicating that the proposed algorithm had a better convergence effect than the other algorithms. Therefore, improving the reference points is essential to improve the algorithm's convergence.

2) SP index results: The SP results of the four algorithms on the DTLZ test functions are presented in TABLE V.

As shown in TABLE V, compared with the other algorithms, most of the SP index values of the proposed algorithm on the test functions were relatively small. Specifically, compared with the AR-NSGA-III algorithm, the SP index

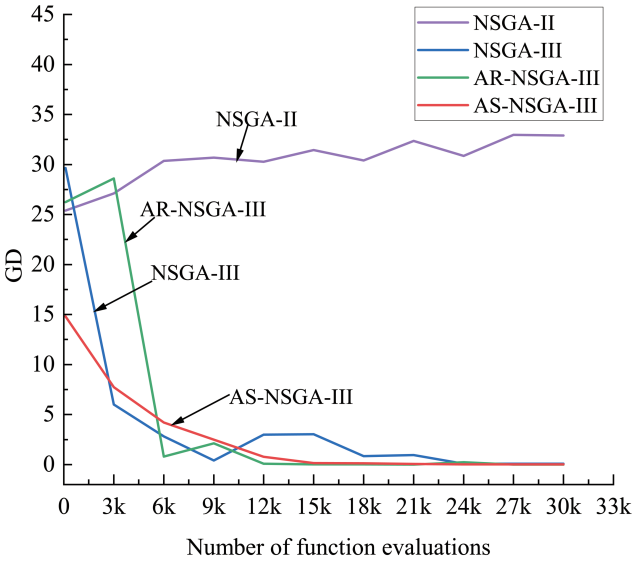

Fig. 3. The comparison results of the GD index of the four algorithms

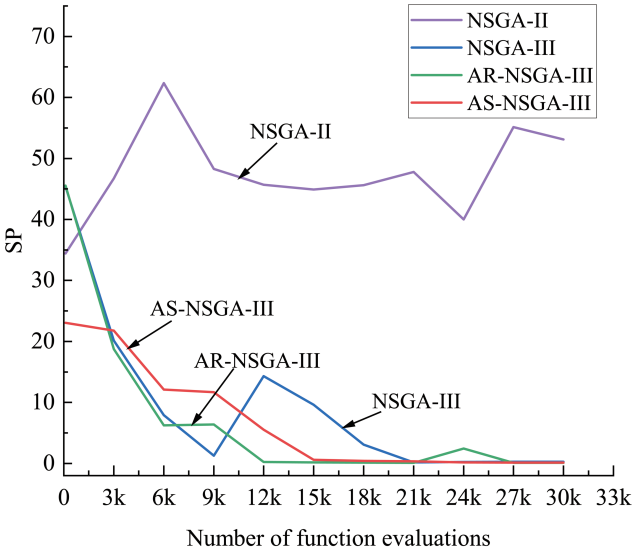

Fig. 4. The comparison results of the SP index of the four algorithms

value of the proposed algorithm was slightly higher on the three- to five-dimensional DTLZ2 and DTLZ4 functions. On the 16 test functions, the proposed algorithm achieved 12 optimal and four suboptimal values; thus, it still had a significant advantage compared to the other algorithms. The adaptive population size of the crossover-mutation operator maintained good population diversity, making the solutions evenly distributed in space and preventing the algorithm from falling into a local optimum.

3) IGD index results: The IGD results of the four algorithms on the DTLZ test functions are presented in TABLE VI.

As shown in TABLE VI, with the increase in the target spatial dimension M in the DTLZ test function, although the IGD index values of the different algorithms increased, they all fluctuated within a minimal range. For instance, the IGD index results of the proposed algorithm on the 16 test functions were the optimal values in all cases. For test function DTLZ4 in particular, which was relatively difficult to converge, the IGD index value of the proposed algorithm

TABLE IV
GD RESULTS OF THE FOUR ALGORITHMS

Function	*M*	*D*	NSGA-II	NSGA-III	AR-NSGA-III	AS-NSGA-III
DTLZ1	3	7	2.64E-02	4.05E-02	1.89E-02	**2.45E-04**
	5	9	4.71E-01	1.68E-01	4.26E-01	**3.54E-02**
	8	12	2.82E+01	9.07E-01	1.50E+00	**2.48E-01**
	10	14	3.07E+01	9.35E-01	2.12E+00	**3.57E-02**
DTLZ2	3	12	1.33E-03	6.09E-04	7.00E-04	**1.56E-04**
	5	14	3.26E-02	5.54E-03	5.38E-03	**8.96E-04**
	8	17	2.09E-01	1.91E-02	1.28E-02	**7.26E-03**
	10	19	2.17E-01	1.23E-02	1.57E-02	**6.94E-03**
DTLZ3	3	12	9.45E-01	1.74E+00	1.81E+00	**1.08E-01**
	5	14	2.41E+01	6.10E+00	6.98E+00	**1.90E+00**
	8	17	1.69E+02	1.70E+01	2.08E+01	**6.02E+00**
	10	19	1.77E+02	2.38E+01	2.30E+01	**1.44E+01**
DTLZ4	3	12	1.17E-03	5.58E-04	6.29E-04	**2.40E-04**
	5	14	2.78E-02	5.32E-03	4.98E-03	**1.87E-03**
	8	17	2.03E-01	1.45E-02	1.28E-02	**8.49E-03**
	10	19	2.12E-01	**1.08E-02**	1.49E-02	1.14E-02

TABLE V
SP RESULTS OF THE FOUR ALGORITHMS

Function	*M*	*D*	NSGA-II	NSGA-II	AR-NSGA-III	AS-NSGA-III
DTLZ1	3	7	4.09E-02	6.55E-02	4.21E-02	**2.17E-03**
	5	9	8.66E-01	7.74E-01	1.80E+00	**3.17E-02**
	8	12	5.09E+01	4.27E+00	9.22E+00	**2.08E+00**
	10	14	7.23E+01	5.38E+00	1.97E+01	**1.72E-01**
DTLZ2	3	12	5.68E-02	5.70E-02	**4.59E-02**	5.73E-02
	5	14	2.28E-01	1.57E-01	**1.37E-01**	1.59E-01
	8	17	9.34E-01	1.79E-01	2.38E-01	**8.23E-02**
	10	19	1.15E+00	3.27E-01	3.01E-01	**1.66E-01**
DTLZ3	3	12	9.55E-01	1.19E+00	1.35E+00	**1.18E-01**
	5	14	3.07E+01	2.72E+01	2.32E+01	**1.93E+01**
	8	17	5.15E+02	6.21E+01	8.00E+01	**1.71E+01**
	10	19	6.54E+02	9.80E+01	1.12E+02	**4.55E+01**
DTLZ4	3	12	5.53E-02	4.48E-02	**3.82E-02**	5.58E-02
	5	14	2.22E-01	1.51E-01	**1.33E-01**	1.56E-01
	8	17	9.01E-01	2.44E-01	2.33E-01	**7.86E-02**
	10	19	1.19E+00	3.05E-01	2.61E-01	**2.32E-01**

decreased significantly compared with the other algorithms. This proved that the Pareto solution sets obtained using the proposed algorithm had the best convergence and distribution effect among all generated solutions. In addition, the proposed algorithm did not fall easily into a local optimum when searching for the optimal solution, and it searched the solution space more effectively than the other algorithms.

To illustrate the population's evolution process in high-dimensional space, one DTLZ test function was selected to analyze the results of the GD, SP, and IGD indexes of the four algorithms. The comparison results are presented in Fig. 3, where the abscissa denotes the number of iterations, and

TABLE VI
IGD RESULTS OF THE FOUR ALGORITHMS

Function	*M*	*D*	NSGA-II	NSGA-III	AR-NSGA-III	AS-NSGA-III
DTLZ1	3	7	2.05E-01	2.48E-01	1.24E-01	**3.19E-03**
	5	9	1.94E+00	5.24E-01	1.25E+00	**2.75E-01**
	8	12	3.29E+01	1.18E+00	2.07E+00	**1.97E-01**
	10	14	3.33E+01	1.26E+00	2.94E+00	**2.13E-01**
DTLZ2	3	12	6.90E-02	5.49E-02	5.57E-02	**7.24E-04**
	5	14	3.35E-01	2.16E-01	2.25E-01	**4.91E-03**
	8	17	1.50E+00	4.30E-01	4.42E-01	**2.91E-02**
	10	19	1.39E+00	5.85E-01	6.04E-01	**4.70E-01**
DTLZ3	3	12	6.27E+00	8.28E+00	8.71E+00	**2.03E+00**
	5	14	6.54E+01	1.89E+01	2.44E+01	**3.41E+00**
	8	17	9.56E+02	3.83E+01	5.49E+01	**1.80E+01**
	10	19	9.19E+02	4.53E+01	5.19E+01	**1.23E+01**
DTLZ4	3	12	1.00E-01	1.85E-01	1.69E-01	**1.10E-03**
	5	14	3.05E-01	2.81E-01	2.58E-01	**9.66E-03**
	8	17	1.45E+00	5.19E-01	4.97E-01	**4.93E-02**
	10	19	1.48E+00	6.02E-01	6.09E-01	**3.55E-02**

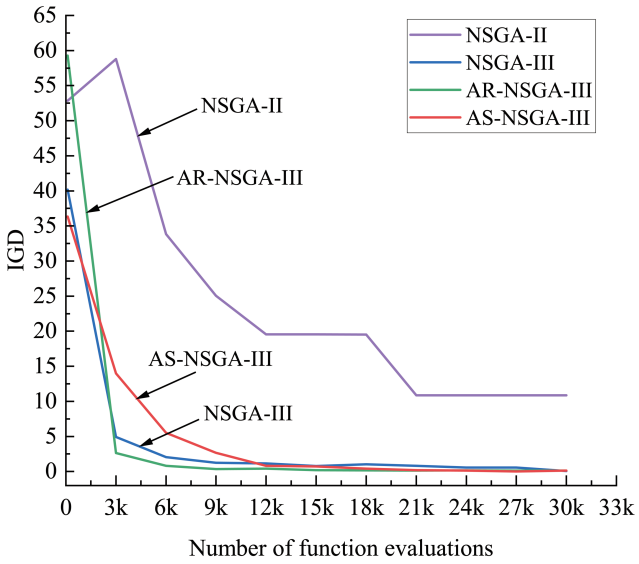

Fig. 5. The comparison results of the IGD index of the four algorithms

the ordinate shows the performance index value. The DTLZ1 function was selected to be used in the test, with M = 8 and D = 12.

As shown in Fig. 3, with the iteration number in the high-dimensional space, the NSGA-II algorithm could not converge. Although the NSGA-III and AR-NSGA-III algorithms converged gradually, their GD index curves fluctuated in the convergence process. However, only the proposed algorithm converged smoothly to zero, which proves that the proposed algorithm had a better convergence effect than the other algorithms.

Further, as shown in Fig. 4, with the increase in the iteration number, N individuals were selected by the proposed algorithm from R_t as a new parent population, and the remaining N individuals were deleted, which decreased the population diversity but increased the SP value. In addition, the SP index curves of the four algorithms all fluctuated, but among them, the SP index curve of the NSGA-II algorithm was the largest and did not converge. In contrast,the SP index values of the NSGA-III and AR-NSGA-III algorithms converged but fluctuated significantly. This was because the crossover-mutation operator of the proposed algorithm used adaptive population change, which maintained the population diversity. Accordingly, the fluctuation range of the SP index curve of the proposed algorithm was small and converged the earliest among all algorithms. Moreover, compared to the other algorithms, the solution distribution of the proposed algorithm was more uniform.

As shown in Fig. 5, with the increase in the iteration number, the IGD index value of the AS-NSGA-III algorithm was lower than that of the other algorithms at the beginning of the iteration. This shows that the convergence and distribution of the solutions obtained by the AS-NSGA-III algorithm were in their optimal state at the beginning of the iterations; they were followed by the NSGA-III and AR-NSGA-III algorithms. The NSGA-II algorithm had the worst performance and did not converge. Considering the whole iterative process, the IGD index curve of the proposed algorithm was the flattest among all algorithms, which is indicative of its good robustness.

V. Conclusion

The paper aims at the problems of low population diversity, high computational complexity, and slow convergence rate of the NSGA-III algorithm in high-dimensional target

spaces. To solve these problems, the AS-NSGA-III algorithm is proposed. The shortcomings of the NSGA-III algorithm are solved by optimizing the initialization cross-mutation operator and introducing the reference point selection mechanism. The proposed algorithm is compared with the classical NSGA-II and NSGA-III algorithms and the AR-NSGA-III algorithm. The experimental results show that compared to the other algorithms, the proposed algorithm can reduce the generation of useless reference points to a certain extent and can improve the diversity of the population and the convergence speed of the Pareto solution set. Although the performance of the proposed algorithm is improved compared to the other algorithms, it is also more time-consuming than the other algorithms.

In future work, the running time of the proposed algorithm will be further reduced, and the proposed algorithm will be applied to practical projects.

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