Efficiency Evaluation of Geo/Geo/1 Queue Performance Measure with Priority using Fuzzy Data Envelopment Analysis

Fatimah Az-Zahra*, Indarsih

Abstract—**The queuing system is a system that consists of a set of customers, servers, and a regulation that regulates the arrival of customers and their services. Queues can be formed for various services. Customers can choose a queue based on the available queue length. Fuzzy logic and queuing theory are used to determine the fuzzy decisions made by service providers (servers) and people who need services (customers). Service providers make fuzzy decisions to manage queues. People who need services also make fuzzy decisions to choose from among the various available service queues. Discrete time fuzzy priority queues with partial buffer distribution are modeled and analyzed by prioritization, namely customers with high priority and customers with low priority. Various alternative options regarding priority coverage and buffer control provide output measures of performance from different queues and are expressed by fuzzy sets. To determine the efficiency of each alternative choice is solved using Fuzzy Data Envelopment Analysis (FDEA).**

Index Terms—**queuing theory, preemptive priority, geometric distribution, fuzzy logic.**

I. Introduction

NATURAL disasters are events or a series caused by nature, including earthquakes, tsunamis, volcanic eruptions, floods, droughts, hurricanes, and landslides. Evacuation strategies for affected people of natural disasters can vary from evacuation based on priority level of emergency, mobility to safe locations and the arrival of rescue teams. Service providers for efforts to rescue the affected people of natural disasters are indispensable and involve handling queues.

After a disaster occurs, rescue crews try to provide different services to the affected people by the disaster based on the level of emergency and the ability to evacuate the affected people. The problem of handling affected people of natural disasters can be viewed as a queue theory problem with the affected people of natural disasters as customers and rescue crews as servers. The problem has parameters and variables that depend on the decisions made by the server and the customer as well as the observed values of the system parameters, such as inappropriate or ambiguous service levels.

In this study, the server prioritizes customers in each queue based on the vulnerability of the customer. Vulnerability can be related to age, health condition, level of injury, pregnancy, and others. For example, although age is a quantitative vulnerability criterion, the idea of each server for a customer being categorized as a child, adult, or elderly is a fuzzy concept. The level of priority given to several customers (priority coverage) and the capacity of the queue length are decision variables that must be optimized by taking into account the performance measures of the queuing system, including customer waiting time and the probability of losing customers. Therefore, the right solution is used for priority setting and buffer control is considered as a possible alternative for queuing system settings, further minimizing the performance measures of the queuing system including customer waiting time and the probability of losing customers due to the congestion of the queue. In Fariborz Jolai's research (2016) problem solving used the CCR model and resulted in an efficiency score exceeding one, while according to Charnes et al [7], the efficiency score has a range of values from zero to one so it needs to be reviewed, either using the CCR model or other models. In this study, solving the problem of handling victims of natural disasters in evaluating the efficiency of a queue system performance measure uses the BCC model.

Manuscript received August 26, 2022; revised April 03, 2023.

*F. Az-Zahra (corresponding author) is a postgraduate student of the Department of Mathematics, Universitas Gadjah Mada, Yogyakarta 55281, Indonesia (email: fatimah.az.z@mail.ugm.ac.id)

Indarsih is a lecturer of Mathematics Department, Universitas Gadjah Mada, Yogyakarta 55281, Indonesia (email: indarsih@ugm.ac.id)

II. Geo/Geo/1 Queue System With Priority Preemptive

A. Model Description

The queuing system that will be used in this study is a Geo/Geo/1 discrete time queue with preemptive priority. There are two types of customers, namely low-priority customers and high-priority customers. High-priority customers have preemptive priority over low-priority customers. In the partial buffer sharing mechanism, lower priority customers will be served if the queue length is less than a predefined k threshold value. The value k is chosen to limit the entry of low-priority customers and utilize more priority queue space capacity for high-priority customers. Inter-arrival times and service times are assumed to be independent and identically distributed according to a geometric distribution. The following is an illustration of the queuing system with priority in Figure 1.

Memoryless is a property of probability that refers to cases when the distribution of the waiting time for a particular event does not depend on how much time has elapsed. There are two types of distribution that are memoryless, namely geometric distribution and exponential distribution. Memoryless refers to the Markov property, the properties of random variables related to the future depend only on relevant information about the current time only.

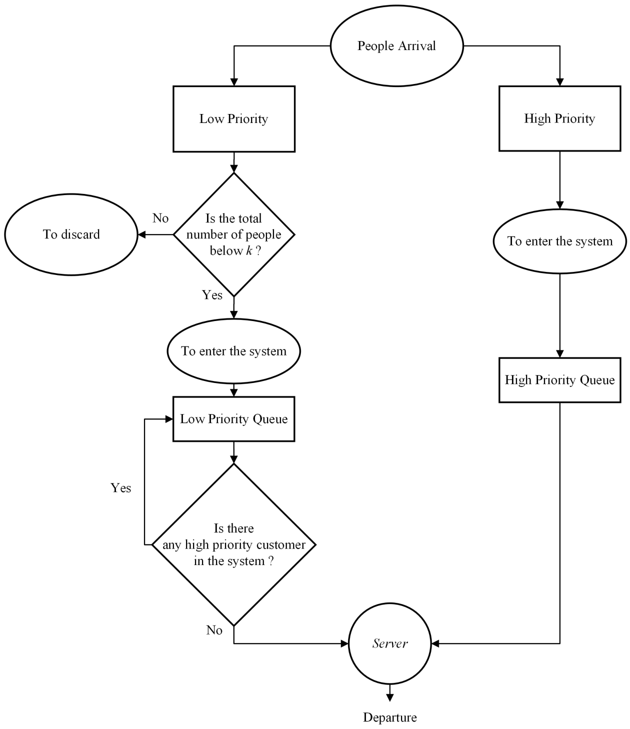

Fig. 1. The Geo/Geo/1 queuing model with priority

B. One-Stage Transistion Probability

The state matrix and one-stage transitions between system states are depicted in Figure 2. The state of the system at time t is defined by $(s_2, s_1)_t$ where s_2 and s_1 represent the number of low-priority and high-priority customers in the system, respectively. In the State Matrix, the state of the system is divided into five sections.

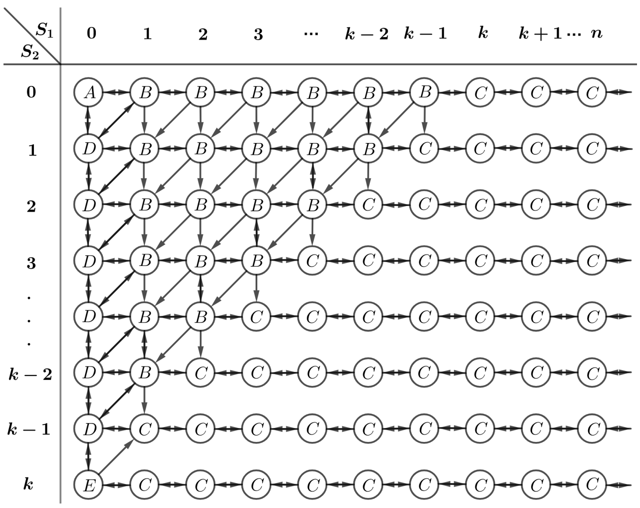

Fig. 2. States Matrix

1) Section A with initial state $(0, 0)$,
2) Section B with the initial state, the number of low-priority and high-priority customers is less than threshold k,
3) Section C with the initial state, the number of low-priority and high-priority customers is greater than or equal to threshold k,
4) Section D with the initial state there are no high-priority customers in the queue and the number of low priority customers is less than threshold k,
5) Section E with the initial state, there are no high-priority customers in the queue and the number of low priority customers is k.

Table I-V shows the destination state for each section. There is a one-stage transition back to self for all states in all sections. At the starting point of each slot a high-priority customer is always in service because a high-priority customer is a preemptive priority. The system state does not change if:

i. The high-priority customer service is completed (with probability) and one high-priority customer enters the system (with probability $\lambda\alpha$).
ii. No arrival occurs (with probability) and high-priority customer service is not completed (with probability μ').

TABLE I
ONE-STAGE TRANSITION PROBABILITIES FOR THE STATE OF SECTION A AS DEPARTURE STATE.

State of Section A (Departure State)	Destination State	One-Stage Transition Probabilies
$(0, 0)$	$(0, 1)$	$\lambda\alpha$
	$(1, 0)$	$\lambda\alpha'$
	$(0, 0)$	λ'

TABLE II
ONE-STAGE TRANSITION PROBABILITIES FOR THE STATE OF SECTION B AS DEPARTURE STATE.

State of Section B (Departure State)	Destination State	One-Stage Transition Probabilies
(m, n),	(m, n)	$\mu\lambda\alpha + \mu'\lambda'$
$m + n < k,\ n > 0$	$(m + 1, n - 1)$	$\mu\lambda\alpha'$
	$(m, n - 1)$	$\mu\lambda'$
	$(m, n + 1)$	$\mu'\lambda\alpha$
	$(m + 1, n)$	$\mu'\lambda\alpha'$

TABLE III
ONE-STAGE TRANSITION PROBABILITIES FOR THE STATE OF SECTION C AS DEPARTURE STATE.

State of Section C (Departure State)	Destination State	One-Stage Transition Probabilies
(m, n),	(m, n)	$\mu\lambda\alpha + \mu'(1 - \lambda\alpha)$
$m + n \geq k,\ n > 0$	$(m, n - 1)$	$\mu(1 - \lambda\alpha)$
	$(m, n + 1)$	$\mu'\lambda\alpha$

C. Equilibrium Equation

The equation for the state of equilibrium in each region is presented in Figure 3. The steady-state probability as the destination state is the sum of the previous steady-state probabilities of each multiplied by the corresponding one-stage transition probability. According to Jolai et al. [2]

TABLE IV
ONE-STAGE TRANSITION PROBABILITIES FOR THE STATE OF SECTION D AS DEPARTURE STATE.

State of Section D (Departure State)	Destination State	One-Stage Transition Probabilies
$(m,0),\ 0<m<k$	$(m-1,1)$	$\mu\lambda\alpha$
	$(m,0)$	$\mu\lambda\alpha'+\mu'\lambda'$
	$(m-1,0)$	$\mu\lambda'$
	$(m,1)$	$\mu'\lambda\alpha$
	$(m+1,0)$	$\mu'\lambda\alpha'$

TABLE V
ONE-STAGE TRANSITION PROBABILITIES FOR THE STATE OF SECTION E AS DEPARTURE STATE.

State of Section E (Departure State)	Destination State	One-Stage Transition Probabilies
$(k,0)$	$(k-1,1)$	$\mu\lambda\alpha$
	$(k-1,0)$	$\mu(1-\lambda\alpha)$
	$(k,1)$	$\mu'\lambda\alpha$
	$(k,0)$	$\mu'(1-\lambda\alpha)$

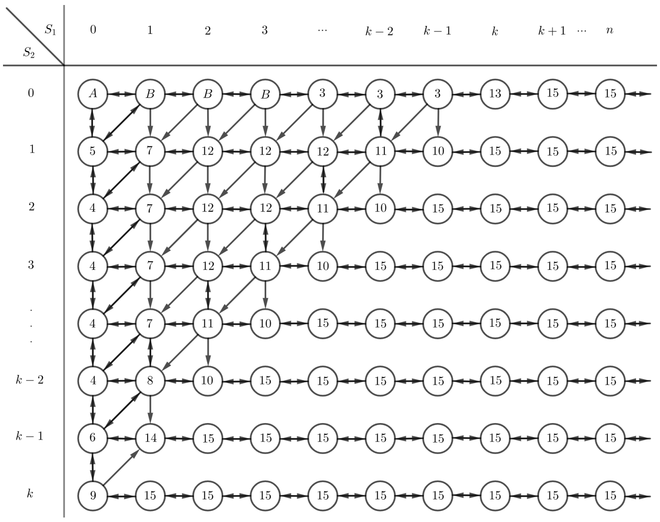

Fig. 3. States Matrix and Regions

Equations 1 to 15 are for the region $1-15$, respectively.

$$\pi_{0,0} = \lambda'\pi_{0,0}+\mu\lambda'\pi_{0,1}+\mu\lambda'\pi_{1,0} \quad (1)$$

$$\pi_{0,1} = (\mu\lambda\alpha+\mu'\lambda')\pi_{0,1}+\lambda\alpha\pi_{0,0}+\mu\lambda\alpha\pi_{1,0}+\mu\lambda'\pi_{0,2} \quad (2)$$

$$\pi_{0,n} = (\mu\lambda\alpha+\mu'\lambda')\pi_{0,n}+\mu'\lambda\alpha\pi_{0,n-1}+\mu\lambda'\pi_{0,n+1},\ 2\le n\le k-1,\ m=0 \quad (3)$$

$$\pi_{m,0} = (\mu\lambda\alpha'+\mu'\lambda')\pi_{m,0}+\mu'\lambda\alpha'\pi_{m-1,0}+\mu\lambda\alpha'\pi_{m-1,1}+\mu\lambda'\pi_{m,1}+\mu\lambda'\pi_{m+1,0},\ n=0,\ 2\le m\le k-2 \quad (4)$$

$$\pi_{1,0} = (\mu\lambda\alpha'+\mu'\lambda')\pi_{1,0}+\lambda\alpha'\pi_{0,0}+\mu\lambda\alpha'\pi_{0,1}+\mu\lambda'\pi_{1,1}+\mu\lambda'\pi_{2,0} \quad (5)$$

$$\pi_{k-1,0} = (\mu\lambda\alpha'+\mu'\lambda')\pi_{k-1,0}+\mu'\lambda\alpha'\pi_{k-2,0}+\mu\lambda\alpha'\pi_{k-2,1}+\mu\lambda'\pi_{k-1,1}+\mu(1-\lambda\alpha)\pi_{k,0} \quad (6)$$

$$\pi_{m,1} = (\mu\lambda\alpha+\mu'\lambda')\pi_{m,1}+\mu'\lambda\alpha'\pi_{m-1,1}+\mu\lambda\alpha'\pi_{m-1,2}+\mu\lambda'\pi_{m,2}+\mu\lambda\alpha\pi_{m+1,0}+\mu'\lambda\alpha\pi_{m,0},\ n=1,\ 1\le m\le k-3 \quad (7)$$

$$\pi_{k-2,1} = (\mu\lambda\alpha+\mu'\lambda')\pi_{k-2,1}+\mu'\lambda\alpha'\pi_{k-3,1}+\mu\lambda\alpha'\pi_{k-3,2}+\mu(1-\lambda\alpha)\pi_{k-2,2}+\mu\lambda\alpha\pi_{k-1,0}+\mu'\lambda\alpha\pi_{k-2,0} \quad (8)$$

$$\pi_{k,0} = \mu'(1-\lambda\alpha)\pi_{k,0}+\mu'\lambda\alpha'\pi_{k-1,0}+\mu'\lambda\alpha'\pi_{k,1} \quad (9)$$

$$\pi_{m,n} = (\mu\lambda\alpha+\mu'(1-\lambda\alpha))\pi_{m,n}+\mu'\lambda\alpha'\pi_{m-1,n}+\mu(1-\lambda\alpha)\pi_{m,n+1}+\mu'\lambda\alpha\pi_{m,n-1},\ 2\le n\le k-1,\ 1\le m\le k-2,\ m+n=k \quad (10)$$

$$\pi_{m,n} = (\mu\lambda\alpha+\mu'\lambda')\pi_{m,n}+\mu'\lambda\alpha'\pi_{m-1,n}+\mu\lambda\alpha'\pi_{m-1,n+1}+\mu(1-\lambda\alpha)\pi_{m,n+1}+\mu'\lambda\alpha\pi_{m,n-1},\ 2\le n\le k-2,\ 1\le m\le k-3,\ n+m=k-1 \quad (11)$$

$$\pi_{m,n} = (\mu\lambda\alpha+\mu'\lambda')\pi_{m,n}+\mu'\lambda\alpha'\pi_{m-1,n}+\mu\lambda\alpha'\pi_{m-1,n+1}+\mu\lambda'\pi_{m,n+1}+\mu'\lambda\alpha\pi_{m,n-1},\ 2\le n\le k-3,\ 1\le m\le k-4,\ n+m\le k-2 \quad (12)$$

$$\pi_{0,k} = (\mu\lambda\alpha+\mu'\lambda')\pi_{0,k}+\mu(1-\lambda\alpha)\pi_{0,k+1}+\mu'\lambda\alpha\pi_{0,k-1} \quad (13)$$

$$\pi_{k-1,1} = (\mu\lambda\alpha+\mu'\lambda')\pi_{k-1,1}+\mu'\lambda\alpha'\pi_{k-2,1}+\mu(1-\lambda\alpha)\pi_{k-1,2}+\mu\lambda\alpha\pi_{k,0}+\mu'\lambda\alpha\pi_{k-1,0} \quad (14)$$

$$\pi_{m,n} = (\mu\lambda\alpha+\mu'(1-\lambda\alpha))\pi_{m,n}+\mu'\lambda\alpha\pi_{m,n-1}+\mu(1-\lambda\alpha)\pi_{m,n+1},\ m+n\ge k+1 \quad (15)$$

Let

$$X=\frac{\mu'\lambda\alpha}{\mu(1-\lambda\alpha)}$$

is the probability of the arrival of a high-priority customer and the absence of service compared to the probability of the absence of a high-priority customer and the presence of service. So that the necessary and sufficient conditions for the stability of this queuing system are $X<1$ or $\lambda\alpha<\mu$.

D. Queue Performance Measures

1) Loss Probability
If the queuing system is in a critical condition so that low-priority customers can not enter the queuing system (lost customers) it is called loss probability (LP). Defined

$$LP=\lim_{z\to\infty}T_c^{(Z)}=\frac{1}{1-X}\sum_{m=0}^{k}\pi_{m,k-m}.$$

2) Average Number of Low Priority Customers in the System (L_2)

$$\begin{aligned} L_2 &= \sum_{m=0}^{k}\left(m\sum_{m=0}^{\infty}\pi_{m,n}\right)\\ &= \sum_{m=1}^{k}\left(m\sum_{n=0}^{k-m-1}\pi_{m,n}+m\sum_{n=k-m}^{\infty}\pi_{m,n}\right)\\ &= \sum_{m=0}^{k}\left(m\left[\sum_{n=0}^{k-m-1}\pi_{m,n}+\frac{\pi_{m,k-m}}{1-X}\right]\right). \end{aligned}$$

3) Average Number of High Priority Customers in the System (L_1)
The probability function of critical and non-critical states is obtained

$$L_1 = \sum_{m=0}^{k-1} \sum_{n=0}^{k-m-1} (n\pi_{m,n}) + \sum_{m=0}^{k} \left(\left[\frac{(k-m)(1-X)+X}{(1-X)^2} \right] \pi_{m,k-m} \right).$$

4) Average Number of Low Priority Customers in Queue (Lq_2)
Low-priority customers get service if there are no high-priority customers in the queue system. When there is one low-priority customer in the service and the remaining low-priority customer is in the queue then there is at least one high-priority customer in the queue system, all the low-priority customers are in line. The probability function of the critical and non-critical threshold states is obtained as follows:

$$Lq_2 = \sum_{m=2}^{k} ((m-1)\pi_{m,0}) + \sum_{m=1}^{k-1} \sum_{n=1}^{k-m} (m\pi_{m,n}) \sum_{m=1}^{k} \left(\frac{mX}{1-X} \pi_{m,k-m} \right).$$

5) Average Number of High Priority Customers in Queue (Lq_1)
There is a relationship between L_1 and Lq_1

$$Lq_1 = L_1 - \left(1 - \sum_{m=0}^{k} \pi_{m,0} \right).$$

6) Average Waiting Time in Queue for High-Priority Customers (Wq_1)
Based on Little Law

$$Wq_1 = \frac{Lq_1}{\lambda\alpha}$$

where $\lambda\alpha$ is the high-priority customer arrival rate.

7) Average Waiting Time in Queue for Low Priority Customers (Wq_2)
When the queuing system is in a critical period, the arrival rate of low-priority customers is zero and in the non-critical period, low-priority customers enter the queuing system with effective rate of $\lambda\alpha'$. So the arrival rate of low-priority customers in the queuing system (AR_2) is

$$AR_2 = \lambda\alpha' \left(\sum_{m=0}^{k-1} \sum_{n=0}^{k-m-1} \pi_{m,n} \right) = \lambda\alpha'(1-LP)$$

$$Wq_2 = \frac{Lq_2}{AR_2}.$$

III. Fuzzy Queue System

A. *Fuzzy Decision Making in Queue*

Capacity status and age of people who need services (customers) are each a fuzzy decision variable. The queue capacity status expressed in the fuzzy decision variable is defined in terms of three fuzzy states (not-crowded, crowded, and full) which describe queue congestion. The queue capacity status is normalized to the largest queue congestion of all available queues. Buffer length is a fixed capacity assigned to queues in the system whereas capacity status is a status variable indicating queue congestion based on customer perception. This state variable always changing when the queue operates. The queue capacity state is illustrated in Figure 4.

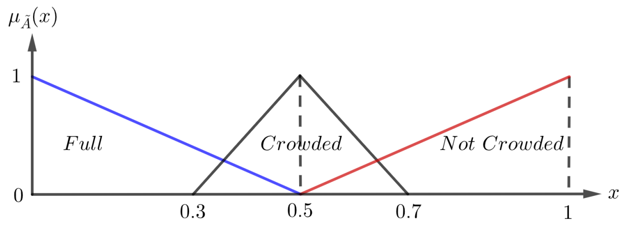

Fig. 4. Normalized Capacity State

Based on Figure 4 the horizontal x axis shows the ratio of customers in line to buffer length and the vertical axis shows the membership function of capacity status. For $x = 0.5$ it means that the customer in the queue is half of the buffer length and the membership function of the crowded capacity status has a value of 1 while the membership function of the full capacity status status and not crowded has a value of 0. Furthermore, the age of customers in the queue is illustrated in Figure 5 as follows

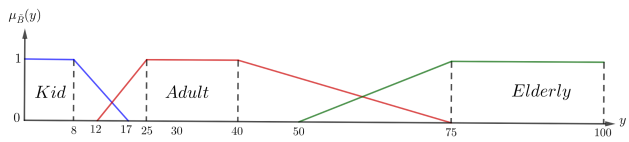

Fig. 5. Age of Customers

B. *Fuzzy Queue Model*

Parents and children are considered as high-priority customers, while adults as low-priority customers. Some customers decide not to join the queue if it is crowded and waiting times are high. Queue capacity is limited for low-priority customers and will be rejected when no capacity is available (referred to as partial buffer sharing). Arrival rate (λ) and service level (μ) are both assumed to be 0.05. The percentage of high-priority arrivals (priority coverage) and capacity state are fuzzy numbers, respectively.

The fuzzy queue will be converted into several preemptive discrete time priority buffer systems by dividing the partial buffer and applying the cut approach. Consider the following fuzzy intervals for fuzzy parameters of buffer length (k), priority coverage, namely the priority level given to several customers (Cov), and performance measures.
Parameter $\tilde{k} = (k^{[1]}, k^{[2]}); \widetilde{Cov} = (Cov^{[1]}, Cov^{[2]})$
Performance measure $\widetilde{Wq_1} = (Wq_1^{[1]}, Wq_1^{[2]}); \widetilde{Wq_2} = (Wq_2^{[1]}, Wq_2^{[2]}); \widetilde{LP} = (LP^{[1]}, LP^{[2]})$.

According to Reza, G. et al. [5] The lower and upper limits of fuzzy intervals for performance measures are as follows

$$Wq_1^{[1]}(\tilde{k}, \widetilde{Cov}) = \frac{Wq_1(k^{[1]}, Cov^{[1]}) + Wq_1(k^{[2]}, Cov^{[1]})}{2}$$

$$Wq_1^{[2]}(\tilde{k},\widetilde{Cov}) = \frac{Wq_1(k^{[1]},Cov^{[2]}) + Wq_1(k^{[2]},Cov^{[2]})}{2}$$
$$Wq_2^{[1]}(\tilde{k},\widetilde{Cov}) = Wq_2(k^{[1]},Cov^{[1]})$$
$$Wq_2^{[2]}(\tilde{k},\widetilde{Cov}) = Wq_2(k^{[2]},Cov^{[2]})$$
$$LP^{[1]}(\tilde{k},\widetilde{Cov}) = LP(k^{[2]},Cov^{[1]})$$
$$LP^{[2]}(\tilde{k},\widetilde{Cov}) = LP(k^{[1]},Cov^{[2]})$$

where $Wq_1^{[1]}(\tilde{k},\widetilde{Cov})$ is the lower bound of Wq_1 for fuzzy parameters $\tilde{k}$ and $\widetilde{Cov}$ and $Wq_1^{[2]}(\tilde{k},\widetilde{Cov})$ is the upper limit of Wq_1 for fuzzy parameters $\tilde{k}$ and $\widetilde{Cov}$. Likewise for Wq_2 and the loss probability (LP). Possible alternatives of fuzzy priority setting and buffer control will be analyzed and the results are presented in Table VI by Reza, G. et al. This table presents the lower and upper limits for system parameters as well as for system performance measures. Value for priority coverage is between 0 and 22%. A value of zero means that no priority is given to people who need the service. Everyone under the age of 17 or over 50 is considered a high-priority customer. The optimal fuzzy value of priority coverage and buffer size will be determined to minimize three system performance measures, namely the average waiting time for high-priority people in the queue (Wq_1), the average waiting time for low-priority people (Wq_2), and the probability of losing a low-priority customer (LP).

C. Fuzzy Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a non-parametric technique for evaluating the efficiency of a particular set of DMUs. The term "DMU" is a decision-making unit entity that converts a set of input into a set of output. This evaluation produces a performance score that ranges between zero and one and represents the "level of efficiency" with a score of one representing the efficient DMU which is obtained from the entity being evaluated. The use of DEA to identify this efficiency relies almost entirely on the input and output data and does not require explicit characterization of the relationship between input and output such as linear or nonlinear, which is commonly used in statistical regression. Assume a sample that includes n DMUs, and each DMU uses m input with the s output property. DEA models can be input or output oriented. In a input oriented model, the rate of all output is constant. Here's a input-oriented DEA model, for all $j = 1,2,3,...,n$. Check the efficiency of the o^{th} DMU with $o \in \{1,2,3,...,n\}$, we get the following fractional program

$$\begin{aligned} &\max \frac{\sum_{r=1}^{s}\rho_r y_{ro}}{\sum_{i=1}^{m}\beta_i x_{io}} \\ &\text{s.t.} \\ &\frac{\sum_{r=1}^{s}\rho_r y_{rj}}{\sum_{i=1}^{m}\beta_i x_{ij}} \le 1 \\ &\rho_r, \beta_i \ge 0 \ \forall i, r. \end{aligned} \tag{16}$$

Under the nonzero of $\beta \ge 0$ and $X > 0$, the denominator of the constraint of (16) is positive for every j, and multiple both side of $\frac{\sum_{r=1}^{s}\rho_r y_{rj}}{\sum_{i=1}^{m}\beta_i x_{ij}} \le 1$ by the denominator. Fractional number is invariant under multiplication of both numerator and denominator by the same nonzero number. Next, set the denominator of objective function $\frac{\sum_{r=1}^{s}\rho_r y_{ro}}{\sum_{i=1}^{m}\beta_i x_{io}}$ equal to 1, move it to constraint, and maxime the numerator. We get the following linear program

$$\begin{aligned} &\max \sum_{r=1}^{s}\rho_r y_{ro} \\ &\text{s.t.} \\ &\sum_{i=1}^{m}\beta_i x_{io} = 1 \\ &\sum_{r=1}^{s}\rho_r y_{rj} - \sum_{i=1}^{m}\beta_i x_{ij} \le 0 \\ &\rho_r, \beta_i \ge 0 \ \forall i, r. \end{aligned} \tag{17}$$

Let optimal solution of (17) be ρ^*, β^* and the optimal objective value θ^*. optimal solution of (16) is also ρ^*, β^*, since the transformation is reversible. Therefor, (16) and (17) have the same optimal objective value θ^*.

Based on the problem of handling disaster victims which involves the use of queue theory in the previous explanation. The percentage of people receiving high-priority services is called priority coverage, which will affect queuing performance measures including the probability of losing customers and the average waiting time in the queue. The higher the priority range the more people waiting in line, therefore the more people with low priority will be lost. Priority coverage is directly related to server decisions, and is a control variable so it is considered as an input denoted by x_1. Meanwhile, the buffer length (k) is not a control variable so it is not an input. Furthermore, the performance measure of the queuing system is output because it shows the output of the queuing system. Therefore, the queuing system performance measures including Wq_1, Wq_2, and LP are the outputs of each denoted by y_1, y_2, and y_3. The inputs and outputs as well as the DMU are shown in the Table VI, with 60 of the alternative being considered the DMU. Based on the (17) model, the linear program is obtained as follows

$$\begin{aligned} &\max \ \rho_1 y_{1,o} + \rho_2 y_{2,o} + \rho_3 y_{3,o} \\ &\text{s.t.} \\ &\beta_1 x_{1,o} = 1 \\ &\rho_1 y_{1,1} + \rho_2 y_{2,1} + \rho_3 y_{3,1} - \beta_1 x_{1,1} \le 0 \\ &\rho_1 y_{1,2} + \rho_2 y_{2,2} + \rho_3 y_{3,2} - \beta_1 x_{1,2} \le 0 \\ &\rho_1 y_{1,3} + \rho_2 y_{2,3} + \rho_3 y_{3,3} - \beta_1 x_{1,3} \le 0 \\ &\vdots \\ &\rho_1 y_{1,60} + \rho_2 y_{2,60} + \rho_3 y_{3,60} - \beta_1 x_{1,60} \le 0 \\ &\rho_r, \beta_i \ge 0 \ \forall i, r. \end{aligned} \tag{18}$$

Primal linear programming (18) is converted to dual linear programming

$$\begin{aligned} &\min \theta \\ &\text{s.t.} \\ &\theta x_{1,o} - \lambda_1 x_{1,1} - \lambda_2 x_{1,2} - \lambda_3 x_{1,3} - ... - \lambda_{60} x_{1,60} \le 0 \\ &\lambda_1 y_{1,1} + \lambda_2 y_{1,2} + \lambda_3 y_{1,3} + ... + \lambda_{60} y_{1,60} \ge y_{1,o} \\ &\lambda_1 y_{2,1} + \lambda_2 y_{2,2} + \lambda_3 y_{2,3} + ... + \lambda_{60} y_{2,60} \ge y_{2,o} \\ &\lambda_1 y_{3,1} + \lambda_2 y_{3,2} + \lambda_3 y_{3,3} + ... + \lambda_{60} y_{3,60} \ge y_{3,o} \\ &\lambda_j \ge 0, \forall j, \ \theta \text{ unrestricted.} \end{aligned} \tag{19}$$

From model (19), the following model is formed

$$\begin{aligned} &\min \theta \\ &\text{s.t.} \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}; \quad i = 1..m \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}; \quad r = 1..s \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda_j \geq 0, \forall j, \theta \text{ unrestricted.} \end{aligned} \tag{20}$$

Model (20) is called the BCC model, proposed by Banker, Charnes, and Cooper in 1984 to measure the relative efficiency. Fuzzy Data Envelopment Analysis (FDEA) is a tool to evaluate the efficiency of the DMU with the fuzzy data set and the FDEA model in the form of a fuzzy linear programming model. The model (21) is a input oriented BCC FDEA model. FDEA can be used to see the efficiency of the DMU on the problem of handling victims of natural disasters because the priority coverage (input FDEA) in the table (VI) is a set of fuzzy resulting in a queue system performance measure output (output FDEA) which is also the fuzzy set.

$$\begin{aligned} &\min \theta \\ &\text{s.t.} \\ &\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \leq \theta \tilde{x}_{io}; \quad i = 1, .., m \\ &\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{ro}; \quad r = 1, ..., s \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda_j \geq 0, \forall j, \ \theta \text{ unrestricted.} \end{aligned} \tag{21}$$

The model (20) is called the BCC model. Since the performance measure of the queuing system is a fuzzy number, FDEA is used to see the efficiency of the DMU. Input and output FDEA and DMU are shown in Table (VI), each alternative is considered a DMU. The percentage of people receiving high-priority service is called priority coverage which will affect queue performance measures including probability of losing customers and average waiting time in line. The higher the priority range the more people waiting in line, therefore the more people with low-priority will be lost. Priority coverage is directly related to the server decision, and is a control variable so it is considered a *input* FDEA. Meanwhile, the length of buffer (k) is not a control variable so it is not a input FDEA. Furthermore, the performance measure of the queuing system is output FDEA because it shows the output of the queuing system. Therefore, the queuing system performance measures of which Wq_1, Wq_2, and LP are output FDEA. Then the FDEA model oriented input is obtained as follows

$$\begin{aligned} &\min \theta \\ &\text{s.t.} \\ &\sum_{j=1}^{n} \lambda_j \widetilde{Cov}_j \leq \theta \widetilde{Cov}_o \\ &\sum_{j=1}^{n} \lambda_j \widetilde{Wq1}_j \geq \widetilde{Wq1}_o \\ &\sum_{j=1}^{n} \lambda_j \widetilde{Wq2}_j \geq \widetilde{Wq2}_o \\ &\sum_{j=1}^{n} \lambda_j \widetilde{LP}_j \geq \widetilde{LP}_o \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda_j \geq 0, \forall j, \theta \text{ unrestricted} \end{aligned} \tag{22}$$

In the 22 model, the index j represents the alternative choices (DMU) in the Table VI and the index o represents the DMU under study. The objective function in the 22 model is to calculate the efficiency score (θ) from DMU_o. According to Jimenez et al. [1] For any pair of *fuzzy* number $\tilde{a}$ and $\tilde{b}$, the degree in which $\tilde{a}$ is bigger than $\tilde{b}$ is the following

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0, \quad a^{[2]} - b^{[1]} < 0 \\ \frac{a^{[2]} - b^{[1]}}{(a^{[2]} - b^{[1]}) - (a^{[1]} - b^{[2]})}, 0 \in (a^{[1]} - b^{[2]}, a^{[2]} - b^{[1]}) \\ 1, a^{[1]} - b^{[2]} > 0 \end{cases}$$

where $[a^{[1]}, a^{[2]}]$ and $[b^{[1]}, b^{[2]}]$ is the expected interval of $\tilde{a}$ and $\tilde{b}$. If $\mu_M(\tilde{a}, \tilde{b}) > \alpha$, it can be stated that $\tilde{a}$ is greater than or equal to $\tilde{b}$ at least to the degree α and the $\tilde{a} \geq \tilde{b}$ constraint is α-feasible. We have

$$\begin{aligned} &\min \ \theta \\ &\text{s.t.} \\ &\sum_{j=1}^{60} \lambda_j [(1-\alpha) Cov_j^{[2]} + \alpha Cov_j^{[1]}] \leq \\ &\theta[\alpha Cov_o^{[2]} + (1-\alpha) Cov_o^{[1]}] \\ &\sum_{j=1}^{60} \lambda_j [(1-\alpha) Wq1_j^{[2]} + \alpha Wq1_j^{[1]}] \geq \\ &\alpha Wq1_o^{[2]} + (1-\alpha) Wq1_o^{[1]} \\ &\sum_{j=1}^{60} \lambda_j [(1-\alpha) Wq2_j^{[2]} + \alpha Wq2_j^{[1]}] \geq \\ &\alpha Wq2_o^{[2]} + (1-\alpha) Wq2_o^{[1]} \\ &\sum_{j=1}^{60} \lambda_j [(1-\alpha) LP_j^{[2]} + \alpha LP_j^{[1]}] \geq \\ &\alpha LP_o^{[2]} + (1-\alpha) LP_o^{[1]} \\ &\sum_{j=1}^{60} \lambda_j = 1 \\ &\lambda_j \geq 0, \forall j, \theta \text{ unrestricted} \end{aligned} \tag{23}$$

where α is the degree of inequality of the satisfied constraint, and $1 - \alpha$ is a measure of the risk of an unfeasible decision vector. Table VII shows the results of the model (23) for the data presented in the Table VI at different α-feasible levels.

The α-feasible level is defined for the best priority coverage and *buffer* control. According to Jimenez et al. [3] The selection of α-level has conflicting objectives, namely to increase the value of the objective function and to increase the level of satisfaction of the constraints. Based on the α-level, alternative 55 and alternative 60 are the efficient DMU at each α-level. In the 1 and 6 alternatives, for $\alpha = 1$ is not α-feasible because it produces a score of $\theta = 0$ which means the DMU is not efficient.

IV. Conclusion

The geometric distribution is used to determine the probability of the first success on a customer's attempt to enter the queuing system. The Geo/Geo/1 queuing model with preemptive priority can be used in the case of people entering the queuing system with different probabilities based on server's decision to provide service based on priority. In Fariborz Jolai's research (2016) problem solving used the CCR model and resulted in an efficiency score exceeding one, while according to Charnes et al [7], the efficiency score has a range of values from zero to one so it needs to be reviewed. In this study, solving the problem of handling victims of natural disasters in evaluating the efficiency of a queue system performance measure uses the BCC model. Fuzzy DEA is used to determine the efficiency score of several alternatives of priority coverage and buffer length, each priority coverage and buffer length is a fuzzy sets and produce a system performance measure is also a fuzzy set. Priority coverage as input FDEA. Customer waiting time in queue and probability of losing customers as output FDEA.

TABLE VI: Alternatives of Priority Coverage and Buffer Length

Alternative	Parameters				Performance Measure					
	Priority Coverage		Buffer Length (k)		Wq_1		Wq_2		LP	
	BB	BA	BB	BA	BB	BA	BB	BA	BA	BB
P. 1	0	4	3	6	0.02	0.80	19.7	52.5	0.138	0.252
P. 2	0	4	5	8	0.02	0.80	39.7	73.3	0.107	0.167
P. 3	0	4	7	10	0.02	0.80	59.6	94.1	0.087	0.125
P. 4	0	4	9	12	0.02	0.80	79.7	115.0	0.074	0.100
P. 5	0	4	11	14	0.02	0.80	99.7	135.8	0.064	0.083
P. 6	0	4	13	16	0.02	0.80	119.7	156.6	0.056	0.071
P. 7	2	6	3	6	0.40	1.23	20.5	54.1	0.140	0.256
P. 8	2	6	5	8	0.39	1.22	40.8	75.3	0.109	0.170
P. 9	2	6	7	10	0.39	1.22	61.2	96.6	0.089	0.128
P. 10	2	6	9	12	0.39	1.22	81.6	117.9	0.075	0.102
P. 11	2	6	11	14	0.39	1.22	102.0	139.1	0.065	0.085
P. 12	2	6	13	16	0.39	1.22	122.4	160.4	0.057	0.073
P. 13	4	8	3	6	0.81	1.67	21.3	55.7	0.143	0.261
P. 14	4	8	5	8	0.80	1.67	42.1	77.4	0.111	0.174
P. 15	4	8	7	10	0.80	1.66	62.9	99.1	0.091	0.130
P. 16	4	8	9	12	0.80	1.66	83.7	120.9	0.077	0.104
P. 17	4	8	11	14	0.80	1.66	104.6	142.6	0.066	0.087
P. 18	4	8	13	16	0.80	1.66	125.4	164.3	0.059	0.074
P. 19	6	10	3	6	1.24	2.14	22.2	57.4	0.146	0.265
P. 20	6	10	5	8	1.23	2.13	43.4	79.6	0.113	0.177
P. 21	6	10	7	10	1.23	2.13	64.7	101.8	0.093	0.133
P. 22	6	10	9	12	1.22	2.12	86.0	124.0	0.078	0.106
P. 23	6	10	11	14	1.22	2.12	107.2	146.2	0.068	0.089
P. 24	6	10	13	16	1.22	2.12	128.5	168.4	0.060	0.076
P. 25	8	12	3	6	1.68	2.62	23.1	59.1	0.149	0.270
P. 26	8	12	5	8	1.68	2.61	44.8	81.8	0.116	0.180
P. 27	8	12	7	10	1.67	2.61	66.5	104.6	0.095	0.136
P. 28	8	12	9	12	1.67	2.61	88.3	127.3	0.080	0.109
P. 29	8	12	11	14	1.66	2.60	110.0	150.0	0.069	0.091
P. 30	8	12	13	16	1.66	2.60	131.7	172.7	0.061	0.078
P. 31	10	14	3	6	2.15	3.12	24.1	61.0	0.152	0.274
P. 32	10	14	5	8	2.14	3.12	46.3	84.2	0.118	0.184
P. 33	10	14	7	10	2.13	3.11	68.5	107.5	0.097	0.138
P. 34	10	14	9	12	2.13	3.11	90.7	130.7	0.082	0.111
P. 35	10	14	11	14	2.13	3.11	112.9	154.0	0.071	0.093
P. 36	10	14	13	16	2.12	3.11	135.1	177.2	0.063	0.079

P. 37	12	16	3	6	2.64	3.65	25.1	62.9	0.155	0.279
P. 38	12	16	5	8	2.62	3.65	47.8	86.7	0.121	0.188
P. 39	12	16	7	10	2.62	3.64	70.5	110.5	0.099	0.141
P. 40	12	16	9	12	2.62	3.64	93.2	134.3	0.084	0.113
P. 41	12	16	11	14	2.61	3.64	115.9	158.1	0.072	0.095
P. 42	12	16	13	16	2.61	3.64	138.6	181.9	0.064	0.081
P. 43	14	18	3	6	3.14	4.21	26.1	64.9	0.158	0.284
P. 44	14	18	5	8	3.13	4.20	49.4	89.3	0.123	0.192
P. 45	14	18	7	10	3.12	4.19	72.6	113.7	0.101	0.144
P. 46	14	18	9	12	3.12	4.19	95.8	138.1	0.085	0.116
P. 47	14	18	11	14	3.11	4.19	119.1	162.5	0.074	0.097
P. 48	14	18	13	16	3.11	4.19	142.3	186.8	0.065	0.083
P. 49	16	20	3	6	3.67	4.79	27.2	67.0	0.161	0.290
P. 50	16	20	5	8	3.66	4.78	51.0	92.0	0.126	0.195
P. 51	16	20	7	10	3.65	4.78	74.8	117.0	0.103	0.148
P. 52	16	20	9	12	3.64	4.77	98.6	142.0	0.087	0.118
P. 53	16	20	11	14	3.64	4.77	122.4	167.0	0.076	0.099
P. 54	16	20	13	16	3.64	4.77	146.2	192.0	0.067	0.085
P. 55	18	22	3	6	4.23	5.40	28.4	69.3	0.165	0.295
P. 56	18	22	5	8	4.21	5.39	52.7	94.9	0.129	0.200
P. 57	18	22	7	10	4.20	5.39	77.1	120.5	0.105	0.151
P. 58	18	22	9	12	4.20	5.38	101.5	146.2	0.089	0.121
P. 59	18	22	11	14	4.19	5.38	125.9	171.8	0.078	0.101
P. 60	18	22	13	16	4.19	5.38	150.3	197.5	0.068	0.087

TABLE VII: FDEA Efficiency Scores for Different α-feasible Level

DMU	FDEA Efficiency Scores for Different α-cut									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	1	1	1	1	1	1	1	1	1	0
2	0.97	0.971	0.973	0.976	0.978	0.981	0.985	0.991	1	0
3	0.966	0.968	0.97	0.972	0.975	0.978	0.983	0.989	1	0
4	0.973	0.974	0.976	0.978	0.98	0.983	0.986	0.992	1	0
5	0.985	0.986	0.987	0.988	0.988	0.99	0.992	0.995	1	0
6	1	1	1	1	1	1	1	1	1	0
7	0.942	0.937	0.93	0.923	0.914	0.902	0.888	0.87	0.846	0.812
8	0.916	0.912	0.907	0.9	0.893	0.883	0.87	0.853	0.829	0.791
9	0.914	0.91	0.905	0.9	0.891	0.882	0.869	0.852	0.829	0.791
10	0.918	0.914	0.908	0.902	0.894	0.883	0.87	0.853	0.829	0.791
11	0.926	0.921	0.915	0.907	0.898	0.887	0.873	0.854	0.829	0.791
12	0.706	0.688	0.668	0.644	0.617	0.586	0.55	0.506	0.455	0.399
13	0.966	0.96	0.952	0.944	0.933	0.921	0.906	0.887	0.863	0.844
14	0.906	0.903	0.899	0.894	0.888	0.881	0.873	0.863	0.851	0.834
15	0.9	0.9	0.893	0.889	0.0.884	0.877	0.87	0.861	0.85	0.834
16	0.904	0.9	0.9	0.891	0.885	0.879	0.871	0.861	0.85	0.834
17	0.91	0.906	0.902	0.896	0.89	0.883	0.874	0.863	0.85	0.834
18	0.921	0.919	0.917	0.916	0.914	0.914	0.915	0.917	0.922	0.931
19	0.941	0.938	0.934	0.93	0.925	0.92	0.914	0.907	0.898	0.9
20	0.905	0.903	0.9	0.897	0.893	0.89	0.884	0.878	0.871	0.862
21	0.904	0.902	0.899	0.896	0.893	0.888	0.884	0.878	0.871	0.862
22	0.903	0.901	0.897	0.894	0.89	0.885	0.88	0.873	0.865	0.855
23	0.909	0.906	0.902	0.898	0.894	0.889	0.883	0.876	0.0.868	0.858
24	0.925	0.923	0.92	0.918	0.916	0.913	0.911	0.909	0.908	0.907
25	0.959	0.956	0.953	0.95	0.946	0.941	0.936	0.931	0.924	0.917
26	0.911	0.91	0.908	0.907	0.905	0.902	0.899	0.896	0.892	0.887
27	0.911	0.909	0.907	0.905	0.902	0.9	0.896	0.892	0.888	0.882
28	0.914	0.912	0.91	0.908	0.905	0.902	0.899	0.894	0.889	0.883
29	0.915	0.913	0.91	0.907	0.904	0.901	0.897	0.892	0.887	0.881
30	0.934	0.931	0.928	0.926	0.922	0.919	0.915	0.911	0.906	0.9
31	0.942	0.941	0.94	0.939	0.938	0.937	0.936	0.935	0.934	0.933
32	0.924	0.923	0.922	0.92	0.919	0.917	0.915	0.913	0.91	0.906
33	0.92	0.92	0.919	0.918	0.916	0.915	0.913	0.91	0.906	0.902
34	0.924	0.923	0.921	0.92	0.919	0.917	0.915	0.912	0.908	0.905

35	0.928	0.926	0.925	0.923	0.921	0.919	0.917	0.914	0.911	0.908
36	0.932	0.932	0.932	0.932	0.933	0.935	0.937	0.941	0.946	0.953
37	0.955	0.954	0.953	0.952	0.951	0.95	0.949	0.947	0.946	0.944
38	0.939	0.938	0.937	0.936	0.935	0.934	0.933	0.931	0.929	0.926
39	0.936	0.936	0.936	0.936	0.935	0.935	0.934	0.932	0.929	0.927
40	0.938	0.938	0.937	0.937	0.936	0.935	0.933	0.931	0.929	0.926
41	0.942	0.941	0.94	0.939	0.938	0.937	0.935	0.933	0.931	0.929
42	0.947	0.946	0.946	0.945	0.945	0.945	0.946	0.946	0.948	0.95
43	0.965	0.964	0.963	0.962	0.961	0.959	0.958	0.956	0.954	0.953
44	0.955	0.955	0.955	0.955	0.955	0.954	0.954	0.953	0.951	0.95
45	0.952	0.953	0.953	0.953	0.953	0.953	0.953	0.951	0.95	0.949
46	0.955	0.955	0.955	0.955	0.955	0.955	0.955	0.953	0.952	0.951
47	0.958	0.957	0.957	0.956	0.956	0.955	0.954	0.953	0.952	0.95
48	0.864	0.861	0.858	0.855	0.852	0.849	0.846	0.842	0.839	0.835
49	0.993	0.991	0.989	0.986	0.983	0.979	0.977	0.977	0.976	0.975
50	0.975	0.975	0.975	0.975	0.975	0.976	0.975	0.975	0.974	0.974
51	0.976	0.976	0.976	0.976	0.976	0.976	0.975	0.975	0.974	0.973
52	0.974	0.974	0.975	0.975	0.975	0.975	0.975	0.974	0.973	0.972
53	0.976	0.976	0.976	0.976	0.976	0.976	0.976	0.976	0.975	0.975
54	0.981	0.981	0.982	0.982	0.983	0.984	0.986	0.987	0.99	0.993
55	1	1	1	1	1	1	1	1	1	1
56	0.999	0.999	0.998	0.998	0.998	0.998	0.998	0.998	0.997	0.997
57	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.997	0.997	0.997
58	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
59	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998
60	1	1	1	1	1	1	1	1	1	1

REFERENCES

[1] Jimenez. et al., 2007, Linear Programming with Fuzzy Parameters: An interactive method resolution, *Operational Research*, 177, pp 1599-1609.

[2] Jolai. et al., 2010, A preemptive discrete-time priority buffer system with partial, *Applied Mathematical Modelling*, 34, pp 2148-2165.

[3] Lertworasirikul. et al., 2003, Fuzzy BCC Model for Data Envelopment Analysis, *Fuzzy Optimization and Decision Making*, 2, pp 337-358.

[4] Prasanna, S., 2013, *Probability and Mathematical Statistics*, Louisville, University of Louisville.

[5] Reza, G. et al, 2016, A Multi-Objective Fuzzy Queuing Priority Assignment Model, *Applied Mathematical Modelling*, 40, pp 9500-9513.

[6] Robert, G. Bartle, Sherbert, 2011, *Introduction to Real Analysis*, University of Illinois, Urbana-Champaign.

[7] William et al., 2006, *Introduction of Data Envelopment Analysis and Its Uses With DEA Solver Software and References*, USA, Springer.