An Efficient Improved Grey Wolf Optimizer for Optimization Tasks

Jisheng Yu, Shengkai Zhang, and Rui Wang

Abstract—**The grey wolf optimizer (GWO) is an efficient meta-heuristic algorithm inspired by hunting mechanism of grey wolf. Although it has successfully solved many engineering problems, it still suffers form premature convergence and poor precision in some cases. This paper presents a hybrid grey wolf optimization algorithm (HGWO) with enhanced exploration and exploitation. In order to boost the efficacy of GWO, a new α, β and δ wolf selected method based on random subgroup strategy is proposed and two new global search formulas are designed to enhance the exploration ability. Meanwhile, the concept of greedy wolf is introduced into HGWO to enhance the exploitation ability. Greedy wolf has no hierarchy concept, and only hunting around the most valuable prey to obtain the maximum profit. A nonlinear factor is designed to control the number of greedy wolves to balance the exploration and exploitation. In order to investigate the effectiveness of the proposed HGWO, it was compared with GWO, CGWO (a varint of GWO) and some recently developed algorithms on CEC2017 benchmark functions and four engineering problems. Statistical tests were also employed to investigate the significance of the results. Experimental results and statistical tests demonstrate that the performance of the proposed HGWO is significantly better than that of GWO and other comparison algorithms.**

Index Terms— **Grey wolf optimizer, Random subgroup strategy, Greedy wolf, Nonlinear factor**

I. Introduction

The process of finding optimal parameters of a given problem to fulfill all design requirements while considering the lowest possible cost is referred to as an optimization [1]. Optimization problems are widespread in scientific computing and engineering applications and different optimization problems expect to achieve alternative and optimization solutions through appropriate optimization methods [2]. Nowadays, some optimization problems can not be solved well or efficiently, so developing new optimization algorithms with stronger adaptability and better performance is a crucial and challenging research task. Meta-heuristics have received widespread attention over the past few decades due to their simplicity, flexibility and derivation-free [3,4]. A variety of meta-heuristics based on diverse natural-based phenomena and philosophies, such as Genetic Algorithm (GA, 1975) [5], Particle Swarm Optimization (PSO, 1995) [6], Gravitational Search Algorithm (GSA, 2009) [7] , Firefly algorithm (FA, 2010) [8], Bat algorithm (BA, 2013) [9], Grey wolf Optimizer (GWO, 2014) [10], Tree Seed Optimization Algorithm (TSA, 2015) [11], Whale Optimization Algorithm (WOA, 2016) [12], Owl Search Algorithm (OSA, 2018) [13], Henry Gas Solubility Optimization (HGSO, 2019) [14] and Harris Hawks Optimization (HHO, 2019) [15], Artificial Ecosystem-based Optimization (AEO, 2019) [16], Chamel Swarm Algorithm (CSA, 2021) [17] and, have been proposed and successfully applied in many fields.

GWO is a population-based algorithm inspired by hunting mechanism of grey wolf. Due to its simple principle, fast running speed, easy implementation and high efficiency[10]. In recent years, some progress has been made in the theoretical research and engineering application of GWO. GWO has a wide range of applications, such as multi-layer perceptron (MLP) training [19], pattern recognition [20], reliable planning of safe smart grid power system [21], path planning of unmanned combat vehicles [22], energy consumption Prediction [23], ontroller parameter tuning [24], flow shop scheduling and feature selection[25]. Although the above applications are successful, like other meta-heuristic algorithms, GWO also suffers from premature convergence and poor precision in solving some complex optimization problems [26]. Hence, some researchers have concerned to alleviate theses shortcomings through improving the standard GWO. Since 2014, some variants have been proposed to enhance the performance of GWO. Rodrigue et al. proposed a new GWO with hierarchical operator and an improved GWO with fuzzy logic, respectively [27, 28]. Heidari and Pahlavani proposed an efficient GWO with Lévy flight (LF) and greedy selection strategies [26]. Joshi and Arora proposed an enhanced grey wolf optimizer with a better hunting mechanism to balance exploration and exploitation [29]. Lu et al. proposed a new grey wolf optimizer with cellular topological structure[4] and named CGWO. In CGWO, each wolf has its own topological neighbors, and interactions among wolves are restricted to their neighbors, which favors exploitation of CGWO. In this study, chaos theory is combined with GWO, and various chaotic maps are used to adjust key parameters.

From the above variants, it is evident that the main direction for improving GWO is to balance exploration and exploitation, so that the algorithm has sufficient local optimal escape potential and improves convergence accuracy. This is consistent with the improvement direction of other meta heuristic algorithms. Researchers addressed this issue by

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suggesting different methods. Lévy flight [31 to 34], cellular topology structure [33,35,36], chaos theory [37, 38], adaptive weight strategy [39], dynamic opposite learning [40 to 42], heterogeneous comprehensive learning [41, 42], multi subpopulaiton strategy [42,44], enhance search mechanism [47 to 48], and hybrid strategy [35,49] are considered potential approaches. These methods have been successfully applied to many variants of meta heuristic algorithms, such as GA, PSO, BA, GSA, WOA, OSA, HGSO, and CSA. Inspired by these methods, an efficient HGWO algorithm with hybrid strategy is proposed in this paper. In HGWO, a new α, β and δ wolf selected method based on random subgroup and two new global search formulas are designed to enhance the exploration. The concept of greedy wolf is introduced to enhance the exploitation. A nonlinear factor is designed to control the number of greedy wolves to balance the exploration and exploitation in different search stages. The proposed HGWO is evaluated and compared with GWO, CGWO and some recently developed meta heuristic algorithms on CEC2017 benchmark functions and four engineering optimization problems. Statistical tests were also employed to investigate the significance of the results.

This paper is organized as follows: standard GWO is introduced in Section II. The detailed presentation of the proposed HGWO is described in Section III. Section IV provides parameter settings and CEC2017 benchmark functions. The comparative experiments and statistical analysis results are also provided in Section IV. In Section V, the proposed HGWO is applied to solve four classical engineering problems. Finally, section VI reports on the main concluding observations and future work.

II. AN OVERVIEW OF GWO ALGORITHM

Grey Wolf Optimizer (GWO) was proposed by S. Mirjalilli in 2014. It is a metaheuristic algorithm inspired by the social hierarchy and hunting strategies of the grey wolf. To establish a social hierarchy of wolves, the grey wolves are classified into four kinds of wolf according to the fitness value. The best wolf (solution) in GWO is denoted as the alpha (α). Similarly, the second and third best wolves are beta (β) and delta (δ), respectively. The rest of wolves are considered to be omega (ω). In GWO, the hunting (optimization) is guided by α, β and δ. The ω wolves obey these three wolves[4,10]. The main behaviors of grey wolves during hunting include encircling prey, hunting prey and attacking prey.

A. Encircling prey

To mathematically model encircling behavior of grey wolves encircle their prey when hunting, the formulas are defined as follows:

$$\vec{D} = \left|\vec{C} \cdot \vec{X}_p(t) - \vec{X}(t)\right| \tag{1}$$

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A} \cdot \vec{D} \tag{2}$$

where t is the current iteration, $\vec{X}_p$ is the position of the prey, $\vec{X}(t)$ is the position of a grey worf, $\vec{A}$ and $\vec{C}$ are coefficient vectors and calculated as follows:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \tag{3}$$

$$\vec{C} = 2\vec{r}_2 \tag{4}$$

where $\vec{r}_1$ and $\vec{r}_2$ are random vectors in [0, 1], and the component of $\vec{a}$ decreases linearly from 2 to 0 during iteration [10].

B. Huntting

During the hunting process of grey wolves, the location of the prey (optimum) is unknown. To simulate the hunting behavior of grey wolves, GWO assumes that the α, β, and δ wolves have better knowledge about the potential location of the prey. Therefore, the three best wolves are used to guide the other wolves to update their positions. In hunting process, the position update formula of a grey wolf in iteration t+1is defined as follows:

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{5}$$

According to (2), $\vec{X}_1$, $\vec{X}_2$ and $\vec{X}_3$ are defined as follow:

$$\begin{cases} \vec{X}_1 = \vec{X}_\alpha - \vec{A}_1 \cdot \vec{D}_\alpha \\ \vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot \vec{D}_\beta \\ \vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot \vec{D}_\delta \end{cases} \tag{6}$$

Where $\vec{X}_1$, $\vec{X}_2$ and $\vec{X}_3$ is affected by the position of α, β and δ wolves, respectively. According to (1), $\vec{D}_\alpha$, $\vec{D}_\beta$ and $\vec{D}_\delta$ are defined as follows:

$$\begin{cases} \vec{D}_\alpha = \left|\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}(t)\right| \\ \vec{D}_\beta = \left|\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}(t)\right| \\ \vec{D}_\delta = \left|\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}(t)\right| \end{cases} \tag{7}$$

C. Attacking prey

The grey wolves finish the hunting by attacking the prey when it stops moving. To mathematically model attacking the prey, the value of vector $\vec{A}$ is decreased with the change of $\vec{a}$. When the random values of $\vec{A}$ are in [-1, 1], the next position of a search agent can be in any position between its current position and the position of the prey. The GWO algorithm allows its search agents to update their positions based on the location of α, β, and δ, and attack towards the prey[10]. The encircling mechanism shows exploration to some extent.

D. Search for prey

The grey wolves diverge from each other to search for prey and converge to attack prey. In order to mathematically model the behavior of divergence, GWO utilizes $\vec{A}$ with random values greater than 1 or less than -1 to oblige the search agent to diverge from the prey. This emphasizes exploration and allows the GWO algorithm to search globally. Another vector $\vec{C}$ that favors exploration provides random weights for prey to stochastically emphasize or deemphasize the effect of prey, which can help to improve exploration and avoid local optimum [10]. For more detailed explanation of GWO, one can refer to [10].

III. THE PROPOSED HGWO ALGORITHM

The standard GWO tends to be locally optimal and stagnant in some cases, resulting in premature convergence. To relive the above mentioned concerns, this paper proposed an improved HGWO algorithm. In HGWO, a new α, β and δ wolf selected method based on random subgroup and two new global search formulas are designed to enhance the exploration. The concept of greedy wolves is introduced to enhance the exploitation. The detailed description of HGWO is as follows.

A. Leader wolves selected based on random subgroup

As mentioned above, the hunting is guided by α, β and δ in GWO. For some complex optimization problems, in the early stage of the iterative search, the three best wolves may gather in a small region, causing other wolves to quickly move towards this region. In this case, the diversity of the population will be lost rapidly, and the algorithm will fall into local optimum. To avoid this case as much as possible, this paper proposes a random subgroup strategy to select the α, β and δ wolf. The random subgroup is composed of S% individuals randomly selected from the whole wolves. S is a parameter that needs to be determined manually in advance. Random subgroup can help to improve the local search ability since a wolf in random subgroup only interacts with part of the wolves for exploitation. Meanwhile, information diffusion mechanism contributes to exploration[4]. The attraction to the first three best solutions is weaker in random subgroup strategy, which can avoids local optimum. In addition, random subgroup strategy in GWO is easy to be implemented due to its simple mechanism.

The flow chart of the proposed random subgroup strategy is shown in Fig. 1. It can be seen from Fig.1 that the selection of the three best wolves is restricted to the subgroup in each iteration. This can help each wolf to perform exploitation inside the subgroup. Meanwhile, the overlapping region provide a migration mechanism from one subgroup to another. It can spread the information of each wolf to the whole wolves, which is conducive to exploring the whole search space.

B. Two new global search formulas

Random subgroup strategy in HGWO can slow down the premature of the algorithm, but also reduce the convergence speed. To alleviate this contradiction, this paper designs two global search formulas in HGWO, as shown in (8) and (9).

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{8}$$

Where $\vec{X}_1, \vec{X}_2$ and $\vec{X}_3$ are calculated by (5) and (6). The first global search formula in (8) is the same as the GWO algorithm. The difference is that the α, β and δ wolf are selected by random subgroup strategy.

$$\begin{aligned}\vec{X}_i(t+1) = \vec{X}_i(t) + c_1 \cdot \vec{r}_p \cdot (\vec{X}_i^p - \vec{X}_i(t)) \\ + c_2 \cdot \vec{r}_g \cdot (\vec{X}^g - \vec{X}_i(t))\end{aligned} \tag{9}$$

Where $\vec{X}_i^p$ is the personal best solution of the ith wolf obtained so far; $\vec{X}^g$ is the global best solution obtained so far; $c_1=2.5-2t/T$, $c_2=0.5+2t/T$, T is the maximum number of iteration. $\vec{r}_p$ and $\vec{r}_g$ are random vectors in [0, 1], and D is the dimension of the problem to be optimized.

It can be seen from (9) , as t increases, the value of c_1 decreases and the value of c_2 increases. The location of grey wolf is increasingly not influenced by its personal best solution, but by the global best solution. The algorithm will gradually converge.

In HGWO, two global search formulas are switched through the following strategies. HGWO algorithm firstly executes (8) to search for prey. If the global best solution is not updated in the latest L iterations, it indicates that the algorithm has trapped in local optimum. At this point, the algorithm switches to (9) to help the algorithm jump out of the local optimum, and vice versa.

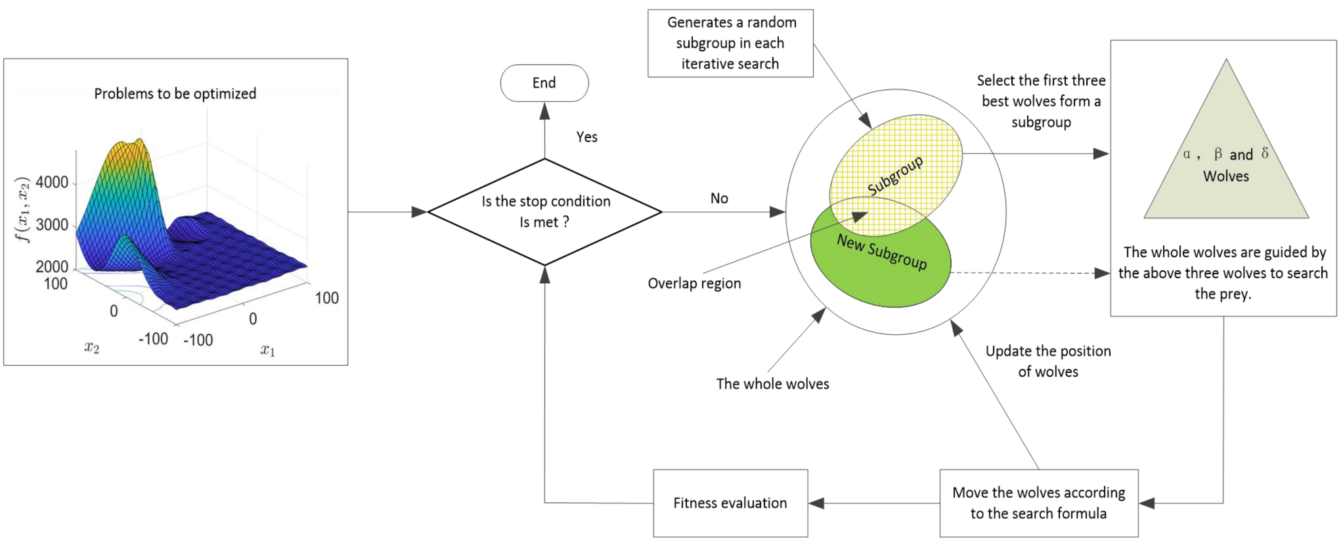


Fig.1. The flow chart of the proposed random subgroup strategy

C. Greedy wolves for local search

The global search formulas proposed in the above subsection are helpful to enhance the exploration ability of HGWO. But this can weaken the exploitation ability. To improve the convergence accuracy of the algorithm (i.e. local search ability), this paper proposes a concept of greedy wolf to enhance the exploitation. Greedy wolf have no hierarchy concept, and only hunt around the most valuable prey to obtain the maximum profit. According to the behavior of greedy wolf, the local search formula is design as follows:

$$\begin{cases} \vec{X}_i(t+1) = \vec{X}_i(t) = \vec{X}^g(t) \\ L = fix(D \cdot \theta) + 1 \\ Ind = randperm(D) \\ \vec{X}_i^{Ind(1:L)}(t+1) = \vec{X}_i^{Ind(1:L)}(t) + \eta \cdot \phi(\kappa)^{Ind(1:L)} \end{cases} \tag{10}$$

Where θ is a random number in [0,1]; $fix(\cdot)$ is a function of rounding towards zero; D is the dimension size of the search space; $randperm(D)$ returns a vector containing a random permutation of the integers 1: D. $\phi(\kappa)$ is the probability density function of standard normal distribution with mean value of 0 and standard deviation of 1; The value of κ is defined in (11); η is a attenuation coefficient, The value of η is defined in (12).

$$\kappa = \frac{1}{20}(X_{\max} - X_{\min}) \cdot (0.5 - rand(1, D)) \tag{11}$$

Where $X_{\max}$ and $X_{\min}$ are upper bound and lower bound of D-dimensional search space, respectively. $rand(1,D)$ represents a D-dimensional random number in [0,1].

$$\eta = 1 - \frac{1}{R} \cdot \sqrt{1 + (R^2 - 1)\frac{t}{T}} \tag{12}$$

Where the value of η refers to the characteristic curve of quick opening valve, and R=30 [51];

It can be seen form (10), (11) and (12), the greedy wolves move randomly around the current best wolf, and with the increase of t, the value of η becomes smaller and smaller, the movement range of the greedy wolf becomes narrower, so the search accuracy gradually improves.

To better balance the exploration and exploitation of HGWO in different search stages, a nonlinear factor is designed in (13) to control the number of greedy wolves.

$$\omega(t) = 1 - \exp\left(\frac{-10(T - t)}{T}\right) \tag{13}$$

Where $\omega(t)$ is an iteration dependent parameter and used to control the number of greedy wolves. For each grey wolf, if $\tau > \omega(t)$ (τ is a randomly number in [0,1]), the wolf becomes greedy wolf, and search the prey according to (10), otherwise it performs a global search according to (8) or (9).

In the early and middle stages of the search, the value of $\omega(t)$ is close to 1, the number of greedy wolves is very small, which can avoid premature convergence. In the later stage, $\omega(t)$ decreases rapidly and finally equals 0, the number of greedy wolves increases rapidly, which will help to improve the final convergence accuracy.

D. Framework of HGWO algorithm

To clearly show the structure of the proposed HGWO, the pseudocodes is listed in Algorithm 1. The pseudocodes can intuitively help understand the program framework of HGWO.

Algorithm 1: HGWO algorithm

Task: Obtain the optimal solution of the problem to be solved.
Input: Initialize Parameters: Wolves size N, Dimension D, Maximum Function Evaluations maxFEs, Maximum iteration number T, Allowable Position Boundary [$X_{\min}$, $X_{\max}$], parameter S and L.
Output: Output the optimal Solution.
Initialize: t=1, $flag$=0, random initialize the whole wolves, calculate the fitness of each grey wolf, record personal best solution $\vec{X}_i^p$ and its fitness Fb_i, global best solution $\vec{X}^g$ and its fitness Fg. Select α, β and δ wolf through random subgroup strategy .

```
While ( t<T)
   for each grey wolf i=1,2,…, N
      If τ>ω(t)
         The wolf becomes greedy wolf, and execute the search strategy by using (10)
      Else
            If flag==0;
               Execute the search strategy by using (8)
            Else
               Execute the search strategy by using (9)
            End If
      End If
      Calculate the fitness of the ith grey wolf; Update X_i^p and Fb_i, X^g and Fg according to greedy selection.
   End for
   Update α, β and δ wolf through random subgroup strategy .
   If The value of Fg has not become better in latest L iterations
      flag=1;
   else
      flag=0;
   End If
   t=t+1;
End while
Output: Optimal solution X^g and its fitness Fg.
```

IV. EXPERIMENTS

In this section, experimental comparative study and statistical analysis are carried out to examine the efficiency, effectiveness and stability of the proposed HGWO algorithm. To make a fair comparison, all experments are implemented in MATLAT 2018a and conducted on the PC with i5-8250U CPU @1.80GHz, 16GB RAM under Microsoft Windows 10 operating system.

A. Benchmark functions

To measure the performance of the proposed HGWO, the CEC2017 benchmark functions [52] will be used for verification. The CEC2017 benchmark functions consist of 30 functions with four types. Due to the instability of f_2 in high dimensions, f_2 is deleted in this article. The reserved functions are unimodal functions (f_1 and f_3,), simple multimodal functions (f_4 to f_{10}), hybrid functions (f_{11} to f_{20}) and composition functions (f_{21} to f_{30}), respectively. Different types of functions can comprehensively and effectively test the optimization ability and stability of an algorithm.

B. Parameters selection of HGWO

In HGWO algorithm, the parameters S and L need to be determined in advance. These two parameters affect the performance of the algorithm. S affects the strength of information exchange of random subgroups, while L determines the switching frequency of two global search formulas. In this subsection, we designed 12 kinds of parameter combinations to study the impact of parameters on the performance of HGWO. See Table I for detailed parameter combinations. In order to determine appropriate parameters of HGWO algorithm from Table I, this paper compare the performance of the algorithm in each parameter combination on 10 selected functions from CEC2017 benchmark functions. These functions include unimodal functions (f_1), simple multimodal functions(f_5 and f_9), hybrid functions (f_{12}, f_{13}, f_{16} and f_{20}), and composition functions (f_{21}, f_{24} and f_{30}). The dimensions of these functions are randomly selected from 10, 30, 50 and 100. See Table II for the information of these functions. The parameter settings of HGWO are as follows: populations size N is 50, maximum evaluations maxFEs is 1×10^6, maximum Iterations T is 2000, and each algorithm independently runs 30 times for each test function. The performance of HGWO with 12 kinds of parameter combinations is list in Table III. In Table III, "total rank" is the cumulative result of "rank", and "rank" is the ascending sorting result of the average value of the optimization results obtained by HGWO for each test function. The smaller the "total rank" value, the better the comprehensive optimization ability of the HGWO algorithm under this parameter combination. It can be seen from Table III that HGWO has best comprehensive performance under P1 parameter combination. Although this parameter combination may not be optimal, it is the best of the 12 optional parameters. Therefore, we choose S=25, L=5 as the initial parameter of HGWO.

TABLE I
DIFFERENT PARAMETER COMBINATION OF HGWO ALGORITHM

NO.	Parameter combination	S	L
1	P1	25	5
2	P2	25	10
3	P3	25	15
4	P4	50	5
5	P5	50	10
6	P6	50	15
7	P7	75	5
8	P8	75	10
9	P9	75	15
10	P10	95	5
11	P11	95	10
12	P12	95	15

TABLE II
FUNCTIONS FOR PARAMETERS SELECTION OF HGWO

Type	$f(x)$	Dimension
Unimodal functions	f_1	100
Simple multimodal functions	f_5	50
	f_9	10
Hybrid functions	f_{12}	30
	f_{13}	10
	f_{16}	100
	f_{20}	50
Composition functions	f_{23}	30
	f_{28}	100
	f_{30}	10

TABLE III
THE PERFORMANCE OF HGWO WITH 12 KINDS OF PARAMETER COMBINATIONS

Index	$f(x)$	Parameter combinations											
		P1	P1	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12
Rank	$f1$	1	6	2	11	3	4	5	9	7	12	8	10
	f_5	3	2	1	6	4	5	9	8	7	11	12	10
	f_9	5	4	3	1	8	6	2	7	9	10	11	12
	f_{12}	1	5	2	4	8	3	6	11	9	10	12	7
	f_{13}	1	10	4	9	8	6	11	2	3	7	12	5
	f_{16}	5	12	1	2	9	8	6	7	3	11	10	4
	f_{20}	5	3	2	1	8	4	9	10	6	12	11	7
	f_{23}	1	2	3	6	5	4	9	7	8	10	12	11
	f_{28}	2	11	6	1	5	4	3	9	7	12	10	8
	f_{30}	4	12	7	1	3	11	10	2	6	9	5	8
Total Rank	—	28	67	31	42	61	55	70	72	65	104	103	82
Final Rank	—	1	7	2	3	5	4	8	9	6	12	11	10

C. Qualitative comparison between GWO and HGWO

To show the difference of HGWO and GWO, this paper compared them qualitatively from the aspects of population diversity and convergence curve on a multimodal Function (f_{10}) and a composition Function (f_{28}) in CEC2017. The 3D figures of the two functions are shown in Fig.2. To observe and compare the results more intuitively, the dimension is set to 2, the maximum iteration number T is set to 200, the population size is set to 20, and the diversity index of the population is defined and shown as (14).

$$Diversity(t)=\frac{1}{N}sqrt(\sum_{i=1}^{N}\left\|X_i(t)-X^g\right\|) \qquad (14)$$

Where N is population size, $X_i(t)$ is the positon of ith wolf at iteration t, X^g is the current best solution.

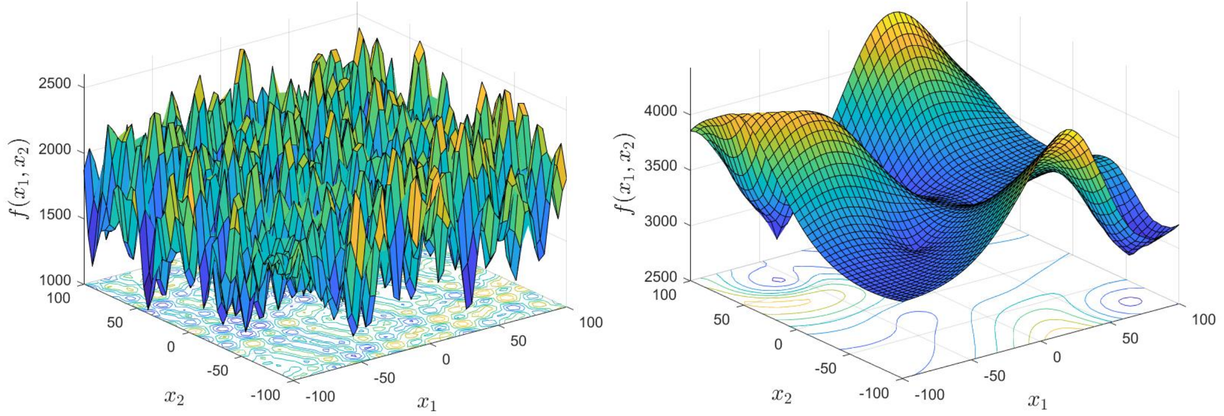
The results of qualitative comparison between HGWO and GWO are graphically shown in Fig. 3 to Fig.6.

For function f_{10}, it can be seen intuitively from Fig.3 that, compared with GWO algorithm, the population distribution of HGWO is more dispersed in the whole search space. Therefore, it has more opportunities to search the potential global optimal region. From Fig.4 (a), it can be seen that in the early and middle stages of the iterative search, the population diversity of HGWO is stronger than that of GWO. While in the late stage, the population diversity of HGWO is worse than that of GWO. This indicates that HGWO has stronger exploration ability in the early and middle stages of the iterative search, and is not easy to fall into local optimum. In the late stage, HGWO's exploitation ability become strong, making it easy to obtain more accurate solutions. From Fig. 4 (b), it can be seen that HGWO has faster convergence speed and better convergence accuracy than that of GWO, and it can obtain the optimal solution of function f_{10}. But GWO algorithm trapped in a local optimum. For function f_{28}, from Fig.5 and Fig.6, we can also get similar analysis results. For complex function, HGWO also has better global search ability and search accuracy than that of GWO .

From the above two examples, we can infer that the strategies proposed in HGWO has a better ability to balance the exploration and exploitation. In the early and middle stages of iterative search, HGWO has more opportunities to jump out of local optimum and search potential global optimum regions. In the late stage, the exploitation ability of HGWO is significantly enhanced, which is conducive to obtaining more accurate solutions. Therefore, the improved strategies of HGWO are effective, and the algorithm is more suitable for solving complex optimization problems.

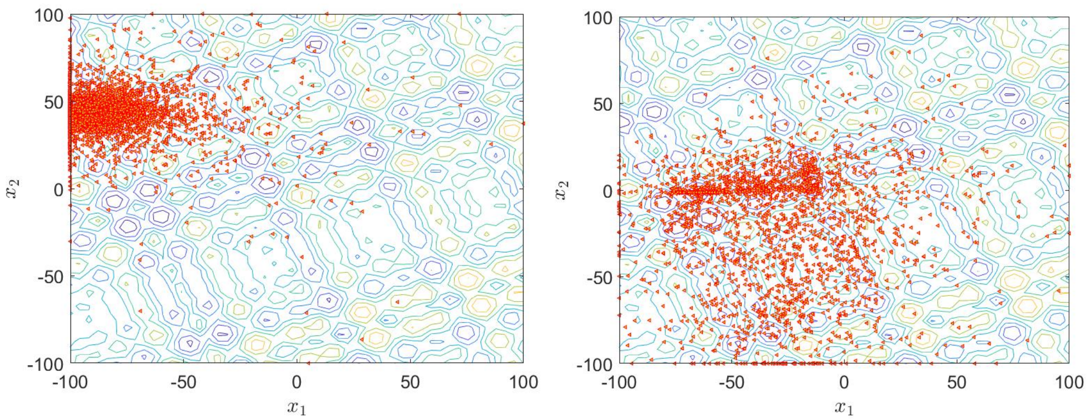
D. Quantitative comparison with other algorithms

To verify the superiority of the proposed HGWO, this paper compares HGWO with GWO, CGWO (A recently variant version), and some newly developed optimization algorithms, including OSA, HHO, HGSO, AEO, CSA and WSO. The parameter configuration for all comparison algorithms is based on the recommendations of the corresponding references and lists in Table IV.



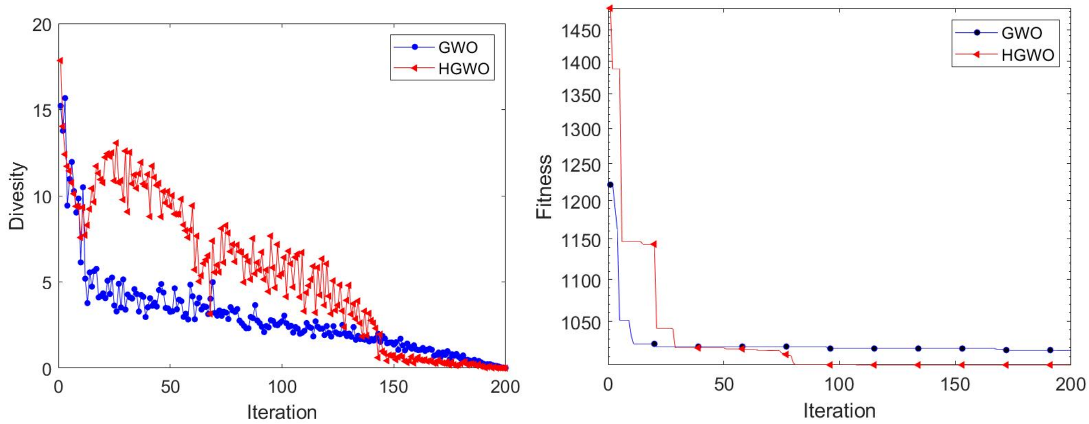
(a) f_{10} (b) f_{28}

Fig.2. 3D figures of f_{10} and f_{28}



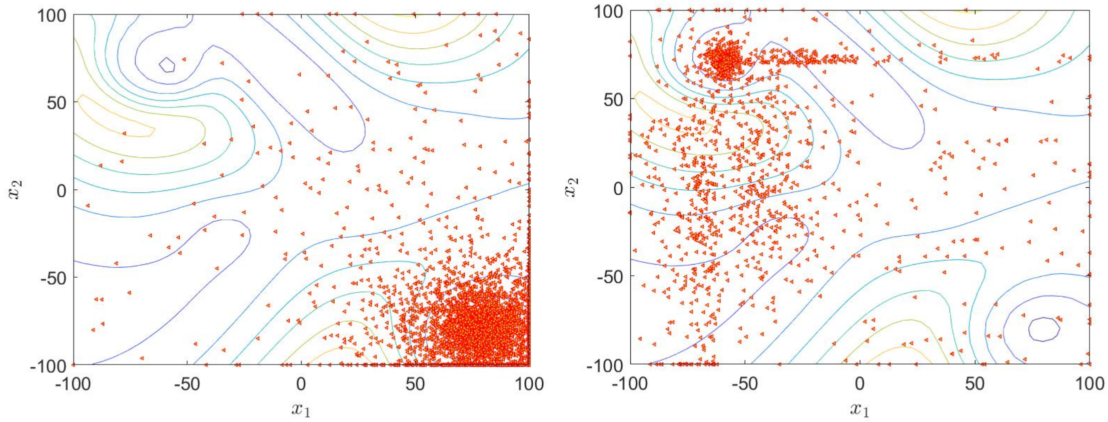
(b) GWO (b) HGWO

Fig.3. Population distribution of GWO and HGWO on f_{10}



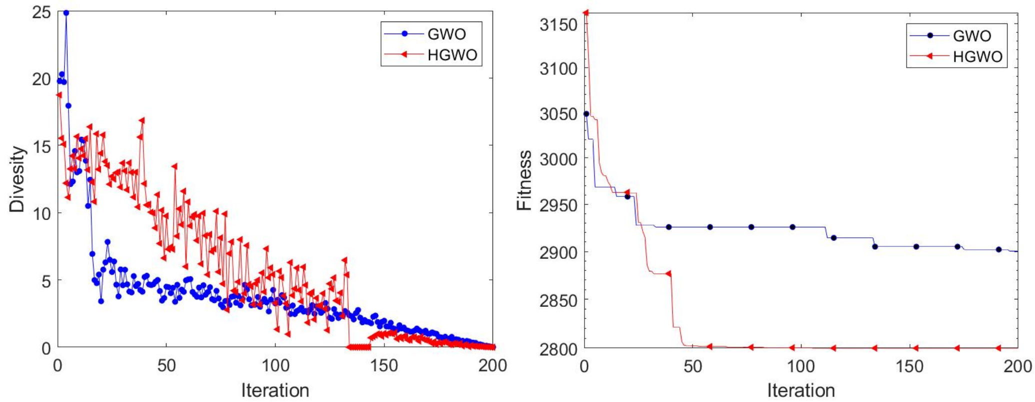
(a) Population diversity (b) Convergence curves

Fig.4. Comparison of population diversity and convergence curves between HGWO and GWO on f_{10}



(a) GWO (b) HGWO

Fig.5. Population distribution of GWO and HGWO on f_{28}



(a) Population diversity (b) Convergence curves

Fig.6. Comparison of population diversity and convergence curves between HGWO and GWO on f_{28}

TABLE IV
PARAMETERS SETTINGS OF THE ALGORITHMS USED IN THE COMPARISONS

Algorithm	Reference	Year	Parameters
GWO	[10]	2014	a decreases linearly from 2 to 0
CGWO	[4]	2018	C25 Cellular Automata, a decreases linearly from 2 to 0
OSA	[13]	2018	$\beta_{\max} = 1.9$, $\beta_{\min}=0$, $\beta = \beta_{\max} - t\,(\beta_{\max} - \beta_{\min})/T$
HHO	[15]	2019	$E=2*(1-t/T)$
HGSO	[14]	2019	$T = 298.15$k, $l_1 = 5$E-02, $l_2 = 100$, $l_3 = 1$E-02
AEO	[16]	2020	$a=(1-t/T)$*rand;
CSA	[17]	2022	$rho=1.0$; $c_1=2.0$, $c_2=1.80$, $gamma=2.0$, $alpha = 4.0$, $beta=3.0$
WSO	[18]	2022	$p_{\min}=0.5$, $p_{\max}=1.5$, $a_0=6.250$, $a_1=100$, $a_2=0.0005$
HGWO	Present	Present	$S=25$, $L=5$, a decreases linearly from 2 to 0

The common parameters of all the algorithms are as follows: populations size N is set to 50, maximum evaluations maxFEs is set to 1×10^6, maximum iterations T is set to 2000, and each algorithm independently runs 30 times for each test function. To more effectively verify the performance of HGWO, we conducted comparative experiments on the dimensions of 30, 50, and 100, respectively. HGWO and other algorithms listed in Table IV are compared with numerical experiments on the CEC2017 benchmark functions. 29 test functions have been test and the comparison results are listed in Table V, Table VI and Table VII. Meanwhile, the "Total rank" and "Final rank" of all algorithms are given at the end of Table V, Table VI and Table VII, respectively. Where "Mean","Std" and "Best" indicate the average, the standard deviation, and the best solution over 30 runs, respectively. The smaller the "Mean" value of an algorithm, the better the ability to avoid local optimum and approach the optimal solution. The smaller the "Std" value of an algorithm, the more stable the performance of the algorithm. The smaller the "Best" value, the better the ability to approach the optimal solution. "Rank" is sorted in ascending order of "Mean", and the "Total rank" is the cumulative result of "Rank". The smaller the "Total rank", the better the overall performance in of an algorithm. The "Final rank" is sorted in ascending order of "Total rank". The best solution of each function is marked in bold in the three Tables. As shown in Table V, Table VI and Table VII, the proposed HGWO is obviously superior to GWO, CGWO and other comparison algorithms, no matter in which dimension.

For the "Mean" values of all test functions, HGWO obtains 14 best results and 9 second best results in 30 dimension, obtains 15 best and 7 second best results in 50 dimension, and obtains 17 best and 7 second best results in 100 dimension. In all dimensions, the "Final Rank" of HGWO is 1, and the value of "Total Rank" is significantly better than that of other algorithms. Meanwhile, The "Total rank" value of HGWO is stable in different dimensions, which indicates that the proposed HGWO algorithm has good stability. By comparing HGWO and GWO separately, we find that except for the "Mean" value of f_6-D30, f_{10}-D30 and f_{16}-D50, the "Best" value of f_6-D50, f_{10}-D50, and f_{10}-D100, HGWO algorithm outperforms GWO algorithm. In other words, For most test functions, HGWO algorithm is superior to GWO algorithm.

In summary, the improved strategies, i.e., random subgroup strategy, two new global search formulas and the concept of greedy wolf, are satisfactory and competitive for enhancing the performance of GWO.

E. Statistic analysis

In order to make a more scientific comparison of all comparison algorithms in a statistical sense, the results of experiments were rigorously analyzed from a mathematical perspective. In this section, two non parametric statistical methods including Friedman test [53] and Wilcoxon signed rank test [54, 55] are employed to analyze the experimental results.

Firstly, the Friedman test was implemented. The mean rank of all comparison algorithms and the corresponding p-values are listed in Table VIII. It can be seen from Table VIII , the p-value of different dimensions calculated by Friedman test are 4.8998e-34, 6.0919e-35 and 8.1920e-36, respectively. Since the p-Value is less than 0.05, we believe that the statistical results are significant. The mean ranks of HGWO are 1.8333, 1.8667, and 1.8000 in 30, 50 and 100 dimension, which are significantly lower other algorithms. Combined with the conclusion that HGWO is superior to other comparative algorithms from the above numerical experiments, Friedman test further confirms that the advantages of HGWO are significant.

Secondly, Wilcoxon signed rank test is used for pairwise comparison to determine which algorithm has better statistical performance. In Wilcoxon signed rank test, the value of significance level α is set to 0.05. Table IX, Table X and TableXI respectively show the Wilcoxon signed rank test results of the "Mean", "Std" and "Best" in the numerical experimental of all test functions in different dimensions. Where 'R+' represents the sum of ranks for the problems in which the proposed HGWO outperforms the other comparison algorithm and 'R-' represents the sum of ranks for the opposite. '+' represents that the proposed HGWO outperforms the comparison algorithm. '−' represents the proposed HGWO algorithm is worse than the comparison algorithm. '=' represents that there is no significant statistical difference between the two algorithms. The last row of Table IX, Table X and TableXI shows the total counts in the (+/ = /−) format. It can be seen from the three tables that HGWO has significant advantages in terms of search ability and stability compared with other comparison algorithms, no matter in which dimension.

Therefore, in a statistical sense, the comprehensive performance of HGWO is obviously superior to GWO, CGWO and other comparison algorithms.

F. Convergence comparisons on the selected functions

Convergence curve is an intuitive way to observe the convergence rate and accuracy of an algorithms. Eight functions are chosen (f_1-D50, f_5-D30, f_8-50, f_9-D100, f_{12}-D30, f_{19}-D100, f_{24}-D50, and f_{26}-D100) and the convergence curves of all comparison algorithms are plotted in Fig.7. It can be seen from Fig.7 that the proposed HGWO has obvious advantages in convergence rate and precision for most functions. It is worth noting that the proposed HGWO obtains better solutions than most comparison algorithms in the early or middle stages of the iterative search, and exceeds most comparison algorithms at the end of the iterative search. By comparing HGWO with GWO, we can also find that the convergence rate and accuracy of HGWO are obviously improved. It indicates that HGWO has stronger ability to jump out of local optimum and obtain better search results than GWO.

TABLE VIII
MEAN RANKS OF DIFFERENT ALGORITHMS OBTAINED BY FRIEDMAN TEST ON ALL TEST FUNCTIONS

Algorithms	Ranked in different dimensions		
	30	50	100
AEO	3.7333	3.5000	3.0667
HHO	5.5000	5.0333	4.7667
CSA	7.0333	7.0000	7.0333
OSA	8.8667	8.8667	8.8333
WSO	2.8667	3.6000	4.2333
HGSO	7.2000	7.6000	7.7333
GWO	4.3667	4.4667	4.4000
CGWO	3.6000	3.0667	3.1333
HGWO	1.8333	1.8667	1.8000
p-value	4.8998e-34	6.0919e-35	8.1920e-36

TABLE V
COMPARATIVE RESULTS OF THE COMPARISON ALGORITHMS ON THE CEC2017 BENCHMARK FUNCTIONS(D=30)

f(*x*)	Index	AEO	HHO	CSA	OSA	WSO	HGSO	GWO	CGWO	HGWO
f1	Mean	4.6292E+03	1.2251E+07	7.9787E+09	5.3819E+10	5.2220E+07	1.3529E+10	1.3321E+09	6.3729E+08	4.5194E+03
	Std	4.7168E+03	2.5552E+06	6.9714E+09	7.4801E+09	1.4363E+08	3.4044E+09	1.2102E+09	7.4506E+08	6.5798E+03
	Best	1.2045E+02	7.8893E+06	7.2141E+08	4.1802E+10	2.1618E+03	6.4584E+09	4.6128E+07	1.1313E+07	**1.0077E+02**
	Rank	2	3	7	9	4	8	6	5	1
f3	Mean	4.3899E+02	1.0560E+04	7.3310E+04	9.1151E+04	1.6641E+04	4.7221E+04	3.3966E+04	2.8178E+04	3.2370E+02
	Std	1.5509E+02	4.1532E+03	1.2147E+04	4.5417E+03	4.9077E+03	7.7295E+03	7.6887E+03	1.0595E+04	3.8347E+01
	Best	3.0938E+02	5.1031E+03	4.8889E+04	7.0452E+04	9.2882E+03	3.3995E+04	1.2672E+04	9.6508E+03	**3.0005E+02**
	Rank	2	3	8	9	4	7	6	5	1
f4	Mean	4.9869E+02	5.3022E+02	2.9740E+03	1.1716E+04	5.0966E+02	2.0106E+03	6.0439E+02	5.3380E+02	5.0189E+02
	Std	2.3972E+01	2.7122E+01	9.9037E+03	1.6492E+03	2.9646E+01	4.7767E+02	8.2244E+01	2.5672E+01	4.3961E+01
	Best	4.5856E+02	4.7302E+02	5.8228E+02	8.1255E+03	4.4835E+02	1.2211E+03	5.0763E+02	5.0667E+02	**4.0000E+02**
	Rank	1	4	8	9	3	7	6	5	2
f5	Mean	6.7277E+02	7.2966E+02	7.3339E+02	9.3428E+02	5.9868E+02	8.0876E+02	6.0744E+02	5.7527E+02	5.4464E+02
	Std	3.3209E+01	2.6454E+01	5.1337E+01	1.5374E+01	2.6476E+01	1.6428E+01	3.8836E+01	2.7200E+01	1.2853E+01
	Best	6.0945E+02	6.5825E+02	6.4423E+02	8.9538E+02	5.4912E+02	7.7120E+02	5.4050E+02	5.3833E+02	**5.2686E+02**
	Rank	5	6	7	9	3	8	4	2	1
f6	Mean	6.3784E+02	6.6239E+02	6.6827E+02	6.8891E+02	6.1544E+02	6.6563E+02	6.0549E+02	6.0351E+02	6.0790E+02
	Std	8.6117E+00	6.7974E+00	9.2690E+00	8.4176E+00	8.6762E+00	6.8748E+00	2.7987E+00	1.5108E+00	2.3257E+00
	Best	6.2547E+02	6.4942E+02	6.4261E+02	6.7599E+02	6.0591E+02	6.5139E+02	6.0218E+02	**6.0153E+02**	6.0462E+02
	Rank	5	6	8	9	4	7	2	1	3
f7	Mean	1.0786E+03	1.2456E+03	1.1637E+03	1.4571E+03	9.6089E+02	1.1391E+03	8.7702E+02	8.2435E+02	7.8570E+02
	Std	8.7749E+01	6.1182E+01	7.5297E+01	4.2561E+01	8.0777E+01	4.3560E+01	4.4794E+01	3.1898E+01	1.7539E+01
	Best	9.2199E+02	1.1220E+03	1.0298E+03	1.2978E+03	8.2644E+02	1.0537E+03	8.0619E+02	7.6646E+02	**7.6573E+02**
	Rank	5	8	7	9	4	6	3	2	1
f8	Mean	9.4409E+02	9.6996E+02	1.0250E+03	1.1374E+03	8.7941E+02	1.0543E+03	8.7811E+02	8.7039E+02	8.3804E+02
	Std	2.5502E+01	2.4375E+01	5.1701E+01	3.0920E+01	1.8383E+01	2.0639E+01	3.5491E+01	2.9254E+01	1.1684E+01
	Best	8.8824E+02	9.2650E+02	9.3526E+02	1.0793E+03	8.4478E+02	1.0092E+03	8.4124E+02	8.4020E+02	**8.2487E+02**
	Rank	5	6	7	9	4	8	3	2	1
f9	Mean	4.3906E+03	6.7836E+03	6.9751E+03	1.1472E+04	3.9723E+03	6.7095E+03	1.8110E+03	1.2025E+03	1.1445E+03
	Std	1.0339E+03	8.8664E+02	2.5849E+03	1.1345E+03	1.0423E+03	9.5277E+02	6.1327E+02	2.7323E+02	1.0923E+02
	Best	2.5212E+03	5.0833E+03	3.6074E+03	8.9795E+03	2.2053E+03	4.6253E+03	1.1153E+03	9.5268E+02	**9.1830E+02**
	Rank	5	7	8	9	4	6	3	2	1
f10	Mean	4.8711E+03	5.3920E+03	6.9135E+03	8.8007E+03	6.5639E+03	7.1770E+03	4.3226E+03	3.6332E+03	4.4369E+03
	Std	5.7277E+02	6.8706E+02	8.1736E+02	4.3924E+02	2.0397E+03	4.7158E+02	1.0754E+03	5.7271E+02	6.8129E+02
	Best	3.8732E+03	4.2986E+03	5.5617E+03	7.8319E+03	2.7143E+03	6.2789E+03	2.9656E+03	**2.6488E+03**	3.2401E+03
	Rank	4	5	7	9	6	8	2	1	3
f11	Mean	1.2392E+03	1.2702E+03	2.3941E+03	1.0785E+04	1.2573E+03	3.3783E+03	1.5863E+03	1.4076E+03	1.1995E+03
	Std	5.6562E+01	5.2918E+01	8.9863E+02	1.4435E+03	5.9040E+01	6.5644E+02	4.9058E+02	1.3429E+02	3.4106E+01
	Best	1.1551E+03	1.1684E+03	1.4584E+03	8.1966E+03	1.1732E+03	2.0426E+03	1.2420E+03	1.2012E+03	**1.1528E+03**
	Rank	2	4	7	9	3	8	6	5	1
f12	Mean	2.3433E+05	1.1667E+07	3.8388E+08	1.4825E+10	1.3517E+06	1.3999E+09	5.6640E+07	5.8403E+07	1.6108E+05
	Std	2.6572E+05	8.3106E+06	3.6405E+08	2.3682E+09	1.4041E+06	5.1686E+08	6.7149E+07	1.0416E+08	2.9288E+05
	Best	3.4385E+04	2.0280E+06	3.4145E+07	1.1417E+10	1.5848E+05	6.1601E+08	3.7987E+06	3.7625E+06	**2.8358E+04**
	Rank	2	4	7	9	3	8	5	6	1
f13	Mean	2.0662E+04	4.7777E+05	3.7203E+07	5.0804E+09	6.6182E+03	4.6458E+08	2.4632E+07	7.9089E+06	2.0117E+04
	Std	2.1695E+04	2.9795E+05	4.8527E+07	9.5713E+08	6.0251E+03	1.4872E+08	5.0306E+07	2.7113E+07	1.4533E+04
	Best	1.8751E+03	1.9629E+05	7.5463E+04	2.6581E+09	**1.5377E+03**	1.8784E+08	1.9216E+04	2.6757E+04	3.9617E+03
	Rank	3	4	7	9	1	8	6	5	2
f14	Mean	3.6647E+03	1.0763E+05	6.8575E+05	2.1069E+07	1.5526E+03	4.8273E+05	2.4228E+05	7.9912E+04	4.3893E+04
	Std	2.8920E+03	1.0459E+05	6.3218E+05	1.0273E+07	6.1466E+01	2.7558E+05	2.8217E+05	8.8190E+04	1.1263E+05
	Best	**1.4664E+03**	2.8876E+03	2.1624E+04	2.6448E+06	1.4700E+03	1.6879E+05	2.7754E+03	3.3136E+03	1.8880E+03
	Rank	2	5	8	9	1	7	6	4	3
f15	Mean	6.7073E+03	6.9762E+04	8.2994E+05	8.3858E+08	1.7403E+03	4.8475E+06	3.7855E+05	1.3441E+05	5.1093E+03
	Std	6.9125E+03	4.5876E+04	1.3879E+06	3.9009E+08	1.1469E+02	2.4754E+06	5.8747E+05	2.9576E+05	3.5602E+03
	Best	1.7750E+03	2.9031E+04	2.5741E+04	6.5137E+08	**1.5944E+03**	9.6590E+05	1.6966E+04	1.2405E+04	1.6357E+03
	Rank	3	4	7	9	1	8	6	5	2

CONTINUED TABLE V

$f(x)$	Index	AEO	HHO	CSA	OSA	WSO	HGSO	GWO	CGWO	HGWO
f16	Mean	2.7660E+03	3.2724E+03	3.5304E+03	5.8263E+03	2.1299E+03	3.8906E+03	2.4428E+03	2.3287E+03	2.2238E+03
	Std	2.5663E+02	3.7149E+02	5.2740E+02	6.9341E+02	1.9172E+02	2.5699E+02	2.7434E+02	2.2007E+02	3.0141E+02
	Best	2.1467E+03	2.2423E+03	2.7758E+03	4.7395E+03	**1.7415E+03**	3.2972E+03	1.9609E+03	1.7921E+03	1.7471E+03
	Rank	5	6	7	9	1	8	4	3	2
f17	Mean	2.2533E+03	2.4639E+03	2.5200E+03	8.1238E+03	1.8511E+03	2.4990E+03	1.9293E+03	1.9647E+03	1.8644E+03
	Std	2.0232E+02	2.8171E+02	3.5620E+02	3.8819E+03	7.6027E+01	1.3273E+02	1.3440E+02	1.2713E+02	9.4049E+01
	Best	1.8796E+03	2.0193E+03	2.1354E+03	4.1312E+03	1.7624E+03	2.2348E+03	1.7562E+03	1.7950E+03	**1.7451E+03**
	Rank	5	6	8	9	1	7	3	4	2
f18	Mean	6.7837E+04	1.2471E+06	4.3502E+06	1.1524E+08	1.3879E+04	3.0696E+06	1.0460E+06	1.0485E+06	1.2872E+05
	Std	3.9406E+04	1.2956E+06	5.3569E+06	1.0461E+08	1.3873E+04	1.3044E+06	1.7714E+06	1.6099E+06	7.9159E+04
	Best	1.0535E+04	7.0951E+04	1.9761E+05	1.4209E+07	**2.8644E+03**	9.2803E+05	5.5624E+04	6.3525E+04	3.2713E+04
	Rank	2	6	8	9	1	7	4	5	3
f19	Mean	1.1790E+04	2.9284E+05	1.6199E+07	4.8315E+08	1.9973E+03	1.5844E+07	1.6772E+06	3.7492E+05	6.0228E+03
	Std	1.0910E+04	2.2607E+05	1.4342E+07	3.0271E+08	5.1228E+01	4.7989E+06	6.2680E+06	6.2118E+05	3.9342E+03
	Best	1.9865E+03	2.5660E+04	6.6106E+05	1.6979E+08	**1.9205E+03**	4.1994E+06	7.5447E+03	3.7141E+03	2.0827E+03
	Rank	3	4	8	9	1	7	6	5	2
f20	Mean	2.5344E+03	2.8173E+03	2.6605E+03	3.0554E+03	2.2068E+03	2.6616E+03	2.3522E+03	2.3002E+03	2.2510E+03
	Std	2.0494E+02	1.8943E+02	2.0145E+02	1.7484E+02	1.0914E+02	8.6787E+01	1.0304E+02	8.5224E+01	9.0715E+01
	Best	2.1433E+03	2.4380E+03	2.3636E+03	2.7728E+03	**2.0850E+03**	2.4696E+03	2.2368E+03	2.1986E+03	2.1018E+03
	Rank	5	8	6	9	1	7	4	3	2
f21	Mean	2.4627E+03	2.5588E+03	2.5404E+03	2.7469E+03	2.3804E+03	2.5601E+03	2.3896E+03	2.3643E+03	2.3423E+03
	Std	3.6607E+01	5.1790E+01	6.2437E+01	4.4414E+01	2.7101E+01	3.3917E+01	3.2271E+01	1.5380E+01	1.2941E+01
	Best	2.3861E+03	2.4504E+03	2.4273E+03	2.6649E+03	2.3313E+03	2.4610E+03	2.3521E+03	2.3372E+03	**2.3236E+03**
	Rank	5	7	6	9	3	8	4	2	1
f22	Mean	5.0476E+03	6.2636E+03	4.8823E+03	9.8025E+03	2.3969E+03	3.8525E+03	4.5620E+03	3.8874E+03	2.6400E+03
	Std	2.1708E+03	2.2778E+03	2.3922E+03	4.6595E+02	1.5912E+02	3.8699E+02	1.9018E+03	1.5143E+03	1.0504E+03
	Best	**2.3000E+03**	2.3189E+03	2.6032E+03	9.0571E+03	2.3005E+03	2.8782E+03	2.4414E+03	2.3869E+03	**2.3000E+03**
	Rank	7	8	6	9	1	3	5	4	2
f23	Mean	2.8032E+03	3.1618E+03	2.9433E+03	3.6669E+03	2.7875E+03	3.1316E+03	2.7687E+03	2.7207E+03	2.7052E+03
	Std	4.0895E+01	9.6533E+01	8.4685E+01	1.6941E+02	3.8590E+01	7.2480E+01	4.5267E+01	2.1164E+01	1.7134E+01
	Best	2.7367E+03	2.9393E+03	2.7650E+03	3.3924E+03	2.7197E+03	2.9998E+03	2.7157E+03	2.6802E+03	**2.6753E+03**
	Rank	5	8	6	9	4	7	3	2	1
f24	Mean	2.9661E+03	3.4256E+03	3.0961E+03	3.8627E+03	2.9509E+03	3.3344E+03	2.9487E+03	2.8833E+03	2.8802E+03
	Std	4.1861E+01	1.5469E+02	6.0222E+01	1.7566E+02	4.4092E+01	6.8621E+01	7.3503E+01	4.3062E+01	2.6477E+01
	Best	2.8948E+03	3.1704E+03	2.9898E+03	3.4800E+03	2.8954E+03	3.2023E+03	2.8573E+03	2.8508E+03	**2.8404E+03**
	Rank	5	8	6	9	4	7	3	2	1
f25	Mean	2.9057E+03	2.9281E+03	3.2689E+03	4.4897E+03	2.9359E+03	3.3396E+03	2.9584E+03	2.9609E+03	2.9047E+03
	Std	2.1530E+01	2.0911E+01	1.7531E+02	2.3101E+02	3.0935E+01	6.8858E+01	2.6262E+01	2.6885E+01	1.6564E+01
	Best	**2.8841E+03**	2.8865E+03	2.9722E+03	4.0657E+03	2.8845E+03	3.1983E+03	2.9198E+03	2.9221E+03	2.8843E+03
	Rank	2	3	7	9	4	8	5	6	1
f26	Mean	5.3578E+03	7.4643E+03	7.1302E+03	1.1779E+04	4.9648E+03	6.7166E+03	4.7620E+03	4.4510E+03	4.1831E+03
	Std	1.0888E+03	7.5250E+02	1.4147E+03	7.5083E+02	1.4848E+03	4.7886E+02	4.7125E+02	3.1703E+02	2.1130E+02
	Best	**2.8000E+03**	5.3585E+03	4.2372E+03	1.0183E+04	2.8773E+03	5.8002E+03	4.1041E+03	4.0539E+03	3.8019E+03
	Rank	5	8	7	9	4	6	3	2	1
f27	Mean	3.2476E+03	3.3678E+03	3.4155E+03	4.5994E+03	3.2513E+03	3.7332E+03	3.2414E+03	3.2398E+03	3.2311E+03
	Std	2.9833E+01	6.4871E+01	1.0749E+02	4.6324E+02	2.7761E+01	1.0017E+02	2.0682E+01	1.5760E+01	1.2245E+01
	Best	**3.2112E+03**	3.2634E+03	3.3175E+03	3.7751E+03	3.2108E+03	3.4996E+03	3.2148E+03	3.2137E+03	3.2124E+03
	Rank	4	6	7	9	5	8	3	2	1
f28	Mean	3.2156E+03	3.2660E+03	3.6599E+03	6.4064E+03	3.2777E+03	4.0956E+03	3.3958E+03	3.3802E+03	3.2398E+03
	Std	2.0182E+01	1.9129E+01	2.2115E+02	5.0254E+02	3.5243E+01	2.1780E+02	4.8497E+01	9.8177E+01	8.6326E+01
	Best	3.1944E+03	3.1954E+03	3.4237E+03	5.5748E+03	3.2058E+03	3.8234E+03	3.3065E+03	3.2641E+03	**3.1000E+03**
	Rank	1	3	7	9	4	8	6	5	2
f29	Mean	4.0224E+03	4.4028E+03	4.8956E+03	7.7021E+03	3.6171E+03	4.9492E+03	3.7702E+03	3.6334E+03	3.7366E+03
	Std	2.5746E+02	3.0194E+02	4.4350E+02	9.6870E+02	1.3167E+02	2.2126E+02	1.8799E+02	1.3036E+02	1.6271E+02
	Best	3.6811E+03	3.8662E+03	4.1244E+03	6.1791E+03	**3.3815E+03**	4.5163E+03	3.5196E+03	3.4760E+03	3.4954E+03
	Rank	5	6	7	9	1	8	4	2	3
f30	Mean	1.6638E+04	2.0945E+06	3.4743E+07	3.2960E+09	1.0092E+04	7.6621E+07	5.2916E+06	5.8430E+06	7.0697E+05
	Std	1.1198E+04	1.1183E+06	2.3463E+07	1.0260E+09	3.3315E+03	2.8056E+07	4.4655E+06	3.5284E+06	1.1149E+06
	Best	6.4690E+03	3.1117E+05	5.3611E+06	1.8029E+09	**5.5765E+03**	4.4071E+07	1.0002E+06	6.4501E+05	1.4413E+04
	Rank	2	4	7	9	1	8	5	6	3
Total Rank		107	160	206	261	81	211	126	103	50
Final Rank		4	6	7	9	2	8	5	3	1

TABLE VI
COMPARATIVE RESULTS OF THE COMPARISON ALGORITHMS ON THE CEC2017 BENCHMARK FUNCTIONS(D=50)

f(*x*)	Index	AE	HHO	CSA	OSA	WSO	HGSO	GWO	CGWO	HGWO
f1	Mean	2.7891E+04	6.3532E+07	2.5700E+10	1.0326E+11	6.7298E+09	4.4256E+10	5.7136E+09	3.3726E+09	2.7181E+03
	Std	7.1701E+04	9.3246E+06	1.1954E+10	8.2874E+09	5.3114E+09	7.1915E+09	2.1622E+09	1.8125E+09	3.8817E+03
	Best	2.6546E+03	4.9706E+07	6.7706E+09	8.8785E+10	4.3141E+07	2.8153E+10	2.0907E+09	5.4799E+08	**1.3283E+02**
	Rank	2	3	7	9	6	8	5	4	1
f3	Mean	2.0238E+04	5.0965E+04	1.9425E+05	2.4178E+05	8.0244E+04	1.4695E+05	8.4775E+04	6.9751E+04	1.5097E+04
	Std	3.7997E+03	1.1677E+04	5.2628E+04	2.8113E+04	1.6440E+04	1.4848E+04	1.4368E+04	1.2691E+04	4.9192E+03
	Best	1.2712E+04	2.7523E+04	1.2686E+05	1.9088E+05	6.2128E+04	1.1197E+05	5.6854E+04	4.5068E+04	**3.6119E+03**
	Rank	2	3	8	9	5	7	6	4	1
f4	Mean	5.6032E+02	6.7778E+02	4.0518E+03	3.7582E+04	8.1206E+02	8.1793E+03	1.2258E+03	7.9171E+02	5.8025E+02
	Std	5.5015E+01	5.8714E+01	2.3831E+03	4.5838E+03	1.9408E+02	2.1826E+03	4.6639E+02	1.1770E+02	6.0594E+01
	Best	**4.4567E+02**	5.8497E+02	1.3279E+03	2.5494E+04	6.8630E+02	4.1932E+03	5.9823E+02	6.6958E+02	4.7198E+02
	Rank	1	3	7	9	5	8	6	4	2
f5	Mean	8.3590E+02	8.9259E+02	1.0350E+03	1.2032E+03	7.4715E+02	1.0832E+03	7.0465E+02	6.5881E+02	6.0285E+02
	Std	5.4601E+01	3.0920E+01	9.2051E+01	3.4998E+01	3.8318E+01	2.4349E+01	5.2839E+01	2.8303E+01	2.1580E+01
	Best	7.5474E+02	8.4737E+02	8.7287E+02	1.1327E+03	6.6420E+02	1.0298E+03	6.3747E+02	6.0206E+02	**5.5572E+02**
	Rank	5	6	7	9	4	8	3	2	1
f6	Mean	6.5410E+02	6.7180E+02	6.8046E+02	7.0404E+02	6.3324E+02	6.8340E+02	6.1608E+02	6.0861E+02	6.1565E+02
	Std	7.0885E+00	4.6346E+00	9.4881E+00	3.4835E+00	8.5621E+00	3.8961E+00	3.4564E+00	3.4566E+00	2.7792E+00
	Best	6.3386E+02	6.5990E+02	6.6723E+02	6.9675E+02	6.1741E+02	6.7497E+02	6.1053E+02	**6.0343E+02**	6.1053E+02
	Rank	5	6	7	9	4	8	3	1	2
f7	Mean	1.5355E+03	1.8092E+03	1.7950E+03	2.0467E+03	1.2939E+03	1.6136E+03	1.0841E+03	9.6932E+02	9.2055E+02
	Std	1.4849E+02	1.0038E+02	1.4403E+02	5.5125E+01	9.0895E+01	7.7258E+01	8.0123E+01	4.4107E+01	4.2790E+01
	Best	1.2762E+03	1.5482E+03	1.4771E+03	1.8734E+03	1.1848E+03	1.5116E+03	9.3001E+02	8.8993E+02	**8.4987E+02**
	Rank	5	8	7	9	4	6	3	2	1
f8	Mean	1.1451E+03	1.1767E+03	1.2906E+03	1.4852E+03	1.0283E+03	1.4035E+03	1.0110E+03	9.6514E+02	8.8706E+02
	Std	4.8761E+01	3.1418E+01	9.8849E+01	4.2626E+01	4.3499E+01	3.0910E+01	2.6290E+01	3.5118E+01	1.7314E+01
	Best	1.0408E+03	1.1211E+03	1.1330E+03	1.3833E+03	9.6420E+02	1.3013E+03	9.4949E+02	9.0032E+02	**8.5870E+02**
	Rank	5	6	7	9	4	8	3	2	1
f9	Mean	1.1306E+04	2.3246E+04	2.3712E+04	3.9716E+04	1.7478E+04	2.9237E+04	7.7552E+03	4.3125E+03	2.4157E+03
	Std	1.7165E+03	2.8300E+03	6.6165E+03	3.9842E+03	3.1493E+03	2.5306E+03	4.3286E+03	2.4285E+03	7.3691E+02
	Best	8.1580E+03	1.7892E+04	1.1484E+04	3.3869E+04	1.2343E+04	2.3167E+04	1.9704E+03	1.7061E+03	**1.5821E+03**
	Rank	4	6	7	9	5	8	3	2	1
f10	Mean	8.1767E+03	9.1507E+03	1.2837E+04	1.5091E+04	1.0591E+04	1.3421E+04	9.5248E+03	6.9317E+03	6.8760E+03
	Std	9.0689E+02	9.4175E+02	1.3193E+03	5.0930E+02	3.8593E+03	6.8531E+02	3.7840E+03	2.5876E+03	8.3463E+02
	Best	6.4899E+03	7.2952E+03	1.0378E+04	1.4291E+04	4.7116E+03	1.0648E+04	**4.1970E+03**	5.2320E+03	5.6257E+03
	Rank	3	4	7	9	6	8	5	2	1
f11	Mean	1.3216E+03	1.5269E+03	7.9011E+03	2.4298E+04	1.7471E+03	6.8181E+03	4.5130E+03	2.8576E+03	1.2641E+03
	Std	4.3281E+01	8.0018E+01	2.6983E+03	2.6323E+03	5.4235E+02	1.4410E+03	2.0438E+03	1.2228E+03	2.8284E+01
	Best	1.2454E+03	1.3877E+03	5.2191E+03	1.7693E+04	1.3999E+03	4.6238E+03	1.6028E+03	1.4170E+03	**1.1985E+03**
	Rank	2	3	8	9	4	7	6	5	1
f12	Mean	3.2828E+06	1.0492E+08	3.3096E+09	8.2709E+10	3.3327E+07	1.2851E+10	1.2261E+09	4.7534E+08	9.3242E+05
	Std	2.2227E+06	5.5841E+07	3.2621E+09	1.3284E+10	3.9013E+07	3.3164E+09	1.2997E+09	6.9277E+08	6.4909E+05
	Best	7.8397E+05	2.7846E+07	5.3668E+08	6.1964E+10	5.4284E+06	8.4140E+09	3.7023E+07	3.0880E+07	**1.4432E+05**
	Rank	2	4	7	9	3	8	6	5	1
f13	Mean	1.6717E+04	3.1225E+06	3.3437E+08	5.6042E+10	1.4365E+04	2.9916E+09	1.5802E+08	7.2748E+07	9.6852E+03
	Std	1.4338E+04	2.4163E+06	4.1811E+08	9.4491E+09	1.3493E+04	1.0110E+09	1.2160E+08	1.0018E+08	3.9442E+03
	Best	**3.3980E+03**	7.2710E+05	1.4966E+07	3.6533E+10	4.0367E+03	1.4075E+09	6.6629E+04	2.3393E+05	3.9108E+03
	Rank	3	4	7	9	2	8	6	5	1
f14	Mean	4.4451E+04	7.3591E+05	2.6244E+06	1.4529E+08	6.6603E+03	5.2090E+06	9.2232E+05	7.0717E+05	5.0389E+04
	Std	4.3563E+04	3.9273E+05	2.1598E+06	1.0197E+08	1.9560E+04	2.2402E+06	9.5813E+05	5.6415E+05	4.9055E+04
	Best	2.7026E+03	1.3930E+05	4.6495E+05	2.0096E+07	**1.6022E+03**	6.9755E+05	8.3565E+04	4.5689E+04	9.6965E+03
	Rank	2	5	7	9	1	8	6	4	3
f15	Mean	1.4073E+04	4.2573E+05	6.1255E+07	8.4694E+09	5.3348E+03	4.1992E+08	1.7264E+07	4.2228E+06	7.5228E+03
	Std	7.4798E+03	1.3832E+05	1.2879E+08	2.2766E+09	4.1421E+03	2.0075E+08	2.2194E+07	1.0944E+07	5.7128E+03
	Best	2.3253E+03	2.0585E+05	4.6262E+04	6.6392E+09	**2.0097E+03**	1.8151E+08	2.1132E+04	1.7023E+04	2.1503E+03
	Rank	3	4	7	9	1	8	6	5	2

CONTINUED TABLE VI

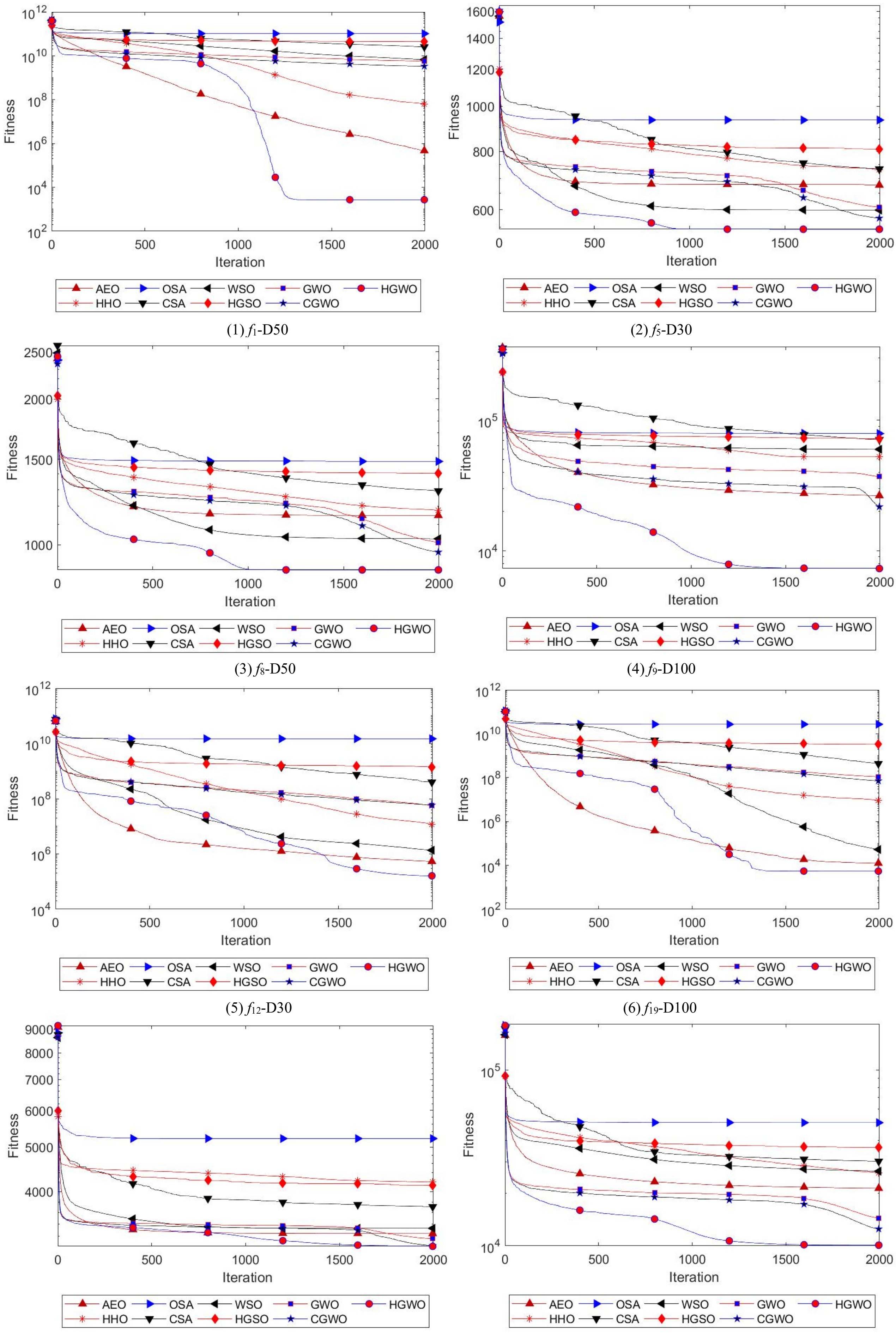
f(*x*)	Index	AEO	HHO	CSA	OSA	WSO	HGSO	GWO	CGWO	HGWO
f16	Mean	3.5614E+03	4.0956E+03	4.7006E+03	9.7146E+03	2.9252E+03	5.6131E+03	2.8284E+03	2.7330E+03	2.8676E+03
	Std	4.5212E+02	5.1016E+02	6.8813E+02	1.6390E+03	2.7054E+02	4.1207E+02	2.7327E+02	3.1991E+02	3.7510E+02
	Best	2.6018E+03	3.4051E+03	3.4013E+03	6.3640E+03	2.3181E+03	4.2137E+03	2.3232E+03	2.1806E+03	**1.9643E+03**
	Rank	5	6	7	9	4	8	2	1	3
f17	Mean	3.3132E+03	3.5774E+03	3.9964E+03	1.3072E+04	2.5898E+03	4.0912E+03	2.8750E+03	2.7035E+03	2.7407E+03
	Std	4.2647E+02	2.8451E+02	4.1133E+02	5.5709E+03	2.0764E+02	3.6094E+02	4.3348E+02	3.5084E+02	2.6183E+02
	Best	2.6996E+03	2.9872E+03	3.1507E+03	6.5553E+03	2.2306E+03	3.3314E+03	2.1826E+03	**2.1466E+03**	2.2609E+03
	Rank	5	6	7	9	1	8	4	2	3
f18	Mean	2.3764E+05	3.6516E+06	1.5439E+07	2.8115E+08	8.5150E+04	1.8550E+07	9.8543E+06	3.2294E+06	2.8293E+05
	Std	1.9419E+05	3.0306E+06	1.3286E+07	9.8216E+07	5.2190E+04	9.1339E+06	1.7512E+07	2.0686E+06	1.8205E+05
	Best	4.5791E+04	1.6809E+05	1.5222E+06	1.6313E+08	**3.7682E+04**	4.9923E+06	4.2752E+05	2.5131E+05	7.3470E+04
	Rank	2	5	7	9	1	8	6	4	3
f19	Mean	2.2187E+04	7.6100E+05	7.8606E+08	3.2608E+09	1.6542E+04	2.3606E+08	1.9706E+06	1.2340E+06	1.5248E+04
	Std	1.3154E+04	4.1153E+05	2.8467E+09	8.5577E+08	1.0332E+04	7.9111E+07	1.4323E+06	1.3841E+06	9.9650E+03
	Best	2.3592E+03	8.2603E+04	1.8216E+06	2.2143E+09	2.1120E+03	1.0416E+08	2.2534E+05	1.2168E+05	**2.0840E+03**
	Rank	3	4	8	9	2	7	6	5	1
f20	Mean	3.4776E+03	3.4599E+03	3.6373E+03	4.2350E+03	2.4708E+03	3.6529E+03	2.8524E+03	2.9119E+03	2.8020E+03
	Std	3.5336E+02	2.6286E+02	3.6959E+02	2.5731E+02	2.9717E+02	1.9574E+02	3.4821E+02	2.5155E+02	3.0933E+02
	Best	2.7857E+03	3.0151E+03	3.0776E+03	3.5583E+03	**2.1884E+03**	3.2478E+03	2.4827E+03	2.4663E+03	2.3147E+03
	Rank	6	5	7	9	1	8	3	4	2
f21	Mean	2.6526E+03	2.8558E+03	2.8446E+03	3.2694E+03	2.5355E+03	2.9030E+03	2.5267E+03	2.4527E+03	2.3870E+03
	Std	8.6063E+01	1.0005E+02	1.0177E+02	1.0384E+02	4.6315E+01	3.6606E+01	8.7360E+01	3.8654E+01	1.9602E+01
	Best	2.5084E+03	2.7010E+03	2.6925E+03	3.1173E+03	2.4469E+03	2.8139E+03	2.4162E+03	2.3854E+03	**2.3460E+03**
	Rank	5	7	6	9	4	8	3	2	1
f22	Mean	1.0190E+04	1.1146E+04	1.4141E+04	1.6630E+04	9.8605E+03	1.2005E+04	9.9215E+03	8.1369E+03	8.7717E+03
	Std	9.6435E+02	9.3400E+02	1.6313E+03	6.1772E+02	4.4810E+03	3.1867E+03	2.9733E+03	1.0031E+03	1.0092E+03
	Best	8.2555E+03	9.2295E+03	1.0976E+04	1.5374E+04	**2.8839E+03**	7.0447E+03	6.6024E+03	6.3085E+03	5.8062E+03
	Rank	5	6	8	9	3	7	4	1	2
f23	Mean	3.1663E+03	3.7541E+03	3.4591E+03	4.4880E+03	3.1217E+03	3.8076E+03	3.0253E+03	2.8930E+03	2.8541E+03
	Std	8.9211E+01	1.7580E+02	1.1870E+02	1.7521E+02	7.2808E+01	1.6123E+02	1.1322E+02	3.6965E+01	3.8448E+01
	Best	2.9954E+03	3.3961E+03	3.2804E+03	4.1602E+03	2.9626E+03	3.5218E+03	2.8849E+03	2.8371E+03	**2.8009E+03**
	Rank	5	7	6	9	4	8	3	2	1
f24	Mean	3.2448E+03	4.1940E+03	3.7105E+03	5.2143E+03	3.3321E+03	4.1243E+03	3.1644E+03	3.0559E+03	3.0454E+03
	Std	8.7385E+01	2.7224E+02	4.7397E+02	2.8762E+02	1.1028E+02	1.1279E+02	1.2349E+02	3.8717E+01	6.2945E+01
	Best	3.0916E+03	3.7295E+03	3.1911E+03	4.5370E+03	3.1627E+03	3.9023E+03	3.0227E+03	2.9853E+03	**2.9585E+03**
	Rank	4	8	6	9	5	7	3	2	1
f25	Mean	3.0904E+03	3.1781E+03	7.5189E+03	1.3735E+04	3.4658E+03	6.8885E+03	3.4550E+03	3.3730E+03	3.1852E+03
	Std	2.5736E+01	4.2386E+01	8.6211E+03	1.9312E+03	2.1129E+02	1.0120E+03	1.2943E+02	2.1082E+02	6.6178E+01
	Best	**3.0291E+03**	3.1087E+03	3.9692E+03	1.1092E+04	3.1680E+03	5.1347E+03	3.2821E+03	3.1197E+03	3.0983E+03
	Rank	1	2	8	9	6	7	5	4	3
f26	Mean	7.6860E+03	1.0022E+04	1.0698E+04	1.6767E+04	9.0068E+03	1.1703E+04	6.5398E+03	5.6186E+03	5.1852E+03
	Std	3.1654E+03	1.8220E+03	1.7338E+03	7.9390E+02	2.0257E+03	9.3537E+02	1.0022E+03	4.6035E+02	3.2607E+02
	Best	**2.9072E+03**	4.8104E+03	7.7981E+03	1.5248E+04	4.7371E+03	1.0135E+04	5.6063E+03	4.9062E+03	4.7179E+03
	Rank	4	6	7	9	5	8	3	2	1
f27	Mean	3.5567E+03	4.2555E+03	4.0779E+03	7.7750E+03	3.6379E+03	5.3702E+03	3.5738E+03	3.4927E+03	3.4961E+03
	Std	1.1402E+02	3.4683E+02	3.1004E+02	9.0008E+02	1.4108E+02	2.4060E+02	8.1794E+01	7.4777E+01	7.1268E+01
	Best	**3.3760E+03**	3.5115E+03	3.5937E+03	5.8105E+03	3.3844E+03	4.8828E+03	3.4067E+03	3.3874E+03	3.4116E+03
	Rank	3	7	6	9	5	8	4	1	2
f28	Mean	3.3516E+03	3.4321E+03	5.9054E+03	1.3068E+04	3.9590E+03	7.0487E+03	4.2298E+03	3.8544E+03	3.4077E+03
	Std	3.1478E+01	5.0515E+01	1.0112E+03	1.6367E+03	2.4932E+02	5.3184E+02	4.1927E+02	2.4977E+02	5.9867E+01
	Best	**3.3003E+03**	3.3327E+03	4.3700E+03	1.0332E+04	3.4915E+03	6.1423E+03	3.6246E+03	3.4658E+03	3.3255E+03
	Rank	1	3	7	9	5	8	6	4	2
f29	Mean	4.8095E+03	5.6241E+03	2.0515E+04	1.9677E+05	4.3222E+03	7.8400E+03	4.5862E+03	4.2251E+03	4.4294E+03
	Std	3.4894E+02	4.9463E+02	6.5346E+04	3.4115E+05	3.1845E+02	6.3101E+02	3.0708E+02	2.7412E+02	2.8407E+02
	Best	4.3011E+03	4.5622E+03	6.4814E+03	3.5011E+04	3.8251E+03	6.9455E+03	4.0986E+03	**3.8182E+03**	3.9515E+03
	Rank	5	6	8	9	2	7	4	1	3
f30	Mean	2.6336E+06	2.7522E+07	4.5183E+08	8.2551E+09	1.4863E+06	7.1055E+08	1.0202E+08	7.1373E+07	5.8360E+07
	Std	1.8910E+06	5.9951E+06	2.0618E+08	2.4007E+09	5.8657E+05	1.5181E+08	3.4318E+07	1.5014E+07	2.1007E+07
	Best	1.0182E+06	1.6974E+07	8.3626E+07	5.7453E+09	**8.8617E+05**	3.1070E+08	4.9723E+07	4.2071E+07	1.9668E+07
	Rank	2	3	7	9	1	8	6	5	4
Total Rank		100	146	205	261	103	223	129	87	51
Final Rank		3	6	7	9	4	8	5	2	1

TABLE VII
COMPARATIVE RESULTS OF THE COMPARISON ALGORITHMS ON THE CEC2017 BENCHMARK FUNCTIONS(D=100)

f(*x*)	Index	AEO	HHO	CSA	OSA	WSO	HGSO	GWO	CGWO	HGWO
f1	Mean	2.1249E+07	6.2725E+08	1.4644E+11	2.5396E+11	5.6330E+10	1.5929E+11	3.5473E+10	2.3214E+10	4.0613E+03
	Std	9.5872E+06	9.6687E+07	2.5176E+10	1.2385E+10	9.8236E+09	1.4559E+10	9.9092E+09	8.9235E+09	4.3494E+03
	Best	1.0795E+07	4.1021E+08	1.1070E+11	2.2753E+11	3.9008E+10	1.2459E+11	1.8002E+10	9.5741E+09	**1.3475E+02**
	Rank	2	3	7	9	6	8	5	4	1
f3	Mean	1.9921E+05	2.2669E+05	4.7121E+05	3.6902E+05	2.3512E+05	3.2331E+05	2.4612E+05	2.4943E+05	2.1090E+05
	Std	3.3986E+04	1.7115E+04	1.5245E+05	7.3748E+03	2.6810E+04	1.0188E+04	2.2568E+04	2.6202E+04	3.9102E+04
	Best	1.5581E+05	1.9817E+05	3.2536E+05	3.5106E+05	1.7780E+05	3.0064E+05	2.1219E+05	1.9474E+05	**1.5067E+05**
	Rank	1	3	9	8	4	7	5	6	2
f4	Mean	9.3442E+02	1.1582E+03	3.0510E+04	1.0512E+05	4.4670E+03	3.3069E+04	3.5782E+03	2.3978E+03	8.9073E+02
	Std	9.2176E+01	9.6609E+01	9.0900E+03	1.3076E+04	1.1233E+03	6.1093E+03	8.0525E+02	4.1876E+02	1.1983E+02
	Best	7.9478E+02	9.6535E+02	1.1816E+04	8.8745E+04	2.8265E+03	2.2166E+04	2.3610E+03	1.5017E+03	**6.3075E+02**
	Rank	2	3	7	9	6	8	5	4	1
f5	Mean	1.3354E+03	1.5380E+03	1.9323E+03	2.1129E+03	1.2359E+03	1.8998E+03	1.0981E+03	1.0019E+03	7.9951E+02
	Std	7.0845E+01	5.6289E+01	1.2269E+02	4.4953E+01	6.4716E+01	5.1127E+01	4.2049E+01	5.0421E+01	5.7256E+01
	Best	1.2158E+03	1.4229E+03	1.6304E+03	2.0126E+03	1.1073E+03	1.7781E+03	1.0135E+03	9.0505E+02	**7.2088E+02**
	Rank	5	6	8	9	4	7	3	2	1
f6	Mean	6.6149E+02	6.8193E+02	6.9456E+02	7.1333E+02	6.5310E+02	6.9981E+02	6.3484E+02	6.2242E+02	6.2938E+02
	Std	3.8743E+00	4.2181E+00	8.3585E+00	4.3215E+00	3.5441E+00	3.3874E+00	5.4740E+00	3.8674E+00	4.8133E+00
	Best	6.5234E+02	6.6757E+02	6.7598E+02	7.0473E+02	6.4681E+02	6.9173E+02	6.2664E+02	**6.1607E+02**	6.2107E+02
	Rank	5	6	7	9	4	8	3	1	2
f7	Mean	3.1098E+03	3.7121E+03	3.7572E+03	4.0409E+03	2.7956E+03	3.5012E+03	1.8675E+03	1.6564E+03	1.6304E+03
	Std	1.8271E+02	7.7099E+01	2.6843E+02	4.9504E+01	2.0015E+02	1.2034E+02	1.2375E+02	1.0724E+02	1.5264E+02
	Best	2.7152E+03	3.5432E+03	3.3439E+03	3.9590E+03	2.5157E+03	3.1457E+03	1.6564E+03	1.4404E+03	**1.3723E+03**
	Rank	5	7	8	9	4	6	3	2	1
f8	Mean	1.7626E+03	1.9705E+03	2.2038E+03	2.6004E+03	1.5746E+03	2.3356E+03	1.4372E+03	1.2896E+03	1.1173E+03
	Std	9.7566E+01	5.9189E+01	1.3116E+02	5.6246E+01	5.8727E+01	5.6982E+01	1.6444E+02	3.9013E+01	7.1173E+01
	Best	1.5899E+03	1.8193E+03	1.8949E+03	2.5178E+03	1.4823E+03	2.1947E+03	1.2649E+03	1.2201E+03	**9.9899E+02**
	Rank	5	6	7	9	4	8	3	2	1
f9	Mean	2.4931E+04	5.2458E+04	7.1129E+04	7.9196E+04	5.9964E+04	7.2331E+04	3.6999E+04	2.1651E+04	7.3193E+03
	Std	1.0590E+03	5.3750E+03	1.3445E+04	5.2493E+03	5.1748E+03	3.6973E+03	9.1670E+03	8.5876E+03	1.6385E+03
	Best	2.2188E+04	4.3203E+04	5.1272E+04	6.8842E+04	5.1712E+04	6.2410E+04	1.4693E+04	9.4592E+03	**4.9456E+03**
	Rank	3	5	7	9	6	8	4	2	1
f10	Mean	1.7048E+04	2.0968E+04	2.7559E+04	3.2347E+04	2.1863E+04	2.9396E+04	1.7709E+04	1.4219E+04	1.4561E+04
	Std	2.1323E+03	1.7549E+03	1.7328E+03	7.6491E+02	7.1761E+03	1.2727E+03	5.3399E+03	1.6773E+03	1.6315E+03
	Best	1.2405E+04	1.8007E+04	2.5150E+04	3.0923E+04	1.4291E+04	2.6025E+04	1.3763E+04	1.1784E+04	**1.1669E+04**
	Rank	3	5	7	9	6	8	4	1	2
f11	Mean	3.9775E+03	1.0672E+04	1.3424E+05	4.1224E+05	2.9674E+04	1.4222E+05	4.4843E+04	3.5671E+04	2.7157E+03
	Std	7.1828E+02	2.9979E+03	3.3681E+04	1.1328E+05	8.1025E+03	1.5029E+04	1.3081E+04	1.0469E+04	1.1228E+03
	Best	2.7246E+03	6.4851E+03	7.2618E+04	2.3771E+05	1.7062E+04	1.1257E+05	1.3082E+04	1.6199E+04	**1.9211E+03**
	Rank	2	3	7	9	4	8	6	5	1
f12	Mean	5.5700E+07	5.9452E+08	3.9740E+10	1.9925E+11	4.1111E+09	6.4315E+10	7.4565E+09	4.1697E+09	2.7444E+07
	Std	3.8306E+07	1.6489E+08	5.3915E+10	1.5554E+10	2.5376E+09	1.2948E+10	4.4675E+09	3.2916E+09	3.6856E+07
	Best	1.8283E+07	2.8535E+08	9.7075E+09	1.7135E+11	8.3056E+08	4.1883E+10	1.7732E+09	1.0886E+09	**2.2711E+06**
	Rank	2	3	7	9	4	8	6	5	1
f13	Mean	1.6727E+04	9.3127E+06	2.8406E+09	4.6895E+10	6.7042E+07	1.0859E+10	9.6720E+08	3.6407E+08	1.4888E+04
	Std	8.1801E+03	3.3476E+06	1.6097E+09	5.3437E+09	1.5792E+08	2.7748E+09	1.4264E+09	4.4270E+08	5.7071E+03
	Best	**7.4162E+03**	5.4239E+06	3.5832E+08	3.5496E+10	6.3052E+04	6.1547E+09	1.6123E+06	4.0567E+05	7.6156E+03
	Rank	2	3	7	9	4	8	6	5	1
f14	Mean	4.6709E+05	2.8755E+06	1.8883E+07	4.9142E+07	5.9233E+05	2.1846E+07	6.3909E+06	5.2444E+06	1.0955E+06
	Std	1.7005E+05	8.9152E+05	1.1467E+07	1.8828E+07	4.7898E+05	3.9683E+06	3.5346E+06	3.2345E+06	2.0259E+06
	Best	2.0146E+05	1.4435E+06	4.2658E+06	2.5999E+07	**1.3873E+05**	1.1941E+07	1.5493E+06	1.1284E+06	1.4298E+05
	Rank	1	4	7	9	2	8	6	5	3
f15	Mean	1.0866E+04	2.3316E+06	6.4782E+08	2.4594E+10	1.4230E+04	2.8457E+09	6.8641E+07	9.0781E+07	6.5618E+03
	Std	1.1159E+04	7.0567E+05	8.1433E+08	3.7855E+09	7.8009E+03	6.5335E+08	1.1314E+08	1.4078E+08	3.5414E+03
	Best	3.2711E+03	8.8842E+05	4.5747E+07	1.7426E+10	3.2212E+03	1.4299E+09	6.8890E+04	6.5834E+04	**1.9316E+03**
	Rank	2	4	7	9	3	8	5	6	1

CONTINUED TABLE VII

$f(x)$	Index	AEO	HHO	CSA	OSA	WSO	HGSO	GWO	CGWO	HGWO
f16	Mean	6.3189E+03	7.4704E+03	1.2515E+04	2.6265E+04	6.1750E+03	1.3344E+04	6.1046E+03	5.2949E+03	5.3495E+03
	Std	8.6254E+02	6.6353E+02	3.7117E+03	2.6924E+03	6.6611E+02	1.0245E+03	1.1644E+03	5.2379E+02	6.8678E+02
	Best	4.7826E+03	6.1310E+03	8.8378E+03	2.1516E+04	5.2911E+03	1.0989E+04	4.7045E+03	**4.2256E+03**	4.4042E+03
	Rank	5	6	7	9	4	8	3	1	2
f17	Mean	5.8809E+03	6.4971E+03	1.1679E+04	2.1342E+07	5.6723E+03	2.2800E+04	4.9452E+03	4.4058E+03	4.7941E+03
	Std	5.6000E+02	6.7756E+02	1.0418E+04	2.0105E+07	7.1236E+02	1.3210E+04	5.8053E+02	5.9904E+02	4.4165E+02
	Best	4.8933E+03	5.4969E+03	6.1259E+03	9.2673E+05	4.7581E+03	9.9938E+03	4.0545E+03	**3.4757E+03**	3.7864E+03
	Rank	5	6	7	9	4	8	3	1	2
f18	Mean	8.6782E+05	4.1221E+06	1.7673E+07	1.2381E+08	6.0110E+05	3.0133E+07	6.5930E+06	4.8624E+06	6.3659E+05
	Std	3.7076E+05	1.4944E+06	1.1752E+07	8.6988E+07	3.4837E+05	7.8232E+06	4.8066E+06	3.8154E+06	2.8541E+05
	Best	3.7322E+05	1.6103E+06	5.2555E+06	3.1472E+07	**1.9571E+05**	1.6766E+07	1.5886E+06	1.4171E+06	2.1215E+05
	Rank	3	4	7	9	1	8	6	5	2
f19	Mean	8.3892E+03	9.1203E+06	4.2591E+08	2.7108E+10	5.2184E+04	3.2777E+09	1.0994E+08	7.0792E+07	5.4412E+03
	Std	5.7689E+03	4.8543E+06	3.0254E+08	3.0666E+09	7.8620E+04	9.9534E+08	1.4873E+08	7.4117E+07	3.3483E+03
	Best	2.7482E+03	3.0117E+06	1.0508E+08	2.2624E+10	5.6225E+03	1.2479E+09	5.2538E+06	7.5415E+06	**2.5540E+03**
	Rank	2	4	7	9	3	8	6	5	1
f20	Mean	5.5625E+03	6.0868E+03	6.7821E+03	7.7640E+03	5.0666E+03	7.1657E+03	5.1157E+03	5.0339E+03	4.8657E+03
	Std	6.3494E+02	4.4464E+02	7.2948E+02	3.3631E+02	1.3460E+03	2.4925E+02	1.0376E+03	1.2028E+03	5.5177E+02
	Best	4.4333E+03	5.2179E+03	5.5869E+03	6.9943E+03	3.4544E+03	6.6605E+03	3.5807E+03	**3.5928E+03**	3.7329E+03
	Rank	5	6	7	9	3	8	4	2	1
f21	Mean	3.2750E+03	4.0336E+03	4.0659E+03	4.7954E+03	3.2441E+03	4.1618E+03	2.9206E+03	2.8528E+03	2.6540E+03
	Std	1.4221E+02	1.9154E+02	2.6871E+02	1.9937E+02	9.1794E+01	1.5322E+02	5.7023E+01	1.1462E+02	5.7382E+01
	Best	3.0354E+03	3.6219E+03	3.6000E+03	4.4014E+03	3.0807E+03	3.7908E+03	2.8013E+03	2.6651E+03	**2.5768E+03**
	Rank	5	6	7	9	4	8	3	2	1
f22	Mean	2.0693E+04	2.4334E+04	3.1699E+04	3.4828E+04	2.7634E+04	3.2330E+04	2.1524E+04	1.7795E+04	1.7631E+04
	Std	1.9022E+03	1.2080E+03	2.4788E+03	8.9622E+02	7.2214E+03	8.5674E+02	6.6434E+03	3.5211E+03	1.2389E+03
	Best	1.7687E+04	2.1239E+04	2.7986E+04	3.3056E+04	**1.1547E+04**	3.0150E+04	1.5733E+04	1.2997E+04	1.4740E+04
	Rank	3	5	7	9	6	8	4	2	1
f23	Mean	3.6816E+03	5.1856E+03	4.7200E+03	7.3982E+03	3.7789E+03	5.9325E+03	3.5072E+03	3.3507E+03	3.1456E+03
	Std	1.1503E+02	2.8623E+02	3.3569E+02	5.1471E+02	1.6618E+02	2.3741E+02	6.7195E+01	7.9868E+01	5.7335E+01
	Best	3.4594E+03	4.7245E+03	4.1323E+03	6.4554E+03	3.6096E+03	5.3853E+03	3.3958E+03	3.2402E+03	**3.0518E+03**
	Rank	4	7	6	9	5	8	3	2	1
f24	Mean	4.4193E+03	6.6290E+03	6.0819E+03	1.1803E+04	4.7906E+03	8.6501E+03	4.1213E+03	3.9031E+03	3.7577E+03
	Std	1.9165E+02	4.3515E+02	4.1628E+02	8.3032E+02	1.8344E+02	7.4658E+02	2.0462E+02	1.2363E+02	1.8751E+02
	Best	4.0164E+03	5.6136E+03	5.1554E+03	9.8586E+03	4.5603E+03	6.8380E+03	3.9092E+03	3.6375E+03	**3.5012E+03**
	Rank	4	7	6	9	5	8	3	2	1
f25	Mean	3.5843E+03	3.8247E+03	1.3459E+04	2.6508E+04	6.2580E+03	1.3509E+04	5.7636E+03	5.1605E+03	3.8181E+03
	Std	6.6393E+01	1.1537E+02	2.6782E+03	1.9161E+03	7.2001E+02	1.5206E+03	6.7316E+02	4.3634E+02	1.6314E+02
	Best	**3.4302E+03**	3.6360E+03	1.0874E+04	2.2928E+04	5.2821E+03	1.0578E+04	5.0273E+03	4.5440E+03	3.5462E+03
	Rank	1	3	7	9	6	8	5	4	2
f26	Mean	2.0792E+04	2.5934E+04	3.0318E+04	5.0327E+04	2.6618E+04	3.6371E+04	1.4408E+04	1.2466E+04	1.0089E+04
	Std	4.4787E+03	3.5014E+03	4.3111E+03	2.4338E+03	2.5457E+03	2.6149E+03	1.6196E+03	1.0636E+03	9.1086E+02
	Best	**4.9014E+03**	1.0027E+04	2.1007E+04	4.5190E+04	2.2845E+04	3.2270E+04	1.1766E+04	1.0594E+04	8.6484E+03
	Rank	4	5	7	9	6	8	3	2	1
f27	Mean	3.8507E+03	4.6195E+03	5.4189E+03	1.5472E+04	4.0477E+03	8.9338E+03	4.0670E+03	3.8232E+03	4.1218E+03
	Std	1.9055E+02	5.2823E+02	6.6452E+02	1.9286E+03	2.0408E+02	8.7387E+02	1.4097E+02	9.5249E+01	2.4598E+02
	Best	**3.6010E+03**	3.9387E+03	4.4193E+03	1.0388E+04	3.7169E+03	7.5939E+03	3.7599E+03	3.6100E+03	3.7286E+03
	Rank	2	6	7	9	3	8	4	1	5
f28	Mean	3.7018E+03	3.8203E+03	1.7082E+04	2.6912E+04	8.9170E+03	1.9652E+04	8.1478E+03	6.7241E+03	4.1923E+03
	Std	5.9204E+01	8.2053E+01	1.2253E+04	1.9551E+03	1.5395E+03	1.7266E+03	1.1445E+03	1.2377E+03	2.9539E+02
	Best	**3.5754E+03**	3.6988E+03	1.0947E+04	2.2934E+04	6.5307E+03	1.6238E+04	5.6649E+03	4.7086E+03	3.6910E+03
	Rank	1	2	7	9	6	8	5	4	3
f29	Mean	7.8821E+03	9.9173E+03	5.4418E+05	5.8798E+05	7.9683E+03	2.0408E+04	8.2097E+03	7.2085E+03	8.1366E+03
	Std	6.8330E+02	8.7616E+02	2.8814E+06	4.5733E+05	5.2439E+02	3.3843E+03	8.9141E+02	5.4400E+02	5.8740E+02
	Best	6.3806E+03	8.3910E+03	1.3021E+04	2.1737E+05	7.2472E+03	1.4883E+04	7.2868E+03	**5.9872E+03**	7.2497E+03
	Rank	2	6	8	9	3	7	5	1	4
f30	Mean	1.4024E+06	5.6927E+07	2.0149E+09	4.3075E+10	1.3233E+07	9.5610E+09	9.4129E+08	2.8022E+08	1.5045E+07
	Std	1.4792E+06	1.9583E+07	1.1373E+09	3.6898E+09	1.9630E+07	2.5628E+09	1.1912E+09	1.6480E+08	4.4679E+07
	Best	1.6981E+05	2.7543E+07	6.9213E+08	3.7605E+10	1.5806E+06	4.5993E+09	1.3992E+08	4.6974E+07	**8.4977E+04**
	Rank	1	4	7	9	2	8	6	5	3
Total Rank		87	138	206	260	122	227	127	89	49
Final Rank		2	6	7	9	4	8	5	3	1



(1) f_1-D50 (2) f_5-D30 (3) f_8-D50 (4) f_9-D100 (5) f_{12}-D30 (6) f_{19}-D100 (7) f_{24}-D50 (8) f_{26}-D100

Fig.7. Convergence curve of comparison algorithms under different functions

TABLE IX
RESULTS OF WILCOXON SIGNED RANK TEST (MEAN)

Index	HGWO vs	Dimension 30				50				100			
		p-Value	*R*+	*R*-	Win	*p*-Value	*R*+	*R*-	Win	*p*-Value	*R*+	*R*-	Win
Mean	AEO	5.1070E-03	24	5	+	4.7751E-03	23	6	+	7.5746E-03	22	7	+
	HHO	2.5631E-06	29	0	+	4.1746E-05	27	2	+	3.9017E-06	28	1	+
	CSA	2.5631E-06	29	0	+	2.5631E-06	29	0	+	2.5631E-06	29	0	+
	OSA	2.5631E-06	29	0	+	2.5631E-06	29	0	+	2.5631E-06	29	0	+
	WSO	7.3750E-01	18	11	=	3.6921E-02	22	7	+	2.5576E-03	24	5	+
	HGSO	2.5631E-06	29	0	+	2.5631E-06	29	0	+	2.5631E-06	29	0	+
	GWO	1.1939E-05	27	2	+	3.1652E-06	28	1	+	3.1652E-06	28	1	+
	CGWO	7.9390E-05	26	4	+	3.1790E-04	23	6	+	4.0717E-04	23	6	+
	+/=/-	7/1/0				8/0/0				8/0/0			

TABLE X
RESULTS OF WILCOXON SIGNED RANK TEST (STD)

Index	HGWO vs	Dimension 30				50				100			
		p-Value	*R*+	*R*-	Win	*p*-Value	*R*+	*R*-	Win	*p*-Value	*R*+	*R*-	Win
Std	AEO	3.8117E-01	20	9	=	1.0397E-02	22	7	+	1.6311E-01	19	10	=
	HHO	1.2398E-04	26	3	+	6.6006E-04	23	6	+	5.2956E-02	16	13	=
	CSA	2.5631E-06	29	0	+	2.5631E-06	29	0	+	2.5631E-06	29	0	+
	OSA	3.1483E-05	27	2	+	2.8631E-05	26	3	+	3.1790E-04	21	8	+
	WSO	9.5689E-01	17	12	=	3.5008E-02	22	7	+	1.1765E-02	20	9	+
	HGSO	1.9148E-04	25	4	+	1.9496E-05	27	2	+	5.1944E-04	21	8	+
	GWO	2.1477E-05	27	2	+	8.8479E-06	28	1	+	1.2398E-04	24	5	+
	CGWO	4.4628E-03	22	7	+	1.9162E-03	22	7	+	1.5360E-03	20	9	+
	+/=/-	6/2/0				8/0/0				6/2/0			

TABLE XI
RESULTS OF WILCOXON SIGNED RANK TEST (BEST)

Index	HGWO vs	Dimension 30				50				100			
		p-Value	*R*+	*R*-	Win	*p*-Value	*R*+	*R*-	Win	*p*-Value	*R*+	*R*-	Win
Std	AEO	1.4441E-01	21	8	=	7.4438E-02	20	9	=	4.4179E-04	23	6	+
	HHO	2.5631E-06	29	0	+	3.8016E-05	28	1	+	2.5631E-06	29	0	+
	CSA	2.5631E-06	29	0	+	2.5631E-06	29	0	+	2.5631E-06	29	0	+
	OSA	2.5631E-06	29	0	+	2.5631E-06	29	0	+	2.5631E-06	29	0	+
	WSO	8.2039E-01	17	12	=	2.3861E-01	19	10	=	1.3225E-03	23	6	+
	HGSO	2.5631E-06	29	0	+	2.5631E-06	29	0	+	2.5631E-06	29	0	+
	GWO	2.1477E-05	27	2	+	4.1746E-05	25	4	+	3.9017E-06	28	1	+
	CGWO	6.0436E-05	26	3	+	1.9148E-04	24	5	+	1.0532E-03	22	7	+
	+/=/-	6/2/0				6/2/0				8/0/0			

G. Cpu-time comparisons on GWO and HGWO

Running speed is also an important indicator to describe the performance of a meta-heuristic algorithm. In this subsection, CPU-time of HGWO and GWO was compared. The Mean CPU-time of all test functions in 30, 50 and 100 dimensions are plotted in Fig. 8 to Fig.10. It can be seen from Fig. 8 to Fig.10 that the CPU-time of HGWO is longer than that of GWO. According to the principle that there is no free lunch [56], the CPU-time increase brought by the performance improvement is reasonable. ood performance achieved by the improvement of GWO does not come for free. But we can also see that, with the increase of dimension, the CPU-time of HGWO and GWO algorithm is closer and closer, which indicates that HGWO is more suitable for solving the high-dimensional optimization problems.

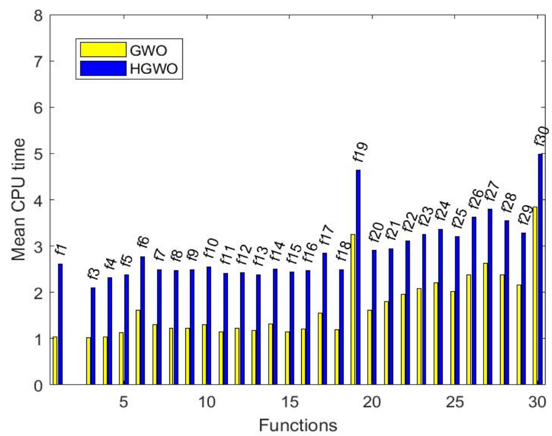


Fig.8. CPU-Time consumption on all test functions(D=30)

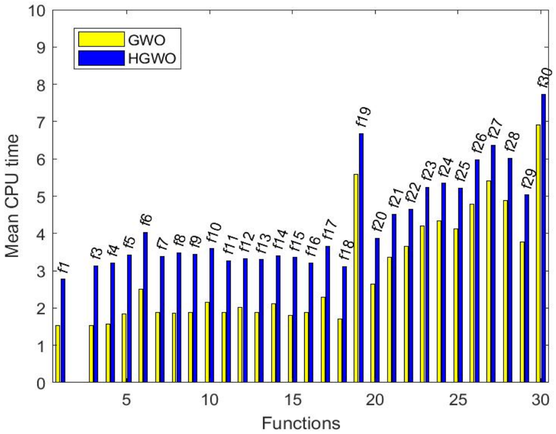


Fig.9. CPU-Time consumption on all test functions (D=50)

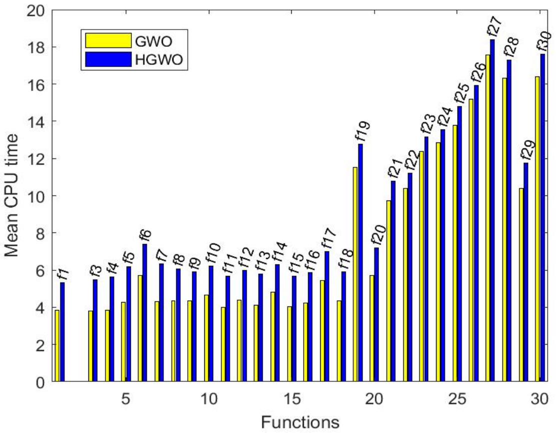


Fig.10. CPU-Time consumption on all test functions(D=100)

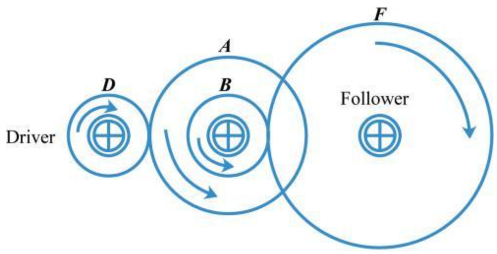


Fig. 11. Gear train design problem

V. Real-world Applications of HGWO in Four Engineering Problems

Engineering design problems can effectively verify the ability of an algorithm to solve real-world problems [41,57]. In this section, the performance of HGWO will be further evaluated on four real-world engineering design problems, including gear train design, pressure vessel design, welded beam design and speed reducer design problem.

A. Gear train design problem

The aim of the gear train design problem is to minimize the square of the difference between the desired gear ratio (1/6.931) and the current design gear ratio [58]. The gear train design model is shown in Fig.11. The main design variables are mainly four, $T_d(x_1)$, $T_b(x_2)$, $T_a(x_3)$, $T_f(x_4)$, where T_d , T_b , T_a and T_f are the number of teeth on gears A, B, D and F, respectively. For each gear, the minimum number of teeth is 12 and the maximum is 60. The objective function of the problem is described in (15).

$$Minimize \quad f(X)=(\frac{1}{6.931}-\frac{x_1x_2}{x_3x_4})^2 \tag{15}$$

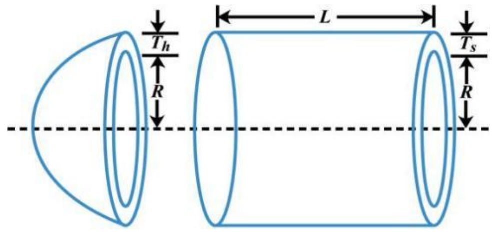


Fig.12. Pressure vessel design problem

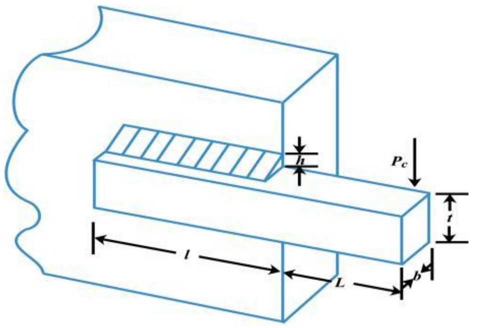


Fig. 13. welded beam design problem

B. Pressure vessel design problem

The aim of the pressure vessel design problem is to minimize the total cost, including the cost of material, forming and welding [59]. The objective function of the problem is described in (16).

$$\begin{aligned} Minimize \quad & f(X)=0.6224x_1x_3x_4+1.7781x_2x_3^2 \\ & \quad +3.1611x_1^2x_4+19.84x_1^2x_3 \\ s.t. \quad & \begin{cases} g_1(X)=-x_1+0.0193x_3\le 0 \\ g_2(X)=-x_2+0.99954x_3\le 0 \\ g_3(X)=-\pi x_3^2x_4-\frac{4}{3}\pi x_3^3+1296000\le 0 \\ g4(X)=x4-240\le 0 \end{cases} \end{aligned} \tag{16}$$

Where $1\leqslant x_1, x_2\leqslant 99$，$10\leqslant x_3, x_4\leqslant 200$.

The pressure vessel model is shown in Fig.12. There are four design variables in this problem: thickness of the shell T_s (x_1), thickness of the headT_h (x_2), inner radius R (x_3), and length of the cylindrical section of the vessel L(x_4). Among these four design variables, T_s and T_h are expected to be integer multiples of 0.0625 inch, and R and L are continuous variables. This problem consists of 3 linear and 1 nonlinear inequality constraints.

C. Welded beam design problem

The welded beam design problem is designed for minimum cost subject to constraints on shear stress (τ), bendding stress (σ) in the beam, buckling load on the bar (P_c), end deflection of the beam (δ), and side constraints. The pressure vessel model is shown in Fig.13. There are four design variables for this problem including h (x_1), l (x_2), t (x_3) and b (x_4) [60]. This problem consists of 4 continuous variables with 2 linear and 5 nonlinear inequality constraints. The objective function of the problem is described in (17).

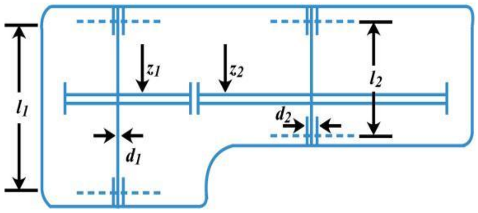


Fig. 14. Speed reducer design problem

$$\text{Minimize} \quad f(X)=1.10471x_1^2x_2+0.04811x_3x_4(14.0+x_2)$$

$$s.t.\begin{cases} g_1(X)=\tau(x)-\tau_{\max}\le 0 \\ g_2(X)=\sigma(x)-\sigma_{\max}\le 0 \\ g_3(X)=x_1-x_4\le 0 \\ g_4(X)=1.10471x_1^2+0.04811x_3x_4(14.0+x_2)-5.0\le 0 \\ g_5(X)=0.125-x_1\le 0 \\ g_6(X)=\delta(X)-\delta_{\max}\le 0 \\ g_7(X)=P-P_c(X)\le 0 \end{cases} \tag{17}$$

Where

$$\tau(X)=\sqrt{(\tau')^2+2\tau'\tau''\frac{x_2}{2R}+(\tau'')^2} \tag{18}$$

$$\tau'=\frac{P}{\sqrt{2}x_1x_2},\tau''=\frac{MR}{J},M=P(L+\frac{x^2}{2}) \tag{19}$$

$$R=\sqrt{\frac{x_2^2}{4}+(\frac{x_1+x_3}{2})^2} \tag{20}$$

$$J=2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12}+(\frac{x_1+x_3}{2})^2\right]\right\} \tag{21}$$

$$\sigma(X)=\frac{6PL}{x_4x_3^2},\delta(X)=\frac{4PL^3}{Ex_3^3x_4} \tag{22}$$

$$P_c(X)=\frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1-\frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \tag{23}$$

Where P=6000 lb, L=14 in, E=20×10^6 psi, G=12×10^6 psi, $\tau_{\max}$=13600 psi, $\sigma_{\max}$=30000 psi, $\delta_{\max}$=0.25 in.

D. Speed reducer design problem

The Speed reducer design problem is a constrained optimization problem in the field of mechanical engineering [60]. As shwon in Fig.14, the weight of the speed reducer is to be minimized subject to constraints on bending stress of the gear teeth, surface stress, transverse deflections of the shafts, and stresses in the shafts. The variables x_1 to x_7 represent the face width $b(x_1)$, module of teeth $m(x_2)$, number of teeth in the pinion $z(x_3)$, length of the first shaft between bearings $l_1(x_4)$, length of the second shaft between bearings $l_2(x_5)$, and the diameter of first $d_1(x_6)$ and second shafts (d_2), respectively. This problem consists of 4 linear and 7 nonlinear inequality constraints with 7 decision variables. The objective function of the problem is described in (24).

$$\text{Minimize} \quad f(X)=0.7854x_1x_2^2(3.3333x_3^2+14.9334x_3-43.0934)-1.508x_1(x_6^2+x_7^2)+7.4777(x_6^3+x_7^3)+0.7854(x_4x_6^2+x_5x_7^2)$$

$$s.t.\begin{cases} g_1(X)=\dfrac{27}{x_1x_2^2x_3}-1\le 0 \\ g_2(X)=\dfrac{397.5}{x_1x_2^2x_3^2}-1\le 0 \\ g_3(X)=\dfrac{1.93x_4^3}{x_2x_3x_6^4}-1\le 0 \\ g_4(X)=\dfrac{1.93x_5^3}{x_2x_3x_7^4}-1\le 0 \\ g_5(X)=\dfrac{\sqrt{\dfrac{745x_4}{x_2x_3}+16.9\times10^6}}{110.0x_6^3}-1\le 0 \\ g_6(X)=\dfrac{\sqrt{(\dfrac{745x_5}{x_2x_3})^2+157.5\times10^6}}{85.0x_7^3}-1\le 0 \\ g_7(X)=\dfrac{x_2x_3}{40}-1\le 0 \\ g_8(X)=\dfrac{5x_2}{x_1}-1\le 0 \\ g_9(X)=\dfrac{x_1}{12x_2}-1\le 0 \\ g_{10}(X)=\dfrac{1.5x_6+1.9}{x_4}-1\le 0 \\ g_{11}(X)=\dfrac{1.1x_7+1.9}{x_5}-1\le 0 \end{cases} \tag{24}$$

Where $2.6\leqslant x_1\leqslant 3.6$, $0.7\leqslant x_2\leqslant 0.8$, $17\leqslant x_3\leqslant 28$, $7.3\leqslant x_4\leqslant 8.3$, $7.8\leqslant x_5\leqslant 8.3$, $2.9\leqslant x_6\leqslant 3.9$ and $5.0\leqslant x_7\leqslant 5.5$.

E. Comparison of results of engineering problems

For engineering problems, it is important to pay more attention to best solutions, because the best solutions are often actually needed [61]. Table XII gives the best solutions of the four engineering problems achieved by all comparison algorithms. The best solutions obtained by comprasion algorithms are marked in bold. Gear train design problem is an unconstrained optimization problem. Eexcept for CSA and OSA, other algorithms achieved the same best solutions, which shows that most algorithms can handle unconstrained optimization problems well. Other three design problems are constrained optimization problems. It can be seen from Table XII that only HGWO can obtain the best solutions of the three problems. GWO can can only obtains the best solution of pressure design problem, while it has poor ability to deal with the other two constrained problems. In fact, constraints are ubiquitous in engineering design. The ability

TABLE XII
BEST RESULTS OF THE FOUR ENGINEERING PROBLEMS ACHIEVED BY ALL COMPARISON ALGORITHMS

Problem	AEO	HHO	CSA	OSA	WSO	HGSO	GWO	CGWO	HGWO
Gear train	**2.7009E-12**	**2.7009E-12**	2.3078E-11	5.1415E-06	**2.7009E-12**	**2.7009E-12**	**2.7009E-12**	**2.7009E-12**	**2.7009E-12**
Pressure vessel	**6.0597E+03**	6.1082E+03	6.1185E+03	6.8795E+03	**6.0597E+03**	6.1446E+03	**6.0597E+03**	6.0598E+03	**6.0597E+03**
Welded beam	1.7784E+00	1.7571E+00	1.7332E+00	1.8061E+00	1.7251E+00	1.9308E+00	1.7251E+00	1.7250E+00	**1.7249E+00**
Speed reducer	2.9968E+03	3.0011E+03	3.0028E+03	3.5554E+03	**2.9963E+03**	3.0601E+03	2.9976E+03	2.9976E+03	**2.9963E+03**

of HGWO to deal with constrained optimization problems is improved compared with GWO, and its comprehensive performance is better than other comparison algorithms. Through the results and analysis of the engineering cases, we can conclude that the proposed HGWO has potential ability to deal with engineering problems, and has certain strength in search for optimal values. In addressing engineering design issues, these strategies proposed in HGWO have greatly improved the performance of the original GWO.

VI. CONCLUSIONS AND PROSPECTS

In this paper, a new GWO variant was proposed to alleviate the stagnation problems of the basic optimizer, which is called HGWO. The proposed HGWO mainly adopts three strategies, random subgroup, two new global search formulas and the concept of greedy wolf. The purpose of the three strategies is to achieve a better balance between exploration and exploitation, and improve the convergence rate and accuracy. Random subgroup strategy can effectively increase population diversity, which can avoids local optimum. The switching of two global search formulas can effectively help the algorithm escaping from local optimum. Greedy wolves only hunt around the most valuable prey to maximize profits, thus effectively improving the convergence accuracy of the algorithm. Nonliner factor can better coordinate the exploration and exploitation. The numerical results on CEC2017 benchmark functions show that the comprehensive performance of HGWO is significantly better than those of comparison algorithms. Friedman and Wilcoxon signed rank test further proved the superiority of HGWO from a statistical sense. Overall, in dealing with optimization problems of different dimensions, the accuracy and stability of HGWO are better than that of comparison algorithms. The convergence comparison shows that HGWO has faster convergence speed than most comparison algorithms. CPU-time cost analysis shows that the good performance achieved by the improvement of GWO is not for free, but HGWO is more suitable for solving high-dimensional optimization problems. No algorithm can get the best results on all optimization problems, but the optimization results of four engineering problems show that the proposed HGWO is more suitable for solving complex engineering optimization problems. Although there are still some gaps between the proposed HGWO and GWO in running speed, and the theoretical analysis still needs to be further strengthened, the improvement strategy actually improves the performance of HGWO and can also provide inspiration for the improvement of other meta-heuristic algorithms.

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