Improved Whale Optimization Algorithm Combined with the Equiangular Spiral Bubble Net Predation

Kui Liu, Lidan Xue, and Sanyang Liu,

Abstract—**At present, whale optimization algorithm (WOA) is one of the hot issues in swarm intelligence algorithm. Since it was proposed, people have done a lot of improvement work for WOA algorithm. To address the shortcomings of WOA, an improved WOA combined with the equiangular spiral bubble net predation (named as IWOA) is proposed in this paper. In IWOA, search agent uses the equiangular spiral rather than 9-shaped path to mimics the foraging trajectory of humpback whale. This rule can increase the convergence speed and exploitation ability of the search agent to an extent. Additionally, with the guidance of the sound wave attenuation steering law, search agent in IWOA can switch back and forth between the actively swim (exploitation) and the randomly swim (exploration) to the goal, hence obtain a better tradeoff between the exploitation and exploration. Numerical experiments are conducted on a set of mathematical benchmark cases. The results show that IWOA has a better performance.**

Index Terms—**whale optimization algorithm, sprint feeding method, equiangular spiral bubble net, selective probability P.**

I. Introduction

ANIMALS have various foraging methods. Some animals use a passive attitude to hunt for food, others take the initiative in catching prey [1-5]. Bubble net feeding is a unique foraging method utilized by humpback whales. Humpback whales are lack of chewing teeth, so they are biased toward hunting school of small fish and shrimp. When the swarm of fish becomes densely packed, the humpback whale just needs to rush into the fish swarm, and open its mouth to swallow the fish and shrimp together with water. However, when the fish swarm is spread out, the effectiveness of this predation will decrease. This is because small fish are agile and able to swim at high speeds. But as an expert in the art of hunting, humpback whale has developed a variety of techniques. Humpback whales will dive into the water and then start to spit small bubbles around the prey. This is the bubble net feeding, which will be introduced in this paper.

Bubble net feeding can be divided into two categories based on the number of whales involved. The first category involves a small group of humpback whales using the "bubble curtain wall" to catch small fish and shrimp. The second category involves a single humpback whale using the "spiral bubble net" to capture its prey. Humpback whale prefers to be alone out of the breeding season, so it needs to use the "spiral bubble net" to hunt alone in most cases. The specific predation process is that humpback whale dived into the water to look for the position of small fish and shrimp. Then the whale swims towards the surface in a spiral drawn. During the acceleration phase, humpback whale constantly releases bubbles of varying sizes to create a spiraling bubble net that drives small fish and shrimp towards the center of the net. Finally, the whale rushes into the center of spiral bubble net, and swallowes the fish and shrimp together.

By mimicking the hunting behavior of humpback whales, Mirjalili et.al proposed a new meta-heuristic optimization algorithm, namely WOA [6]. Due to its simplicity and ease of implementation, WOA algorithm has been applied to solve many kinds of problems besides numerical function optimization. Unfortunately, like other evolutionary algorithms, WOA also has some insufficiencies. For example, WOA can easily get trapped in local optima when solving complex multimodal function problems and its exploitation ability is also an issue in some cases. These weaknesses have restricted the applications of the WOA. Consequently, more and more researchers [7-24] are paying close attention to the improvement of WOA so as to overcome these shortages. With the aid of chaotic local search and l$\widetilde{e}$vy flight, Chen et.al [16] proposed a balanced whale optimization algorithm (BWOA), which can avoid search agent being stuck at local optima. In [17], an enhanced whale optimization algorithm (EWOA) was proposed by Kaveh. This algorithm can achieve a better performance in terms of reliability and solution accuracy. Based on l$\widetilde{e}$vy flight trajectory, Ling et al. [18] proposed a l$\widetilde{e}$vy flight trajectory based whale optimization algorithm (LWOA) for global optimization. Since l$\widetilde{e}$vy flight trajectory is helpful for increasing the diversity of the population against, LWOA can jump out of the local optimal optima. Combine with simulated annealing strategy (SA), Mafarja et.al proposed two hybrid whale optimization algorithms called WOASA-1 and WOASA-2 in [19]. These hybridizations can enhance the exploitation property of the search agent to an extent. Rajathi successfully uses these hybrids WOASA algorithm to classify chronic liver disease in [20]. In order to enhance the diversity of the population, Fan [21] et. al uses an opposition-based learning mechanism and an adaptive inertia weight rule to update the individuals of JSWOA. This multi-mechanisms whale optimization algorithm can improve the solution accuracy at the expense of computation complexity. By modifying and integrating

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the mutualism phase of Symbiotic Organisms Search with WOA, Chakraborty proposed an enhanced whale algorithm (WOAmM) in [22]. Liu proposes a reinforced exploration mechanism whale optimization algorithm (REM-WOA) for continuous optimization problems in [23]. Experiment result shows that this algorithm can enhance whale population global exploration efficiency.

In this paper, we develop an improved whale optimization algorithm (IWOA) combined with the equiangular spiral bubble net predation. As we know, the shaped of spiral bubble net looks more like an equiangular spiral rather than 9-shaped path, so search agent uses equiangular spiral to mimic the foraging trajectory of humpback whale, which can enhance the exploitation ability of IWOA. Meanwhile, with the guidance of a sound wave attenuation steering law, search agent can switch back and forth between the actively swim (exploitation) and the randomly swim (exploration) to the goal, hence obtain a better tradeoff between the exploitation and exploration of the algorithm. To very $IWOA$'s performance, some numerical experiments are carried on. The corresponding test result is compared with WOA, LWOA, BWOA, EWOA, WOASA-1, WOASA-2, PSO [24], DE [25], GA [26], IHS [27], and ES [28].

The structure of this paper is designed as follows: Section I briefly introduces the research status of whale optimization algorithm. The process of basic whale optimization algorithm is explained in Section II, and the detail of IWOA algorithm combined with the equiangular spiral bubble net predation is given in Section III. Experiment and related results are discussed in Section IV, and finally the conclusion is given in Section V.

II. Standard Whale Optimization Algorithm

In standard WOA, humpback whales only have two different position updating rules to choose for preying. Depending on the density of fish swarm, humpback whales can choose to use either the "$Shrinking\ encircling\ mechanism$" or the "$Spiral\ updating\ strategy$" to prey. In order to simulate the automatic selection behavior of humpback whale, a control factor p was introduced. if the value of p is less than 0.5, the "$Shrinking\ encircling\ mechanism$" is employed by whale to search a virgin territory; else, the "$Spiral\ updating\ strategy$" is used by whale to exploit a promising candidate solution.

There are two search equations in "$Shrinking\ encircling\ mechanism$". If the module of coefficient vector $\overrightarrow{A}$ is less than 1, humpback whale will swim to the target source actively and use the optimal solution X_{best} in current population to guide the exploitation process; else, humpback whale will swim passively and randomly and use X_{rand} (selected randomly from the whole population) to guide the exploration process.

The location of humpback whale is continuous updated according to the result produced by search agent. Calculating the fitness value of each solution and selecting the solution with the minimum fitness value as the optimal solution for this iteration. If the number of iterations is not satisfied, repeat the above steps. The framework of the basic WOA is described in Algorithm 1.

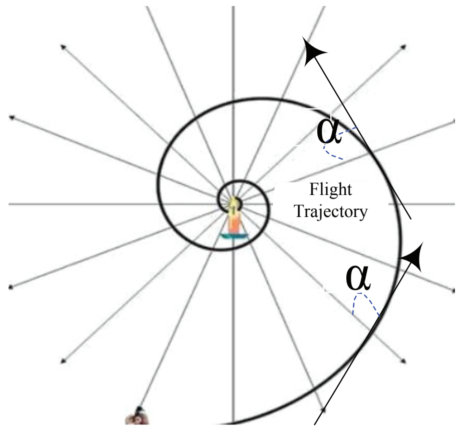

Fig. 1. The phenomenon of moths flying into flames

III. Improved WOA Combined with the Equiangular Spiral Bubble Net Predation

The fundamental principle of "$Spiral\ updating\ strate-gy$" is to ensure that the initial and final bubbles released by the humpback whale ascend to the surface simultaneously. Thus forming a spiral bubble net that tightly surrounds the prey and forces them toward the center of bubble net. The shaped of spiral bubble net looks more like an equiangular spiral rather than 9-shaped path. It is important to highlight that "$equiangular\ spiral$" can more accurately mimic the foraging trajectory of humpback whale, and then help to identify the optimal solution from the original set. Equiangular spirals are found in various natural phenomena, such as the nautilus shell's stripe which closely resembles the shape of this spiral. Equiangular spiral also can be observed in certain unusual natural occurrences, such as the phenomenon of moths flying into flames. Fig. 1 is the phenomenon of moths flying into flames.

A. Standard Equation of Equiangular Spiral

The equiangular spiral has a particular nature that the angle between the tangent vector and the polar radius is a constant. Suppose P is an arbitrary point on equiangular spiral. The angle between the tangent vector and the polar radius is marked as φ ($\varphi \neq \pi/2$). Set ordinate origin o as the polar pole, hence the polar coordinates equation of equiangular spiral can be expressed as $r = f(\theta)$; Point P can be marked as $(f(\theta), \theta)$; The tangent vector at point P can be expressed as:

$$(f^{'}(\theta)\cos\theta - f(\theta)\sin\theta, f^{'}(\theta)\sin\theta + f(\theta)\cos\theta)$$

Use the vectorial angle formula to calculate cosine value of the angle φ.

$$cot\varphi = \frac{cos\theta[f^{'}(\theta)cos\theta - f(\theta)sin\theta]}{\sqrt{(f^{'}(\theta))^2+(f(\theta))^2}} + \frac{sin\theta[f^{'}(\theta)sin\theta + f(\theta)cos\theta]}{\sqrt{(f^{'}(\theta))^2+(f(\theta))^2}} = \frac{f^{'}(\theta)}{\sqrt{(f^{'}(\theta))^2+(f(\theta))^2}} \tag{1}$$

Through proper simplification, we can easily obtain the following result: $cot\varphi = \frac{f^{'}(\theta)}{f(\theta)}$

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Algorithm 1: Pseudo-code of WOA
Initialize the whales population x_i (i = 1, 2, 3, ...n).
Calculate the fitness of each search agent, set x* = the best search agent
While (t < maximum number of iterations)
  For each search agent
  Update a, A, C, l and p
    If1 (p < 0.5)
      If2 (|A| < 1)
        Updater the position of the current search agent by x(t+1) = x*(t) − A D
      Else If2 (|A| >= 1)
        Select a random search agent x_rand
        Updater the position of the current search agent by x(t+1) = x_rand − A D
      End if2
    Else If1 (p >= 0.5)
      Updater the position of the current search agent by x(t+1) = D' e^{bl} cos(2πl) + x*(t)
    End If1
  End For
  Check if any search agent goes beyond the search space and amend it
  Calculate the fitness of each search agent and update x* if there is a better solution
  t=t+1
End while
Output best solution x*
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And then, by solving this differential equation, the standard equation of equiangular spiral can be expressed simply:

$$r = f(\theta) = \alpha e^{\theta cot\varphi}, \tag{2}$$

where α is a constant for defining the shape of equiangular spiral.

B. Equiangular Spiral Updating Position

The equiangular spiral is a special curve that can achieve congruent stretching continuation. No matter how many times it performed enlarge or shrink transformation, the obtained curve is still an equiangular spiral. This self-perpetuating feature can make equiangular spiral more suited to exhibit the character of spiral bubble net. Hence, an equiangular spiral update formula is created as follows:

$$\vec{X}(t+1) = \left|\vec{X}^*(t) - \vec{X}(t)\right| e^{\theta cot\varphi} + \vec{X}^*(t) \tag{3}$$

where $\left|\vec{X}^*(t) - \vec{X}(t)\right|$ is the distance between whale and prey; φ is a constant angle; θ is a random polar angle. If φ is less than $\pi/2$, θ is a random number in $[-\infty, 0]$; Otherwise, θ is a random number in $[0, \infty]$.

Due to the advantage of equiangular spiral in mimicking the foraging patterns of humpback whales, the proposed equiangular spiral position updating rule can increase the exploitation ability of the search agent.

C. Steering Law of Sound Wave Attenuation

Humpback whales used to live in the deepest part of the ocean, so they may use ultrasound to complete information exchange. When a humpback whale finds fish swarm, it will send out ultrasonic signals to other whales nearby. The intensity of ultrasound will be attenuate in the process of spread. Usually, the transmission attenuation formula can be expressed as $\rho = \rho_o e^{-\eta d}$. Where ρ_o is the initial strength; η is the loss coefficient; d is the distance to the wave source. The value of loss coefficient η is depending on the physical characteristics of the media. For function optimization problems, parameter η is depending on dimensions, multi-peak distribution, domain of objective function, and search operator coverage metrics. Hence, parameter η needs to be set various values depending on the different functions. To be operational, suppose ultrasonic intensity maybe attenuate down to the level of $25\%\rho_o$, when the transmission distance is equal to the one twentieth of the max-width of the search space. Thus, the loss coefficient η can be simplified calculation as follow.

$$\rho_o e^{-\eta d_{max}/20} = 25\%\rho_o \tag{4}$$

where d_{max} is the max-width of the search space. Through proper simplification, the loss coefficient η can be expressed simply:

$$\eta = \frac{20ln4}{d_{max}} = ln \sqrt[d_{max}]{4^{20}} \tag{5}$$

According to the strength of ultrasound, humpback whales may dynamically determine the next search locations. When the whale receives ultrasound information from faraway search location (the strength of received ultrasound is smaller than threshold), due to the accuracy of this information is uncertain, it will just be passively and randomly swim toward goal. This is because the information may be distorted after the long distance transmission. However, if the whale is close to the information source (the strength of received ultrasound is bigger than threshold), it will be actively swim to the target location. Base on this phenomenon, a sound wave attenuation steering law is given follow.

$$\vec{X}(t+1) = \begin{cases} \vec{X}(t) + \delta[D\vec{X}^* - \vec{X}(t)], & d_{xx^*} < d_o, \\ \vec{X}(t) + \delta[D\vec{X}_{rand} - \vec{X}(t)], & otherwise, \end{cases} \tag{6}$$

where δ is a random number in $(e^{-\eta d_{xx^*}}, 1)$; D is a random number in $(1 - e^{-\eta d_{xx^*}}, 1 + e^{-\eta d_{xx^*}})$. According to the strength of received ultrasound, humpback whale timely calculates the distance to the wave source, and then makes a dynamic determination on choosing between either randomly swim (exploration) or actively swim (exploitation) to the target location. If the distance between whale and the wave source is smaller than d_o (d_o is the adaptive threshold), the actively swim model (exploitation phase) is chose to update the position of whale during optimization. Otherwise, the randomly swim model (exploration phase) is selected to update the position of whale.

D. Proposed Whale Optimization Algorithm

Similar to the standard WOA algorithm, we also use a selective probability parameter P to control the frequency of introducing $''Equiangular\ spiral\ updatingposition''$ and $''The\ steering\ law\ of\ sound\ wave\ attenuation''$. The threshold value of parameter P is set to 0.5 . Based on the above explanation, the flowchart of the proposed method (denoted as $IWOA$ Algorithm) is given in Fig. 2.

IV. Experimental Results and Analysis

A. Test Function and Parameter Settings

To test the performance of the IWOA algorithm, we have carried out different experiments using various mathematical benchmark cases. The first kind of mathematical cases are 18 common continuous functions and 5 composite benchmark functions. The second kind is 3 constrained engineering design problems. The third kind is 18 machine leaning datasets from UCI database, which can be used to confirm the efficiency of the IWOA algorithm in improving the classification accuracy. The performance of IWOA algorithm with respect to solution accuracy is first compared with the BWOA[16], EWOA[17], LWOA[18] and standard WOA algorithms on 23 test functions provided in Table 1 and Table 2. Then, the effective of IWOA algorithm is further compared with PSO[24], DE[25], GA[26], IHS[27], ES[28], and BWOA on three constrained engineering design problems. Finally, the comparison on the classification accuracy is performed between the hybrid version IWOA algorithm and the hybrid WOA algorithm [19] marked as $WOASA-1$, and $WOASA-2$.

To make a fair comparison, all test functions are conducted for 30 runs, and the means and standard deviations of the statistical experimental data are reported. Meanwhile, in this section, all the algorithms are coded in Matlab 7.0 and the simulations are run under a Windows 10 with Intel (R) Core $i7-4790$ CPU @$3.6GHz$ with 8GB memory capacity.

B. Effects of Parameter d_o on the Performance of IWOA

As introduced in Section 3, the balance between exploration and exploitation strongly depends on the parameter d_o, which controls the switch between the exploratory and exploitative patterns. To obtain better coordination relation, the proper value of d_o needed be investigated. We investigate the impact of parameter d_o on the IWOA algorithm. It is evident that if d_o is small, the whales may tend to explore uncharted search space. Hence IWOA algorithm will be good at exploration but pool at exploitation. Conversely, large values of d_o may prompt whales to perform the local search frequently. This situation can easily make IWOA algorithm trapped in a local optimum, thereby cutting down the performance of exploration. In conclusion, an appropriate value of d_o must be used. Different types of test functions are used to investigate the impact of d_o. They are $Sphere$, $Step$, $Schwefei$ 1.2, $Ackley$, $Griewank$, and $Penalized$ 1 functions, as defined in Table 1. The swarm size is set to be 20, and the maximum iteration number is set to 800. The IWOA algorithm runs 30 times separately on different values of d_o, and the average values of the test results are plotted in Fig. 3. We can clearly observe that different landscapes have dissimilar responses to d_o values. Specifically, in functions $Sphere$, $Step$ and $Ackley$, smaller test results and higher convergence speed are obtained when parameter d_o is set to $0.05d_{max}$. Function $Schwefei$ 1.2 is not sensitive to the value of parameter d_o, because smaller test results are obtained for all values of d_o. For the remaining functions, i.e., $Griewank$, and $Penalized$ 1, smaller test results are obtained when $d_o = 0.1d_{max}$; However, these results are not significantly different when compared to those obtained with $d_o = 0.05d_{max}$. Therefore, in our experiments, the parameter d_o is set to $0.05d_{max}$ for all test functions. Under this setting, exploration function and the exploitation function of IWOA algorithm can be coordinated and balanced to achieve satisfactory results on different optimization problem.

C. Performance of IWOA on Different Benchmark Functions

In the first part of experiment, all benchmark functions are minimization problems and widely adopted to test the performance of evolutionary algorithms. These functions are of different types such as: unimodal functions, multimodal functions, composite functions, noisy quartic functions, discontinuous step functions, shifted functions, and rotated functions. In particular, f_1, f_2, and f_4 are unimodal functions, f_7, f_8, f_9, f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}, and f_{18} are multimodal functions, f_{19}, f_{20}, f_{21}, f_{22}, and f_{23} are composite benchmark functions, f_3 and f_{10} are shifted functions, f_5 is a discontinuous step function, f_6 is a noisy quartic function. Simulation result obtained by the $IWOA$, $LWOA$, $BWOA$, $EWOA$, and WOA on different test functions is used to analyze the performance of $WOAs$ algorithm. Comparison results are shown in Tables 3-6 in terms of the best, median, worst, mean, and standard deviation of the solutions. In addition, to further test the efficiency of the $IWOA$ algorithm, the convergence curves of the $WOAs$ algorithm on different test functions are shown in Fig. 4.

For the unimodal functions, the swarm size is set to be 20, and the maximum iteration number is set to 800 for each $WOAs$ algorithm. Simulation results of different $WOAs$ algorithm are reported in Table 3. From Table 3, we can see that the results of $IWOA$ algorithm do not differ significantly from those of the $LWOA$, $BWOA$, $EWOA$, and stand WOA algorithms. This is because the optimal value of these test functions is easy to find out. But, from the standard deviation rows of Tables 3, we can see that the standard deviations abstained by $IWOA$ algorithm are relatively small. It implies that the solution quality of the $IWOA$ algorithm is considerable competitive with other WOA algorithms. In particular, $IWOA$ is the most efficient optimizer in text function f_1, f_2, f_3, f_4 and f_6. For text function f_5 and f_{10}, all algorithms do not find satisfactory results, but the $IWOA$ converges to a smaller value than

TABLE I
18 BENCHMARK FUNCTIONS IN EXPERIMENT 1

Functions	D	Search range	Optimum
$f_1=\sum_{i=1}^{D} x_i^2$	30	[-100,100]	0
$f_2=\sum_{i=1}^{D} \mid x_i \mid + \prod_{i=1}^{D} \mid x_i \mid$	30	[-10,10]	0
$f_3=\sum_{i=1}^{D} (\sum_{j=1}^{i} x_j)^2$	30	[-100,100]	0
$f_4=\max\{\mid x_i \mid, 1 \le i \le D\}$	30	[-100,100]	0
$f_5=\sum_{i=1}^{D} (\lfloor x_i+0.5 \rfloor)^2$	30	[-100,100]	0
$f_6=\sum_{i=1}^{D} ix_i^4 + random[0,1)$	30	[-1.28,1.28]	0
$f_7=\sum_{i=1}^{D} \mid x_i sin(x_i)+0.1x_i \mid$	50	[-10,10]	0
$f_8=\sum_{i=1}^{D} (x_i^2-10\cos(2\pi x_i)+10)$	30	[-5.12,5.12]	0
$f_9=-20\exp(-0.2*\sqrt{\sum_{i=1}^{D} x_i^2/D})-\exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i))+20+e$	30	[-32,32]	0
$f_{10}=\frac{1}{4000}\sum_{i=1}^{D} x_i^2-\prod_{i=1}^{D} cos(\frac{x_i}{\sqrt{i}})+1$	30	[-600,600]	0
$f_{11}=\frac{\Pi}{D}10\sin^2(\Pi y_1)+\frac{\Pi}{D}\sum_{i=1}^{D-1}(y_i-1)^2[1+10sin^2(\Pi y_{i+1})]$ $+\frac{\Pi}{D}(y_D-1)^2+\sum_{i=1}^{D} u(x_i,10,100,4)$ $y_i=1+\frac{x_i+1}{4},\ \ u(x_i,a,k,m)=\begin{cases} k(x_i-a)^m, & x_i>a \\ 0, & -a\le x_i\le a \\ k(-x_i-a)^m, & x_i<-a \end{cases}$	30	[-50,50]	0
$f_{12}=(\frac{1}{500}+\sum_{j=1}^{25}(j+\sum_{i=1}^{2}(x_i+a_{ij})^6)^{-1})^{-1}$	2	[-65,65]	1
$f_{13}=\sum_{i=1}^{11}[a_i-\frac{x_1(b_i^2+b_i x_2)}{b_i^2+b_i x_3+x_4}]^2$	4	[-5,5]	0.0003
$f_{14}=(x_2-\frac{5.1x_1^2}{4\Pi^2}+\frac{5x_1}{\Pi}-6)^2+10(1-\frac{1}{8\Pi})cosx_1+10$	2	[-5,5]	0.398
$f_{15}=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)]$ $*[30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	2	[-5,5]	3
$f_{16}=-\sum_{i=1}^{5}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	[0,10]	-10.1532
$f_{17}=-\sum_{i=1}^{7}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	[0,10]	-10.4028
$f_{18}=-\sum_{i=1}^{10}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	[0,10]	-10.5364

TABLE II
COMPOSITE BENCHMARK FUNCTIONS IN EXPERIMENT 1.

Test function	D	Search range	Optimum	Mathematical representation
CF1	30	[-5, 5]^D	0	$f_1,f_2,\cdots,f_{10}=SphereFunction, [\sigma_1,\sigma_2,\cdots,\sigma_{10}]=[1,1,\cdots,1]$ $[\lambda_1,\lambda_2,\cdots,\lambda_{10}]=[0.05,0.05,\cdots,0.05]$
CF2	30	[-5, 5]^D	0	$f_1,f_2,\cdots,f_{10}=Griewank^{'}sFunction, [\sigma_1,\sigma_2,\cdots,\sigma_{10}]=[1,1,\cdots,1]$ $[\lambda_1,\lambda_2,\cdots,\lambda_{10}]=[0.05,0.05,\cdots,0.05]$
CF3	30	[-5, 5]^D	0	$f_1,f_2,\cdots,f_{10}=Griewank^{'}sFunction, [\sigma_1,\sigma_2,\cdots,\sigma_{10}]=[1,1,\cdots,1]$ $[\lambda_1,\lambda_2,\cdots,\lambda_{10}]=[1,1,\cdots,1]$
CF4	30	[-5, 5]^D	0	$f_1,f_2=Weierstrass^{'}s\ Function,\ \ f_3,f_4=Griewank^{'}s\ Function,$ $f_5,f_6=Ackley^{'}s\ Function,\ \ f_7,f_8=Rastrigin^{'}s\ Function,$ $f_9,f_10=Sphere\ Function,\ \ [\sigma_1,\sigma_2,\cdots,\sigma_{10}]=[1,1,\cdots,1]$ $[\lambda_1,\lambda_2,\cdots,\lambda_{10}]=[10,10,0.05,0.05,5/32,5/32,1,1,0.05,0.05]$
CF5	30	[-5, 5]^D	0	$f_1,f_2=Weierstrass^{'}s\ Function,\ \ f_3,f_4=Griewank^{'}s\ Function,$ $f_5,f_6=Ackley^{'}s\ Function,\ \ f_7,f_8=Rastrigin^{'}s\ Function,$ $f_9,f_10=Sphere\ Function,\ \ [\sigma_1,\sigma_2,\cdots,\sigma_{10}]=[1,1,\cdots,1]$ $[\lambda_1,\lambda_2,\cdots,\lambda_{10}]=[10,10,0.05,0.05,5/32,5/32,0.2,0.2,0.05,0.05]$

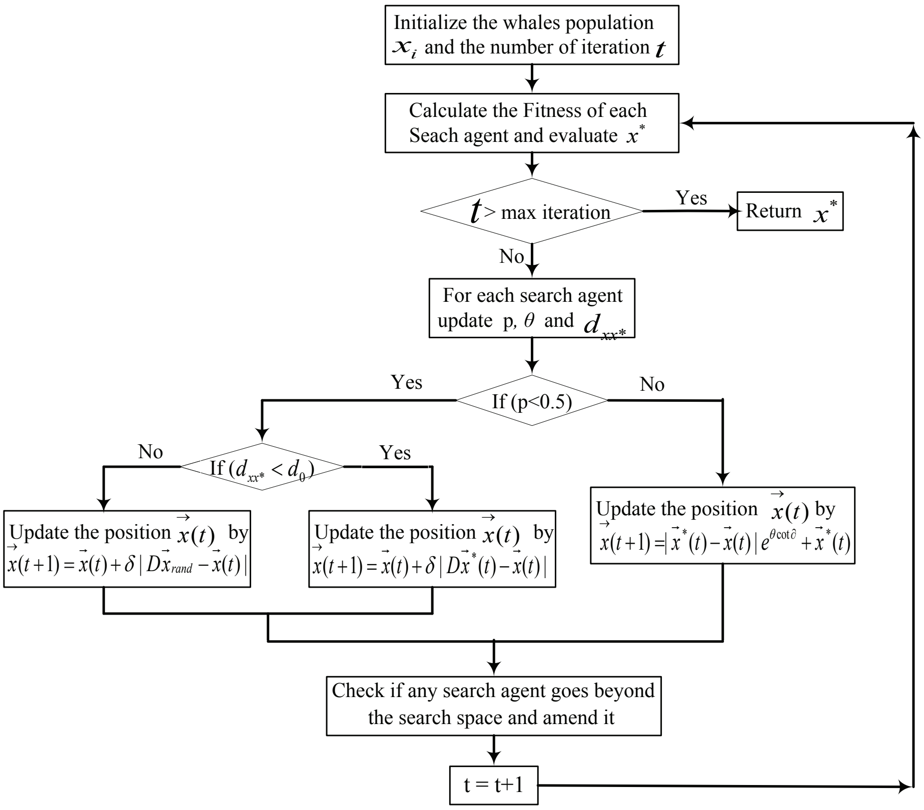

Fig. 2. The flowchart for the IWOA algorithm

TABLE III
COMPUTATIONAL RESULTS OF DIFFERENT WOAS ON UNIMODAL FUNCTIONS.

Function	Result	IWOA	WOA	LWOA	EWOA	BWOA
f_1	Best	**0**	1.93e-144	**0**	3.42e-185	**0**
	Mean	**0**	3.48e-131	**0**	4.42e-172	**0**
	Worst	**0**	9.53e-127	**0**	6.77e-160	**0**
	Std	**0**	4.73e-130	**0**	3.28e-169	**0**
f_2	Best	3.25e-297	5.26e-105	5.71e-288	4.37e-202	6.57e-292
	Mean	4.28e-264	4.22e-096	9.33e-240	3.18e-182	4.55e-259
	Worst	5.37e-261	8.85e-094	2.53e-236	5.42e-167	3.75e-256
	Std	1.75e-260	2.55e-096	2.15e-239	2.75e-190	1.87e-257
f_4	Best	3.36e-237	14.526434	8.55e-222	4.12e-198	7.76e-236
	Mean	1.75e-210	73.577285	5.24e-192	6.37e-189	3.28e-208
	Worst	2.44e-203	95.406733	3.87e-187	3.16e-182	3.44e-197
	Std	6.38e-204	25.068562	1.75e-190	5.57e-190	8.42e-203
f_3	Best	**0**	111980.48	**0**	100802.75	**0**
	Mean	**0**	165755.38	**0**	143723.18	**0**
	Worst	**0**	236779.22	**0**	194537.22	**0**
	Std	**0**	33455.339	**0**	12431.227	**0**
f_{10}	Best	**0**	**0**	4.37e-107	7.46e-087	2.83e-126
	Mean	3.75e-125	0.0567574	2.28e-105	5.35e-084	4.62e-113
	Worst	1.22e-110	0.8905877	1.89e-103	3.13e-079	6.55e-107
	Std	1.83e-126	0.0820968	1.76e-104	4.28e-082	3.11e-110
f_5	Best	0.4331728	0.5978572	0.7835524	0.8022356	0.6100235
	Mean	1.6537825	1.9526035	2.0692323	2.8243133	1.8926541
	Worst	2.8743224	3.0285035	3.9788245	4.1034225	2.8824733
	Std	0.5804226	0.7044416	0.9152733	0.9927357	0.9021475
f_6	Best	1.375e-09	2.687e-04	5.387e-06	8.795e-05	1.757e-08
	Mean	2.546e-07	0.0046875	4.679e-04	0.0018225	2.014e-06
	Worst	3.687e-06	0.0354670	0.0087455	0.0077561	8.421e-05
	Std	4.783e-06	0.0058894	7.874e-05	0.0043152	4.885e-06

other WOA algorithms. Hence, $IWOA$ algorithm provides very excellent performance.

For the multimodal functions, the maximum number of swarm size is set to 30, and the maximum iteration number

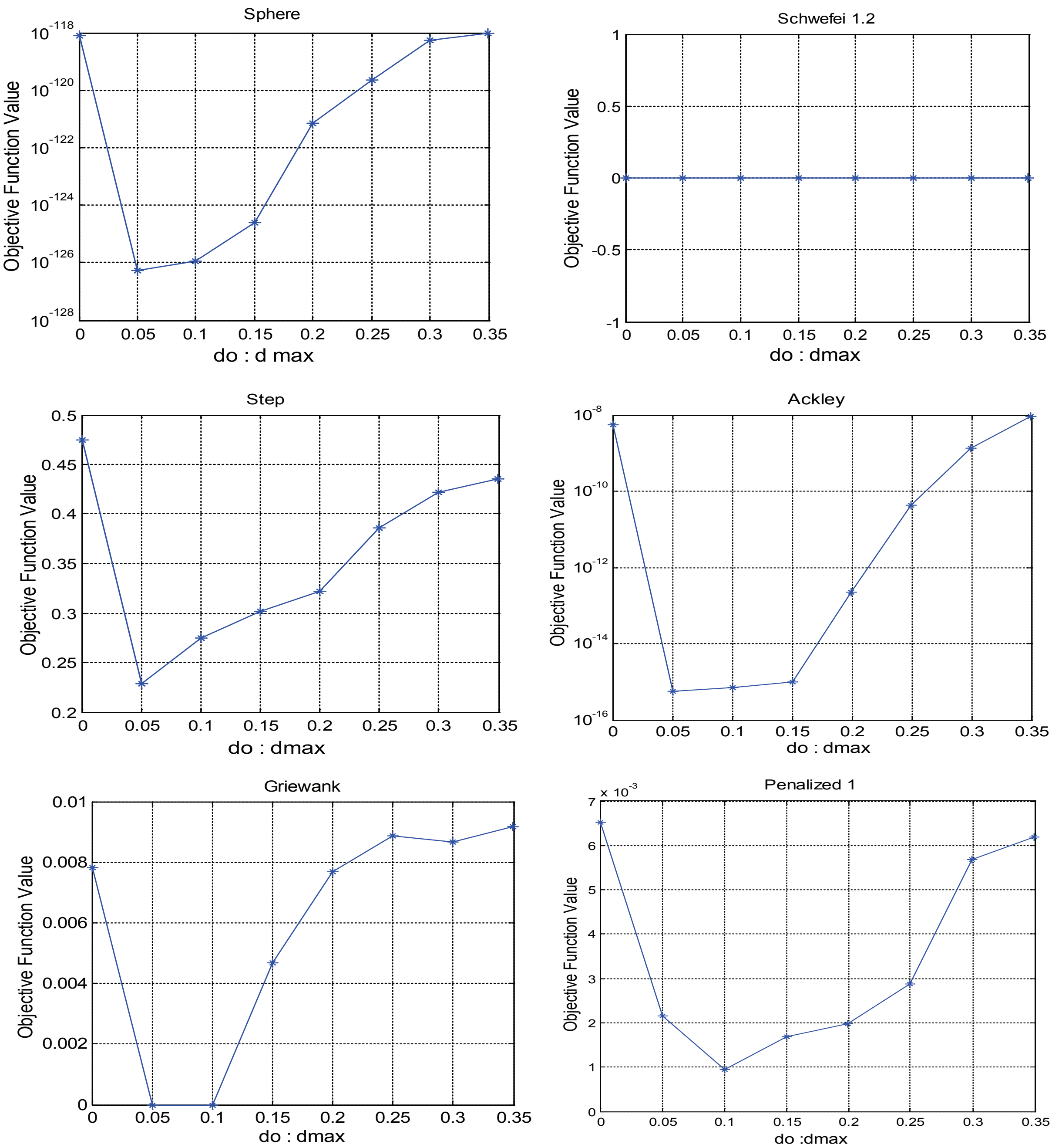

Fig. 3. Performance of IWOA algorithm for different values of d_o

is set to 1000 for each swarm intelligence algorithm. In fact, $IWOA$ algorithm works better in almost all cases and achieves better result than $LWOA$, $BWOA$, $EWOA$, and stand WOA algorithms. More specifically, the $IWOA$ algorithm can find the global optimal solutions in text functions f_8, f_{15}, f_{16}, f_{17}, and f_{18}, and obtain highly accurate solutions that are extremely close to the optimum values in text functions f_7, f_9, f_{12}, f_{13} and f_{14}. The results reported in Table 4 and Table 5 suggests that the exploration capability of $IWOA$ algorithm is the best one among the $WOAs$ algorithm. This is due to Eq. 6 enforces humpback whales to switch randomly between the actively swim (exploitation) and the randomly swim (exploration) toward the goal according to the strength of received ultrasound. This integrated mechanism of exploration can lead IWOA algorithm jumping out of the local optimal optima and terminating by finding the global optimum value.

For the composite functions and other representative test functions, the number of swarm size is set to 10, and the maximum iteration number is set to 100 for each enhanced version $WOAs$ algorithm. Optimization results of different $WOAs$ algorithm are reported in Table 6. These results prove that $IWOA$ algorithm is the best optimizer in most cases and can efficiently avoid local optima optimal. In the concrete causes, a sound wave attenuation steering law obtain a better tradeoff between the exploitation and exploration of the WOA algorithm. Then, in the rest of iterations, excellent diversity and high convergence are emphasized which originate from equiangular spiral updating position mechanism. Meanwhile, this update mechanism allows the

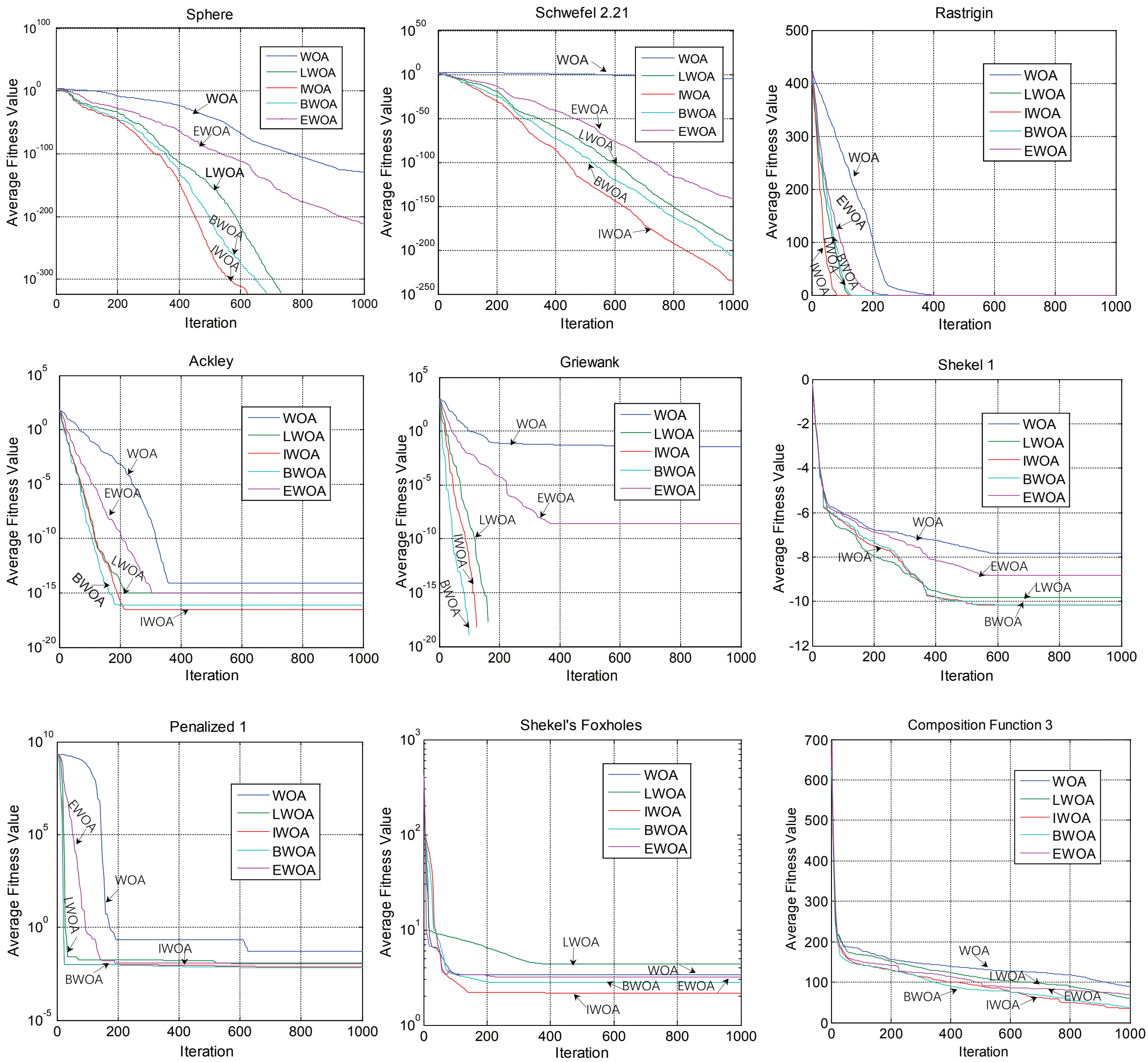

Fig. 4. Convergence performance of different WOAs on 9 test functions

humpback whales to rapidly re-position themselves around the superior individuals produced in different generation, and finally terminates by getting the satisfactory results.

The convergence characteristics of WOA algorithms can be observed in Fig. 4. The convergence speed of $IWOA$ algorithm is higher than that of WOA, $BWOA$, $EWOA$, and $LWOA$ algorithms on most of test functions. More specially, during the early evaluation, there is no significant difference among the convergence performance of the $WOAs$ algorithm. But, in the subsequent iteration, $IWOA$ algorithm exhibits better convergence performance than the WOA, $EWOA$ and $BWOA$ algorithms on most test functions.

To sum up, we can come to the conclusion that "$Equiangular\ spiral\ updating\ mechanism$" can be used to guide the further foraging of humpback whales, and lead to a more efficient search procedure than other $WOAs$ algorithm. Meanwhile, the "$steering\ law\ of\ sound\ wave\ attenuation$" of $IWOA$ algorithm can achieve the right equilibrium between the exploration and exploitation, and ensure the depth and extent of the global search. Therefore, $IWOA$ can increase the search precision, put down the number of failed search procedures and find a better solution at a higher speed. In order to increase the diversity of population and enhance the capability of exploratory pattern, $LWOA$ algorithm employs the l$\tilde{e}$vy flight mechanism to guide the swarm of humpback whales. This factor may put down the exploitation ability, and hence, reduce the promised search fruits of whales. $BWOA$ algorithm also uses two novel effective strategies (l$\tilde{e}$vy flight and chaotic local search) to coordinate the relation between the exploration and exploitation, but these strategies increase the computational burden of the $BWOA$ algorithm.

The non-parametric statistical Wilcoxon signed-rank test at 0.05 significant levels is used to estimate the statisti-

TABLE IV
COMPUTATIONAL RESULTS OF DIFFERENT WOAS ON MULTIMODAL FUNCTIONS.

Function	Result	IWOA	WOA	LWOA	EWOA	BWOA
f_7	Best	7.85e-293	4.96e-112	5.89e-285	1.38e-156	3.42e-292
	Mean	4.25e-257	7.06e-086	4.25e-235	8.63e-135	4.39e-257
	Worst	8.16e-241	3.55e-081	3.57e-229	6.22e-114	6.75e-242
	Std	3.59e-250	2.95e-083	1.55e-232	4.55e-127	1.82e-251
f_8	Best	**0**	**0**	**0**	**0**	**0**
	Best	**0**	**0**	**0**	**0**	**0**
	Best	**0**	**0**	**0**	**0**	**0**
	Best	**0**	**0**	**0**	**0**	**0**
f_9	Best	7.66e-016	8.92e-013	4.68e-014	3.26e-013	2.22e-015
	Mean	1.75e-015	2.46e-011	8.98e-012	2.14e-011	1.45e-013
	Worst	2.38e-014	1.45e-009	9.85e-010	9.97e-010	4.67e-012
	Std	3.01e-015	5.33e-010	8.45e-011	8.45e-010	4.35e-013
f_{11}	Best	0.0013262	0.0115472	0.0047225	0.0095344	0.0022473
	Mean	0.0565751	0.0527662	0.0874250	0.0934226	0.0463722
	Worst	0.0750144	0.3829915	0.1025898	0.1562175	0.0935881
	Std	0.0620146	0.0996435	0.0954250	0.1025614	0.0875647
f_{12}	Best	0.9999876	0.9979482	0.9985416	0.9984344	0.9999577
	Mean	1.5206538	2.6348402	4.9504671	5.8632752	2.2930442
	Worst	6.2033485	12.641925	13.026507	10.523942	4.3685241
	Std	1.9526419	2.9214228	4.0322405	3.9265624	3.8124021
f_{13}	Best	0.0003965	0.0004125	0.0004846	0.0004962	0.0004013
	Mean	0.0005863	0.0011392	0.0005538	0.0008875	0.0005916
	Worst	0.0006614	0.0027716	0.0008766	0.0022138	0.0006542
	Std	0.0003325	0.0008244	0.0003541	0.0007526	0.0003022

TABLE V
COMPUTATIONAL RESULTS OF DIFFERENT WOAS ON MULTIMODAL FUNCTIONS.

Function	Result	IWOA	WOA	LWOA	EWOA	BWOA
f_{14}	Best	0.3978873	0.3754226	0.3773076	0.3761932	0.3897556
	Mean	0.3978873	0.3567944	0.3675234	0.3691443	0.3753826
	Worst	0.3978873	0.3388245	0.3461538	0.3402675	0.3602435
	Std	0	1.795e-04	3.154e-03	1.232e-04	2.625e-03
f_{15}	Best	3.1592842	3.6003568	3.4665872	3.6012516	3.1162837
	Mean	3.2601473	3.9561057	3.8294364	3.9706221	3.2193456
	Worst	3.4003576	4.7625013	4.6704385	4.7345644	3.3235562
	Std	0.0392285	1.4461055	1.3628427	1.4530332	0.0375487
f_{16}	Best	-10.12350	-9.680575	-10.02968	-9.853446	-10.105223
	Mean	-9.214733	-8.042236	-8.709271	-8.107783	-9.026048
	Worst	-5.227436	-0.950811	-0.832497	-0.812675	-3.870434
	Std	1.216532	2.401735	2.232854	2.400247	1.433729
f_{17}	Best	-10.38175	-10.06573	-10.18775	-10.03471	-10.307418
	Mean	-10.16509	-8.530527	-9.931806	-8.630273	-10.162283
	Worst	-10.04238	-3.674533	-9.054278	-3.759052	-9.5287464
	Std	0.002304	2.629745	0.083144	2.741153	0.0726140
f_{18}	Best	-10.41832	-9.874283	-10.20644	-9.920653	-10.240317
	Mean	-9.968437	-9.508242	-9.816507	-9.573184	-10.037150
	Worst	-3.174463	-2.114735	-2.336240	-2.403362	-3.407526
	Std	1.864353	1.872533	2.053627	2.061031	1.763708

TABLE VI
COMPUTATIONAL RESULTS OF DIFFERENT WOAS ON COMPOSITE FUNCTIONS.

Function	Result	IWOA	WOA	LWOA	EWOA	BWOA
CF_1	Mean	0.073364	0.512635	0.205344	0.504331	0.278325
	Std	0.057365	0.495532	0.195613	0.487576	0.194263
CF_2	Mean	56.84637	69.78465	65.47352	69.15223	62.86775
	Std	37.06552	40.58632	39.37265	40.52164	39.01844
CF_3	Mean	38.47133	46.35272	41.55372	46.73162	42.64273
	Std	18.97417	20.13752	19.15137	20.08474	19.62415
CF_4	Mean	35.02546	43.27235	40.26443	42.85523	40.65208
	Std	18.20552	19.07341	18.52713	18.75261	18.66372
CF_5	Mean	52.14335	65.42735	59.87745	63.12752	60.56143
	Std	41.52750	47.30642	46.50644	48.06224	47.02571

cally significant difference between the $IWOA$ algorithm and other competitors. If the result of the corresponding algorithm is statistically significantly better than that of the $IWOA$ algorithm, this situation is represented by the "$-$" symbol. If the result is statistically comparable to that of the $IWOA$ algorithm, this situation is recorded by the "$\approx$" symbol. If the result of the $IWOA$ algorithm is statistically significantly better than that of the corresponding algorithm, this situation is represented by the "$+$" symbol. If the result of the corresponding algorithm and the $IWOA$ algorithm both achieve the same accuracy results, this situation does not need to estimate the statistically significant difference, and represented by the "NA" symbol which stands for Not Applicable.

In this test, the swarm size is set to be 30, and the iteration number is set to 500. The run results of the WOA algorithm is taken from [6]. Table 7 and Table 8 report the statistical significance level of the difference between the means of two algorithms. From Table 7 and Table 8, we can see that $IWOA$ algorithm is statistically better than $LWOA$, $EWOA$, and $BWOA$ algorithm in functions f_1, f_2, f_5, f_6, f_{10} and f_{12}. In functions f_{11}, $IWOA$ algorithm is statistically better than WOA, $LWOA$, and $EWOA$ algorithm, but statistically worse than $BWOA$. In functions f_4 and f_{16}, $IWOA$ algorithm is statistically better than WOA, $EWOA$, and $LWOA$ algorithm, but statistically comparable to $BWOA$. In functions f_3, f_8 and f_{14}, all algorithms achieve comparably performances.

In the second part of experiment, $IWOA$, standard PSO[24], $FPSO$[29], standard DE[25], and $JADE$[30] algorithms are compared, with a maximum of 1.5×10^4 fitness evaluations for each test function. The test functions used in this experimental are chosen from table 1. The run results of PSO, DE, WOA algorithms are taken from [6]. The comparison of solution accuracy is listed in Table 9. For test functions f_1, f_3, f_4, f_6, f_7, and f_8, the run result of $IWOA$ algorithm is obviously superior to PSO, DE, $FPSO$, and $JADE$ algorithm. In addition, the $IWOA$ algorithm works better in the standard deviation, which implies that the solution quality of $IWOA$ is stable. Other representative test functions, i.e., f_{10}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}, f_{18}, which contain shifted functions and rotated functions are listed for further testing the efficiency of the $IWOA$ algorithm. The result of comparison demonstrates that the "$steering\ law\ of\ sound\ wave\ attenuation\ mechanism$" can help humpback whales to produce higher quality solutions than state-of-the-art evolutionary algorithms, and increase exploitation and convergence ability of $IWOA$ to an extent. Meanwhile, "$Equiangular\ spiral\ updating\ mechanism$" is performed near the superior individuals produced in different generation. This rule can pull numerous humpback whales to swarm toward the different regions, and obtain a set of solutions with excellent diversity. As a summary, the results of this section revealed different characteristics of the proposed $IWOA$ algorithm.

D. Performance of IWOA on Engineering Design Problems

To very the performance of $IWOA$, it is used to solve 3 constrained optimization problems. And the best solution obtained by $IWOA$ is compared with some other intelligent algorithms. The first engineering problem is to design a tension compression spring (TCS) with minimum weight. This design must satisfy constraints on shear stress, surge frequency, and deflection. This problem can be modeled as follows:

$$\min f(d, D, N) = d^2DN + 2d^2D$$

subject to

$$1 - \frac{D^3N}{71785d^4} \le 0,$$

$$\frac{4D^2 - dD}{12566(Dd^3 - d^4)} + \frac{1}{5108d^2} \le 1,$$

$$1 - \frac{140.45d}{D^2N} \le 0,$$

$$\frac{d + D}{1.5} - 1 \le 0,$$

where

$$0.05 \le d \le 2.00, \quad 0.25 \le D \le 1.30, \quad 2.00 \le N \le 15.0.$$

In this model, there are three variables, wire diameter (d), mean coil diameter (D), and number of active coils (N). This problem has been optimized by many researchers using different methods like PSO[24], DE[25], GA[26], IHS[27], RO[31], CCM[32], MOM[33], $JSWOA$[21], $WOAmM$[22], $REM-WOA$[23],$BWOA$. The best solution and the optimal value obtained by each algorithm are recorded in Table 10. During the running process, $IWOA$ algorithm sets the swarm size to be 10, and the maximum iteration number is set to 500. The comparisons show that $IWOA$ outperforms other methods. The optimum weight obtained by $BWOA$ is less than $IWOA$, but to be clear,this design is a infeasible solution than that of all other comparison algorithms.

The second engineering problem is to design a pressure vessel, whose objective is to minimize the total cost (material, forming and welding) subjected to 4 inequalities constraints. Meanwhile, the both ends of vessel are covered, and the head has hemi-spherical shape. The model of this problem can be expressed as follows:

$$\begin{aligned} \min f(T_s, T_h, R, L) = 0.6224T_sRL + 1.7781R^2T_h \\ + 3.1661T_s^2L + 19.84T_s^2R \end{aligned}$$

subject to

$$\begin{aligned} &-T_s + 0.0193R \le 0, \\ &-T_h + 0.00954R \le 0, \\ &-\pi LR^2 - \frac{4}{3}\pi R^3 + 1296000 \le 0, \\ &-240 + L \le 0, \end{aligned}$$

where

$$0 \le T_s \le 99, \ 0 \le T_h \le 99, \ 10 \le R \le 200, \ 10 \le L \le 200,$$

In this model, there are four variables, the depth of the shell (T_s) and the head (T_h), the inner radius (R), and the length of the cylindrical section without considering the head

TABLE VII
STATISTICAL SIGNIFICANCE LEVEL OF THE DIFFERENT OF THE MEANS OF WOAS.

Function	Result	IWOA	WOA	LWOA	EWOA	BWOA
f_1	Mean	**4.55e-142**	1.41e-030	2.64e-099	1.35e-030	3.75e-110
$D = 30$	Std	**2.73e-137**	4.91e-030	1.78e-097	4.55e-030	2.33e-105
	Sign		+	+	+	+
f_2	Mean	**3.24e-183**	1.06e-021	2.44e-159	8.75e-022	5.53e-167
$D = 30$	Std	**1.75e-180**	2.39e-021	2.22e-155	2.13e-021	4.86e-162
	Sign		+	+	+	+
f_3	Mean	1.77e-007	5.39e-007	3.12e-007	4.25e-007	2.34e-007
$D = 30$	Std	1.31e-006	2.91e-006	2.65e-007	2.93e-006	2.00e-006
	Sign		≈	≈	≈	≈
f_4	Mean	**8.87e-202**	0.072581	3.27e-183	0.042581	1.02e-201
$D = 30$	Std	**3.45e-199**	0.397472	4.32e-199	0.362573	2.45e-200
	Sign		+	+	+	≈
f_5	Mean	**1.55443**	3.11626	2.82164	3.02535	2.10375
$D = 30$	Std	**0.27542**	0.53229	0.35276	0.38423	0.31642
	Sign		+	+	+	+
f_6	Mean	**3.74e-018**	0.001425	1.45e-004	8.95e-004	2.77e-012
$D = 30$	Std	**1.85e-016**	0.001149	7.87e-005	6.44e-004	3.46e-010
	Sign		+	+	+	+
f_8	Mean	0	0	0	0	0
$D = 30$	Std	0	0	0	0	0
	Sign		NA	NA	NA	NA

TABLE VIII
STATISTICAL SIGNIFICANCE LEVEL OF THE DIFFERENT OF THE MEANS OF WOAS.

Function	Result	IWOA	WOA	LWOA	EWOA	BWOA
f_{10}	Mean	**3.44e-113**	2.89e-055	2.45e-080	8.32e-069	6.32e-107
$D = 30$	Std	**2.07e-110**	1.59e-054	2.17e-075	3.87e-068	3.77e-104
	Sign		+	+	+	+
f_{11}	Mean	0.0704622	0.0753967	0.0784563	0.0816522	**0.0649335**
$D = 30$	Std	0.0253041	0.3314864	0.0306714	0.0344632	**0.0221704**
	Sign		+	+	+	-
f_{12}	Mean	**1.406214**	2.111973	1.774562	1.873645	1.584406
$D = 30$	Std	**1.220435**	2.498594	1.431543	2.125745	1.284361
	Sign		+	+	+	+
f_{13}	Mean	4.21e-004	5.72e-004	4.12e-004	4.87e-004	4.19e-004
$D = 30$	Std	2.04e-004	3.24e-004	3.06e-004	3.75e-004	2.16e-004
	Sign		+	-	+	≈
f_{14}	Mean	0.398	0.397914	0.397958	0.395714	0.398
$D = 30$	Std	0	2.70e-005	1.73e-005	2.45e-005	0
	Sign		≈	≈	≈	NA
f_{16}	Mean	-8.95374	-7.04918	-8.65804	-7.82758	-8.91732
$D = 30$	Std	1.46524	3.62955	2.74283	4.06321	2.13584
	Sign		+	+	+	≈

(L). This problem has been optimized by many methods like PSO, GA, DE, IHS[27], ES[28], $Branch-bound$[34], $Lagrangian-mul$[35], $JSWOA$, $WOAmM$, $REM-WOA$, $BWOA$. The optimum solution obtained by $IWOA$ algorithm is compared with other design results. During the running process of $IWOA$, the population size is set to 20, and the maximum iteration number is set to 500. Statistical optimization results obtained by different algorithm are listed in Table 11. The comparisons show that $IWOA$ outperforms all other methods. The minimum cost of pressure vessel can be 5946.3845 when the variables T_s, T_h, R, and L are set as $0.812361, 0.401551, 42.091257$, and 176.725695.

The third is a cantilever beam design problem, whose objective is to minimize the weight of a cantilever beam, subjected to 1 inequalities constraints. The five variable x_1, x_2, x_3, x_4, x_5 in this model are heights of the cross-section of each hollow blocks. This problem can be described as follows:

$$\min f(x_1, x_2, x_3, x_4, x_5) = 0.6224(x_1 + x_2 + x_3 + x_4 + x_5)$$

subject to

$$\frac{61}{x_1^3} + \frac{27}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0,$$

where

$$0.01 \leq x_1, x_2, x_3, x_4, x_5 \leq 100,$$

This problem has been optimized by many methods like PSO, GA, DE, $JSWOA$, $WOAmM$, and $BWOA$. The optimum sloution obtained by IWOA algorithm is compared with other design results in Table 12. During the running process, $IWOA$ algorithm sets the swarm size to be 15, and the maximum iteration number is set to 700. The minimum

TABLE IX
RESULT COMPARISONS OF DIFFERENT EVOLUTIONARY ALGORITHM.

Function	Result	IWOA	WOA	FPSO	PSO	JADE	DE
f_1	ave	0	1.41e-030	2.76e-064	1.36e-004	4.26e-086	8.20e-014
	Std	0	4.91e-030	3.24e-061	2.02e-004	3.47e-083	5.90e-014
f_2	ave	3.75e-246	1.06e-021	5.64e-178	0.047441	2.73e-239	1.50e-009
	Std	4.28e-242	2.39e-021	6.48e-173	0.045421	1.64e-233	9.90e-010
f_3	ave	0	5.39e-007	3.38e-012	70.12562	5.26e-087	6.80e-011
	Std	0	2.93e-006	4.76e-011	22.11924	6.63e-085	7.40e-011
f_4	ave	6.45e-165	0.07258	5.84e-035	1.086481	0	0
	Std	4.73e-160	0.39747	2.11e-033	0.317039	0	0
f_5	ave	0.506275	3.116266	1.25e-014	0.000102	0	0
	Std	0.26513	0.53249	4.86e-014	8.28e-005	0	0
f_6	ave	2.37e-006	0.001425	0.007425	0.122854	3.83e-006	0.00463
	Std	2.15e-006	0.001149	0.005846	0.044957	3.33e-006	0.0012
f_8	ave	0	0	13.79455	46.70423	23.46573	69.2
	Std	0	0	9.46574	11.62938	12.26833	38.8
f_9	ave	4.76e-012	7.4043	3.72e-016	0.276015	6.67e-022	9.70e-008
	Std	2.34e-009	9.897572	2.45e-012	0.50904	5.32e-021	4.20e-008
f_{10}	ave	2.22e-033	2.78e-004	5.64e-024	0.009215	0	0
	Std	1.96e-030	1.63e-003	3.43e-022	0.007724	0	0
f_{11}	ave	3.43e-035	0.339676	3.75e-031	0.006917	5.37e-040	7.90e-015
	Std	2.83e-030	0.214864	2.77e-031	0.026301	4.44e-042	8.00e-015
f_{12}	ave	1.356274	2.111973	2.015423	3.627168	1	0.998004
	Std	1.417358	2.498594	2.053342	2.560828	1.21e-020	3.30e-016
f_{13}	ave	3.44e-004	5.72e-004	4.27e-004	5.77e-004	3.35e-004	4.50e-014
	Std	2.15e-004	3.24e-004	1.98e-004	2.22e-004	1.88e-004	3.30e-004
f_{14}	ave	0.398	0.397914	0.398	0.397887	0.398	0.397887
	Std	3.45e-008	2.70e-005	0	0	4.35e-015	9.90e-009
f_{15}	ave	3	3	3	3	3	3
	Std	2.17e-034	4.22e-015	1.85e-027	1.33e-015	1.95e-022	2.00e-015
f_{16}	ave	-10.1322	-7.04918	-9.8557	-6.8651	-10.1532	-10.1532
	Std	1.75e-006	3.629551	2.35e-004	3.019644	1.23e-007	0.0000025
f_{17}	ave	-10.3557	-8.18178	-10.2251	-8.45653	-10.4028	-10.4029
	Std	2.11e-005	3.829202	1.95e-002	3.081094	2.49e-011	3.90e-007
f_{18}	ave	-10.5283	-9.34238	-10.5054	-9.95291	-10.5636	-10.5364
	Std	1.54e-003	2.414737	1.68e-003	1.782886	1.72e-009	1.90e-007

TABLE X
OPTIMUM DESIGNS OBTAINED BY DIFFERENT ALGORITHMS FOR TCS DESIGN PROBLEM.

Algorithm	Optimum Variables			Optimum	Feasible
	d	D	N	weight	solution
PSO	0.051728	0.357644	11.244543	0.0126747	Y
GA	0.051480	0.351661	11.632201	0.0127048	Y
DE	0.051609	0.354714	11.410837	0.0126702	Y
RO	0.051370	0.349096	11.762790	0.0126788	N
IHS	0.051154	0.349871	12.076432	0.0126706	N
CCM	0.050000	0.315900	14.250000	0.0128334	Y
MOM	0.053396	0.399180	9.1854000	0.0127303	Y
$BWOA$	0.051602	0.357488	11.244198	0.0126654	N
$JSWOA$	0.051611	0.354734	11.410945	0.0126720	Y
$WOAmM$	0.051608	0.354724	11.412445	0.0126716	Y
$REM-WOA$	0.051617	0.354753	11.413734	0.0126783	Y
$IWOA$	0.051606	0.354720	11.407158	0.0126655	Y

TABLE XI
OPTIMUM DESIGNS OBTAINED BY DIFFERENT ALGORITHMS FOR PV DESIGN PROBLEM.

Algorithm	Optimum Variables				Optimum	Feasible
	T_s	T_h	R	L	cost	solution
PSO	0.812500	0.437500	42.091266	176.746500	6061.0777	Y
GA	0.812500	0.437500	40.323900	200.000000	6288.7445	Y
DE	0.812500	0.437500	42.098411	176.637690	6059.7340	Y
ES	0.812500	0.437500	42.098370	176.637146	6059.7144	N
IHS	1.125000	0.625000	58.290150	43.6926800	7197.7300	N
$Branch\text{-}bound$	1.125000	0.625000	47.700000	117.701000	8129.1036	N
$Laagrangian\text{-}mul$	1.125000	0.625000	58.291000	43.6900000	7198.0428	N
$BWOA$	1.258663	0.621865	65.179120	10.1987370	7318.1690	Y
$JSWOA$	0.954722	0.510223	48.859274	107.661401	6485.7758	Y
$WOAmM$	1.258900	0.100000	65.220000	10.000000	3368.5000	N
$REM-WOA$	0.812500	0.437500	41.955027	178.422179	6077.2635	Y
$IWOA$	0.812361	0.401551	42.091257	176.725695	5946.3845	Y

TABLE XII
OPTIMUM DESIGNS OBTAINED BY DIFFERENT ALGORITHMS FOR CANTILEVER BEAM DESIGN PROBLEM.

Algorithm	Optimum Variables					Optimum	Feasible
	x_1	x_2	x_3	x_4	x_5	cost	solution
PSO	5.9513284	4.9214222	4.5581397	3.4261537	2.0907730	13.037921	Y
GA	5.8566353	4.8469272	4.8109691	3.9480581	1.9114557	13.303206	Y
DE	5.8087225	4.7114432	4.6782256	3.8852335	2.0451168	13.150529	Y
WOA	5.8255731	4.6981373	4.6578377	3.8682607	2.0184437	13.112880	Y
$BWOA$	5.9875344	4.8723062	4.4712103	3.4800182	2.1287477	13.032942	Y
$JSWOA$	5.8774185	4.7068225	4.6354451	3.7985503	2.0142775	13.090637	Y
$WOAmM$	5.9705657	4.8718027	4.4804933	3.4868702	2.1296397	13.032665	Y
$IWOA$	5.9712011	4.8871107	4.4782235	3.4775665	2.1254001	13.032745	Y

weight of cantilever beam can be 13.032745, when the variables x_1, x_2, x_3, x_4, and x_5, are set as 5.9712011, 4.8871107, 4.4782235, 3.4775665, and 2.1253999.

Based on the simulation results and analyses above, $IWOA$ outperforms all other comparative algorithms and it can offer a more efficient solution on three optimization problems. Therefore, $IWOA$ is capable and effective in solving these practical problems under a better tradeoff between the exploitation and exploration.

E. Performance of IWOA on Feature Selection

Feature selection is one of the major preprocessing steps in data mining since it aims to elimimate the redundant irrelevant variaables within a dataset. In this section, several different algorithms[36-39] are used to design different feature selection techniques and evaluate on 18 standard benchmark dataset from UCI repository. Classification accuracy and average selected attributes of $IWOA$, WOA, $WOASA-1$, $WOASA-2$, GA[37], PSO[38], ALO[39] algorithms are compared in table 13 and 14. In this section, the maximum fitness evaluations is set to 1.0×10^3. All datasets are conducted 5 times with random seed, and the means and standard deviations of the statistical experimental data are reported.

In the first part of experiment, the $IWOA$, standard WOA, $WOASA-1$, $WOASA-2$ algorithms are compared. The results of the $WOASA-1$, $WOASA-2$ algorithms are taken from [19]. For making a fair comparison, we also employ SA to further improve the best solution, found after each iteration of $IWOA$ performs. This rule can increace the exploitation capability by searching the most promising regions. The comparison of classification accuracy and selection attributes among the WOA, $IWOA$, $WOASA-1$, and $WOASA-2$ is listed in Tables 12. In the second part of experiment, other three representative evaluation algorithms, which contain ALO, GA, PSO algorithms are used for further testing the efficiency of the $IWOA$ algorithm. Table 13 show the experimental results of comparison with other three evaluation algorithms. The corresponding experimental results of the ALO, GA, PSO algorithms are excerpted from [37-39]. The results show that $IWOA$ performs better than other evaluation algorithms, and has strong superiority in terms of classification accuracy, convergence speed, and search precision on most of datasets. For example, in penglungew dataset, the classification accuracy of $IWOA$ has increase about 2%∼30%; in sonarew dataset, the classification accuracy of $IWOA$ has increase about 2%∼23%, comparing to the other algorithms. As a summary, the results of this section revealed good property of local feature selection behave of $IWOA$ algorithm. This property of $IWOA$ can be well used to solve feature selection problems by decreasing the redundant attributes in a dataset and reducing the search space.

TABLE XIII
CLASSIFICATION ACCURACY AND AVERAGE SELECTED ATTRIBUTES OBTAINED BY DIFFERENT WOA ALGORITHMS.

Dataset	Attributes	Instance	Accuracy				Attributes			
			WOA	IWOA	WOASA−1	WOASA−2	WOA	IWOA	WOASA−1	WOASA−2
Breastcancer	9	699	0.96	0.97	0.97	0.96	6.4	5.0	5.6	5.2
BreastEW	30	569	0.93	0.98	0.96	0.97	23.8	12.5	13.6	12.6
CongressEW	16	435	0.93	0.97	0.97	0.97	10	4.9	4.4	5.2
Exactly	13	1000	0.77	1.00	1.00	1.00	9.2	6.0	6.0	6.0
Exactly2	13	1000	0.74	0.74	0.73	0.72	4.8	1.8	1.0	1.4
HeartEW	13	270	0.79	0.88	0.79	0.84	9.4	6.1	6.2	7.2
ConosphereEW	34	351	0.87	0.98	0.92	0.96	22.4	11.5	11.4	11.8
KrvshpEW	36	3196	0.93	0.98	0.98	0.98	24.2	17.0	19.4	17.0
Lymphography	18	148	0.78	0.92	0.90	0.87	10.8	6.5	6.8	7.6
M-of-n	13	1000	0.91	1.00	1.00	1.00	8.6	6.0	6.0	6.0
PenglungEW	325	73	0.84	0.93	0.85	0.91	188.4	130.5	138	128.8
SonarEW	60	208	0.86	0.97	0.94	0.95	46.4	28.2	26.6	26.4
SpectEW	22	267	0.81	0.88	0.82	0.84	9.4	9.2	9.6	9.4
Tic-tac-toe	9	958	0.76	0.81	0.79	0.76	8.4	5.8	5.8	5.8
Vote	16	300	0.92	0.95	0.97	0.96	9.4	3.6	3.8	5.8
WaveformEW	40	5000	0.71	0.73	0.69	0.68	33.6	18.7	21.6	19.4
WineEW	13	178	0.95	1.00	0.99	0.99	7.4	6.5	6.8	6.8
Zoo	16	101	0.96	1.00	0.99	0.97	8.8	5.3	5.8	5.4

TABLE XIV
CLASSIFICATION ACCURACY AND AVERAGE SELECTED ATTRIBUTES OBTAINED BY DIFFERENT EVOLUTIONARY ALGORITHMS.

Dataset	Attributes	Instance	Accuracy				Attributes			
			ALO	IWOA	GA	PSO	ALO	IWOA	GA	PSO
Breastcancer	9	699	0.96	0.97	0.96	0.95	6.28	5.0	5.09	5.72
BreastEW	30	569	0.93	0.98	0.94	0.94	16.08	12.5	16.35	16.56
CongressEW	16	435	0.93	0.97	0.94	0.94	6.98	4.9	6.62	6.83
Exactly	13	1000	0.66	1.00	0.67	0.68	6.62	6.0	10.82	9.75
Exactly2	13	1000	0.75	0.74	0.76	0.75	10.70	1.8	6.18	6.18
HeartEW	13	270	0.83	0.88	0.82	0.78	10.31	6.1	9.49	7.94
ConosphereEW	34	351	0.87	0.98	0.83	0.84	9.42	11.5	17.31	19.18
KrvshpEW	36	3196	0.96	0.98	0.92	0.94	24.70	17.0	22.43	20.81
Lymphography	18	148	0.79	0.92	0.71	0.69	11.05	6.5	11.05	8.98
M-of-n	13	1000	0.86	1.00	0.93	0.86	11.08	6.0	6.83	9.04
PenglungEW	325	73	0.63	0.93	0.70	0.72	164.13	130.5	177.13	178.75
SonarEW	60	208	0.74	0.97	0.73	0.74	37.92	28.2	33.30	31.20
SpectEW	22	267	0.80	0.88	0.78	0.77	16.15	9.2	11.75	12.50
Tic-tac-toe	9	958	0.73	0.81	0.71	0.73	6.99	5.8	6.85	6.61
Vote	16	300	0.92	0.95	0.89	0.89	9.52	3.6	6.62	8.80
WaveformEW	40	5000	0.77	0.73	0.77	0.76	35.72	18.7	25.28	22.72
WineEW	13	178	0.91	1.00	0.93	0.95	10.70	6.5	8.63	8.36
Zoo	16	101	0.91	1.00	0.88	0.83	13.97	5.3	10.11	9.74

V. Conclusion

In this paper, we develop an improved algorithm, IWOA, to solve global numerical optimization problems by introducing an equiangular spiral search mechanism and an sound wave attenuation steering law. Equiangular spiral, which can better mimic the foraging trajectory of humpback whale and increase exploitation ability of the search agent, is employed to generate candidate solutions in IWOA. Additionally, with the guidance of sound wave attenuation steering law, IWOA algorithm can switch back and forth between the actively swim (exploitation) and the randomly swim (exploration), hence obtain a better tradeoff between the exploitation and exploration. Numerical experiments show the IWOA is very useful. In the next step, this method will be used to solve some practical problems.

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