A Comprehensive Analysis of Total and Semi-Total Graphs

Surekha Ravishankar Bhat, Ravishankar Bhat and Smitha Ganesh Bhat*

***Abstract*—Six new graphs are defined viz - Edge-Edge graph, total vertex edge graph, total edge vertex graph, semitotal and total clique polycliqual vertex graph, semitotal and total clique vertex graph and semitotal and total clique vertex edge graph arising from the given graph. Expressions for number of edges in the newly defined graphs are derived.**

***Index Terms*—VE graph, EV graph, EE graph, Semitotal and total graphs.**

I. Introduction

The line graph $L(H)$ of a graph H is a graph where each vertex represents an edge of H, and two vertices in $L(H)$ are adjacent if and only if the corresponding edges in H share a common endpoint. The elements of graph H, consisting of its edges and vertices, are collectively referred to as its constituents. The semitotal graph $t(H)$ defined by Sampathkumar and Chikodimath[11] is constructed from the graph H by considering a vertex set comprising both the vertices $V(H)$ and the edges $X(H)$. Two vertices in $t(H)$ are adjacent if and only if the corresponding vertices in H are adjacent or if the corresponding elements (vertex and edge) are incident. M. Behzad's [4] total graph $T(H)$ of a graph H is formed by considering a vertex set comprising both the vertices $V(H)$ and the edges $X(H)$. Two vertices in $T(H)$ are adjacent if and only if the corresponding elements (vertex and edge) in H are adjacent or incident.

V.R. Kulli [10] introduced and studied the properties of semitotal and total-block graphs. The semitotal-block graph $T_b(H)$ of a graph H is a graph that combines the vertex set of the original graph H, denoted by $V(H)$, with an additional set of blocks, denoted by $B(H)$. Each block vertex represents a connected subgraph, i.e., a block, of the original graph H. The vertex set of $T_b(H)$ is given by the union of the original vertices and the block vertices, i.e., $V(T_b(H)) = V(H) \cup B(H)$. The complete block graph $T_B(H)$ of a graph H is a graph that combines the vertex set of the original graph H, denoted by $V(H)$, with an additional set of blocks, denoted by $B(H)$. Each block vertex represents a connected subgraph, i.e., a block, of the original graph H. The vertex set of $T_B(H)$ is given by the union of the original vertices and the block vertices, i.e., $V(T_B(H)) = V(H) \cup B(H)$. The edges in $T_B(H)$ are determined based on the following condition: Two vertices, either from the set of block vertices $B(H)$ or the original vertex set $V(H)$, are adjacent in $T_B(H)$ if and only if they are adjacent or incident in the original graph H.

V.R.Kulli [10] defined the block vertex tree of a graph H, the block vertex tree $b_p(g)$ of a graph H is a tree with a vertex set consisting of the union of the set of block vertices $B(H)$ and the set of original vertices $V(H)$. Each vertex in $b_p(g)$ corresponds to either a block from $B(H)$ or an original vertex from $V(H)$. In other words, there is an edge between a vertex representing an original vertex and a vertex representing a block if and only if the original vertex is part of the corresponding block in the graph H. Later, Surekha R Bhat [13] created six new graphs originating from the given graph: semitotal block cutvertex graph, total block cutvertex graph, semitotal block vertex graph, total block vertex graph, semitotal block vertex edge graph, and total block vertex edge graph. The number of edges in the newly created graphs is expressed as an expression. We now define new graphs that arise from the provided graph, inspired by these definitions.

Furthermore, a comprehensive investigation into the characteristics of cliques in graph structures has been conducted by Surekha et.al [14], Sayinath Udupa N. V. [12] and Tana et. al [15]. In a parallel line of research, Isabel Cristina Lopes et. al [7] have also explored this intriguing topic.

II. Different types of graphs

Assume $H = (V, X)$ is a graph. For a vertex v and an edge x, v *m-dominates* x if $x \in \langle N[v] \rangle$ and x *m-dominates* v if $v \in N[x]$.

Remark II.1. *If v m-dominates x then x m-dominates v; but not conversely.*

For any $v \in V$ the *open neighborhood* $N(v) = \{u \in V | u \text{ is adjacent to } v\}$ and the *closed neighborhood* $N[v] = N(v) \cup \{v\}$. An induced subgraph of a graph is another graph, formed from a subset of the vertices of the graph and all of the edges (from the original graph) connecting pairs of vertices in that subset and is denoted by $\langle S \rangle$.

A set $S \subseteq V$ is a *vertex-edge dominating set* ($VED-set$) if every edge in H is m-dominated by a vertex in S. A set $F \subseteq X$ is an *edge-vertex dominating set* ($EVD-set$) if every vertex in H is m-dominated by an edge in F. A set D of elements of H is a *mixed dominating* set of H if every element not in D is m-dominated by an element in D. The *vertex-edge domination number* $\gamma_{ve} = \gamma_{ve}(H)$, *edge-vertex domination number* $\gamma_{ev} = \gamma_{ev}(H)$ and the *mixed domination number* $\gamma_m = \gamma_m(H)$ of a graph H are respectively the cardinality of a minimum VED-set, EVD-set and a mixed dominating set of H. R. S. Bhat et al. [5] defined the edge -edge domination number as follows. We say that an edge y is ee-adjacent to the edge x if $y \in \langle N[x] \rangle$. A set

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$L \subseteq X(H)$ is said to be *edge-edge dominating set* (EED-set) if every edge in $X - L$ is ee- dominated by an edge in L. The *edge-edge domination number* $\gamma_{ee} = \gamma_{ee}(H)$ is the cardinality of a minimum EED set of H.

A. Handhshaking Lemma

The following are some well known theorems in Graph Theory for our reference which gives an expression for the sum of all vertex degrees.

Theorem II.2. *For any (p, q) graph H,*

$$\sum_{v \in V(H)} d(v) = 2q.$$

The edge analogue of the handshaking lemma is discussed in [1], [2] and [3].

Theorem II.3. *For any (p, q) graph H,*

$$\begin{aligned}\sum_{x \in X(H)} d_e(x) &= \sum_{x=uv, x \in X(H)} (d(u) + d(v) - 2) \\ &= \sum_{u \in V(H)} (d(u))^2 - 2q.\end{aligned}$$

The vertex-edge degree, edge-vertex degree and edge-edge degree is defined by R. S. Bhat et al. [9]

B. VE- Degree

The $VE - degree$ of a vertex $v \in V(H)$, $d_{ve}(u)$ is the number of edges m-dominated by v or equivalently $d_{ve}(u)$ is the number of edges in $\langle N[u] \rangle$. Then $\Delta_{ve}(H)$ and $\delta_{ve}(H)$ denote the maximum and minimum VE-degrees of H repectively.

We observe that for any triangle free graph H, $d(u) = d_{ve}(u)$, $u \in V(H)$ and hence, $\Delta(H) = \Delta_{ve}(H)$.

We now give an expresion for the sum of the VE-degrees of all vertices (a result similar to the Handshaking Lemma)

Proposition II.4. *For any graph H of order p and size q with t triangles*

$$\sum_{u \in V(H)} d_{ve}(u) = 2q + 3t.$$

Corollary II.4.1. *For any triangle free graph H of order p and size q,*

$$\sum_{u \in V(H)} d_{ve}(u) = 2q.$$

C. EV- Degree

The $EV - degree$ of an edge $x \in X(H)$, $d_{ev}(x)$ is the number of vertices m-dominated by x or equivalently $d_{ev}(x)$ is the number of vertices in $N[x]$. Then $\Delta_{ev}(H)$ and $\delta_{ev}(H)$ denote the maximum and minimum EV-degrees of H respectively.

We observe that for any triangle free graph H, $d_{ev}(x) = d_e(x) + 2$, $x \in X(H)$ and hence, $\Delta_{ev}(H) = \Delta_e(H) + 2$.

We now give an expresion for the sum of the EV-degrees of all edges (a result similar to the Handshaking Lemma)

Proposition II.5. *For any graph H of order p and size q with t triangles*

$$\sum_{x \in X(H)} d_{ev}(x) = \sum_{u \in V(H)} (d(u))^2 - 3t.$$

Corollary II.5.1. *For any triangle free graph H of order p and size q,*

$$\sum_{x \in X(H)} d_{ev}(x) = \sum_{u \in V(H)} (d(u))^2.$$

D. EE- Degree

The $EE - degree$ of an edge $x \in X(H)$, $d_{ee}(x)$ is the number of edges e-dominated by x together with x or equivalently $d_{ee}(x)$ is the number of edges in $\langle N[x] \rangle$. Then $\Delta_{ee}(H)$ and $\delta_{ee}(H)$ denote the maximum and minimum EE-degrees of H repectively.

Lemma II.6.

$$\sum_{x=uv, x \in X(H)} [d_{ve}(u) + d_{ve}(u)] = \sum_{u \in V(H)} [d_{ve}(u) d(u)].$$

Proposition II.7. *For any graph H of order p and size q. Let t_1 be the number of triangles in H and t_2 be the number of quadrilaterals without induced triangles. Then,*

$$\sum_{x \in X(H)} d_{ee}(x) = \sum_{u \in V(H)} [d_{ve}(u) d(u)] - q - 6t_1 + 4t_2.$$

Corollary II.7.1. *If H is a graph which is free from triangles and quadrilaterals, then*

$$\sum_{x \in X(H)} d_{ee}(x) = \sum_{u \in V(H)} (d(u))^2 - q.$$

First of all consider VE-graph H_{ve} and EV-Graph H_{ev} which has been defined by S. S. Kamath [8]

E. VE- Graph H_{ve}

The vertex set of $H_{ve} = V(H) \cup X(H)$ and a vertex $v \in V(H)$ and an edge $x \in X(H)$ are adjacent in H_{ve} if and only if x is m-dominated by v.

We find the number of edges in VE-graph. R.S.Bhat and S.S.Kamath [9] obtained an expression for sum of VE degrees. Hence the total number of edges in H_{ve} is $q(H_{ve}) = 2q + 3t$ where t is the number of triangles in H.

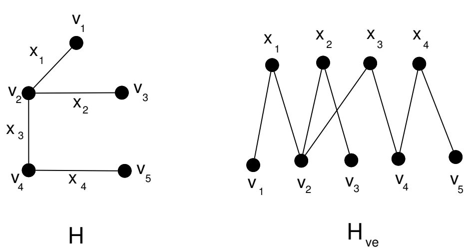

Fig. 1. A Graph H and its VE-Graph H_{ve}

Example II.1. *For the graph H of Fig. 1,* $q(H_{ve}) = 8 = 2(4) + 3(0)$.

F. EV- Graph H_{ev}

The vertex set of $H_{ev} = V(H) \cup X(H)$ and a vertex $v \in V(H)$ and an edge $x \in X(H)$ are adjacent in H_{ev} if and only if v is m-dominated by x.

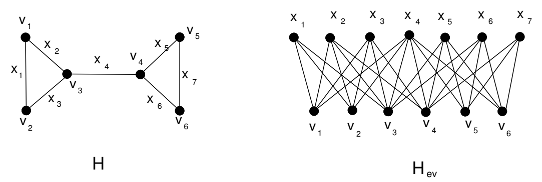

Fig. 2. A graph H and its EV- Graph H_{ev}

The total number of edges in H_{ev} is $q(H_{ev}) = \left[\sum_{u\in V(H)}(d(u))^2\right] - 3t$ where t is the number of triangles in H.

Example II.2. *For the graph H of Fig. 2,* $q(H_{ev}) = 28 = 34 - 3(2)$.

Definition II.8. The *line graph* of H denoted by $L(H)$ is a graph with vertex set as edges of H. Two vertices of $L(H)$ are adjacent whenever corresponding edges are adjacent in H.

We define the new graphs as follows.

G. EE- Graph H_{ee}

An edge x is ee-adjacent to y if and only if $y \in \langle N[x]\rangle$. The edges of H are vertices of H_{ee} and any two vertices in H_{ee} are adjacent if they are ee-adjacent. Note that if x is ee-adjacent to y then y need not be ee-adjacent to x. Therefore H_{ee} is a digraph.

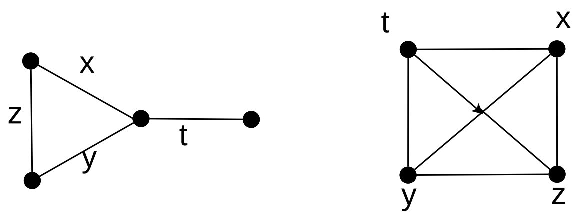

Fig. 3. A graph H and its EE- Graph H_{ee}

Lemma II.9. *For any graph H, the number of unidirectional edges k, in H_{ee} is equal to number of induced $K_3 \circ K_2$ in H.*

Proof: Let H has a $K_3 \circ K_2$ as induced subgraph and x, y, z be the edges of the triangle in $K_3 \circ K_2$ and t be the edge of K_2. Then t ee-dominate z but z doesnot ee-dominate t. Therefore H_{ee} has a directed edge from t to z.

k =*number of directed edges in* H_{ee}=*number of* $K_3 \circ K_2$. ■

Note II.10. *If H is free from $K_3 \circ K_2$ then H_{ee} is a graph.*

Proposition II.11.

$$\sum_{x\in X(H)} d_{ee}(x) = \sum_{v\in H_{ee}} d(v) = 2q_{ee} + \overrightarrow{q_{ee}}.$$

Note II.12. *In The graph H_{ee} of Fig. 4, x_2 is ee-adjacent to x_5 but the edge x_5 is not ee-adjacent to x_2. Similarly x_4 is ee-adjacent to x_1 but the edge x_1 is not ee-adjacent to x_4.*

Proposition II.13. *For any graph H of order p and size q with t_1 triangles and t_2 quadrilateral without induced*

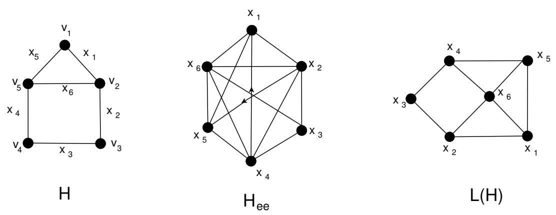

Fig. 4. A Graph H and its EE- Graph H_{ee} and Line Graph $L(H)$

triangles, the number of arcs in H_{ee},

$$\overrightarrow{q}_{ee} = \overrightarrow{q}(H_{ee}) = \left[\sum_{u\in V(H)} d_{ve}(u)d(u) - 2q - 6t_1 + 4t_2\right].$$

Proof:

$$\overrightarrow{q_{ee}} = \sum_{v\in H_{ee}} d(u),$$

$$= \sum_{x\in X(H)} d_{ee}(x),$$

$$= \left[\sum_{u\in V(H)} d_{ve}(u)d(u) - 2q - 6t_1 + 4t_2\right].$$

■

Example II.3. *For the graph H of Fig. 4,* $\overrightarrow{q}(H_{ee}) = 24 {=} \sum d_{ee}(x)$.

Total block cutvertex graph has been defined and studied by Surekha R Bhat et.al. [13]. On similar lines we define Total VE-graph.

H. Total Vertex Edge Graph

The total vertex edge graph $T_{ve}(H)$ has vertex set $V(H) \cup X(H)$ and two vertices of $T_{ve}(H)$ are adjacent if and only if the corresponding constituents (vertices and edges) are ee-adjacent or ve-adjacent or adjacent. It is immediate that $T_{ve}(H) = H_{ee} \cup H_{ve} \cup H$.

Proposition II.14. *For any graph H of order p and size q with t_1 triangles and t_2 quadrilateral without induced triangles, the number of arcs in T_{ve} is,*

$$\overrightarrow{q}(T_{ve}(H)) = \sum_{u\in V(H)} d_{ve}(u)d(u) + 4t_2 + 4q.$$

Proof:

Then the total number of edges in $T_{ve}(H)$ is

$$\begin{aligned} q(T_{ve}) &= q(H_{ee}) + q(H_{ve}) + q(H), \\ \overrightarrow{q}(T_{ve}) &= \left[\sum_{u\in V(H)} d_{ve}(u)d(u) - 2q - 6t_1 + 4t_2\right] \\ &\quad + (2q + 3t_1 + q)2, \\ &= \sum_{u\in V(H)} d_{ve}(u)d(u) + 4t_2 + 4q. \end{aligned}$$

■

Corollary II.14.1. *For any graph H of order p and size q free from triangles and quadrilaterals,*

$$\overrightarrow{q}(T_{ve}(H)) = \sum_{u\in V(H)} (d(u))^2 + 4q.$$

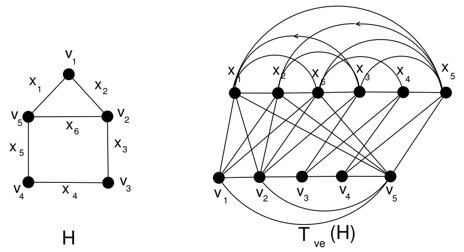

Fig. 5. A Graph H and its Total Vertex Edge Graph $T_{ve}(H)$

Example II.4. *For the graph H of Fig. 5, the number of triangles=1, the number of quadrilaterals without induced triangles=1, $q = 6$, $\sum_{u\in V(H)} d_{ve}(u)d(u) = 38$. Therefore $q(T_{ve}) = 66 = 38+4+24 = \sum_{u\in V(H)} d_{ve}(u)d(u)+4t_2+4q$.*

I. Total Edge Vertex Graph

The constituents are vertices and edges. The total edge vertex graph $T_{ev}(H)$ has vertex set $V(H) \cup X(H)$ and two vertices of $T_{ev}(H)$ are adjacent if and only if the corresponding constituents are ee-adjacent or ev-adjacent or adjacent. It is immediate that $T_{ev}(H) = H_{ee} \cup H_{ev} \cup H$.

Proposition II.15. *For any graph H of order p and size q with t_1 triangles and t_2 quadrilaterals without induced triangles,*

$$q(T_{ev}(H)) = \frac{1}{2}\left[\sum_{u\in V(H)} d_{ve}(u)d(u) + 4t_2\right] + \frac{1}{2}\left[\sum_{u\in V(H)} (d(u))^2\right] - 6t_1.$$

Proof: Then the total number of edges in $T_{ev}(H)$ is

$$q(T_{ev}) = q(H_{ee}) + q(H_{ev}) + q(H).$$

$$\begin{aligned} q(T_{ev}) &= \frac{1}{2}\left[\sum_{u\in V(H)} d_{ve}(u)d(u) - q - 6t_1 + 4t_2\right] \\ &\quad - \frac{q}{2} + \frac{1}{2}\left[\sum_{u\in V(H)} (d(u))^2\right] - 3t_1 + q \\ &= \frac{1}{2}\left[\sum_{u\in V(H)} d_{ve}(u)d(u) + 4t_2\right] - \frac{q}{2} \\ &\quad + \frac{1}{2}\left[\sum_{u\in V(H)} (d(u))^2\right] + \frac{q}{2} \\ &= \frac{1}{2}\left[\sum_{u\in V(H)} d_{ve}(u)d(u) + 4t_2\right] \\ &\quad + \frac{1}{2}\left[\sum_{u\in V(H)} (d(u))^2\right] - 6t_1. \end{aligned}$$

■

Corollary II.15.1. *For any graph H of order p and size q free from triangles and quadrilaterals,*

$$q(T_{ev}(H)) = \frac{3}{2}\left[\sum_{u\in V(H)} (d(u))^2\right].$$

Proof:

The total number of edges in $T_{ev}(H)$ is $q(T_{ev}) = q(H_{ee}) + q(H_{ev}) + q(H)$

$$\begin{aligned} q(T_{ev}) &= \frac{1}{2}\left[\sum_{u\in V(H)} (d(u))^2 - q\right] - \frac{q}{2} \\ &\quad + \left[\sum_{u\in V(H)} (d(u))^2\right] + q, \\ &= \frac{3}{2}\left[\sum_{u\in V(H)} (d(u))^2\right]. \end{aligned}$$

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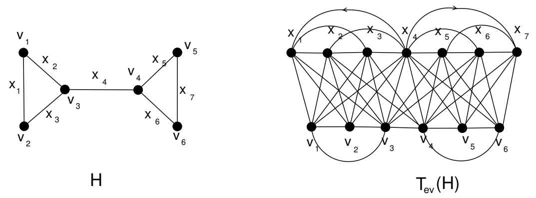

Fig. 6. A Graph H and its Total Edge Vertex Graph $T_{ev}(H)$

Example II.5. *For the graph H of Fig. 6, the number of triangles=2, the number of quadrilaterals without induced triangle=0 and $\sum_{u\in V(H)} d_{ve}(u)d(u) = 48$, $\sum_{u\in V(H)}(d(u))^2 = 34$. Therefore $q(T_{ev}) = 47 = \frac{1}{2}(48) + 34 - 12 = \frac{1}{2}\left[\sum_{u\in V(H)} d_{ve}(u)d(u) + 4t_2\right] + \sum_{u\in V(H)}(d(u))^2 - 6t_1$.*

J. EC Degree

The *ec-degree* (*edge clique-degree*) $d_{ec}(x)$ of an edge x is the number of cliques containing the edge x.

A vertex of H is called *unicliqual* if it is incident to only one clique in H. If v is incident on more than one clique we call it a *polyclical vertex*. Let $P_C(H)$ denote the set of all polycliqual vertices of H. By a *polycliqual vertex graph* $PV(H)$ we mean a graph with vertex set $P_C(H)$ and any two vertices in $PV(H)$ are adjacent if corresponding vertices in H have a clique in common.

Proposition II.16. *The number of edges in polycliqual vertex graph is*

$$q(PV(H)) = \sum_{k\in K(H)} \binom{d_{p_c}(k)}{2} - \sum_{x\in X(H)} [d_{ec}(x) - 1].$$

Proof: Since all the polycliqual vertices incident to a clique are mutually adjacent, every clique k yields $\binom{d_{p_c}(k)}{2}$ edges in $PV(H)$. Thus $q(PV(H)) = \sum_{k\in K(H)} \binom{d_{p_c}(k)}{2}$. But each polycliqual edge is counted twice. Thus we subtract one from $\sum_{x\in X(H)} d_{ec}(x)$. Thus total number of edges in PV(H) is $\sum_{k\in K(H)} \binom{d_{p_c}(k)}{2} - \sum_{x\in X(H)}[d_{ec}(x)-1]$. ■

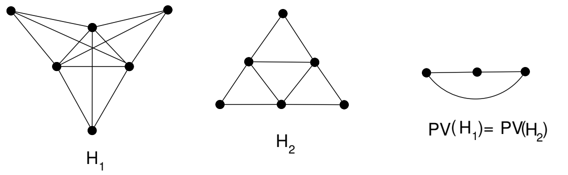

Fig. 7. Graphs H_1, H_2 and its Polycliqual Vertex Graph

Example II.6. *For the graph H_1 of Fig. 7, $d_{p_c}(k)^2 = 21$, $d_{p_c}(k) = 9$ and $d_{ec}(x) = 2$. Thus $\sum_{k\in K(H)} \binom{d_{p_c}(k)}{2} - \sum_{x\in X(H)}[d_{ec}(x)-1] = 6-3 = 3$. Number of edges in PV(H) is also 3.*

Similarly for the graph H_2 of Fig. 7, $d_{p_c}(k) = 3$ and $d_{ec}(x) = 3$. Thus $\sum_{k\in K(H)} \binom{d_{p_c}(k)}{2} - \sum_{x\in X(H)}[d_{ec}(x)-1] = 9-6 = 3$. Number of edges in PV(H) is 3.

K. Semitotal Clique-Polycliqual Vertex Graph and Total-Clique-Polycliqual Vertex Graph

The Semitotal clique polycliqual vertex graph $T_{kp_c}(H)$ of a graph H is a graph with vertex set $K(H)\cup P_C(H)$ and any two vertices in $T_{kp_c}(H)$ are adjacent if and only if the corresponding polycliqual vertices are adjacent or the corresponding constituents are incident. It is immediate that $T_{kp_c}(H) = CPV(H)\cup PV(H)$.

The total clique polycliqual vertex graph $T_{KP_C}(H)$ of a graph H is a graph with vertex set $K(H)\cup P_C(H)$ and any two vertices in $T_{KP_C}(H)$ are adjacent if and only if the corresponding constituents are adjacent or incident. Again we note that $T_{KP_C}(H) = CV(H)\cup PV(H)\cup K_H(H)$.

Theorem II.17. *Let H be a graph with k cliques and p_c polycliqual vertices. Let q_{kp_c} denote number of edges in $T_{kp_c}(H)$. Then,*

$$k+p_c-1+\frac{1}{2}\sum_{k\in K(H)}[(d_{p_c}(k))^2-d_{p_c}(k)] - \left[\sum_{x\in X(H)} d_{ec}(x)-1\right] \le q_{kp_c} \le kp_c + \frac{1}{2}\sum_{k\in K(H)}[(d_{p_c}(k))^2-d_{p_c}(k)] - \left[\sum_{x\in X(H)} d_{ec}(x)-1\right].$$

Proof: For a clique complete graph, the clique polycliqual vertex graph, CPV(H) is a complete bipartite graph. The number of edges in a bipartite graph is kp_c. This yields the upper bound.

If the given graph is a clique tree, then the graph is a block graph. Thus the number of edges in a clique tree is $k+p_c-1$. This yields the lower bound. Thus

$$\begin{aligned} q_{kp_c} &= q(CPV(H)) + q(PV(H)) \\ &\ge k+p_c-1+\sum_{k\in K(H)}\binom{d_{p_c}(k)}{2} \\ &\quad -\sum_{x\in X(H)}[d_{ec}(x)-1] \\ &\ge k+p_c-1+\sum_{k\in K(H)}[(d_{p_c}(k))^2-d_{p_c}(k)] \\ &\quad -\sum_{x\in X(H)}[d_{ec}(x)-1]. \end{aligned}$$

Similarly,

$$\begin{aligned} q_{kp_c} &= q(CPV(H)) + q(PV(H)), \\ &\le kp_c + \sum_{k\in K(H)}\binom{d_{p_c}(k)}{2} - \sum_{x\in X(H)}[d_{ec}(x)-1], \\ &\le kp_c + \sum_{k\in K(H)}[(d_{p_c}(k))^2-d_{p_c}(k)] \\ &- \sum_{x\in X(H)}[d_{ec}(x)-1]. \end{aligned}$$

■

Theorem II.18. *Let H be a graph with k cliques and p_c polycliqual vertices. Let q_{KP_C} denote number of edges in $T_{KP_C}(H)$. Then,*

$$\begin{aligned} &k+p_c-1+\frac{1}{2}\left[\sum(d_{vc}(p_c))^2-d_{vc}(p_c)\right] \\ &+\frac{1}{2}\left[\sum(d_{p_c}(k)^2-d_{p_c}(k)\right] - \left[\sum_{x\in X(H)} d_{ec}(x)-1\right] \le \\ &q_{KP_C} \le kp_c + \frac{1}{2}\left[\sum(d_{vc}(p_c))^2-d_{vc}(p_c)\right] \\ &+\frac{1}{2}\left[\sum(d_{p_c}(k)^2-d_{p_c}(k)\right] - \left[\sum_{x\in X(H)} d_{ec}(x)-1\right]. \end{aligned}$$

Proof: The number of edges in $T_{KP_C}(H)$ is,

$$\begin{aligned} q_{KP_C} &= q(CPV(H)) + q(K_H(H)) + q(PV(H)) \\ &\le kp_c + \sum_{p_c\in P_C(H)}\binom{d_{vc}(p_c)}{2} + \sum_{k\in K(H)}\binom{d_{p_c}(k)}{2} \\ &\qquad - \left[\sum_{x\in X(H)} d_{ec}(x)-1\right] \\ &\le kp_c + \frac{1}{2}\left[\sum(d_{vc}(p_c))^2-d_{vc}(p_c)\right] \\ &\qquad +\frac{1}{2}\left[\sum(d_{p_c}(k)^2-d_{p_c}(k)\right] \\ &\qquad - \left[\sum_{x\in X(H)} d_{ec}(x)-1\right]. \end{aligned}$$

Similarly,

$$\begin{aligned}
q_{KP_C} &\geq k + p_c - 1 + \sum_{p_c \in P_C(H)} \binom{d_{vc}(p_c)}{2} \\
&+ \sum_{k \in K(H)} \binom{d_{p_c}(k)}{2} - \left[\sum_{x \in X(H)} d_{ec}(x) - 1 \right] \\
&\geq k + p_c - 1 + \frac{1}{2}\left[\sum (d_{vc}(p_c))^2 - d_{vc}(p_c)\right] \\
&+ \frac{1}{2}\left[\sum (d_{p_c}(k)^2 - d_{p_c}(k)\right] - \\
&\left[\sum_{x \in X(H)} d_{ec}(x) - 1 \right].
\end{aligned}$$

■

Corollary II.18.1. *For a block graph the above theorem holds.*

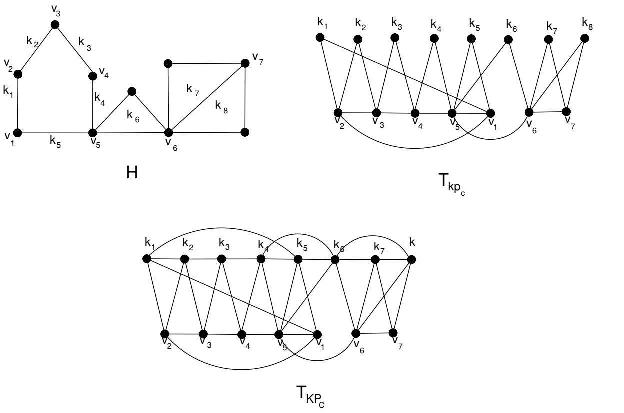

Fig. 8. A Graph H and its Semitotal and Total clique-polycliqual vertex graph

Example II.7. For the graph T_{kp_c} of Fig. 8, $q_{kp_c} = 23$. $k = 8, p_c = 7, \sum_{k \in K(H)} (d_{p_c}(k))^2 = 32, \sum_{k \in K(H)} d_{p_c} = 16, \left[\sum_{x \in X(H)} d_{ec}(x) - 1\right] = 1$. Therefore $k + p_c - 1 + \frac{1}{2}\sum_{k \in K(H)}[(d_{p_c}(k))^2 - d_{p_c}(k)] - \left[\sum_{x \in X(H)} d_{ec}(x) - 1\right] \leq q_{kp_c} \leq kp_c + \frac{1}{2}\sum_{k \in K(H)}[(d_{p_c}(k))^2 - d_{p_c}(k)] - \left[\sum_{x \in X(H)} d_{ec}(x) - 1\right]$ gives $8 + 7 - 1 + \frac{1}{2}[32 - 16] - 1 = 21 \leq 23 \leq 8 * 7 + \frac{1}{2}[32 - 16] - 1 = 69$.

For the graph T_{KP_C} of Fig. 8, $q_{KP_C} = 33. k = 8, p_c = 7, \sum_{k \in K(H)} (d_{p_c}(k))^2 = 32, \sum_{k \in K(H)} d_{p_c} = 16, \left[\sum_{x \in X(H)} d_{ec}(x) - 1\right] = 1$ *and* $\sum_{p_c \in P_C(H)} (d_{vc}(p_c))^2 = 38, \sum_{p_c \in P_C(H)} d_{vc}(p_c) = 16$.
Therefore, $k + p_c - 1 + \frac{1}{2}\left[\sum (d_{vc}(p_c))^2 - d_{vc}(p_c)\right] + \frac{1}{2}\left[\sum (d_{p_c}(k)^2 - d_{p_c}(k)\right] - \left[\sum_{x \in X(H)} d_{ec}(x) - 1\right] \leq q_{KP_C} \leq kp_c + \frac{1}{2}\left[\sum (d_{vc}(p_c))^2 - d_{vc}(p_c)\right] + \frac{1}{2}\left[\sum (d_{p_c}(k)^2 - d_{p_c}(k)\right] - \left[\sum_{x \in X(H)} d_{ec}(x) - 1\right]$ gives $8 + 7 - 1 + \frac{1}{2}[38 - 16] + \frac{1}{2}[32 - 16] - [2 - 1] = 32 \leq 33 \leq 8 * 7 + \frac{1}{2}[38 - 16] + \frac{1}{2}[32 - 16] - [2 - 1] = 74$.

L. Semitotal Clique Vertex Graph and Total Clique Vertex Graph.

The Semitotal clique vertex graph $T_{kv}(H)$ of a graph H is a graph with vertex set $K(H) \cup V(H)$ and any two vertices in $T_{kv}(H)$ are adjacent if and only if the corresponding vertices are vv-adjacent or the corresponding constituents (cliques and vertices) are incident. It is immediate that $T_{kv}(H) = P_H(H) \cup CV(H)$.

The total clique vertex graph $T_{KV}(H)$ of a graph H is a graph with vertex set $K(H) \cup V(H)$ and any two vertices in $T_{KV}(H)$ are adjacent if and only if the corresponding constituents are vv-adjacent or adjacent or incident. It is immediate that $T_{KV}(H) = P_C(H) \cup CV(H) \cup K_H(H)$.

Theorem II.19. *Let H be a graph with k cliques and p vertices. Let q_{kv} denote number of edges in $T_{kv}(H)$. Then,*

$$\begin{aligned}
&k + p - 1 + \frac{1}{2}\left[\sum_{h \in B(H)} (d_{bv}(h)^2 - (p + m - 1)\right] \\
&\leq q_{kv} \leq kp + \frac{1}{2}\left[\sum_{h \in B(H)} (d_{bv}(h)^2 - (p + m - 1)\right].
\end{aligned}$$

Proof:

$$\begin{aligned}
q_{kv} &= q(P_H(H)) + q(CV(H)) \\
&\geq \sum_{h \in B(H)} \binom{d_{bv}(h)}{2} + k + p - 1 \\
&\geq \frac{1}{2}\left[\sum_{h \in B(H)} (d_{bv}(h)^2 - (p + m - 1)\right] + k + p - 1.
\end{aligned}$$

To establish the upper bound we have

$$\begin{aligned}
q_{kv} &= q(P_H(H)) + q(CV(H)), \\
&\leq \sum_{h \in B(H)} \binom{d_{bv}(h)}{2} + kp, \\
&\leq \frac{1}{2}\left[\sum_{h \in B(H)} (d_{bv}(h)^2 - (p + m - 1)\right] + kp.
\end{aligned}$$

■

Theorem II.20. *Let H be a graph with k cliques and p vertices. Let q_{KV} denote number of edges in $T_{KV}(H)$. Then*

$$\begin{aligned}
&k + p - 1 + \frac{1}{2}\left[\sum (d_{bv}(h))^2 - d_{bv}(h)\right] \\
&\quad + \frac{1}{2}\left[\sum (d_{vc}(p_c))^2 - d_{vc}(p_c)\right] \\
&\leq q_{KV} \leq kp + \frac{1}{2}\left[\sum (d_{bv}(h))^2 - d_{bv}(h)\right] \\
&\quad + \frac{1}{2}\left[\sum (d_{vc}(p_c))^2 - d_{vc}(p_c)\right].
\end{aligned}$$

Proof:

$$\begin{aligned} q_{KV} &= q(P_H(H)) + q(CV(H)) + q(K_H(H)), \\ &\geq \sum_{h\in B(H)} \binom{d_{bv}(h)}{2} + k + p - 1, \\ &\quad + \sum_{p_c\in P_C(H)} \binom{d_{vc}(p_c)}{2} \\ &\geq \frac{1}{2}\left[\sum_{h\in B(H)} (d_{bv}(h)^2 - (p+m-1))\right] \\ &\quad + k + p - 1 \\ &\quad + \frac{1}{2}\left[\sum_{p_c\in P_C(H)} (d_{vc}(p_c))^2 - d_{vc}(p_c)\right]. \end{aligned}$$

To establish the upper bound we have

$$\begin{aligned} q_{KV} &= q(P_H(H)) + q(CV(H)) + q(K_H(H)) \\ &\leq \sum_{h\in B(H)} \binom{d_{bv}(h)}{2} + kp + \sum_{p_c\in P_C(H)} \binom{d_{vc}(p_c)}{2} \\ &\leq \frac{1}{2}\left[\sum_{h\in B(H)} (d_{bv}(h)^2 - (p+m-1)\right] + kp \\ &\quad + \frac{1}{2}\left[\sum_{p_c\in P_C(H)} (d_{vc}(p_c))^2 - d_{vc}(p_c)\right]. \end{aligned}$$

■

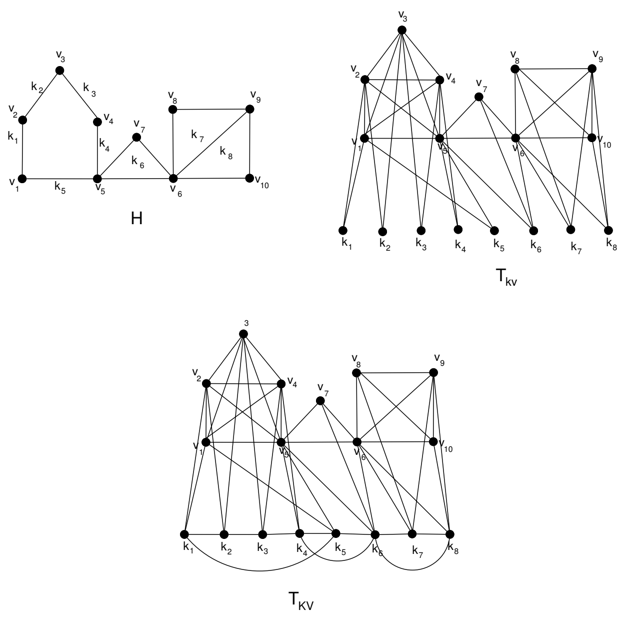

Fig. 9. A Graph H and its Semitotal and Total Clique Vertex Graph

Example II.8. For the graph T_{kv} of Fig. 9, $q_{kv} = 38$. $k = 8, p = 10, \sum_{h\in B(H)}(d_{bv}(h))^2 = 52$ and $m = 3$. Therefore $k+p-1+\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-(p+m-1)\right] \leq q_{kv} \leq kp+\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-(p+m-1)\right]$ gives $8+10-1+\frac{1}{2}[52-12] = 37 \leq 38 \leq 8*10+\frac{1}{2}[52-12] = 100$.

For the graph T_{KV} of Fig. 9, $q_{KV} = 48$, $k = 8$, $p = 10$, $\sum_{h\in B(H)}(d_{bv}(h))^2 = 52$, $m = 3$ and $\sum_{p_c\in P_C(H)}(d_{vc}(p_c))^2 = 32$, $\sum_{p_c\in P_C(H)} d_{vc}(p_c) = 16$. Therefore $k + p - 1 + \frac{1}{2}\left[\sum(d_{bv}(h))^2 - d_{bv}(h)\right] + \frac{1}{2}\left[\sum(d_{vc}(p_c))^2 - d_{vc}(p_c)\right] \leq q_{KV} \leq kp + \frac{1}{2}\left[\sum(d_{bv}(h))^2 - d_{bv}(h)\right] + \frac{1}{2}\left[\sum(d_{vc}(p_c))^2 - d_{vc}(p_c)\right]$ gives $8+10-1+\frac{1}{2}[52-12]+\frac{1}{2}[32-16] = 45 \leq 48 \leq 8*10+\frac{1}{2}[52-12]+\frac{1}{2}[32-16] = 108$.

Definition II.21. A *clique edge graph* $C_e(H)$ is a bigraph with vertex set as $K(H)\cup X(H)$ and a clique $k \in K(H)$ and an edge $x \in X(H)$ are adjacent in $C_e(H)$ if and only if the edge x is contained in the clique k.

Thus the number of edges in $C_e(H)$ is $\sum_{x\in X(H)} d_{ec}(x)$

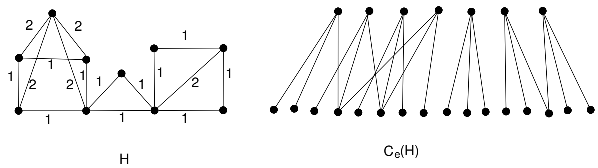

Fig. 10. A Graph H and its Clique Edge Graph $C_e(H)$

Example II.9. *The EC-degree of different edges is shown in the Fig. 10. Adding all those we get* $\sum_{x\in X(H)} d_{ec}(x) = 21$. *The total number of edges in* $C_e(H)$ *is also 21.*

Definition II.22. A *vertex edge graph* $V_e(H)$ is a bigraph with vertex set as $V(H)\cup X(H)$ and a vertex $v \in V(H)$ and an edge $x \in X(H)$ are adjacent in $V_e(H)$ if and only if the vertex v is incident on the edge x.

M. Semitotal Clique Vertex Edge Graph and Total Clique Vertex Edge Graph

The cliques, vertices and edges are called its constituents. The Semitotal clique vertex edge graph $T_{kve}(H)$ of a graph H is a graph with vertex set $K(H)\cup V(H)\cup X(H)$ and any two vertices in $T_{kve}(H)$ are adjacent if and only if the corresponding vertices are vv-adjacent or the corresponding constituents are incident. It is immediate that $T_{kve}(H) = P_H(H)\cup C_e(H)\cup V_e(H)\cup CV(H)$.

The total clique vertex edge graph $T_{KVE}(H)$ of a graph H is a graph with vertex set $K(H)\cup V(H)\cup X(H)$ and any two vertices in $T_{KVE}(H)$ are adjacent if and only if the corresponding constituents are vv-adjacent or adjacent or incident. It is immediate that $T_{KVE}(H) = T_{kve}\cup L(H)\cup K_H(H)$.

Theorem II.23. *Let* H *be a graph with* k *cliques and* p *vertices. Let* q_{kve} *denote number of edges in* $T_{kve}(H)$*. Then,*

$$\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-(p+m-1)\right]+\sum_{x\in X(H)}d_{ec}(x)$$
$$+2q+k+p-1\leq q_{kve}\leq$$
$$\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-(p+m-1)\right]$$
$$+\sum_{x\in X(H)}d_{ec}(x)+2q+kp.$$

Proof: Since each edge is incident on two vertices, there are $2q$ edges in vertex edge graph $V_e(H)$. Then

$$\begin{aligned}q_{kve}&=q(P_H(H))+q(C_e(H))+q(V_e(H))+q(CV(H)),\\&\geq\sum_{h\in B(H)}\binom{d_{bv}(h)}{2}+\sum_{x\in X(H)}d_{ec}(x)+2q\\&\quad+(k+p-1),\\&\geq\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-d_{bv}(h)\right]+\sum_{x\in X(H)}d_{ec}(x)\\&\quad+2q+k+p-1.\end{aligned}$$

Similarly

$$\begin{aligned}q_{kve}&\leq\sum_{h\in B(H)}\binom{d_{bv}(h)}{2}+\sum_{x\in X(H)}d_{ec}(x)+2q+(kp)\\&\leq\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-d_{bv}(h)\right]+\sum_{x\in X(H)}d_{ec}(x)\\&\quad+2q+kp.\end{aligned}$$

■

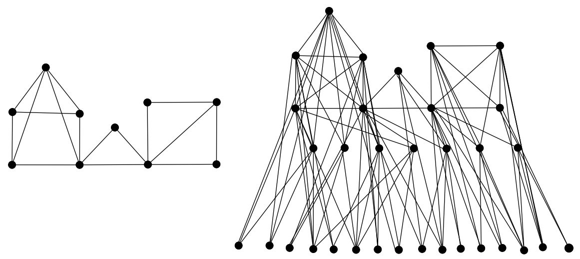

Fig. 11. A Graph H and its Semitotal Clique Vertex Edge Graph

Example II.10. *For the graph H of Fig. 11* $q(P_H(H))=19$, $q(C_e(H))=21$, $q(V_e(H))=32$, $q(CV(H))=21$ *and* $q_{kve}=93$. $\sum_{h\in B(H)}(d_{bv}(h))^2=50$, $\sum_{h\in B(H)}d_{bv}(h)=12$, $\sum_{x\in X(H)}d_{ec}(x)=21$, $q=16$, $k=7$ *and* $p=10$. *Thus* $\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-(p+m-1)\right]+\sum_{x\in X(H)}d_{ec}(x)+2q+k+p-1\leq q_{kve}\leq\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-(p+m-1)\right]+\sum_{x\in X(H)}d_{ec}(x)+2q+kp$ *gives* $19+21+32+7+10-1=88\leq 93\leq 19+21+32+70=142$.

Theorem II.24. *Let H be a graph with k cliques and p vertices. Let q_{KVE} denote number of edges in $T_{KVE}(H)$. Then,*

$$\frac{1}{2}\sum_{h\in B(H)}(d_{bv}(h))^2+\frac{1}{2}\left[\sum_{p_c\in P_C(H)}(d_{vc}(p_c))^2-d_{vc}(p_c)\right]$$
$$+\frac{1}{2}\sum_{u\in V(H)}d(u))^2+\frac{1}{2}(p-m-1)+\sum_{x\in X(H)}d_{ec}(x)$$
$$+q+k\leq q_{KVE}\leq\frac{1}{2}\sum(d_{bv}(h))^2$$
$$+\frac{1}{2}\left[\sum(d_{vc}(p_c))^2-d_{vc}(p_c)\right]+\frac{1}{2}\sum_{u\in V(H)}d(u)^2$$
$$-\frac{1}{2}(p-m+1)+\sum_{x\in X(H)}d_{ec}(x)+q+kp.$$

Proof: We know that $q(L(H))=\sum_{u\in V(H)}\binom{d(u)}{2}$ (see Harary [6]). Then,

$$\begin{aligned}q_{KVE}&=q_{kve}(H))+q(K_H(H))+q(L(H))\\&\geq\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-(p+m-1)\right]\\&\quad+\sum_{x\in X(H)}d_{ec}(x)+2q+k+p-1+\\&\quad\sum_{p_c\in P_C(H)}\binom{d_{vc}(p_c)}{2}+\frac{1}{2}\left[\sum_{u\in V(H)}(d(u))^2-2q\right],\\&\geq\frac{1}{2}\left[\sum_{h\in B(H)}(d_{bv}(h)^2-(p+m-1)\right]\\&\quad+\sum_{x\in X(H)}d_{ec}(x)+2q+k+p-1+\\&\quad\sum_{p_c\in P_C(H)}\left[(d_{vc}(p_c)^2-d_{vc}(p_c)\right]\\&\quad+\frac{1}{2}\left[\sum_{u\in V(H)}(d(u))^2-2q\right],\\&\geq\frac{1}{2}\sum_{h\in B(H)}(d_{bv}(h))^2+\\&\quad\frac{1}{2}\left[\sum_{p_c\in P_C(H)}(d_{vc}(p_c))^2-d_{vc}(p_c)\right]\\&\quad+\frac{1}{2}\sum_{u\in V(H)}d(u))^2+\frac{1}{2}(p-m-1)+\\&\quad\sum_{x\in X(H)}d_{ec}(x)+q+k.\end{aligned}$$

Similarly

$$
\begin{aligned}
q_{KVE} &= q_{kve}(H)) + q(K_H(H)) + q(L(H)) \\
&\leq \frac{1}{2}\left[\sum_{h\in B(H)} (d_{bv}(h)^2 - d_{bv}(h)\right] \\
&+ \sum_{x\in X(H)} d_{ec}(x) + 2q + kp + \sum_{p_c\in P_C(H)} \binom{d_{vc}(p_c)}{2} + \\
&\qquad \frac{1}{2}\left[\sum_{u\in V(H)} (d(u))^2 - 2q\right] \\
&\leq \frac{1}{2}\left[\sum_{h\in B(H)} (d_{bv}(h)^2 - d_{bv}(h)\right] + \\
&\sum_{x\in X(H)} d_{ec}(x) + 2q + kp + \\
&\sum_{p_c\in P_C(H)} \left[(d_{vc}(p_c)^2 - d_{vc}(p_c)\right] \\
&+ \frac{1}{2}\left[\sum_{u\in V(H)} (d(u))^2 - 2q\right] \\
&\leq \frac{1}{2}\sum (d_{bv}(h))^2 + \frac{1}{2}\left[\sum (d_{vc}(p_c))^2 - d_{vc}(p_c)\right] \\
&+ \frac{1}{2}\sum_{u\in V(H)} d(u))^2 - \frac{1}{2}(p+m-1) + q + kp.
\end{aligned}
$$

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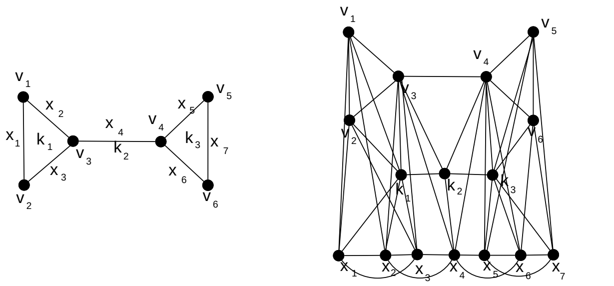

Fig. 12. A graph H and its Total Clique Vertex Edge Graph

Example II.11. *For the graph H of Fig. 12, $q(P_H(H)) = 7$, $q(C_e(H)) = 7$, $q(V_e(H)) = 14$, $q(CV(H)) = 8$, $q(K_H(H)) = 2$ and $q(L(H)) = 10$. Thus $q_{KVE} = 48$. $\sum_{h\in B(H)} (d_{bv}(h))^2 = 22$, $\sum_{h\in B(H)} d_{bv}(h) = 8$, $\sum_{x\in X(H)} d_{ec}(x) = 7$, $q = 7$, $k = 3$, $p = 6$, $\sum_{u\in V(H)} d(u)^2 = 34$, $\sum_{p_c\in P_C(H)} (d_{vc}(p_c))^2 = 8$ and $\sum_{p_c\in P_C(H)} d_{vc}(p_c) = 4$.*

Thus,

$$
\begin{aligned}
&\frac{1}{2}\sum_{h\in B(H)} (d_{bv}(h))^2 + \frac{1}{2}\left[\sum_{p_c\in P_C(H)} (d_{vc}(p_c))^2 - d_{vc}(p_c)\right] \\
&+ \frac{1}{2}\sum_{u\in V(H)} d(u))^2 + \frac{1}{2}(p-m-1) + \sum_{x\in X(H)} d_{ec}(x) \\
&+ q + k \leq q_{KVE} \leq \frac{1}{2}\sum (d_{bv}(h))^2 + \\
&\frac{1}{2}\left[\sum (d_{vc}(p_c))^2 - d_{vc}(p_c)\right] + \frac{1}{2}\sum_{u\in V(H)} d(u)^2 \\
&- \frac{1}{2}(p-m+1) + \sum_{x\in X(H)} d_{ec}(x) + q + kp
\end{aligned}
$$

$$
\begin{aligned}
&gives \\
&\frac{1}{2}[22 + 8 - 4 + 34 + 6 - 3 - 1] + 7 + 7 + 3 = 48 = 48 \\
&\leq \frac{1}{2}[22 + 8 - 4 + 34 - 6 - 3 + 1] + 7 + 7 + 18 = 58.
\end{aligned}
$$

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