A Computer-Aided Algorithm to Determine Distance Antimagic Labeling of Some Graphs

Anjali Yadav and Minirani S.

***Abstract*—Let $f\colon V(G) \to \{1, 2, \ldots, n\}$ be a bijection for a graph G of order n. The weight of a vertex v of G denoted by $w(v)$ is defined as sum of labels of all vertices adjacent to vertex v in G. If $w(u) \neq w(v)$ for every pair of distinct vertices $u, v \in V(G)$ then labeling f is called distance antimagic. Any graph G which admits such a labeling is called a distance antimagic graph. In this paper, we determine the distance antimagic labeling of Kneser graph $K(2n, n)$, bipartite Kneser graph $H(2n+1, n)$ using computer-aided algorithm. We investigate the existence of distance antimagic labeling for cycle related graphs having application in surveillance systems of civil engineering and urban planning and verify that vertex weights are distinct using computer algorithm. Also, we present some families of disconnected graphs that admit distance antimagic labeling and later show that Kneser graph $K(n, 2)$ does not admit (a, d)-distance antimagic labeling.**

***Index Terms*—distance magic labeling; distance antimagic labeling; (a, d)-distance antimagic labeling; Kneser graph.**

I. Introduction

GRAPH labeling involves establishing a one-to-one correspondence between the edge set or vertex set of a graph and the set of integers, subject to certain conditions. This area of study holds significance due to its broad applications in telecommunication, networking, civil engineering, urban planning, coding theory, cryptography and crystallography.

The Advanced Encryption Standard (AES), a block cipher cryptographic encryption algorithm employs antimagic labeling to encrypt data in blocks [1], [2]. X-Ray crystallography is analogous to assigning labels to a graph with predetermined arc labels [3] while adjacent vertex reducible edge labeling [4] finds application in transportation network systems. The versatility of graph labeling is evident in its numerous applications across various engineering fields. Projects related to engineering, networking or industry that involve constructing connections analogous to the graphs explored in this research can utilize the distance antimagic labeling of graphs proposed in this paper as a readily applicable labeling method. Specifically, the study delves into distance magic and distance antimagic labeling, extensively investigating their direct applications in scheduling fair, equalized and handicap incomplete tournaments. Additional insights into recent surveys and open problems on labeling can be found in Gallian [5] and Arumugam et al. [6].

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This research contributes computer programs (implemented in Python) and algorithms designed to generate combinations of subsets of sets in a specific sequence and assign labels in a defined order for computing weights in antimagic labeling. These algorithms proved instrumental in establishing the existence of distance antimagic labeling for certain graphs, a task that would have been cumbersome to accomplish manually. Also, algorithms are established to verify that the vertex weights are distinct.

All graphs considered in this paper are finite, undirected, connected graphs devoid of loops or multiple edges. The order $|V|$ and size $|E|$ of a graph $G = (V, E)$ are denoted by n and m respectively. For graph theory-related terminologies and notations, Chartrand and Lesniak [7] serve as a reference.

The concept of distance magic labeling was initially introduced by Vilfred [8] under the term "sigma labelings" inspired by the construction of magic squares. Miller et al. [9] later explored the same notion naming it "1-vertex magic vertex labeling". The terminology "distance magic labeling" was subsequently coined by Sugeng et al. [10]. Distance antimagic labeling, introduced by Kamatchi and Arumugam [11] represents another type of distance magic labeling where all vertex weights are distinct integers. Moreover, distance antimagic labeling in which the set of all vertex weights forms an arithmetic progression with an initial term a and common difference d is referred to as (a, d)-distance antimagic labeling [6]. Patel and Vasava [12] extended these studies to explore (a, d)-distance antimagic labeling of specific graph classes, establishing bounds on a and d and demonstrating that certain graph families are not (a, d)-distance antimagic graphs. Kamatchi et al. [13] further expanded on distance antimagic labeling, proving its applicability to the hypercube and certain families of disconnected graphs.

The paper is organized into six sections beginning with an introduction in Section I, followed by preliminary definitions in Section II. Section III presents results on graphs that admit distance antimagic labeling while Section IV discusses distance antimagic labeling of cycle-related graphs. Section V focuses on graphs that do not admit distance antimagic labeling and the conclusion is presented in the final section.

II. Preliminary Definitions

Definition 1 ([11]). *Let $f\colon V \to \{1, 2, ..., n\}$ be a bijection and vertex weight $w(u) = \sum_{v \in N(u)} f(v)$ for all $u \in V$ where $N(u) = \{v \in V : v \text{ is adjacent to } u\}$ is the open neighbourhood of u. A graph G is said to be (a, d)-distance antimagic if the set of all vertex weights forms an arithmetic progression with difference d and initial term a and the set of all vertex weights formed is $\{a, a+d, a+2d, \ldots, a+(n-1)d\}$.*

Definition 2 ([6]). *For a graph G the bijection $f\colon V \to \{1,2,\ldots,n\}$ is called distance antimagic labeling if $w(x) \neq w(y)$ for every pair of distinct vertices $x, y \in V(G)$ where $w(u) = \sum_{x \in N(u)} f(x)$ is weight of vertex u and $N(u)$ is open neighbourhood of $u \in V$. If such a labeling exists, then G is said to be distance antimagic graph.*

Definition 3. *Consider the cycle $C_n = (u_1, \ldots, u_n, u_1)$ of order n. A wheel is the graph obtained from C_n by adding an additional vertex u_{n+1} and joining it to all vertices of C_n. It is denoted by W_{n+1}.*

Definition 4. *The graph obtained by attaching corresponding vertices of an outer cycle to vertices of cycle in a wheel W_n is called the double wheel. It is denoted by WW_n.*

Definition 5. *A graph G with vertex set $\{p_1, p_2, ..., p_n, q_1, q_2, ..., q_n\}$ and edge set $\{(p_r, q_s) : 1 \le r, s \le n, r \neq s\}$ is called the n-crown graph for $n \ge 3$.*

Definition 6. *Let k, n be positive integers with $k \le n$. The Kneser graph $G = K(n,k)$ is a graph with ${}^{n}C_k$ vertices and vertex set consists of k-element subsets of $\{1, 2, ..., n\}$. Two vertices in G are connected if and only if they belong to distinct subsets.*

Definition 7. *Let k, n be positive integers with $k \le n$. Bipartite Kneser graph $H(n,k)$ is the graph whose two sets of vertices that are bipartite consists of k-subsets and $(n-k)$-subsets of $\{1, 2, ..., n\}$ and where two vertices are connected with an edge if and only if they are in distinct sets and one is a subset of the other.*

Corollary 8 ([6]). *$a = n + 2$ and $d = 1$ if G is 3-regular (a,d)-distance antimagic graph of order n.*

Corollary 9 ([12]). *If the number of vertices of degree δ is $t(> 1)$ for an (a,d)-distance antimagic graph G of order n,then $d \le \frac{\delta(n-\delta)}{t-1}$.*

Lemma 10 ([12]). *If the number of vertices of degree Δ is $t(> 1)$ for an (a,d)-distance antimagic graph G of order n, then $a \ge \frac{\Delta(\Delta+1)}{2} - (n - t)d$.*

Theorem 11 ([13]). *Let H be the graph attained from the cycle C_3 by assigning a vertex of degree 1 at one vertex. Then G is distance antimagic where G is the union of t copies of H*

III. Graphs that admit Distance antimagic labeling

The Kneser graph and bipartite Kneser graph serve as combinatorial structures offering coding solutions to disjointness. These structures generate diverse sequences, which, when derived from Kneser graphs, find application in the creation of continuously evolving music produced by pairs of bells in church towers and handbells.

Additionally, the computation of the number of Hamiltonian cycles in a crown graph has been a focal point in graph theory. This computation proves invaluable in determining the various possible seating arrangements of guests at a dinner table. This section of the research establishes the distance antimagic labeling of the Kneser graph $K(2n,n)$ and the bipartite Kneser graph $H(2n+1,n)$ through the implementation of a computer algorithm. The research also extends to prove the distance antimagic labeling of the crown graph and the disjoint union of specific classes of graphs.

Theorem 1. *The Kneser graph $K(2n,n)$ is $(1,1)$-distance antimagic.*

Proof: Let G=$K(2n,n)$ be a Kneser graph of order ${}^{2n}C_n$. We generate the weight of vertices of graph G using the following algorithm.

Algorithm 1 Algorithm to find weight of vertices of G=$K(2n,n)$

```
Input : Matrix of n elements V = {1, 2, ..., n}
Output: Weight matrix W
Generate TotalNoOfSets = 2nC_n   /* each set
    will have n elements and sets will be
    generated according to the sequence of
    vertices                                */
S = [ ]
for i ← 0 to 2^n do
    Templist=[ ]
    for j ← 0 to 2^n do
        if (i & (1 << j)! = 0) then
            Templist.append(V[j])
        end
    end
    if (Templist contains n elements) then
        S[i].append(Templist)
    end
end
sort (S)
    /* Create weight matrix W_1XTotalNoOfSets
                                            */
for i ← 1 to (TotalNoOfSets) do
    W(i) = i
end
/* Update weight matrix W_1XTotalNoOfSets
    */
for i ← 1 to (TotalNoOfSets) do
    for j ← 1 to (TotalNoOfSets) do
        if ( set(i) ∩set(j) = ∅) then
            W(i) = W(j)
        end
    end
end
```

The above algorithm is verified using Python programming and we observe that the vertex weights are strictly decreasing from ${}^{2n}C_n$, ${}^{2n}C_n - 1$, …,1. Hence G is $(1,1)$-distance antimagic. ∎

Example 2. *Figure 1 shows distance antimagic labeling of Kneser graph $K(6,3)$ where vertices given by 3- element subsets of the set {1, 2, 3, 4, 5, 6} are indicated in curly brackets and the vertex labels are given in brackets.*

Theorem 3. *The bipartite Kneser graph $H(2n+1,n)$ is distance antimagic for all n.*

Proof: Let G=$H(2n+1,n)$ be a bipartite Kneser graph of order $2\,{}^{2n+1}C_n$. We generate the weight of vertices of graph G using the following algorithm.

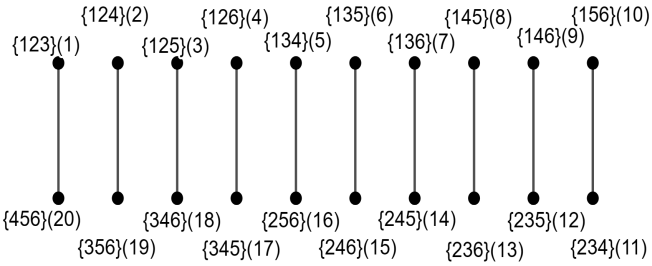

Fig. 1: Distance antimagic labeling of Kneser graph $K(6, 3)$.

Algorithm 2 Algorithm to find weight of vertices of $G = H(2n + 1, n)$

Input : k
Output: Weight matrix W
$n = 2k + 1$
Matrix of n elements $V = \{1, 2, \ldots, n\}$

For a given input of k, create two sets namely S_1 and S_2 of size ${}^nC_k$.
S$_1$ = generate_Set(n, k)
S_2 = generate_Set$(n, n - k)$
Function (*generate_Set(n, t)*)
$S = [\,]$
for $i \leftarrow 0$ **to** 2^n **do**
$Templist = [\,]$
for $j \leftarrow 0$ **to** 2^n **do**
if *(i & $(1 << j)! = 0)$* **then**
Templist.append($V[j]$)
end
end
if *(Templist contains t elements)* **then**
$S[i]$.*append*$(Templist)$
end
end
sort *(S)*
▷ Create two weight matrices W_1 and W_2 of size ${}^nC_k$ each.
$size = {}^nC_k$

for $i \leftarrow 1$ **to** $(size + 1)$ **do**
$W_2(i) = i$
end
for $i \leftarrow (size + 1)$ **to** $(2 * size + 1)$ **do**
$W_1(i) = i$
end
Function (*update_weights*($S_1, S_2, W, Flag$))
updatedweight= [ ]
for i *in range* S_1 **do**
$index = 0$
$sum = 0$
for j *in range* S_2 **do**
if *(flag)* **then**
if *(set(i).issubset(set(j))* **then**
$sum = sum + W(index)$
else
if *(set(i).issuperset(set(j))* **then**
$sum = sum + W(index)$
end
$index = index + 1$
end
$updatedweight.append(sum)$
end
return $updatedweight$
$W_1 = update\_weight(S_1, S_2, W_1, true)$
$W_2 = update\_weight(S_1, S_2, W_2, false)$
if $(len(W_1) == len(set(W_1)))and\ (len(W_2) == len(set(W_2)))$ **then**
distinct elements
else
similar elements

The above algorithm is verified using Python programming and we observe that the vertex weights are strictly increasing. Hence G is distance antimagic. ■

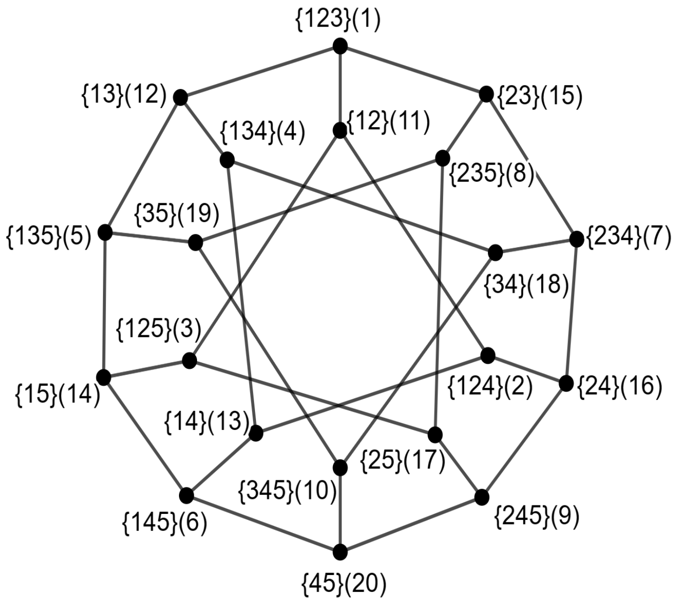

Fig. 2: Distance antimagic labeling of bipartite Kneser graph $H(5, 2)$.

Example 4. *Figure 2 shows distance antimagic labeling of bipartite Kneser graph $H(5, 2)$ where vertices consisting of 3-element and 2-element subsets of the set {1, 2, 3, 4, 5} are given in curly brackets and the vertex labels are given in brackets.*

Theorem 5. *The n-crown graph is distance antimagic for all $n \geq 3$.*

Proof: Let G be an n-crown graph with vertex set $\{x_1, x_2, ..., x_n, y_1, y_2, ..., y_n\}$. Define a function $f\colon V(G) \to \{1, 2, ..., 2n\}$

$$f(x_i) = i \qquad \text{if } 1 \leq i \leq n$$
$$f(y_i) = n + i \qquad \text{if } 1 \leq i \leq n$$

Clearly f is a bijection. The vertex weights are given by

$$w(x_i) = \frac{n(3n-1)}{2} - i \qquad \text{if } 1 \leq i \leq n$$
$$w(y_i) = \frac{n(n+1)}{2} - i \qquad \text{if } 1 \leq i \leq n$$

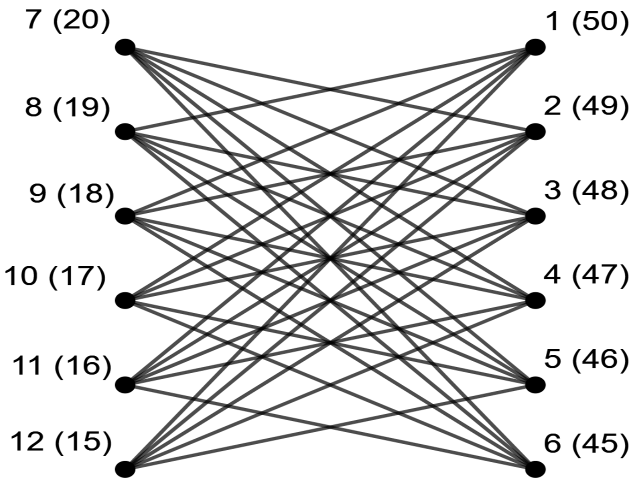

Fig. 3: Distance antimagic labeling of 6-crown graph.

We observe that the vertex weights are distinct as they are monotonically decreasing. Hence G is distance antimagic. ■

Example 6. *Figure 3 shows distance antimagic labeling of* 6*-crown graph labels are assigned in usual text and corresponding weights are given in brackets.*

Theorem 7. *Let $H^{'}$ be the graph attained from the cycle C_3 by assigning a vertex of degree* 1 *at each vertex. Let G be the union of t copies of $H^{'}$. Then G is distance antimagic.*

Proof: Let $H_i^{'}$ be the i^{th} copy of $H^{'}$ in G. Let $V(H_i^{'}) = \{v_{i1}, v_{i2}, v_{i3}, v_{i4}, v_{i5}, v_{i6}\}$ and $E(H_i^{'}) = \{(v_{i1}, v_{i2}), (v_{i1}, v_{i3}), (v_{i1}, v_{i4}), (v_{i2}, v_{i3}), (v_{i2}, v_{i5}), (v_{i3}, v_{i6})\}$
Define $f^{'} : V(G) \to \{1, 2, ..., 6t\}$ by

$$f^{'}(v_{ij}) = \begin{cases} 3(i-1)+j & \text{if } 1 \leq j \leq 3 \\ 3t+3i & \text{if } j = 4 \\ 3t+3i-1 & \text{if } j = 5 \\ 3t+3i-2 & \text{if } j = 6 \end{cases}$$

where $1 \leq i \leq t$. The vertex weights are given by

$$w(v_{ij}) = \begin{cases} 3t+9i-2j+1 & \text{if } 1 \leq j \leq 3 \\ 3i+j-6 & \text{if } 4 \leq j \leq 6 \end{cases}$$

Clearly, the vertex weights are distinct. Hence, $f^{'}$ is a distance antimagic labeling of G. ■

Theorem 8. *Let $H^{'}$ be union of t copies of G where G is an r-regular graph of order n . $H^{'}$ is distance antimagic if G is distance antimagic.*

Proof: Let f_G be a *distance antimagic labeling* of G. Let $G_1, G_2, \ldots, G_t$ be t copies of G in $H^{'}$.
Define $f_{H'} : V(H^{'}) \to \{1, 2, ..., tn\}$ by

$$f_{H'}(u) = \begin{cases} f_G(u) & \text{if } u \in V(G_1) \\ f_G(u) + (i-1)n & \text{if } u \in V(G_i),\ 2 \leq i \leq t \end{cases} \quad (1)$$

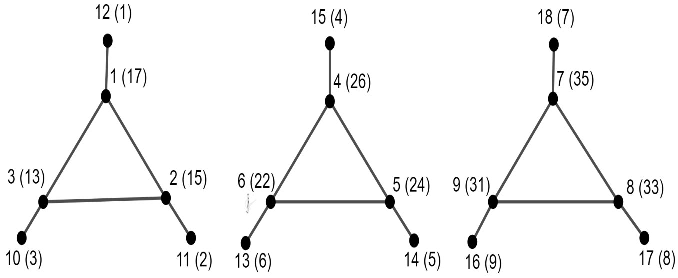

Fig. 4: Distance antimagic labeling of union of 3 copies of $H^{'}$ where the vertex labels are given and corresponding weights are indicated in brackets.

Then vertex weights of $H^{'}$ are given by

$$w_{f_{H'}}(u) = \begin{cases} w_{f_G}(u) & \text{if } u \in V(G_1) \\ w_{f_G}(u) + (i-1)rn & \text{if } u \in V(G_i),\ 2 \leq i \leq t \end{cases} \quad (2)$$

Let $u, v \in V(H^{'})$. As f_G is a distance antimagic labeling of G, we obtain $w_{f_{H'}}(u) \neq w_{f_{H'}}(v)$ if $u, v \in V(G_1)$ or $u, v \in V(G_i)$ for $2 \leq i \leq t$. Now, let $u \in V(G_1)$ and $v \in V(G_i)$ for some $2 \leq i \leq t$. We assume that $w_{f_G}(u) < w_{f_G}(v)$ without loss of generality since $w_{f_G}(u) \neq w_{f_G}(v)$. So, $w_{f_{H'}}(u) = w_{f_G}(u) < w_{f_G}(v) < w_{f_G}(v) + (i-1)rn = w_{f_{H'}}(v)$. Thus, $f_{H'}$ is distance antimagic labeling of $H^{'}$. ■

IV. Distance Antimagic Labeling of Cycle Related Graphs

The utilization of distance antimagic labeling in cycle-related graphs, specifically centerless wheel $WC2_n$ and double wheel WW_n holds potential applications in civil engineering and urban planning. In scenarios such as the supervised vigilance of sensitive buildings where rooms are represented as vertices and paths between rooms as edges, antimagic labeling becomes relevant. In this context, any attempt to enter a room results in a complete derangement of labeling disrupting the unique weights assigned to each room. Consequently, antimagic labeling proves effective in enhancing security systems. Thus, the antimagic labeling of these graphs serves as a model for surveillance in various types of buildings or areas [14]. This section substantiates the proof of distance antimagic labeling for the centerless wheel $WC2_n$ and double wheel WW_n and further verifies that the vertex weights are distinct using Python programming.

Theorem 1. *The double wheel WW_n is distance antimagic for all n.*

Proof: Let $G = WW_n$ be the double wheel. Let the set of vertices of inner cycle be $\{v_1, v_2, \ldots, v_n\}$ and the set of corresponding vertices of outer cycle be $\{u_1, u_2, \ldots, u_n\}$ such that v_i and u_i are adjacent for $1 \leq i \leq n$. Let v_0 be the centre vertex of G. Define a function $h\colon V \to \{1, 2, \ldots, 2n+1\}$ as follows

$$h(u_i) = i$$
$$h(v_i) = 2n + 1 - i$$
$$h(v_0) = 2n + 1$$

where $1 \le i \le n$. Clearly, h is a bijection and the vertex weights are given by

$$w(u_i) = \begin{cases} 3n+2 & :i=1 \\ 2n+1+i & :2 \le i \le n-1 \\ 2n+1 & :i=n \end{cases}$$

$$w(v_i) = \begin{cases} 5n+2 & :i=1 \\ 6n+3-i & :2 \le i \le n-1 \\ 6n+3 & :i=n \\ \frac{n(3n+1)}{2} & :i=0 \end{cases}$$

Clearly, the vertex weights are distinct and so the double wheel WW_n is distance antimagic. ■

The algorithm to verify that the vertex weights are distinct using Python is given below.

Algorithm 3 Algorithm to verify vertex weights of $G = WW_n$ are distinct

Input : n
Output: Weight matrix W_u and W_v
Matrix of vertices $V \rightarrow \{1, 2, \ldots, 2n+1\}$
For a given input of n, create two sets of vertices namely u_i and v_i for outer and inner cycle respectively.
for $i \leftarrow 1$ **to** $(n+1)$ **do**
  $u_i = i$
end
for $i \leftarrow 0$ **to** $(n+1)$ **do**
  $v_i = 2*n+1-i$
end
▷ `Create two weight matrices` W_u `and` W_v `of size` n `and` $n+1$ `respectively.`
for $i \leftarrow 1$ **to** $(n+1)$ **do**
  if $i == 1$ **then**
    $W_u(i) = 3*n+2$
  else if $(i \ge 2)$ *and* $(i \le n-1)$ **then**
    $W_u(i) = 2*n+1+i$
  else if $i == n$ **then**
    $W_u(i) = 2*n+1$
end
for $i \leftarrow 0$ **to** $(n+1)$ **do**
  if $i == 1$ **then**
    $W_u(i) = 5*n+2$
  else if $(i \ge 2)$ *and* $(i \le n-1)$ **then**
    $W_u(i) = 6*n+3-i$
  else if $i == n$ **then**
    $W_u(i) = 6*n+3$
  else if $i == 0$ **then**
    $W_u(i) = (n*(3*n+1))/2$
end
if $(len(W_u) == len(set(W_u)))$ *and* $(len(W_v) == len(set(W_v)))$ **then**
  Distinct weights
else
  Duplicate weights

Example 2. *Figure 5 shows distance antimagic labeling of double wheel* WW_{10}.

Theorem 3. *The centreless wheel* $WC2_n$ *is distance antimagic for all* n.

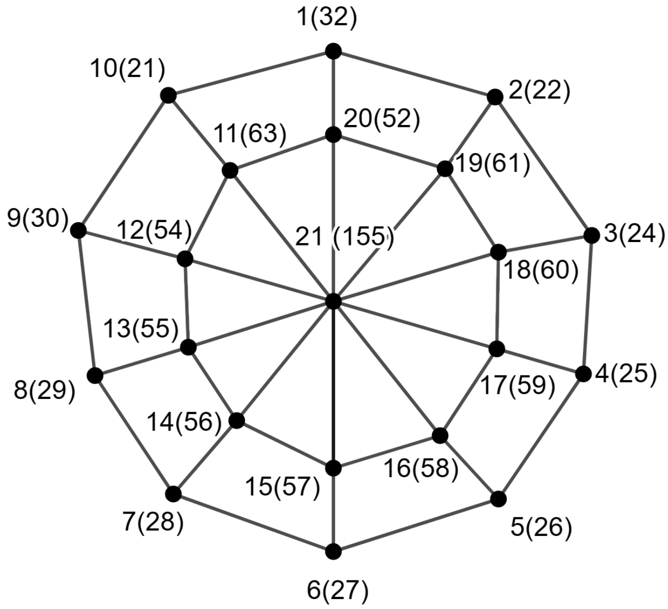

Fig. 5: Distance antimagic labeling of double wheel WW_{10} where the vertex labels are given and corresponding weights are indicated in brackets.

Proof: Let $G = WC2_n$ be the centreless wheel. Let the set of vertices of inner cycle be $\{v_1, v_2, \ldots, v_n\}$ and the set of corresponding vertices of outer cycle be $\{u_1, u_2, \ldots, u_n\}$ such that v_i and u_i are adjacent for $1 \le i \le n$. Define a function $h \colon V \rightarrow \{1, 2, \ldots, 2n\}$ as follows

$$h(u_i) = i$$
$$h(v_i) = 2n+1-i$$

where $1 \le i \le n$. Clearly, h is a bijection and the vertex weights are given by

$$w(u_i) = \begin{cases} 3n+2 & :i=1 \\ 2n+1+i & :2 \le i \le n-1 \\ 2n+1 & :i=n \end{cases}$$

$$w(v_i) = \begin{cases} 3n+1 & :i=1 \\ 4n+2-i & :2 \le i \le n-1 \\ 4n+2 & :i=n \end{cases}$$

Clearly, the vertex weights are distinct and so the centreless wheel $WC2_n$ is distance antimagic. ■

The algorithm to verify that the vertex weights are distinct using Python is given below.

Algorithm 4 Algorithm to verify vertex weights of $G = WC2_n$ are distinct

Input : n
Output: Weight matrix W_u and W_v
Matrix of vertices $V \rightarrow \{1, 2, \ldots, 2n\}$
For a given input of n, create two sets of vertices namely u_i and v_i for outer and inner cycle respectively.

for $i \leftarrow 1$ **to** $(n+1)$ **do**
  $u_i = i$
end
for $i \leftarrow 1$ **to** $(n+1)$ **do**
  $v_i = 2 * n + 1 - i$
end
▷ Create two weight matrices W_u and W_v of size n each.
for $i \leftarrow 1$ **to** $(n+1)$ **do**
  if $i == 1$ **then**
    $W_u(i) = 3 * n + 2$
  else if $(i \geq 2)$ *and* $(i \leq n-1)$ **then**
    $W_u(i) = 2 * n + 1 + i$
  else if $i == n$ **then**
    $W_u(i) = 2 * n + 1$
end
for $i \leftarrow 1$ **to** $(n+1)$ **do**
  if $i == 1$ **then**
    $W_u(i) = 3 * n + 1$
  else if $(i \geq 2)$ *and* $(i \leq n-1)$ **then**
    $W_u(i) = 4 * n + 2 - i$
  else if $i == n$ **then**
    $W_u(i) = 4 * n + 2$
end
if $(len(W_u) == len(set(W_u)))$ *and* $(len(W_v) == len(set(W_v)))$ **then**
  Distinct weights
else
  Duplicate weights

Example 4. *Figure 6 shows distance antimagic labeling of double wheel $WC2_{16}$.*

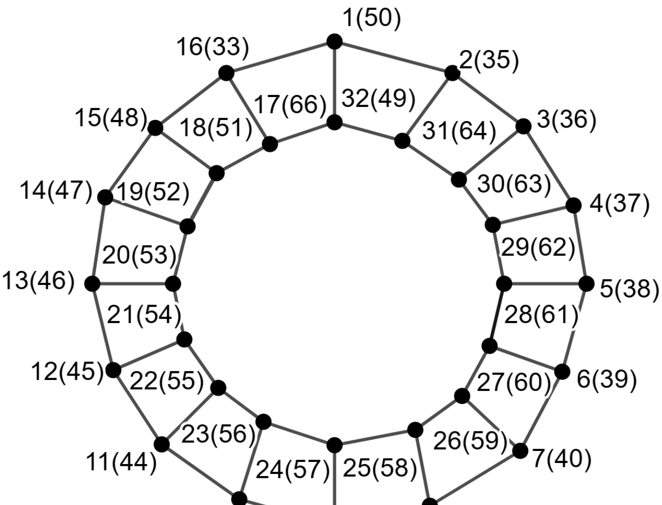

Fig. 6: Distance antimagic labeling of centreless wheel $WC2_{16}$ where the vertex labels are given and corresponding weights are indicated in brackets.

V. Graph that does not admits (a, d)-Distance Antimagic Labeling

Theorem 1. *The Kneser Graph $K(n, 2)$ is not (a, d)-distance antimagic for any n, $n > 4$.*

Proof: Let x_{ij} be the vertices of the graph $G = K(n, 2)$ where x_{ij}=$\{i, j\}$ for $1 \leq i \leq (n-1)$ and $2 \leq j \leq n$ are 2-element subsets of a set of n elements. We assume on the contrary that G is (a, d)-distance antimagic for some a and d. Then there exists a bijection $f : V \to \{1, 2, 3, \ldots, {}^nC_2\}$ and we obtain the set of all vertex weights $\{a, a+d, \ldots, a + ({}^nC_2 - 1)d\}$. For any arbitrary vertex say x_{12} without loss of generality, we see that

$$\begin{aligned} w(x_{12}) &= f(x_{34}) + f(x_{35}) + \cdots + f(x_{(n-1)n}) \\ &\geq a + (a+d) + \cdots + [a + ({}^{n-2}C_2 - 1)d] \\ &= {}^{n-2}C_2 a + \left[\frac{({}^{n-2}C_2 - 1)\,{}^{n-2}C_2}{2}\right] d \end{aligned}$$

Also, $a + ({}^nC_2 - 1)d \geq w(x_{12})$ and hence

$$a + ({}^nC_2 - 1)d \geq {}^{n-2}C_2 a + \left[\frac{({}^{n-2}C_2 - 1)\,{}^{n-2}C_2}{2}\right] d \quad (1)$$

$$0 \geq ({}^{n-2}C_2 - 1)a + \left[\frac{({}^{n-2}C_2 - 1)\,{}^{n-2}C_2}{2} - ({}^nC_2 - 1)\right] d \quad (2)$$

Now, $({}^{n-2}C_2 - 1) \geq 0$ for $n \geq 4$ and $\frac{({}^{n-2}C_2 - 1)\,{}^{n-2}C_2}{2} - ({}^nC_2 - 1) > 0$ for $n > 5$. So, (2) is not possible if $n > 5$ because a and d are positive integers. Thus, G is not (a, d)-distance antimagic for $n > 5$. Now we prove the same when $n = 5$.

It follows from corollary 9 that

$$d \leq \frac{\delta(n-\delta)}{t-1}$$

$$d \leq \frac{{}^{n-2}C_2({}^nC_2 - {}^{n-2}C_2)}{{}^nC_2 - 1}$$

So, $d \leq \frac{7}{3}$ for $n = 5$.

As the Kneser graph $G = K(5, 2)$ is 3-regular (a, d)-distance antimagic graph for some a and d, so $d = 1$ using corollary 8.

Case 1. $n = 5$ and $d = 1$

When $n = 5$, order of graph G is 10 and each vertex is of degree 3. So, it follows from lemma 10 that $a \geq \frac{\Delta(\Delta+1)}{2} - (n-t) \implies$ a ≥ 6. On the other hand when $n = 5$ and $d = 1$, equation (1) reduces to $a + 9d \geq 3a + 3d \implies 2a \leq 3$ and so G is not $(a, 1)$-distance antimagic.

■

VI. Conclusion

This paper presents the determination of distance antimagic labeling for the Kneser graph $K(2n, n)$ and the bipartite Kneser graph $H(2n+1, n)$ through the implementation of a computer-assisted algorithm. Additionally, the paper delves into the discussion of distance antimagic labeling for a family of disconnected graphs and provides proofs for distance antimagic labeling in cycle-related graphs. The research establishes the non-existence of (a, d)-distance antimagic labelling for $K(n, 2)$. The distance antimagic labeling achieved in this study serves as a model applicable to fields with similar connection schemes particularly in surveillance or security systems. Furthermore, the paper poses questions regarding the existence or non-existence of distance antimagic labeling for other classes of Kneser graphs, bipartite Kneser graphs, various graph products paving the way for further investigations.

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