Improvement for a Stochastic Newsboy Problem with Fuzzy Shortages Cost

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Abstract—**We study the paper of Ishii and Konno for a stochastic newsboy problem with fuzzy shortages cost. The purpose of this note is threefold. First, we review their approach and point out their questionable results. Second, we revise their system. Third, by the same numerical example with fuzzy shortage cost in their paper, we provide an illustrative example to demonstrate our findings. We suggest researchers do not adopt the comparison between fuzzy numbers proposed by Ishii and Konno.**

Index Terms—**Optimal ordering quantity, Fuzzy min order, Fuzzy shortage cost, Fuzzy sets, Inventory system**

I. Introduction

In the past, there were many methods to compare two fuzzy numbers. Most researchers are eager to develop new systems, and only some previously published methods have been scrutinized by authors and following readers. In this article, we will study a paper to show that their source model (traditional model) has an optimal solution. Still, on the other hand, their new fuzzy model needed an optimal solution that would reveal that in their development, the comparison approach between fuzzy numbers contains severe problems.
Petrovic et al. [1] considered the newsboy problem in a fuzzy environment with imprecise demand, overage cost, and shortage cost. Ishii and Konno [2] extended Petrovic et al. [1]. This paper will prove that the theoretical results contained questionable derivations such that Theorems 2 and 3 of Ishii and Konno [2] are invalid. Our findings will help researchers realize inventory models under a fuzzy environment.

II. Review of Their Results

Ishii and Konno [2] considered a newsboy problem with the unit purchasing cost b, unit selling cost $a+b$, and the unit lost sale penalty, which is the unit shortage cost c.

The daily demand is a random variable, say Y, with a probability density function, $p(y)$.

When a newsboy purchases newspapers, the actual demand is y. His total profit, say $e(x,y)$, is denoted as follows:

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$$e(x,y)=\begin{cases}(a+b)y-bx, & y\le x,\\ ax-c(y-x), & y\ge x.\end{cases} \tag{2.1}$$

They mentioned that the expected profit, say $E(x)$, would be derived as

$$\begin{aligned}\mathrm{E}(\mathrm{x}) &= \textstyle\sum_{\mathrm{y}=0}^{\infty} \mathrm{e}(\mathrm{x},\mathrm{y})\mathrm{p}(\mathrm{y})\\ &= \textstyle\sum_{\mathrm{y}=0}^{\mathrm{x}}[(\mathrm{a}+\mathrm{b})\mathrm{y}-\mathrm{bx}]\,\mathrm{p}(\mathrm{y})\\ &+ \textstyle\sum_{\mathrm{y}=\mathrm{x}+1}^{\infty}[\mathrm{ax}-\mathrm{c}(\mathrm{y}-\mathrm{x})]\mathrm{p}(\mathrm{y}).\end{aligned} \tag{2.2}$$

For later development, we rewrite equation (2.2) as

$$\begin{aligned}\mathrm{E}(\mathrm{x}) &= \mathrm{ax}+(\mathrm{a}+\mathrm{b})\textstyle\sum_{\mathrm{y}=0}^{\mathrm{x}}(\mathrm{y}-\mathrm{x})\mathrm{p}(\mathrm{p})\\ &-\mathrm{c}\textstyle\sum_{\mathrm{y}=\mathrm{x}+1}^{\infty}(\mathrm{y}-\mathrm{x})\mathrm{p}(\mathrm{p}).\end{aligned} \tag{2.3}$$

and then

$$E(x)=\beta(x)+c\alpha'(x), \tag{2.4}$$

with

$$\beta(x)=ax+(a+b)\sum_{y=0}^{x}(y-x)\,p(y), \tag{2.5}$$

and

$$\alpha'(x)=-\sum_{y=x+1}^{\infty}(y-x)\,p(y). \tag{2.6}$$

They tried to find a local maximum, say at x, then

$$E(x)\ge E(x-1), \tag{2.7}$$

and

$$E(x)\ge E(x+1). \tag{2.8}$$

They found the conditions to ensure that x is a local maximum:

$$\begin{aligned}&\mathrm{E}(\mathrm{x}) \ge \mathrm{E}(\mathrm{x}-1)\\ &\Leftrightarrow \textstyle\sum_{\mathrm{y}=0}^{\mathrm{x}-1}\mathrm{p}(\mathrm{y}) \le \frac{\mathrm{a}+\mathrm{c}}{\mathrm{a}+\mathrm{b}+\mathrm{c}},\end{aligned} \tag{2.9}$$

and

$$E(x)\ge E(x+1) \quad \Leftrightarrow \quad \sum_{y=0}^{x}p(y)\ge\frac{a+c}{a+b+c}. \tag{2.10}$$

This paper has revised typo errors in Ishii and Konno [2]. They mentioned that the optimal solution, say x^*, satisfies the following condition

$$\sum_{y=0}^{x^*-1}p(y)\le\frac{a+c}{a+b+c}\le\sum_{y=0}^{x^*}p(y), \tag{2.11}$$

and it is uniquely determined.
Next, they considered the newsboy model with fuzzy shortage cost, say $\tilde{c}$, and its membership function $\mu_{\tilde{c}}(t)=\max\{L(t-m),0\}$ where L is a shape function with (a) $L(t)=L(-t)$ for t in the universe of disclose, (b) $L(t)=1$ if and only if $t=0$, (c) $L(t)$ is non-increasing

on $[0,\infty)$, and (d) Let $t_0 = \inf\{t>0 \mid L(t)=0\}$ be the zero point of L. Then $0<t_0<\infty$.

Based on equation (4), they found the expected profit function, say $\tilde{E}(x)$, as follows:

$$\tilde{E}(x)=\beta(x)+\tilde{c}\alpha'(x). \tag{2.12}$$

They assumed that

$$\tilde{C}(x)=-\tilde{E}(x), \tag{2.13}$$

and a new auxiliary function denoted as α(x), with

$$\alpha(x)=-\alpha'(x)=\sum_{y=x+1}^{\infty}(y-x)p(y), \tag{2.14}$$

to imply that

$$\tilde{C}(x)=\tilde{c}\alpha(x)-\beta(x), \tag{2.15}$$

such that the maximum problem of $\tilde{E}(x)$ becomes the minimum problem of $\tilde{C}(x)$. The membership function $\mu_{\tilde{C}(x)}(t)$ of $\tilde{C}(x)$ is given by

$$\max\left\{L\left(\frac{t-m\alpha(x)+\beta(x)}{\alpha(x)}\right),0\right\}. \tag{2.16}$$

They defined a fuzzy number $\tilde{A}$ with its membership function, $\mu_{\tilde{A}}(t)$, such that (a) there is a unique point, say $m_{\tilde{A}}$, defined as the center of $\tilde{A}$, with $\mu_{\tilde{A}}(m_{\tilde{A}})=1$, (b) $\mu_{\tilde{A}}(t)$ is non-decreasing on $(-\infty,m_{\tilde{A}}]$, and (c) $\mu_{\tilde{A}}(t)$ is non-increasing on $[m_{\tilde{A}},\infty)$.

For two fuzzy numbers $\tilde{A}$ and $\tilde{B}$, $\tilde{A}\prec\tilde{B}$ if and only if the following holds: (i) $m_{\tilde{A}}\le m_{\tilde{B}}$, (ii) there is a number, say d, such that (a) $m_{\tilde{A}}\le d\le m_{\tilde{B}}$, (b) $\mu_{\tilde{A}}(t)\ge\mu_{\tilde{B}}(t)$ for all $t\le d$, and (c) $\mu_{\tilde{A}}(t)\le\mu_{\tilde{B}}(t)$ for all $t\ge d$.

A fuzzy number $\tilde{A}$ with its membership function $\mu_{\tilde{A}}(t)=\max\left\{L\left(\frac{t-m}{\alpha}\right),0\right\}$, is then denoted by $\tilde{A}=(m,\alpha)_L$ such that m is the center of $\tilde{A}$.

For two fuzzy numbers $\tilde{A}=(m,\alpha)_L$, and $\tilde{B}=(n,\beta)_L$, Ishii and Konno [2] mentioned the following theorem.

Theorem 1 of Ishii and Konno [2]. If $\tilde{A}=(m,\alpha)_L$, and $\tilde{B}=(n,\beta)_L$, then

$$\tilde{A}\prec\tilde{B} \quad\Leftrightarrow\quad t_0|\alpha-\beta|\le n-m. \tag{2.17}$$

They also defined a new order relation as follows. Let $0\le\lambda\le1$ be an arbitrary but fixed number. For any two fuzzy numbers $\tilde{A}=(m,\alpha)_L$, and $\tilde{B}=(n,\beta)_L$, they assumed an order with a parameter λ by

$$\tilde{A}\prec_\lambda\tilde{B}$$
$$\Leftrightarrow\begin{cases}(a)\quad t_0|\alpha-\beta|\le n-m,\ or\\ (b)\ \lambda t_0|\alpha-\beta|\le n-m<t_0|\alpha-\beta|,\ or\\ (c)|n-m|<\lambda t_0|\alpha-\beta|\quad and\quad \beta>\alpha.\end{cases} \tag{2.18}$$

By equation (1.15), we know that

$$C(x)=m\alpha(x)-\beta(x), \tag{2.19}$$

is the center of $\tilde{C}(x)$, and then we may denote $\tilde{C}(x)$ as

$$\tilde{C}(x)=\big(C(x),\alpha(x)\big)_L. \tag{2.20}$$

Ishii and Konno [2] tried to find the conditions to ensure that $\tilde{C}(x-1)\succ\tilde{C}(x)$ and $\tilde{C}(x+1)\succ\tilde{C}(x)$ such that x being a local minimum. Under the assumption in Section II, $F(x)=\sum_{y=0}^{x}p(y)$ is the accumulated probability, from equations (1.5), (1.14), and (1.19), they derived that

$$\begin{aligned}&C(x)-C(x-1)\\&=(a+b+m)F(x-1)-(a+m),\end{aligned} \tag{2.21}$$

$$\begin{aligned}&C(x+1)-C(x)\\&=(a+b+m)F(x)-(a+m),\end{aligned} \tag{2.22}$$

and

$$\alpha(x)-\alpha(x-1)=F(x-1)-1. \tag{2.23}$$

We will point out their questionable derivation in the following section.

III. Their Questionable Results

We quote their theorem 2 in the following.

Theorem 2 of Ishii and Konno [2]. The optimal purchasing quantity, x^f, of the above fuzzy version problem exists between x^l and x^u, where x^l is the greatest integer x satisfying $F(x-1)\le\frac{a+m}{a+b+m}$ and x^u the smallest integer x satisfying $F(x)\ge\frac{a+m+t_0}{a+b+m+t_0}$.

Proof. Since

$$\begin{aligned}&C(x)-C(x-1)-t_0|\alpha(x-1)-\alpha(x)|\\&=b\textstyle\sum_{y=0}^{x-1}p(y)-(a+m+t_0)\sum_{y=x}^{\infty}p(y)\\&=a+m+t_0\\&-(a+b+m+t_0)\textstyle\sum_{y=0}^{x-1}p(y),\end{aligned} \tag{3.1}$$

and

$$\frac{a+m+t_0}{a+b+m+t_0}>\frac{a+m}{a+b+m}, \tag{3.2}$$

it holds that

$$\tilde{C}(x)\prec\tilde{C}(x-1), \tag{3.3}$$

by Theorem 1 if

$$F(x-1) \le \frac{a+m}{a+b+m}. \tag{3.4}$$

We have derived that

$$C(x+1) - C(x) - t_0|\alpha(x+1) - \alpha(x)| = (a+b+m+t_0)F(x) - (a+m+t_0). \tag{3.5}$$

Thus, according to Theorem 1,

$$\tilde{C}(x) \prec \tilde{C}(x+1), \tag{3.6}$$

if

$$F(x) \ge \frac{a+m+t_0}{a+b+m+t_0}. \tag{3.7}$$

These two inequalities imply that x^f exists between x^l, and x^u.

Next, Ishii and Konno [2] considered the optimal purchasing quantity, x_λ^f, in the sense of λ fuzzy minimum order, and then we quote their theorem 3.

Theorem 3 of Ishii and Konno [2]. Let x_λ^f be the smallest integer x satisfying $F(x) \ge \frac{a+m+\lambda t_0}{a+b+m+\lambda t_0}$. Then x_λ^f exists between x^l, and x^u.

Proof. We can show the result of Theorem 3 by replacing t_0 with λt_0.

IV. Our Improvements

We point out that equation (3.1) is false. The corrected expression should be revised as follows,

$$\begin{aligned} &C(x-1) - C(x) - t_0|\alpha(x-1) - \alpha(x)| \\ &= a+m-t_0 \\ &-(a+b+m-t_0)F(x-1), \end{aligned} \tag{4.1}$$

such that if

$$\frac{a+m-t_0}{a+b+m-t_0} \ge F(x-1), \tag{4.2}$$

then

$$\tilde{C}(x-1) \succ \tilde{C}(x), \tag{4.3}$$

holds.

Equation (3.5) is correct, and then it yields that if

$$F(x) \ge \frac{a+m+t_0}{a+b+m+t_0}, \tag{4.4}$$

then

$$\tilde{C}(x) \prec \tilde{C}(x+1), \tag{4.5}$$

holds.

If we follow their approach, then it derives that

$$\begin{aligned} F(x-1) &\le \frac{a+m-t_0}{a+b+m-t_0} < \frac{a+m}{a+b+m}, \\ &< \frac{a+m+t_0}{a+b+m+t_0} \le F(x). \end{aligned} \tag{4.6}$$

Now, we compare equations (2.11) and (4.6) to discover that equation (2.11) has a unique solution for the newsboy problem. However, equation (4.6) does not have a solution. However, from

$$F(x-1) \le \frac{a+m-t_0}{a+b+m-t_0}, \tag{4.7}$$

that means there is an upper bound, x^U, such that for

$$x \in \{0, 1, ..., x^U\}, \tag{4.8}$$

the condition

$$F(x-1) \le \frac{a+m-t_0}{a+b+m-t_0}, \tag{4.9}$$

holds.

On the other hand, from

$$\frac{a+m+t_0}{a+b+m+t_0} \le F(x), \tag{4.10}$$

that means there is a lower bound, x^L, such that for

$$x \in \{x^L, 1+x^L, ...\}, \tag{4.11}$$

the condition

$$\frac{a+m+t_0}{a+b+m+t_0} \le F(x), \tag{4.12}$$

holds.

In general, t_0 is defined as $t_0 = \inf\{t > 0 | L(t) = 0\}$, for a fuzzy number such that

$$t_0 > 0, \tag{4.13}$$

and then it yields that

$$\{0, 1, ..., x^U\} \cap \{x^L, 1+x^L, ...\} = \varnothing, \tag{4.14}$$

such that there is no solution for equation (4.6). Hence, their Theorem 2 is false.

Next, we consider their Theorem 3. According to their new fuzzy order, we may claim that Ishii and Konno [2] believed that

$$n - m < t_0|\alpha - \beta|, \tag{4.15}$$

holds, and then they tried to find conditions to ensure that

$$\lambda t_0 |\alpha - \beta| \le n - m. \tag{4.16}$$

Table 1. Demand distribution and accumulative distribution

p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}
0.03	0.03	0.05	0.08	0.11	0.12	0.14	0.14	0.12	0.10	0.05	0.03	0.03
0.03	0.06	0.11	0.19	0.30	0.42	0.56	0.70	0.82	0.92	0.97	1.00	1.03

Based on Theorem 2, x^l is independent of t_0 such that they accepted that x_λ^l is also independent of λt_0. However, based on our revision of Theorem 2, the improved version of Theorem 3 will be expressed as

$$F(x-1) \le \frac{a+m-\lambda t_0}{a+b+m-\lambda t_0} \le \frac{a+m}{a+b+m} \le \frac{a+m+\lambda t_0}{a+b+m+\lambda t_0} \le F(x). \tag{4.17}$$

Under the restriction of $0 < \lambda \le 1$, equation (4.17) has no solution.

For the case with $\lambda = 0$, that is, the fuzzy newsboy model is reduced to the classic newsboy model, then equation (4.17) is simplified to equation (2.11), and then there is a unique solution. Therefore, we may conclude that Theorem 3 of Ishii and Konno [2] is false for the fuzzy newsboy model.

V. Review of Their Numerical Examples

We consider the same numerical example as in Ishii and Konno [2] with the following data, $a = 200$, $b = 300$, $t_0 = 100$, $c = m = 100$ and the demand distribution is listed in the second row of Table 1. To help the comparison when applying Theorem 2, we also list the accumulative distribution, $F(x)$, with $F(x) = \sum_{y=0}^{x} p(y)$, for $x = 0,1,...,12$ in the third row of Table 1. The unit shortage cost is given by the *L* fuzzy number $\max\{L(t-100), 0\}$. The third row of Table 1 reveals that the total probability is exceeded 1 such that there must be a minor modification of the values for p_j, with $j = 0,1,...,12$.

For the deterministic unit shortage cost, by equation (2.11), with $c = m$, Ishii and Konno [2] examined that

$$\sum_{y=0}^{x^m-1} p(y) \le \frac{m+a}{m+b+a} = \frac{300}{600} \le \sum_{y=0}^{x^m} p(y), \tag{5.1}$$

to find the optimal ordering quantity,

$$x^m = 6. \tag{5.2}$$

For the model with fuzzy unit shortage cost, they applied Theorem 3 with

$$\frac{a+m+t_0}{a+b+m+t_0} = \frac{400}{700} \approx 0.571, \tag{5.3}$$

to imply that

$$x^f = 7. \tag{5.4}$$

VI. Revisions for Their Numerical Example

Following our revision, with the condition,

$$F(x-1) \le \frac{a+m-t_0}{a+b+m-t_0} = \frac{2}{5} = 0.4, \tag{6.1}$$

then the solution is

$$\{0,1,2,3,4,\} \tag{6.2}$$

and for

$$0.571 = \frac{4}{7} = \frac{a+m+t_0}{a+b+m+t_0} \le F(x), \tag{6.3}$$

then the solution is

$$\{7,8,9,10,11,12\}, \tag{6.4}$$

such that

$$\{0,1,2,3,4,\} \cap \{7,8,9,10,11,12\} = \varnothing, \tag{6.5}$$

as we proposed in equation (4.14).

For completeness, we may change $p_{11} = 0.02$, and $p_{12} = 0.01$ such that the total probability, $\sum_{j=0}^{12} p_j$, becomes 1. Moreover, we point out that this modification will not influence the previous discussion to illustrate that Theorems 2 and 3 of Ishii and Konno [2] are false.

VII. Directions for Future Research

We studied some recently published articles to reveal possible directions for future studies. Cheng and Chen [3] examined Kwiesielewicz and van Uden [4], Chang et al. [5], and Xu [6] to point out their unsolved problems and then provided revisions. Wang et al. [7] studied Deng et al. [8], Murphy [9], Ardajan [10], Chu et al. [11], Wang and Chen [12], and Aguaron and Moreno-Jimenez [13] to provide improvements. Chen and Cheng [14] reviewed Mitra et al. [15], Xiao and Qi [16], and Tung et al. [17] to show several amendments. Wang and Chiang [18] considered Filev and Yager [19], Mandal et al. [20], Bustince et al. [21], and Cárdenas-Barrón [22] to offer new derivations. Wu [23] showed Glock et al. [24], Liberatore and Nydick [25], and Finan and Hurley [26] that can be revised by a mathematical approach. Wang et al. [27] provided new findings for D'Urso et al. [28], VanDeWater and DeVries [29], and Karapetrovic and Rosenbloom [30]. Lin [31] presented improvements for Saaty and Vargas [32], Lin [33], Chu et al. [34], and Yen et al. [35]. Wang and Lin [36] studied Hung et al. [37], Ronald et al. [38], and Lin et al. [39] to present several new results. Wang and Chen [12] examined Aguaron and Moreno-Jimenez [13], and Yen [40] to obtain novel derivations. Yang and Chen [41] solved questionable results in Yen [40], Osler [42], Çalışkan [43], and Çalışkan [44] to help researchers to realize the genuine mathematical procedure. Yen [45] revised Çalışkan [46] and Çalışkan [47], and then he demonstrated his findings can be applied to Wee et al. [48], Çalışkan [49], and Minner [50]. Yen [40] reviewed Luo and Chou [51], Chang et al. [5], Cárdenas-Barrón [52], and Grubbström and Erdem [53] to explain their solution process, and then Yen [40] constructed a new algebraic method to handle inventory models. Yen [40] also obtained new findings for Chang and Schonfeld [54]. Our literature review shows that practitioners can locate hot spots for future studies.

VIII. Revision of a Related Problem

The second part of this article is a further study of the article by Karapetrovic and Rosenbloom [55] that presented several examples to show that a decision-maker might seem reasonable in making the pairwise comparison of the analytic hierarchy process. Still, they may need to pass the consistency test. In this discussion, we will show that there are many questionable derivations when they set up their comparison matrix and derived their result. Slight revisions in the judgments provide more accurate priorities that do not violate the inconsistency bounds. Thus, their claim of examples that violate consistency and are paradoxes of the analytic hierarchy process is unjustified. According to these findings, the consistency test of the analytic hierarchy process is still a viable way to check the consistency of a comparison matrix in decision-making.

Decision problems in planning, resource allocation, and conflict resolution, involve many criteria, sub-criteria, and alternatives. The analytic hierarchy process helps to achieve consensus in tasks involving the ranking of alternatives by aggregating heterogeneous opinions from decision-makers using a paired comparisons approach. The analytic hierarchy process is an effective and efficient multicriteria decision-making tool for synthesizing group preference. However, sometimes, the pairwise comparison matrices are inconsistent and do not pass the consistency test proposed by Saaty [56]. Karapetrovic and Rosenbloom [55] provided three examples to illustrate judgments that are neither illogical nor random violate the consistency test established in the AHP. In addition, six papers that we know of, Kwiesielewicz and van Uden [57], Chakraborty and Banik [58], VanDeWater and DeVries [59], Ma et al. [60], Pramod et al. [61], and Cho and Cho [62] have cited Karapetrovic and Rosenbloom [55] in their references. However, none of them pointed out that the work of Karapetrovic and Rosenbloom [55] is questionable. In this study, we will provide detailed explanations to indicate how these comparison matrices are generated and then point out questionable conclusions obtained from their derivation, which, in turn, explains how the three examples in Karapetrovic and Rosenbloom [55] do not represent paradoxes in the consistency test of the analytic hierarchy process. In the next section, we consider each example separately.

IX. Review and Revision of Three Examples

Karapetrovic and Rosenbloom [55] provide three examples of pairwise comparison matrices that appear reasonable, logical, and non-random but do not pass the consistency test. As a result, these authors criticize the consistency test of the analytic hierarchy process and propose using a control approach.

In what follows, we (a) review their examples, (b) point out questionable results in their derivation, and (c) propose improvements.

9.1 The first example

(i) Review

In their first Example, Karapetrovic and Rosenbloom [55] assumed that there are three alternatives c, b and a. A decision maker believes the following relationships: (i) a is weakly more important (denoted by 3) than b, (ii) a is weakly more important (denoted by 3) than c, (iii) and b is weakly more important (denoted by 3) than c. The pairwise comparison matrix can be written as

$$A=\begin{bmatrix} 1 & 3 & 3 \\ 1/3 & 1 & 3 \\ 1/3 & 1/3 & 1 \end{bmatrix}. \tag{9.1}$$

We found the random index 0.58, consistency index $CI=0.068$, and maximum eigenvalue $\lambda_{\max}=3.136$, so the comparison matrix A did not pass the consistency test of around 0.05.

Karapetrovic and Rosenbloom [55] mention that the priority vector $(w_a, w_b, w_c)=(0.584, 0.281, 0.135)$ is consistent and reasonable with the views of the decision-maker.

(ii) Our revision

Based on the priority vector of Karapetrovic and Rosenbloom [55], we obtain a new pairwise comparison matrix. If the priority vector (w_a, w_b, w_c) = $(0.584, 0.281, 0.135)$ is vaguely contained in the mind of the decision maker, according to $w_a/w_b=2.078$, $w_a/w_c=4.326$ and $w_b/w_c=2.081$, then we may posit that the pairwise comparison matrix should be revised as follows

$$A=\begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix}, \tag{9.2}$$

with $\lambda_{\max}=3$ and $CI=0$ to pass the consistency test. Based on the comparison matrix of Equation (6.2), the new priority vector is $(w_a, w_b, w_c)=(0.571, 0.286, 0.143)$ which is comparable with the priority vector above. However, our new pairwise comparison matrix passes the consistency test. If we compare the (1, 2) entries in Equations (6.1) and (6.2) above, we find that "a is weakly more important than b" has been overestimated. Similarly, the comparison of the (1, 3) entries, "a is weakly more important than *c*" is underestimated. We observe that the decision maker has not used the intermediate values, 2 and 4, in the 1-9 scale proposed by Saaty [56]. Consequently, their calculation is artificially problematic.

From a decision maker's viewpoint, if we check their derivation, that a is weakly more important than b and c, then the relative ratio between b and c should range from very close to equally important. If the decision maker still accepts that b is more important than *c*, then the intermediate value "2" would be an appropriate choice. Hence, a revised comparison matrix should be as follows

$$A=\begin{bmatrix} 1 & 3 & 3 \\ 1/3 & 1 & 2 \\ 1/3 & 1/2 & 1 \end{bmatrix}, \tag{9.3}$$

with $\lambda_{\max}=3.054$ and $CI=0.027$ thus passing the consistency test. The modified priority vector $(w_a, w_b, w_c)=(0.594, 0.229, 0.157)$ is comparable with the previous results. The preceding shows that their first example is not a counterexample to the consistency test of the analytic hierarchy process if the intermediate values of the 1-9 scale are used.

9.2 The second example

(i) Review

For the children's game "Stone, Scissors, and Paper," Karapetrovic and Rosenbloom [55] claim that a logical practitioner will rank paper over stone by "*b*" (where $b \geq 1$), and rank paper over scissors by "$1/b$". Therefore, the pairwise comparison matrix is

$$B = \begin{bmatrix} 1 & b & 1/b \\ 1/b & 1 & b \\ b & 1/b & 1 \end{bmatrix}. \tag{9.4}$$

They discovered that if $1.4 \geq b \geq 1$, then B would pass the consistency test. On the other hand, if $9 \geq b > 1.4$, then B would not pass the consistency test. The priority vector

$$\left(w_{paper}, w_{scissors}, w_{stone}\right) = \left(1/3, 1/3, 1/3\right), \tag{9.5}$$

is reasonable and correct.

(ii) Our revision

We accept their assertion that the priority vector is suitable since paper beats stone, scissors beat paper, and stone beats scissors. That directly implies that they have the same weights. Hence, the priority vector is

$$\left(w_{paper}, w_{scissors}, w_{stone}\right) = \left(1/3, 1/3, 1/3\right). \tag{9.6}$$

Equation (9.6) can be directly decided by game's rules, so creating a comparison matrix to evaluate the priority vector is unnecessary.

Let us reconsider their pairwise comparison matrix. They used "*a*" to express paper beats stone and used "$1/a$" to denote paper was beaten by scissors. It indicates that Karapetrovic and Rosenbloom [55] assumed that "*a*" stands for "wins" and "$1/a$" represents "defeats". $a\left(1/a\right) = 1$ yields the following results:

"wins" multiplied by "defeats" becomes one. (9.7)

In the 2007 baseball season, the Yankee baseball team played 162 games with 94 wins and 68 defeats. Usually, it yields the winning rate being

$$\frac{94}{162} = 58.02\%. \tag{9.8}$$

We may say that the winning rate $\frac{94}{162} = 58.02\%$ and the losing rate

$$\frac{68}{162} = 41.98\%, \tag{9.9}$$

so the ratio of wins added to the ratio of defeats equals one. However, if we still want to derive that "wins" multiplied by "defeats" become one, then the winning rate becomes

$$\frac{94}{68} = 1.382, \tag{9.10}$$

and the losing rate becomes

$$\frac{68}{94} = 0.723. \tag{9.11}$$

The above winning rate, 1.382, and losing rate, 0.723, need further explanation. In the "Paper, Scissors, Stone" game, paper always beats stone. On the other hand, paper always loses to scissors. We observe that this implies that the value of $1/a$ tends to zero and the value of a tends to ∞. However, our observation indicates that their construction of a comparison matrix in equation (4) will violate the 1-9 scale bounded set proposed by Saaty [56]. Hence, a fundamental problem in their second example is that it does not have the required conditions of equation (5). Accordingly, we conclude that their second example did not offer a counterexample for the consistency test of AHP.

9.3 The third example

(i) Review

They considered the next dice game. Two players each have four dice where the six faces of Dice A: two zeros and four ones, the six faces of Dice B: all threes, the six faces of Dice C: four twos and two sevens, and the six faces of Dice D: three ones and three fives. Each player rolls his chosen dice, and the player with the highest face number wins.

Dice A defeats dice B two-thirds of the time; Dice A defeats dice C four-ninths of the time, and Dice A defeats dice D one-third of the time. Hence, the pairwise comparison matrix is

$$A = \begin{bmatrix} 1 & 2 & 4/5 & 1/2 \\ 1/2 & 1 & 2 & 1 \\ 5/4 & 1/2 & 1 & 2 \\ 2 & 1 & 1/2 & 1 \end{bmatrix}, \tag{9.12}$$

where the random index is 0.90, with consistency index $CI = 0.169$, and the maximum eigenvalue $\lambda_{\max} = 4.507$, so that A did not pass the consistency test. They mentioned that the priority vector

$$\left(w_A, w_B, w_C, w_D\right) = \left(0.239, 0.253, 0.262, 0.246\right), \tag{9.13}$$

is quite reasonable in this scenario.

(ii) Our revision

We point out that entries $a_{13} = 4/5$ and $a_{31} = 5/4$ are not in the permissible range of a 1-9 bounded set, $\{1/9, 1/8, ..., 1/2, 1, 2, ..., 9\}$, as proposed by Saaty [56]. Therefore, the revised entries should be $a_{13} = 1$, and $a_{31} = 1$.

After our revision, the winning rates of A over B, B over C, C over D, and D over A are all the same so that, based on their second example, we assume that the winning rate of A over B is "g". Hence, the winning rates of A over D, B over A, C over B, and D over C are assumed as "$1/g$". It implies that the revised pairwise comparison matrix can be abstractly expressed as

$$G=\begin{bmatrix}1 & g & 1 & 1/g\\ 1/g & 1 & g & 1\\ 1 & 1/g & 1 & g\\ g & 1 & 1/g & 1\end{bmatrix}, \quad (9.14)$$

with the maximum eigenvalue $\lambda_{\max}=2+g+(1/g)$, where $CI=(g+(1/g)-2)/3$ and the priority vector

$$(w_A, w_B, w_C, w_D) = (0.25, 0.25, 0.25, 0.25). \quad (9.15)$$

Since the random index for four-by-four comparison matrices is 0.90, our revised pairwise comparison matrix will pass the consistency test if

$$(g+(1/g)-2)/3 \le (0.1)(0.90), \quad (9.16)$$

holds. From

$$g^2-2.27g+1\le 0, \quad (9.17)$$

we imply that

$$0.598 \le g \le 1.672. \quad (9.18)$$

Because winning is preferred by almost everyone, the natural restriction of "*a*" should be that $a\ge 1$. Hence, we know that when

$$1 \le g \le 1.672, \quad (9.19)$$

then our revised pairwise comparison matrix will pass the consistency test.

From our result of the condition $1\le g\le 1.672$, we point out the fundamental assumption in their third example as follows

$$\text{winning rate of A over B} = 2, \quad (9.20)$$

Karapetrovic and Rosenbloom [55] implicitly assumed that

$$w_A/w_B = 2. \quad (9.21)$$

If their approach for the third example, of equations (9.20) and (9.21) is valid, then for their second example, the corrected winning rate of paper over stone should be ∞. Hence, the entry of a_{12} in equation (9.4) should be changed to ∞, which is beyond the 1-9 bounded scale proposed by Saaty [56]. Here, we point out that their Examples 2 and 3 contradict each other.

Moreover, if we reconsider the four dice problem under the restriction of 1-9 bounded scale proposed by Saaty [56] and equation (9.12), it yields the following ratios among dices A, B, C, and D, where w_A, w_B, w_C and w_D denote their priority weight, respectively,

$$w_A/w_A=1,\ w_A/w_B=2,\ w_A/w_C=1,$$
$$w_A/w_D=1/2,\ w_B/w_B=1,\ w_B/w_C=2,$$
$$w_B/w_D=1,\ w_B/w_A=1/2,\ w_C/w_C=1,$$
$$w_C/w_D=2,\ w_C/w_A=1,\ w_C/w_B=1/2,$$
$$w_D/w_D=1,\ w_D/w_A=2,\ w_D/w_B=1,$$

and

$$w_D/w_C=1/2. \quad (9.22)$$

We may list these four dices, repeated in a row, C, D, A, B, C, D, A, B, C, D, A, B, …, to discover that a dice will beat the right dice by 2, and lose to the left dice by 1/2, and make it even with the following two right (that is the next two left) dice.

If we recall their second example, to list the "Paper, Scissors, Stone" repeated in a row, … paper, scissors, stone, paper, scissors, stone, paper, scissors, stone, … to illustrate that paper defeats the left one, stone, and loses to the right one, scissors. It indicates that their Example 3 is an extension of their Example 2. In their Example 2, paper, stone, and scissors are equally important. Similarly, in their Example 3, after we revised the entries of a_{13} and a_{31}, those four dices should be equally important. It implies that constructing a pairwise comparison matrix to derive a questionable priority vector in equation (13), or an improved priority vector in equation (15), becomes redundant.

Besides, if the relative winning percentage is already known by a decision maker, then we may provide a probabilistic approach to derive the priority vector without referring to AHP. From

$$w_A/w_A=1,\ w_A/w_B=2,\ w_A/w_C=1,$$

and

$$w_A/w_D=1/2, \quad (9.23)$$

it yields that winning percentages of A over $A, B, C,$ and D are $1/2$, $2/3$, $1/2$, and $1/3$, respectively. Next, we directly use the probabilistic method to compute the winning percentage of A as

$$\frac{1}{4}\left(\frac{1}{2}\right)+\frac{1}{4}\left(\frac{2}{3}\right)+\frac{1}{4}\left(\frac{1}{2}\right)+\frac{1}{4}\left(\frac{1}{3}\right)=\frac{1}{2}, \quad (9.24)$$

since four dices $A, B, C,$ and D have the same probability, $1/4$, that will be selected by a player. Similarly, the winning percentage of $B, C,$ and D is also $1/2$.

After normalization, the priority vector, equation (9.15), can be directly obtained.

Based on the above discussion, we explained in detail that all three examples in Karapetrovic and Rosenbloom [55] contained questionable results. They did not provide evidence that paradoxes exist in the consistency test of Saaty [56]. On the basis of these results, we discuss that one can continue to be confident in using the consistency tests of AHP.

X. Application to a Related Model

We examined a related model to show our mathematical analysis can help researchers realize their problems. Xiao et al. [63] used the analytic method to solve the maximum problem of the following inventory model,

$$\pi_{T1}(p_1, L_1)=(p_1-c_1)\frac{2}{t}(r-p_1-\alpha L_1)-\frac{\beta_1}{L_1}, \quad (10.1)$$

where p_1 is the price and L_1 is the lead time.

We will apply a hybrid method that consists of algebraic approaches and analytic procedures to find the optimal solution of equation (10.1). We rewrite equation (10.1) in the descending order of p_1 to imply that

$$\pi_{T1}(p_1, L_1)=\frac{-2}{t}p_1^2+\frac{2}{t}(c_1+r-\alpha L_1)p_1,$$

$$+\frac{2}{t}c_1(\alpha L_1 - r) - \frac{\beta_1}{L_1}. \tag{10.2}$$

We complete the square for p_1 to yield that

$$\pi_{T1}(p_1, L_1) = \frac{-2}{t}\left(p_1 - \frac{c_1 + r - \alpha L_1}{2}\right)^2 - \frac{\beta_1}{L_1}$$

$$+\frac{1}{2t}\left[4c_1(\alpha L_1 - r) + (c_1 + r - \alpha L_1)^2\right]. \tag{10.3}$$

Owing the coefficient of $\left(p_1 - \frac{c_1 + r - \alpha L_1}{2}\right)^2$ is $\frac{-2}{t}$ which is a negative number, for the maximum problem we know the optimal solution for p_1 is derived as

$$p_1 = \frac{c_1 + r - \alpha L_1}{2}. \tag{10.4}$$

We simplify the expression to assume that

$$\pi_{T_1}(L_1) = \pi_{T1}\left(p_1 = \frac{c_1 + r - \alpha L_1}{2}, L_1\right), \tag{10.5}$$

to derive that

$$\pi_{T_1}(L_1) = \frac{\alpha^2}{2t}L_1^2 + \frac{\alpha(c_1 - r)}{t}L_1$$

$$+\frac{(c_1 - r)^2}{2t} - \frac{\beta_1}{L_1}. \tag{10.6}$$

Now we are facing the following problem: How did researchers find the maximum value of Equation (10.6)?
Based on equation (10.6), we derive that

$$\frac{d}{dL_1}\pi_{T1}(L_1) = \frac{1}{tL_1^2}f(L_1), \tag{10.7}$$

where $f(L_1)$ is an auxiliary function, which is defined as follows,

$$f(L_1) = \alpha^2 L_1^3 + \alpha(c_1 - r)L_1^2 + t\beta_1. \tag{10.8}$$

We examine the first and the second derivative of $f(L_1)$ to obtain that

$$\frac{d}{dL_1}f(L_1) = 3\alpha^2 L_1^2 + 2\alpha(c_1 - r)L_1, \tag{10.9}$$

and

$$\frac{d^2}{dL_1^2}f(L_1) = 6\alpha^2 L_1 + 2\alpha(c_1 - r). \tag{10.10}$$

Owing to equation (10.9), we solve $\frac{d}{dL_1}f(L_1) = 0$ to locate two solutions:

$$L_1 = 0, \tag{10.11}$$

and

$$L_1 = \frac{2(r - c_1)}{3\alpha}. \tag{10.12}$$

Referring to equation (10.10), we solve $\frac{d^2}{dL_1^2}f(L_1) = 0$ to derive an inflection point,

$$L_1 = \frac{r - c_1}{3\alpha}. \tag{10.13}$$

Consequently, we show that $f(L_1)$ is concave down for $\left(-\infty, \frac{r - c_1}{3\alpha}\right)$ and concave up for $\left(\frac{r - c_1}{3\alpha}, \infty\right)$, with a local maximum at $L_1 = 0$, and a local minimum at $L_1 = \frac{r - c_1}{3\alpha}$.

Based on equation (10.8), we obtain that

$$f(0) = t\beta_1 > 0. \tag{10.14}$$

We recall that Xiao et al. [63] already found the condition to guarantee $f\left(\frac{2(r - c_1)}{3\alpha}\right) < 0$ as $\beta < \beta_1$.

Based on our above discussion, we show that $f(L_1)$ decreases from $f(0) > 0$ to $f\left(\frac{2(r - c_1)}{3\alpha}\right) < 0$, and then $f(L_1)$ increases to infinite such that there are two points, denoted as L_1^a and L_1^b, that satisfies

$$f(L_1^a) = 0, \tag{10.15}$$

$$f(L_1^b) = 0, \tag{10.16}$$

and

$$0 < L_1^a < \frac{2(r - c_1)}{3\alpha} < L_1^b, \tag{10.17}$$

We recall equation (10.7) to know that $\frac{d}{dL_1}\pi_{T1}(L_1)$ and $f(L_1)$ have the same sign. Hence, we derive the following three sub-domain for the monotonic property of $\pi_{T1}(L_1)$:

(i) For $0 < L_1 < L_1^a$,

$$\frac{d}{dL_1}\pi_{T1}(L_1) > 0, \tag{10.18}$$

(ii) For $L_1^a < L_1 < L_1^b$,

$$\frac{d}{dL_1}\pi_{T1}(L_1) < 0, \tag{10.19}$$

(iii) For $L_1^b < L_1 < \infty$,

$$\frac{d}{dL_1}\pi_{T1}(L_1) > 0. \tag{10.20}$$

Based on our results of equations (10.18-10.20), we imply that $L_1 = L_1^a$ is a local maximum point, and $L_1 = L_1^b$ is a local minimum point. On the other hand, $L_1 = \infty$ is a boundary point which is a candidate for a local maximum point.

However, we recall equation (10.4) that indicate that there is an upper bound for L_1 as

$$\frac{c_1 + r}{\alpha} > L_1, \tag{10.21}$$

to guarantee the positivity of the price, $p_1 > 0$. Consequently, $L_1 = \infty$ will not be a candidate for a local maximum point.

XI. A New Open Question

In this section, we will proposed a new open question for future study. We claim that to solve the maximum problem of $\pi_{T_1}(L_1)$ in equation (10.6) by a pure algebraic method that will be an interesting research topic.

Based on our past experience, we implicitly accept the first critical point is the global maximum point, that is denoted as L_1^*.

We compute that

$$\begin{aligned}
&\pi_{T1}(L_1^*) - \pi_{T1}(L_1^* + \Delta L_1) \\
&= \frac{\alpha^2 L_1^*}{2t\, L_1^*(L_1^* + \Delta L_1)}(\Delta L_1)^3 \\
&+ \frac{3\alpha^2 (L_1^*)^2 - 2\alpha(r - c_1)L_1^*}{2t\, L_1^*(L_1^* + \Delta L_1)}(\Delta L_1)^2 \\
&+ \frac{2\left[\alpha^2 (L_1^*)^3 - \alpha(r - c_1)(L_1^*)^2 + t\beta_1\right]}{2t\, L_1^*(L_1^* + \Delta L_1)}\Delta L_1 . \quad (11.1)
\end{aligned}$$

Next, if we only concern the term with ΔL_1 and neglect those terms containing $(\Delta L_1)^2$ or $(\Delta L_1)^3$. Consequently, we will try to solve the following equation:

$$\alpha^2 (L_1^*)^3 - \alpha(r - c_1)(L_1^*)^2 + t\beta_1 = 0 . \quad (11.2)$$

Now, we compare equations (10.8) and (11.2) that are identical to indicate that using algebraic methods to solve the maximum problem of equation (10.6) is possible.

In the following, we may provide a possible decomposition of $\pi_{T_1}(L_1)$ to help researchers.

$$\begin{aligned}
\pi_{T1}(L_1) = &\frac{1}{2tL_1}\left[\alpha^2 (L_1)^3 - \alpha(r - c_1)(L_1)^2 + t\beta_1\right] \\
&+ \frac{(c_1 - r)^2}{2t} + \frac{(-1)}{2t\, L_1}\left[\alpha(r - c_1)L_1^2 + 3t\,\beta_1\right] . \quad (11.3)
\end{aligned}$$

XII. Conclusion

We studied the paper of Ishii and Konno for a newsboy problem with fuzzy shortage cost and then explained that their approach did not solve the problem. Here, we may point out that one possible direction to solve the problem is to consider another different approach to comparing two fuzzy numbers. Our contribution will help researchers delete one questionable comparison method between fuzzy numbers from the competition list. We also discussed three examples of Karapetrovic and Rosenbloom [55] to reveal their questionable results, and then we presented our revisions.

References

[1] D. Petrovic, R. Petrovic, M. Vujosevic, "Fuzzy Models for the Newsboy Problem," *International Journal of Production Economics*, vol. 45, 1996, pp. 435-441.

[2] H. Ishii, T. Konno, "A Stochastic Inventory Problem with Fuzzy Shortage Cost," *European Journal of Operational Research,* vol. 106, 1998, PP. 90-94.

[3] Y. N. Cheng, K. H. Chen, "Study for Contradictory Pairwise Comparison Matrices," *IAENG International Journal of Applied Mathematics*, vol. 53, no. 3, 2023, pp. 1138-1147.

[4] M. Kwiesielewicz, E. van Uden, "Inconsistent and Contradictory Judgments in Pairwise Comparison Method in the AHP," *Computers and Operations Research*, vol. 31, no. 5, 2004, pp. 713-719

[5] S. K. J. Chang, J. P. C. Chuang, H. J. Chen, "Short Comments on Technical Note the EOQ and EPQ Models with Shortages Derived Without Derivatives," *International Journal of Production Economics*, vol.97, no. 2, 2005, pp. 241-243.

[6] Z. Xu, "Incomplete Linguistic Preference Relations and Their Fusion," *Information Fusion*, vol. 7, 2006, pp. 331-337.

[7] C. M. Wang, Y. L. Wang, M. L. Chen, "Discussion for Consistency Test in Analytic Hierarchy Process," IAENG International Journal of Applied Mathematics, vol. 53, no.3, 2023, pp. 1123-1131.

[8] P. S. Deng, K. Chen, P. Chu, "Estimation of Priority Vectors," *International Journal of Operations Research,* vol. 2, no. 1, 2005, pp. 83−84.

[9] C. K. Murphy, "Limitations on the Analytic Hierarchy Process from Its Consistency Index," *European Journal of Operational Research,* vol. 65, 1993, pp. 138-139.

[10] A. Ardajan, "Optimal Ordering Policies in Response to a Sale," *IIE Transactions*, vol. 20, 1988, pp. 292-294.

[11] P. Chu, P. S. Chen, T. Niu, "Note on Supplier-restricted Order Quantity under Temporary Price Discounts," *Mathematical Methods of Operations Research*, vol. 58, 2003, pp. 141-147.

[12] Y. L. Wang, M. L. Chen, "Study for Local Stability Intervals in Analytic Hierarchy Process," *IAENG International Journal of Applied Mathematics*, vol. 53, no.1, 2023, pp. 194-201.

[13] J. Aguaron, J. M. Moreno-Jimenez, "The Geometric Consistency Index: Approximated Thresholds," *European Journal of Operational Research*, vol. 147, 2003, pp. 137–145.

[14] K. H. Chen, Y. N. Cheng, "A Simple Algorithm to Estimate the Order Time Interval with a Linear Demand," IAENG International Journal of Applied Mathematics, vol. 53, no.3, 2023, pp. 985-993.

[15] A. Mitra, J. F. Cox, R. R. Jr. Jesse, "A Note on Determining Order Quantities with a Linear Trend in Demand," *Journal of Operations Research Society*, vol.35, 1984, pp. 141-144.

[16] T. Xiao, X. Qi, "Price Competition, Cost and Demand Disruptions and Coordination of a Supply Chain with One Manufacturer and Two Competing Retailers," *Omega*, vol. 36, no. 5, 2008, pp. 741-753.

[17] C. T. Tung, Y. W. Wou, S. W. Lin, P. Deng, "Technical note on (Q, r, L) inventory model with defective items," *Abstract and Applied Analysis*, vol. 2010, Article ID 878645, 8 pages, doi:10.1155/2010/878645.

[18] Y. L. Wang, T. Y. Chiang, "Further Study for the Ordered Weighted Averaging Operator," *IAENG International Journal of Applied Mathematics*, vol. 53, no.3, 2023, pp. 810-817.

[19] D. Filev, R. R. Yager, "On the Issue of Obtaining OWA Operator Weights," *Fuzzy Sets and Systems,* vol. 94, 1998, pp. 157–169.

[20] N. K. Mandal, T. K. Roy, M. Maiti, "Multi-objective Fuzzy Inventory Model with Three Constraints: a Geometric Programming Approach," *Fuzzy Sets and Systems,* vol. 150, 2005, pp. 87–106.

[21] H. Bustince, E. Barrenechea, M. Pagola, "Restricted Equivalence Functions," *Fuzzy Sets and Systems,* vol. 157, 2006, pp. 2333–2346.

[22] L. E. Cárdenas-Barrón, "The Derivation of EOQ/EPQ Inventory Models with Two Backorders Costs Using Analytic Geometry and Algebra," *Applied Mathematical Modelling*, vol. 35, no. 5, 2011, pp. 2394-2407.

[23] L. C. Wu, "Formulated Optimal Solution for EOQ Model with Fuzzy Demand," IAENG International Journal of Computer Science, vol. 50, no.3, 2023, pp. 890-898.

[24] C. H. Glock, K. Schwindl, M. Y. Jaber, "An EOQ Model with Fuzzy Demand and Learning in Fuzziness," *International Journal Services and Operations Management,* vol. 12, no. 1, 2012, pp. 90-100.

[25] M. J. Liberatore, R. L. Nydick, "Wash Criteria and the Analytic Hierarchy Process," *Computers and Operations Research*, vol. 31, 2004, pp. 889-892.

[26] J. S. Finan, W. J. Hurley, "The Analytic Hierarchy Process: Can Wash Criteria be Ignored?," *Computers and Operations Research*, vol. 29, 2002, pp. 1025-1030.

[27] Y. L. Wang, C. L. Wen, M. L. Chen, "A Minimum Problem of Cepstral-based Clustering for Time Series," *IAENG International Journal of Applied Mathematics*, vol. 53, no.2, pp724-733, 2023.

[28] P. D'Urso, L. De Giovanni, R. Massari, R. L. D'Ecclesia, E. A. Maharaj, "Cepstral-based Clustering of Financial Time Series," *Expert Systems with Applications,* vol. 161, 2020, 113705.

[29] H. VanDeWater, J. DeVries, "Choosing a Quality Improvement Project Using the Analytic Hierarchy Process," *International Journal of Quality and Reliability Management*, vol. 23, no. 4, 2006, pp. 409-425.
[30] S. Karapetrovic, E. S. Rosenbloom, "A Quality Control Approach to Consistence Paradoxes in AHP," *European Journal of Operational Research*, vol. 119, 1999, pp. 704-718.
[31] S. S. C. Lin, "Comparison of Methods to Derive Relative Weights," *IAENG International Journal of Applied Mathematics*, vol. 53, no. 2, 2023, pp. 507-517.
[32] T. L. Saaty, L. G. Vargas, "Comparison of Eigenvalue, Logarithmic Least Square and Least Square Methods in Estimating Ratios," *Mathematical Modelling*, vol. 5, 1984, 309-324.
[33] S. S. C. Lin, "Analytic Hierarchy Process by Least Square Method Revisit," *Mathematical Problems in Engineering*, vol. 2019, 2019, Article ID 2797515, 5 pages.
[34] C. H. Chu, K. C. Hung, P. Julian, "A Complete Pattern Recognition Approach under Atanassov's Intuitionistic Fuzzy Sets," *Knowledge-Based Systems*, vol. 66, 2014, pp. 36-45.
[35] P. C. P. Yen, K. C. Fan, H. C. J. Chao, "A New Method for Similarity Measures for Pattern Recognition," *Applied Mathematical Modelling*, vol. 37, 2013, pp. 5335-5342.
[36] Y. L. Wang, C. T. Lin, "Intersection-union Operation to Divide a Network into Communities," *Engineering Letters*, vol. 31, no. 2, 2023, pp.756-763.
[37] K. C. Hung, J. Lin, P. Chu, "An Extended Algorithm of Similarity Measures and Its Application to Radar Target Recognition Based on Intuitionistic Fuzzy Sets," *Journal of Testing and Evaluation*, vol. 43, no. 4, 2015, pp. 1–10.
[38] R. J. Ronald, G. K. Yang, P. Chu, "Technical Note—The EOQ and EPQ Models with Shortages Derived without Derivatives," *International Journal of Production Economics,* vol. 92, no. 2, 2004, pp.181–184.
[39] S. C. Lin, H. W. Tuan, C. T. Tung, P. Julian, "Partition Network into Communities Based on Group Action on Sets," *Physica A*, vol. 491, 2018, pp. 361–376.
[40] C. P. Yen, "Solving Inventory Models by the Intuitive Algebraic Method," *IAENG International Journal of Applied Mathematics*, vol. 51, no. 2, 2021, pp. 341-345.
[41] T. C. Yang, Y. C. Chen, "Solution for Open Questions in Yen (2021) Osler (2001) and Caliskan (2020, 2022)," *IAENG International Journal of Applied Mathematics*, vol. 53, no.1, 2023, pp48-57.
[42] T. J. Osler, "Cardan Polynomials and the Reduction of Radicals," *Mathematics Magazine*, vol. 47, no. 1, 2001, pp. 26–32.
[43] C. Çalışkan, "A Derivation of the Optimal Solution for Exponentially Deteriorating Items without Derivatives," *Computers & Industrial Engineering*, vol. 148,2020, 106675.
[44] C. Çalışkan, "Derivation of the Optimal Solution for the Economic Production Quantity Model with Planned Shortages without Derivatives," *Modelling*, vol. 2022, no. 3, 2022, pp. 54–69.
[45] C. P. Yen, "Further Study for Inventory Models with Compounding," *IAENG International Journal of Applied Mathematics*, vol. 52, no. 3, 2022, pp. 684-691.
[46] C. Çalışkan, "On the Economic Order Quantity Model with Compounding," *American Journal of Mathematical and Management Sciences*, 2020, DOI: 10.1080/01966324.2020.1847224.
[47] C. Çalışkan, "A Simple Derivation of the Optimal Solution for the EOQ Model for Deteriorating Items with Planned Backorders," *Applied Mathematical Modelling*, vol. 89,2021, pp. 1373-1381.
[48] H. M. Wee, W.T. Wang, C.J. Chung, "A Modified Method to Compute Economic Order Quantities without Derivatives by Cost-Difference Comparisons," *European Journal of Operational Research*, vol. 194, 2009, pp. 336-338.
[49] C. Çalışkan, "A Note on "A Modified Method to Compute Economic Order Quantities without Derivatives by Cost-Difference Comparisons"", *Journal of Statistics and Management Systems*, 2021, DOI: 10.1080/09720510.2020.1859809.
[50] S. Minner, "A Note on How to Compute Economic Order Quantities without Derivatives by Cost Comparisons," *International Journal of Production Economics*, vol. 105, 2007, 293–296.
[51] X. R. Luo, C. S. Chou, "Technical Note: Solving Inventory Models by Algebraic Method," *International Journal of Production Economics*, vol. 200, 2018, pp. 130-133.
[52] L. E. Cárdenas-Barrón, "The Economic Production Quantity (EPQ) with Shortage Derived Algebraically," *International Journal of Production Economics*, vol. 70, 2001, pp. 289-292.
[53] R. W. Grubbström, A. Erdem, "The EOQ with Backlogging Derived without Derivatives," *International Journal of Production Economics*, vol. 59, 1999, pp. 529-530.
[54] S. K. Chang, P. M. Schonfeld, "Multiple Period Optimization of Bus Systems," *Transportation Research Part B*, vol. 25, no. 6, 1991, pp. 453-478.
[55] S. Karapetrovic, E.S. Rosenbloom, "A Quality Control Approach to Consistence Paradoxes in AHP," *European Journal of Operational Research,* vol. 119, 1999, pp. 704–718.
[56] T. L. Saaty, *The Analytic Hierarchy Process.* McGraw-Hill, New York, 1980.
[57] M. Kwiesielewicz, E. van Uden, "Inconsistent and Contradictory Judgments in Pairwise Comparison Method in the AHP," *Computers and Operations Research,* vol. 31, no. 5, 2004, pp. 713–719.
[58] S. Chakraborty, D. Banik, "Design of a Material Handling Equipment Selection Model Using Analytic Hierarchy Process," *International Journal of Advanced Manufacturing Technology,* vol. 28, no. 11-12, 2006, pp. 1237–1245.
[59] H. VanDeWater, J. DeVries, "Choosing a Quality Improvement Project Using the Analytic Hierarchy Process," *International Journal of Quality and Reliability Management,* vol. 23, no. 4, 2006, pp. 409–425.
[60] J. Ma, Z. Fan, Y. Jiang, J. Mao, L. Ma, "A Method for Repairing the Inconsistency of Fuzzy Preference Relations," *Fuzzy Sets and Systems,* vol. 157, no. 1, 2006, pp. 20–33.
[61] V. R. Pramod, K. Sampath, S. R. Devadasan, V. P. JagathyRaj, M. G. Dakshina, "Multicriteria Decision Making in Maintenance Quality Function Deployment through the Analytical Hierarchy Process," *International Journal of Industrial and Systems Engineering,* vol. 2, no. 4, 2007, pp. 454–478.
[62] Y. Cho, K. Cho, "A Loss Function Approach to Group Preference Aggregation in the AHP," *Computers and Operations Research,* vol. 35, no. 3, 2008, pp. 884–892.
[63] T. Xiao, J. Shi, G. Chen, " Price and Leadtime Competition, and Coordination for Make-to-order Supply Chains," *Computers & Industrial Engineering*, vol. 68, 2014, pp. 23–34.

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