Pairwise Neutrosophic Supra Pre-Open Set in Neutrosophic Supra Bi-topological Spaces

Suman Das, Rakhal Das, Prasanna Poojary, Vadiraja Bhatta G. R.*, and Varsha Jarali.

Abstract— **This paper presents the innovative notion of a "neutrosophic supra bi-topological space," which serves as an expansion of both neutrosophic supra topological space and bi-topological space. Furthermore, we delve into various categories of open and closed sets within this framework, including concepts like pairwise neutrosophic supra open sets, pairwise neutrosophic supra semi-open sets, and pairwise neutrosophic supra pre-open sets. Additionally, we formulate a few outcomes in the form of theorems, propositions, and lemmas.**

Index Terms— **Neutrosophic Supra Topology; Neutrosophic Supra Bi-topology; Pairwise Neutrosophic Supra Open Set.**

I. INTRODUCTION

Set theory is a tool for representing the various objects of the real world in mathematical expressions. G. Cantor (1845-1918) formally named the concept of set theory. Cantors set theory was insufficient for dealing with and working with a variety of real-world situations, including uncertainty. In 1965, the late Prof. L.A. Zadeh proposed fuzzy set theory, in which each element has a membership value. Later on, K. Atanassov believed that the non-membership of a mathematical expression played a part in mathematically solving problems. This inspired him to develop the notion of an intuitionistic fuzzy set (IFS). Many uncertainty events will include an indeterminacy component that cannot be represented using the concepts of crisp set, fuzzy set, or intuitionistic fuzzy set. Keeping indeterminacy in mind, Smarandache [36] proposed the neutrosophic set as an extension of IFS, with each element having the degree of truth-membership, indeterminacy-membership, and false membership. The notion of a neutrosophic set and its extensions has been employed in many theoretical ([3], [9], [10], [13], [14], [17], [34]) and practical research ([5], [11], [12], [25], [31]).

In 2012, Salama and Alblowi [32] introduced the concept of neutrosophic topological space, which is a natural generalization of intuitionistic fuzzy topological space. The concepts of generalised neutrosophic set and generalised neutrosophic topological space were then investigated further by Salama and Alblowi [33]. Following them, several researchers contributed to this field of study and developed applications. Arokiarani et al. [1] later procured the concept of neutrosophic semi-open functions and established a relation between them. Iswaraya and Bageerathi [21] proposed the notion of neutrosophic semi-closed set and neutrosophic semi-open set in the context of neutrosophic topological spaces. Afterwards, Rao and Srinivasa [30] further investigated the concept of neutrosophic pre-open set through neutrosophic topological space. Later on, the idea of neutrosophic generalized semi-closed set through neutrosophic topological space was presented by Shanthi et al. [35]. Thereafter, Ebenanjar et al. [20] introduced the notion of neutrosophic *b*-open set in neutrosophic topological spaces. In neutrosophic topological space, Maheswari et al. [24] developed the concept of neutrosophic generalized *b*-closed sets. Mohammed Ali Jaffer and Ramesh [26] established the notion of neutrosophic generalized pre-regular closed set through neutrosophic topological space in 2019. Pushpalatha and Nandhini [28] further investigated the concept of generalized neutrosophic closed set through neutrosophic topological space. Afterward, Das and Pramanik [7] presented the notion of generalized neutrosophic *b*-open sets in the context of neutrosophic topological spaces in 2020. In addition, Das and Pramanik [8] investigated neutrosophic ϕ-open sets and neutrosophic ϕ-continuous functions. Ramesh [29] then established the concept of Ngpr-homomorphism using neutrosophic topological space. Suresh and Palaniammal [37] further introduced the concept of neutrosophic weakly generalized open set and neutrosophic weakly generalized closed set via neutrosophic topological space. Das and Tripathy [16] developed the notion of neutrosophic simply *b*-open set through neutrosophic topological space in 2021. Das et al. [4] recently established neutrosophic separation axioms via neutrosophic topological space.

Jayaparthasarathy et al. [22] grounded the notion of neutrosophic supra topological space and demonstrated its use in data mining. Dhavanseelan et al. [19] used neutrosophic supra topological spaces to establish the concept of neutrosophic semi-supra open set and neutrosophic semi-supra continuous functions. Moreover, Dhavaseelan et al. [18] investigated the notion of

Manuscript received December 5, 2022; revised November 24, 2023.

Suman Das is a research scholar at the Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India. (e-mail: sumandas18842@gmail.com, dr.suman1995@yahoo.com).

Rakhal Das is an assistant professor at the Department of Mathematics, The ICFAI University, Agartala, 799210, Tripura, India. (e-mail: rakhaldas95@gmail.com, rakhaldas@iutripura.edu.in).

Prasanna Poojary is an assistant professor at the Department of Mathematics, Manipal Institute of Technology Bengaluru, Manipal Academy of Higher Education, Manipal, India. (e-mail: poojary.prasanna@manipal.edu, poojaryprasanna34@gmail.com).

Vadiraja Bhatta G. R is an associate professor at the Department of Mathematics, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal, India. (Corresponding author e-mail: vadiraja.bhatta@manipal.edu).

Varsha Jarali is a research scholar at the Department of Mathematics, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal, India. (email: varshapjarali@gmail.com).

neutrosophic α-supra open set. In 2021, Das [2] introduced the concept of neutrosophic supra simply open set via neutrosophic supra topological space.

In 1963, Kelly [23] presented the notion of bi-topological space. Later on, Ozturk and Ozkan [27] grounded the idea of neutrosophic bi-topologoical space. In the recent past, Das and Tripathy [15] introduced pairwise neutrosophic *b*-open sets in neutrosophic bi-topological spaces. Thereafter, Tripathy and Das [38] studied the notion of pairwise neutrosophic *b*-continuous functions in neutrosophic bi-topological spaces. Recently, the notion of a neutrosophic pre-*I*-open set via neutrosophic ideal bi-topological space was studied by Das et al. [6].

The primary goal of this paper is to develop the concept of neutrosophic supra bi-topological spaces by extending the notions of bi-topological space and neutrosophic supra topological space. We also present different types of open set and closed set, such as pairwise neutrosophic supra open set, pairwise neutrosophic supra semi-open set, and so on via neutrosophic supra bi-topological spaces.

The following parts comprise the rest of this article:

Preliminaries and definitions are covered in Section 2. In this part, we provide several definitions and theorems that will be quite helpful in preparing the major findings of this article. Section 3 introduces the concepts of neutrosophic supra bi-topology and neutrosophic supra bi-topological space, as well as proofs of several theorems and propositions based on neutrosophic supra bi-topological space. Finally, in Section 4, we provide the concluding remarks on the work done in this article.

Throughout this article, we use the following short terms which are listed in Table 1, for the clarity of the presentation.

Table I. Nomenclatures

Short Terms	
Neutrosophic Set	NS
Neutrosophic Topological Space	NTS
Neutrosophic Bi-topological Space	NBTS
Neutrosophic Supra Topology	NST
Neutrosophic Supra Topological Space	NSTS
Neutrosophic Supra Open Set	NSOS
Neutrosophic Supra Closed Set	NCOS
Neutrosophic Supra α-Open Set	NS-α-OS
Neutrosophic Supra Semi-Open Set	NSSOS
Neutrosophic Supra Pre-Open Set	NSPOS
Neutrosophic Supra *b*-Open Set	NS-*b*-OS
Neutrosophic Supra Bi-topological Space	NSBTS
Neutrosophic Supra Bi-Open Set	NS-bi-O-Set
Neutrosophic Supra Bi-Closed Set	NS-bi-C-Set
Neutrosophic Supra Bi-Semi-Open Set	NS-bi-S-O-Set
Neutrosophic Supra Bi-Pre-Open Set	NS-bi-P-O-Set
Neutrosophic Supra Bi-*b*-Open Set	NS-bi-*b*-O-Set
Pairwise Neutrosophic Supra Open Set	Pairwise-NSOS
Pairwise Neutrosophic Supra Closed Set	Pairwise-NSCS
Pairwise Neutrosophic Supra Semi-Open Set	Pairwise-NSSOS
Pairwise Neutrosophic Supra Semi-Closed Set	Pairwise-NSSCS
Pairwise Neutrosophic Supra Pre-Open Set	Pairwise-NSPOS
Pairwise Neutrosophic Supra Pre-Closed Set	Pairwise-NSPCS
Pairwise Neutrosophic Supra *b*-Open Set	Pairwise-NS-*b*-OS
Pairwise Neutrosophic Supra *b*-Closed Set	Pairwise-NS-*b*-CS
Pairwise Neutrosophic Supra-Continuous Mapping	Pairwise-NS-C-Mapping
Pairwise Neutrosophic Supra Semi-Continuous Mapping	Pairwise-NSS-C-Mapping
Pairwise Neutrosophic Supra Pre-Continuous Mapping	Pairwise-NSP-C-Mapping
Pairwise Neutrosophic Supra *b*-Continuous Mapping	Pairwise-NS-*b*-C-Mapping

II. Preliminaries and Definitions

In this section, we present some preliminaries and definitions that are very helpful for the preparation of the main results of this article.

Definition 2.1.[36] An NS L over a fixed set Ψ is defined as follows:

$$L=\{(x, T_L(x), I_L(x), F_L(x)): x\in\Psi\},$$

where $T_L(x)$, $I_L(x)$, $F_L(x)$ ($\in[0, 1]$) are the truth-membership, indeterminacy-membership and false-membership values of each $x\in\Psi$. So, $0 \leq T_L(x) + I_L(x) + F_L(x) \leq 3$, $\forall x\in\Psi$.

Definition 2.2.[36] The null NS (0_N) and whole NS (1_N) over Ψ are defined as follows:

$0_N=\{(x, 0, 0, 1): x\in\Psi\}$ & $1_N=\{(x, 1, 0, 0): x\in\Psi\}$.

Clearly, $0_N \subseteq R \subseteq 1_N$, for any NS R over Ψ.

One can also represent the null NS (0_N) and whole NS (1_N) in the following way:

(*i*) $0_N=\{(x, 0, 1, 0): x\in\Psi\}$ & $1_N=\{(x, 1, 1, 0): x\in\Psi\}$;

(*ii*) $0_N=\{(x, 0, 1, 1): x\in\Psi\}$ & $1_N=\{(x, 1, 0, 1): x\in\Psi\}$.

Definition 2.3.[36] Let $A=\{(x, T_A(x), I_A(x), F_A(x)): x\in\Psi\}$ be an NS over Ψ. Then, the complement of A is defined by $A^c=\{(x, 1\text{-}T_A(x), 1\text{-}I_A(x), 1\text{-}F_A(x)): x\in\Psi\}$.

Definition 2.4.[36] An NS $A=\{(x, T_A(x), I_A(x), F_A(x)): x\in\Psi\}$ is contained in the other NS $B=\{(x, T_B(x), I_B(x), F_B(x)): x\in\Psi\}$ (i.e., $A\subseteq B$) if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$, for each $x\in\Psi$.

Definition 2.5.[36] Let $A=\{(x, T_A(x), F_A(x), I_A(x)): x\in\Psi\}$ and $B=\{(x, T_B(x), F_B(x), I_B(x)): x\in\Psi\}$ be two NSs over X. Then, their union is defined as follows:

$$A\cup B=\{(x, T_A(x)\vee T_B(x), F_A(x)\wedge F_B(x), I_A(x)\wedge I_B(x)): x\in\Psi\}.$$

Definition 2.6.[36] Let $A=\{(x, T_A(x), F_A(x), I_A(x)): x\in\Psi\}$ and $B=\{(x, T_B(x), F_B(x), I_B(x)): x\in\Psi\}$ be two NSs over X. Then, their intersection is defined as follows:

$$A\cap B=\{(x, T_A(x)\wedge T_B(x), F_A(x)\vee F_B(x), I_A(x)\vee I_B(x)): x\in\Psi\}.$$

Definition 2.7.[32] A family τ of NSs over a fixed set Ψ is said to be an neutrosophic topology on Ψ if the following condition holds:

(*i*) $0_N, 1_N \in \tau$;

(ii) $X_1, X_2 \in \tau \Rightarrow X_1 \cap X_2 \in \tau$;

(iii) $\{X_i : i \in \Delta\} \subseteq \tau \Rightarrow \cup_{i\in\Delta} X_i \in \tau$.

The pair (Ψ, τ) is called an neutrosophic topological space. If $X \in \tau$, then X is called an neutrosophic open set in (Ψ, τ), and the complement of X i.e., X^c is called an neutrosophic closed set in (Ψ, τ).

Definition 2.8.[27] Let (Ψ, τ_1) and (Ψ, τ_2) be two different neutrosophic topological spaces. Then, the triplet (Ψ, τ_1, τ_2) is called an neutrosophic bi-topological space.

Definition 2.9.[22] A non-empty collection τ of NSs over a fixed set Ψ is said to be an NST on Ψ if the following holds:

(i) 0_N, $1_N \in \tau$;

(ii) $\{X_i : i \in \Delta\} \subseteq \tau \Rightarrow \cup X_i \in \tau$.

Then, the pair (Ψ, τ) is called an NSTS. If $X \in \tau$, then X is called an NSOS and its complement (X^c) is called an NSCS in (Ψ, τ).

Definition 2.10.[22] Let (Ψ, τ) be an NSTS. Then, the neutrosophic supra interior (in short N^s_{int}) and neutrosophic supra closure (in short N^s_{cl}) of an NS Y over Ψ are defined by

$N^s_{int}(Y) = \cup\{R: R$ is an NSOS in Ψ and $R \subseteq Y\}$;

and $N^s_{cl}(Y) = \cap\{P: P$ is an NSCS in Ψ and $Y \subseteq P\}$.

Definition 2.11. Let (Ψ, τ) be an NSTS. Then Y, an NS over Ψ is called as

(i) NS-α-OS [18] if $Y \subseteq N^s_{int}(N^s_{cl}(N^s_{int}(Y)))$;

(ii) NSSOS [19] if $Y \subseteq N^s_{cl}(N^s_{int}(Y))$;

(iii) NSPOS [19] if $Y \subseteq N^s_{int}(N^s_{cl}(Y))$;

(iv) NS-b-OS [2] if $Y \subseteq N^s_{int}(N^s_{cl}(Y)) \cup N^s_{cl}(N^s_{int}(Y))$.

Definition 2.12. [18] Let ξ be a bijective mapping from an NSTS (Ψ, τ_1) to another NSTS (Ω, τ_2). Then, ξ is called as

(i) neutrosophic supra continuous function if $\xi^{-1}(K)$ is an NSOS in Ψ, whenever K is an NSOS in Ω.

(ii) neutrosophic supra α-continuous function if $\xi^{-1}(K)$ is an NS-α-OS in Ψ, whenever K is an NSOS in Ω.

III. Pairwise Neutrosophic Supra Bi-topological Space

In this section, we procure the notion of pairwise neutrosophic supra pre-open set, pairwise neutrosophic supra semi-open set, etc. via neutrosophic supra bi-topological spaces, and establish several interesting results on them.

Definition 3.1. Let (Ψ, τ_1) and (Ψ, τ_2) be two different NSTSs. Then, the triplet (Ψ, τ_1, τ_2) is said to be an NSBTS.

Example 3.1. Let Ψ={a, b} be a fixed set. Let
P={(a,0.6,0.5,0.6), (b,0.7,0.5,0.6): a, b∈Ψ},
Q={(a,0.4,0.7,0.8), (b,0.3,1.0,0.9): a, b∈Ψ},
R={(a,0.3,0.8,0.9), (b,0.2,1.0,1.0): a, b∈Ψ},
X={(a,0.4,0.6,0.8), (b,0.7,0.8,0.7): a, b∈Ψ},
Y={(a,0.5,0.5,0.5), (b,0.8,0.5,0.6): a, b∈Ψ} and
Z={(a,0.6,0.4,0.2), (b,1.0,0.2,0.3): a, b∈Ψ} are six NSs over Ψ. Then, $\tau_1=\{0_N, 1_N, P, Q, R\}$ and $\tau_2=\{0_N, 1_N, X, Y, Z\}$ are two different NSTs on Ψ. Hence, (Ψ, τ_1, τ_2) is an NSBTS.

Remark 3.1. Since every neutrosophic topology is an NST, so every NBTS is also an NSBTS. But the converse is not true in general. This follows from the following example.

Example 3.2. From Example 3.1, it is clear that (Ψ, τ_1, τ_2) is an NSBTS, but it is not an NBTS.

Definition 3.2. Let (Ψ, τ_1, τ_2) be an NSBTS. Then, an NS R over Ψ is called an NS-bi-O-Set if $R \in \tau_1 \cup \tau_2$. If R is a NS-bi-O-Set, then R^c is called an NS-bi-C-Set in (Ψ, τ_1, τ_2).

Remark 3.2. The collection of all NS-bi-O-Sets and NS-bi-C-Sets in (Ψ, τ_1, τ_2) may be denoted by NS-bi-O(Ψ) and NS-bi-C(Ψ) respectively.

Theorem 3.1. Every NSOS in one of the NSTSs (Ψ, τ_i) (i=1, 2) is an NS-bi-O-Set in (Ψ, τ_1, τ_2).

Proof. Let W be an NSOS in (Ψ, τ_1). So, $W \in \tau_1$. This implies, $W \in \tau_1 \cup \tau_2$. Hence, W is an NS-bi-O-Set in (Ψ, τ_1, τ_2). Therefore, every NSOS in (Ψ, τ_1) is an NS-bi-O-Set in (Ψ, τ_1, τ_2).

Similarly, it can be shown that, every NSOS in (Ψ, τ_2) is an NS-bi-O-Set in (Ψ, τ_1, τ_2).

Therefore, every NSOS in one of the NSTSs (Ψ, τ_i) (i=1,2) is also an NS-bi-O-Set in (Ψ, τ_1, τ_2).

Remark 3.3. In an NSBTS (Ψ, τ_1, τ_2), the union of two NS-bi-O-Sets may not be an NS-bi-O-Set in general. This follows from the following example.

Example 3.3. Let us consider an NSBTS (Ψ, τ_1, τ_2) as shown in Example 3.1. Clearly, P and Y are two NS-bi-O-Sets in (Ψ, τ_1, τ_2). But their union $P \cup Y$={(a,0.6,0.5,0.5), (b,0.8,0.5,0.6)} is not an NS-bi-O-Set, because $P \cup Y \notin \tau_1 \cup \tau_2$. Therefore, the union of any two NS-bi-O-Sets may not be an NS-bi-O-Set in general.

Remark 3.4. In an NSBTS (Ψ, τ_1, τ_2), the intersection of two NS-bi-O-Sets may not be an NS-bi-O-Set. This follows from the following example.

Example 3.4. Suppose (Ψ, τ_1, τ_2) be an NSBTS as shown in Example 3.1. Clearly, P and Y are two NS-bi-O-Sets in (Ψ, τ_1, τ_2). Since $P \cap Y \notin \tau_1 \cup \tau_2$, so $P \cap Y$={(a,0.5,0.5,0.6), (b,0.7,0.5,0.6)} is not an NS-bi-O-Set. Therefore, the intersection of any two NS-bi-O-Sets may not be an NS-bi-O-Set in general.

Theorem 3.2. Every NSCS in one of the NSTSs (Ψ, τ_i) (i=1,2) is an NS-bi-C-Set in (Ψ, τ_1, τ_2).

Proof. Let W be an NSCS in (Ψ, τ_1). So, W^c is an NS-bi-O-Set in (Ψ, τ_1). Therefore, $W^c \in \tau_1$. This implies, $W^c \in \tau_1 \cup \tau_2$. Hence, W^c is an NS-bi-O-Set in (Ψ, τ_1, τ_2). This implies, W is an NS-bi-C-Set in (Ψ, τ_1, τ_2). Hence, every NSCS in (Ψ, τ_1) is an NS-bi-C-Set in (Ψ, τ_1, τ_2).

Similarly, it can be shown that, every NSCS in (Ψ, τ_2) is an NS-bi-C-Set in (Ψ, τ_1, τ_2).

Hence, every NSCS in one of the NSTSs (Ψ, τ_i) (i=1, 2) is an NS-bi-C-Set in (Ψ, τ_1, τ_2).

Definition 3.3. Let (Ψ, τ_1, τ_2) be an NSBTS. An NS G over Ψ is called an NS-bi-S-O-Set in (Ψ, τ_1, τ_2) if G is an NSSOS in at least one of two NSTSs (Ψ, τ_1) and (Ψ, τ_2).

Example 3.5. Let us consider an NSBTS (Ψ, τ_1, τ_2) as shown in Example 3.1. Clearly, S={(a,1.0,0.3,0.3), (b,1.0,0.5,0.5)} is an NSSOS in (Ψ, τ_1). Therefore, S is an NS-bi-S-O-Set in (Ψ, τ_1, τ_2).

Definition 3.4. In an NSBTS (Ψ, τ_1, τ_2), an NS G over Ψ is said to be an NS-bi-P-O-Set in (Ψ, τ_1, τ_2) if G is an

NSPOS in at least one of two NSTSs (Ψ, τ_1) and (Ψ, τ_2).

Example 3.6. Let (Ψ, τ_1, τ_2) be an NSBTS as shown in Example 3.1. Clearly, $K=\{(a,1.0,0.1,0.1), (b,1.0,0.2, 0.2)\}$ is an NSPOS in (Ψ, τ_1). Hence, K is an NS-bi-P-O-Set in (Ψ, τ_1, τ_2).

Definition 3.5. In an NSBTS (Ψ, τ_1, τ_2), an NS G over Ψ is called an NS-bi-*b*-O-Set in (Ψ, τ_1, τ_2) if G is an NS-*b*-OS in at least one of two NSTSs (Ψ, τ_1) and (Ψ, τ_2).

Example 3.7. Let us consider an NSBTS (Ψ, τ_1, τ_2) as shown in Example 3.1. Clearly, $G=\{(a,1.0,0.6,0.6), (b,1.0,0.6,0.6)\}$ is an NSSOS in (Ψ, τ_1). Since every NSSOS is an NS-*b*-OS, so G is an NS-*b*-OS in (Ψ, τ_1). Therefore, S is an NS-bi-*b*-O-Set in (Ψ, τ_1, τ_2).

Remark 3.5.

(i) Every NS-bi-S-O-Set in an NSBTS (Ψ, τ_1, τ_2) is also NS-bi-*b*-O-Set.

(ii) Every NS-bi-P-O-Set in an NSBTS (Ψ, τ_1, τ_2) is also NS-bi-*b*-O-Set.

From the above results, we draw the following figure Fig. 1:

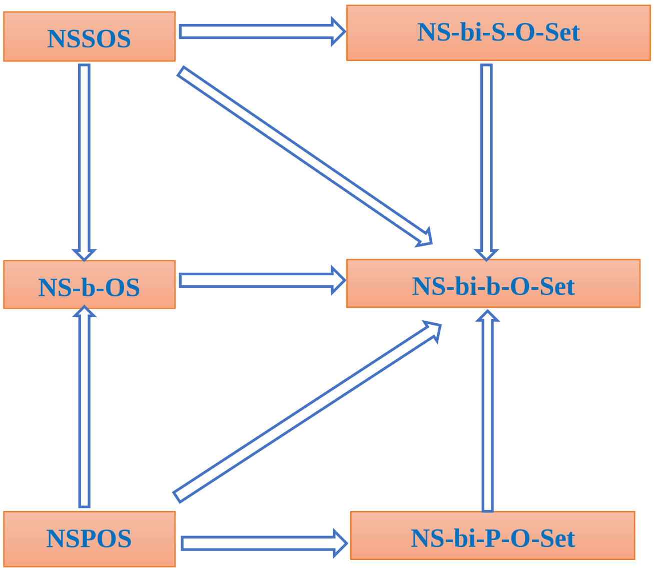

Fig. 1. Relationship between Different types of Neutrosophic Open Sets.

Remark 3.6. Let (Ψ, τ_1, τ_2) be an NSBTS, and $\tau_{1,2}=\tau_1\cup\tau_2$. Then, $\tau_{1,2}$ may not be an neutrosophic supra topology on Ψ in general. This follows from the following example.

Example 3.8. Let (Ψ, τ_1, τ_2) be an NSBTS as shown in Example 3.1. Clearly, $P=\{(a,0.6,0.5,0.6), (b,0.7,0.5, 0.6)\}$ and $Y=\{(a,0.5,0.5,0.5), (b,0.8,0.5,0.6)\} \in\tau_{1,2}$, but their union $P\cup Y=\{(a,0.6,0.5,0.5), (b,0.8,0.5,0.6)\}\notin\tau_{1,2}$. Therefore, $\tau_{1,2}$ is not an neutrosophic supra topology on Ψ.

Definition 3.6. Let (Ψ, τ_1, τ_2) be an NSBTS. Then, an NS R over Ψ is called a Pairwise-NSOS in (Ψ, τ_1, τ_2) if there exist NSOSs R_1 in τ_1 and R_2 in τ_2 such that $R = R_1\cup R_2$.

Example 3.9. Let us consider an NSBTS (Ψ, τ_1, τ_2) as shown in Example 3.1. Consider an neutrosophic set $K=\{(a,0.4,0.6,0.8), (b,0.7,0.7,0.7)\}$ over Ψ. Now, K can be written as $K=Q\cup X$, where $Q=\{(a,0.4,0.7,0.8), (b,0.3,1.0,0.9)\}$ is an NSOSs in (Ψ, τ_1) and $X=\{(a,0.4,0.6,0.8), (b,0.7,0.8,0.7)\}$ is an NSOSs in (Ψ, τ_2). Therefore, K is a Pairwise-NSOS in (Ψ, τ_1, τ_2).

Remark 3.7. In an NSBTS (Ψ, τ_1, τ_2), an NS G is said to be a Pairwise-NSCS if G^c is a Pairwise-NSOS in (Ψ, τ_1, τ_2).

Theorem 3.3. In an NSBTS (Ψ, τ_1, τ_2), every NS-bi-O-Set is also a Pairwise-NSOS.

Proof. Let (Ψ, τ_1, τ_2) be an NSBTS. Let X be an NS-bi-O-Set in (Ψ, τ_1, τ_2). Therefore, $X\in\tau_1\cup\tau_2$. Then, there are three cases.

Case-1: $X\in\tau_1$

Case-2: $X\in\tau_2$

Case-3: $X\in\tau_1$ and $X\in\tau_2$

In case 1, we can write, $X=X\cup 0_N$. Therefore, X is the union of NSOSs X (in (W, τ_1)) and 0_N (in (W, τ_2)). Hence, X is a Pairwise-NSOS in (Ψ, τ_1, τ_2).

In case 2, we can write, $X=0_N\cup X$. Therefore, X is the union of NSOSs 0_N (in (W, τ_1)) and X (in (W, τ_2)). Hence, X is a Pairwise-NSOS in (Ψ, τ_1, τ_2).

In case 3, we can write, $X=X\cup X$. Therefore, X is the union of NSOSs X (in (W, τ_1)) and X (in (W, τ_2)). Hence, X is a Pairwise-NSOS in (Ψ, τ_1, τ_2).

Hence, every NS-bi-O-Set in (Ψ, τ_1, τ_2) is a Pairwise-NSOS.

Theorem 3.4. Let (Ψ, τ_1, τ_2) be an NSBTS. Then, the following holds:

(*i*) 0_N and 1_N are both Pairwise-NSOS and Pairwise-NSCS in (Ψ, τ_1, τ_2);

(*ii*) Every NSOS in one of the NSTS (Ψ, τ_1) and (Ψ, τ_2) are Pairwise-NSOS in (Ψ, τ_1, τ_2);

(*iii*) Every NSCS in one of the NSTS (Ψ, τ_1) and (Ψ, τ_2) are Pairwise-NSCS in (Ψ, τ_1, τ_2).

Proof. (*i*) Suppose that (Ψ, τ_1, τ_2) be an NSBTS. Now, one can express the neutrosophic null set (0_N) as $0_N=W\cup M$, where $W=0_N$ and $M=0_N$ are NSOSs in (Ψ, τ_1) and (Ψ, τ_2) respectively. Therefore, 0_N is a Pairwise-NSOS in (Ψ, τ_1, τ_2). Hence, $(0_N)^c=1_N$ is a Pairwise-NSCS in (Ψ, τ_1, τ_2).

Similarly, one can write the neutrosophic whole set (1_N) as $1_N=W\cup M$, where $W=1_N$ and $M =1_N$ are NSOSs in (Ψ, τ_1) and (Ψ, τ_2) respectively. Therefore, 1_N is a Pairwise-NSOS in (Ψ, τ_1, τ_2). Hence, $(1_N)^c=0_N$ is a Pairwise-NSCS in (Ψ, τ_1, τ_2).

(*ii*) Let W be an NSOS in (Ψ, τ_1). Now, one can write $W=W\cup 0_N$. Therefore, there exist NSOSs W and 0_N in (Ψ, τ_1) and (Ψ, τ_2) respectively such that $W=W\cup 0_N$. Hence, W is a Pairwise-NSOS in (Ψ, τ_1, τ_2).

Similarly, let W be an NSOS in (Ψ, τ_2). Now, one can express $W=0_N\cup W$. Therefore, there exist NSOSs 0_N and W in (Ψ, τ_1) and (Ψ, τ_2) respectively such that $W=0_N\cup W$. Hence, W is a Pairwise-NSOS in (Ψ, τ_1, τ_2).

(*iii*) Let W be an NSCS in (Ψ, τ_1). Therefore, W^c is a NSOS in (Ψ, τ_1). By the second part of this theorem, W^c is a Pairwise-NSOS in (Ψ, τ_1, τ_2). Hence, W is a Pairwise-NSCS in (Ψ, τ_1, τ_2).

Suppose that W is an NSCS in (Ψ, τ_2). Therefore, W^c is an NSOS in (Ψ, τ_2). By the second part of this theorem, W^c is a Pairwise-NSOS in (Ψ, τ_1, τ_2). Hence, W is a Pairwise-NSCS in (Ψ, τ_1, τ_2).

Theorem 3.5. In an NSBTS (Ψ, τ_1, τ_2), the union of two Pairwise-NSOSs is a Pairwise-NSOS.

Proof. Suppose that X and Y are two Pairwise-NSOSs in an NSBTS (Ψ, τ_1, τ_2). Therefore, there exist NSOSs X_1, Y_1 in (Ψ, τ_1) and X_2, Y_2 in (Ψ, τ_2) such that X=X_1∪X_2 and Y=Y_1∪Y_2. Now, we have $X\cup Y=(X_1\cup X_2)\cup(Y_1\cup Y_2)$ $=(X_1\cup Y_1)\cup(X_2\cup Y_2)$. Since X_1 and Y_1 are NSOSs in (Ψ, τ_1), so $X_1\cup Y_1$ is an NSOS in (Ψ, τ_1). Since X_2 and Y_2 are NSOSs in (Ψ, τ_2), so $X_2\cup Y_2$ is an NSOS in (Ψ, τ_2). Therefore, $X\cup Y$ is a Pairwise-NSOS in (Ψ, τ_1, τ_2).

Remark 3.8. In an NSBTS (Ψ, τ_1, τ_2), the intersection of two Pairwise-NSOSs may not be a Pairwise-NSOS in general.

Definition 3.7. Let (Ψ, τ_1, τ_2) be an NSBTS. Then, an NS Q over Ψ is called a Pairwise-NSSOS in (Ψ, τ_1, τ_2) if there exist two NSSOSs Q_1 in (W, τ_1) and Q_2 in (W, τ_2) such that Q=Q_1∪Q_2.

Theorem 3.6. Every NS-bi-S-O-Set in an NSBTS (Ψ, τ_1, τ_2) is also a Pairwise-NSSOS.

Proof. Let X be an NS-bi-S-O-Set in (Ψ, τ_1, τ_2). Therefore, X must be an neutrosophic supra semi-open set in at least one of the NSTSs (Ψ, τ_1), (Ψ, τ_2). So, there will be three cases.

Case 1: X is an NSSOS in (Ψ, τ_1).

Case 2: X is an NSSOS in (Ψ, τ_2).

Case 3: X is an NSSOS in (Ψ, τ_1) and (Ψ, τ_2).

In case 1, we can write, X=$X\cup 0_N$. Therefore, X is the union of NSSOSs X (in (W, τ_1)) and 0_N (in (W, τ_2)). Hence, X is a Pairwise-NSSOS in (Ψ, τ_1, τ_2).

In case 2, we can write, X=$0_N\cup X$. Therefore, X is the union of NSSOSs 0_N (in (W, τ_1)) and X (in (W, τ_2)). Hence, X is a Pairwise-NSSOS in (Ψ, τ_1, τ_2).

In case 3, we can write, X=$X\cup X$. Therefore, X is the union of NSSOSs X (in (W, τ_1)) and X (in (W, τ_2)). Hence, X is a Pairwise-NSSOS in (Ψ, τ_1, τ_2).

Hence, every NS-bi-S-O-Set in (Ψ, τ_1, τ_2) is a Pairwise-NSSOS.

Definition 3.8. Let (Ψ, τ_1, τ_2) be an NSBTS. Then, an NS Q is called a Pairwise-NSPOS in (Ψ, τ_1, τ_2) if there exist two NSPOSs Q_1 in τ_1 and Q_2 in τ_2 such that Q=Q_1∪Q_2.

Theorem 3.7. In an NSBTS (Ψ, τ_1, τ_2), every NS-bi-P-O-Set is a Pairwise-NSPOS.

Proof. Let X be an NS-bi-P-O-Set in an NSBTS (Ψ, τ_1, τ_2). Therefore, X must be a neutrosophic supra pre-open set in at least one of the NSTSs (Ψ, τ_1) and (Ψ, τ_2). So, there will be three cases.

Case 1: X is an NSPOS in (Ψ, τ_1).

Case 2: X is an NSPOS in (Ψ, τ_2).

Case 3: X is an NSPOS in (Ψ, τ_1) and (Ψ, τ_2).

In case 1, we can express, X=$X\cup 0_N$. This implies, X is the union of NSPOSs X (in (W, τ_1)) and 0_N (in (W, τ_2)). Therefore, X is a Pairwise-NSPOS in (Ψ, τ_1, τ_2).

In case 2, we can express, X=$0_N\cup X$. This implies, X is the union of NSPOSs 0_N (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a Pairwise-NSPOS in (Ψ, τ_1, τ_2).

In case 3, we can express, X=$X\cup X$. This implies, X is the union of NSPOSs X (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a Pairwise-NSPOS in (Ψ, τ_1, τ_2).

Hence, every NS-bi-P-O-Set is a Pairwise-NSPOS in (Ψ, τ_1, τ_2).

Definition 3.9. Let (Ψ, τ_1, τ_2) be an NSBTS. Then, an NS Q over Ψ is said to be a Pairwise-NS-*b*-OS in (Ψ, τ_1, τ_2) if there exist two NS-*b*-OSs Q_1 in τ_1 and Q_2 in τ_2 such that Q=Q_1∪Q_2.

Theorem 3.8. Every NS-bi-*b*-O-Set in an NSBTS (Ψ, τ_1, τ_2) is also a Pairwise-NS-*b*-OS.

Proof. Let X is an NS-bi-*b*-O-Set in an NSBTS (Ψ, τ_1, τ_2). Therefore, X must be an NS-*b*-OS in at least one of the NSTSs (Ψ, τ_1) and (Ψ, τ_2). So, there will be three cases.

Case 1: X is an NS-*b*-OS in (Ψ, τ_1).

Case 2: X is an NS-*b*-OS in (Ψ, τ_2).

Case 3: X is an NS-*b*-OS in (Ψ, τ_1) and (Ψ, τ_2).

In case 1, we can express, X=$X\cup 0_N$. This implies, X is the union of NS-*b*-OSs X (in (W, τ_1)) and 0_N (in (W, τ_2)). Therefore, X is a Pairwise-NS-*b*-OS in (Ψ, τ_1, τ_2).

In case 2, we can express, X=$0_N\cup X$. This implies, X is the union of NS-*b*-OSs 0_N (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a Pairwise-NS-*b*-OS in (Ψ, τ_1, τ_2).

In case 3, we can express, X=$X\cup X$. This implies, X is the union of NS-*b*-OSs X (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a Pairwise-NS-*b*-OS in (Ψ, τ_1, τ_2).

Hence, every NS-bi-*b*-O-set is a Pairwise-NS-*b*-OS in (Ψ, τ_1, τ_2).

From the above results, we draw the following figure Fig. 2:

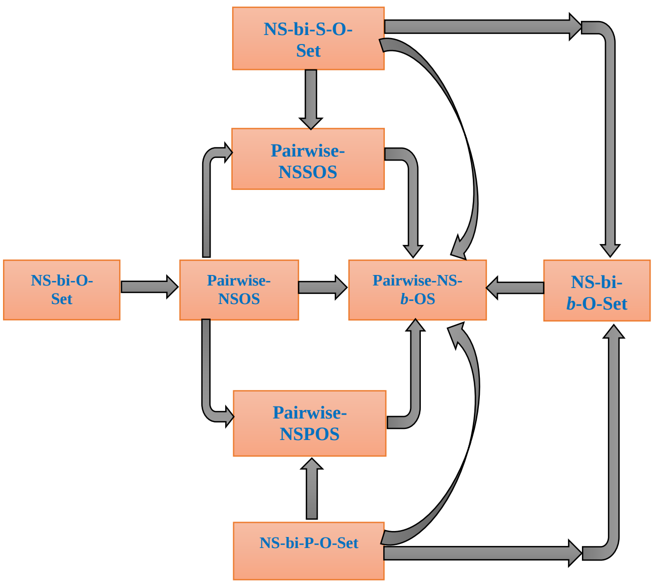

Fig. 2. Relationship between Neutrosophic b-Open, bi-Open, Semi Open and P-Open Sets.

Definition 3.10. Let (Ψ, τ_1, τ_2) be an NSBTS. Then, the pairwise neutrosophic supra interior (Pairwise-NS_{int}) and pairwise neutrosophic supra closure (Pairwise-NS_{cl}) of an NS X is defined as follows:

Pairwise-$NS_{int}(X)$ = $\cup\{L$: L is a Pairwise-NSOS and $L\subseteq X\}$;

Pairwise-$NS_{cl}(X)$ = $\cap\{L$: L is a Pairwise-NSCS and $X\subseteq L\}$.

Clearly, the Pairwise-$NS_{int}(X)$ is the largest Pairwise-NSOS which is contained in X and Pairwise-$NS_{cl}(X)$ is the smallest Pairwise-NSCS which contains X.

Theorem 3.9. Let (Ψ, τ_1, τ_2) be an NSBTS. Let X and Y be two neutrosophic sets over Ψ. Then, the following holds:

(i) Pairwise-$NS_{int}(X) \subseteq X$;

(ii) $X \subseteq Y \Rightarrow$ Pairwise-$NS_{int}(X) \subseteq$ Pairwise-$NS_{int}(Y)$;

(iii) If X is a Pairwise-NSOS, then Pairwise-$NS_{int}(X) = X$;

(iv) Pairwise-$NS_{int}(0_N) = 0_N$ and Pairwise-$NS_{int}(1_N) = 1_N$.

Proof. (i) It is known that, Pairwise-$NS_{int}(X) = \cup\{B$: B is a Pairwise-NSOS, and $B \subseteq X\}$.

Since $B \subseteq X$, so $\cup\{B$: B is a Pairwise-NSOS and $B \subseteq X\} \subseteq X$.

Therefore, Pairwise-$NS_{int}(X) \subseteq X$.

(ii) Let X and Y be two neutrosophic sets over Ψ such that $X \subseteq Y$.

We have, Pairwise-$NS_{int}(X)$

$= \cup\{B$: B is a Pairwise-NS-O-set and $B \subseteq X\}$;

$\subseteq \cup\{B$: B is a Pairwise-NS-O-set and $B \subseteq Y\}$ [Since $X \subseteq Y$]

= Pairwise-$NS_{int}(Y)$

$\Rightarrow$ Pairwise-$NS_{int}(X) \subseteq$ Pairwise-$NS_{int}(Y)$.

Hence, $X \subseteq Y \Rightarrow$ Pairwise-$NS_{int}(X) \subseteq$ Pairwise-$NS_{int}(Y)$.

(iii) Let X be a Pairwise-NSOS in an NSBTS (Ψ, τ_1, τ_2).

Now, Pairwise-$NS_{int}(X) = \cup\{B$: B is a Pairwise-NSOS and $B \subseteq X\}$.

Since X is a Pairwise-NSOS in (Ψ, τ_1, τ_2), so X is the largest Pairwise-NSOS in (Ψ, τ_1, τ_2), which is contained in X. Therefore, $\cup\{B$: B is a Pairwise-NSOS and $B \subseteq X\}=X$. Hence, Pairwise-$NS_{int}(X) = X$.

(iv) It is known that, both 0_N and 1_N are Pairwise-NSOSs in (Ψ, τ_1, τ_2). Hence, by the third part of this theorem, we have Pairwise-$NS_{int}(0_N) = 0_N$, and Pairwise-$NS_{int}(1_N) = 1_N$.

Theorem 3.10. Let (Ψ, τ_1, τ_2) be an NSBTS. Let X and Y be two NSs over Ψ. Then, the following holds:

(i) $X \subseteq$ Pairwise-$NS_{cl}(X)$;

(ii) $X \subseteq Y \Rightarrow$ Pairwise-$NS_{cl}(X) \subseteq$ Pairwise-$NS_{cl}(Y)$;

(iii) If X is a Pairwise-NSCS, then Pairwise-$NS_{cl}(X) = X$;

(iv) Pairwise-$NS_{cl}(0_N)=0_N$, and Pairwise-$NS_{cl}(1_N)=1_N$;

Proof. (i) It is known that, Pairwise-$NS_{cl}(X)= \cap\{B$: B is a Pairwise-NSCS and $X \subseteq B\}$. Since each $X \subseteq B$, so $X \subseteq \cap\{B$: B is a Pairwise-NSCS and $X \subseteq B\}$. Hence, $X \subseteq$ Pairwise-$NS_{cl}(X)$.

(ii) Let X and Y be two neutrosophic sets over Ψ such that $X \subseteq Y$. Then,

Pairwise-$NS_{cl}(X)=\cap\{B$: B is a Pairwise-NSCS and $X \subseteq B\}$.

$\subseteq \cap\{B$: B is a Pairwise-NSCS and $Y \subseteq B\}$ [Since, $X \subseteq Y$]

= Pairwise-$NS_{cl}(Y)$.

Therefore, $X \subseteq Y \Rightarrow$ Pairwise-$NS_{cl}(X) \subseteq$ Pairwise-$NS_{cl}(Y)$.

(iii) Let X be a Pairwise-NSCS in an NSBTS (Ψ, τ_1, τ_2). Now, Pairwise-$NS_{cl}(X) = \cap\{B$: B is a Pairwise-NSCS and $X \subseteq B\}$. Since X is a Pairwise-NSCS in (Ψ, τ_1, τ_2), so X is the smallest Pairwise-NSCS in (Ψ, τ_1, τ_2), which contains X. Therefore, $\cap\{B$: B is a Pairwise-NSCS and $X \subseteq B\} = X$. Therefore, Pairwise-$NS_{cl}(X)=X$.

(iv) It is known that, both 0_N and 1_N are Pairwise-NSCSs in an NSBTS (Ψ, τ_1, τ_2). Hence, by the third part of this theorem, we have Pairwise-$NS_{cl}(0_N) = 0_N$,

Pairwise-$NS_{cl}(1_N) = 1_N$.

Theorem 3.11. In an NSBTS (Ψ, τ_1, τ_2), τ_i-$NS_{int}(X)$ = Pairwise-$NS_{int}(X)$ for any NS X over Ψ.

Proof. Let X be an neutrosophic sub-set of an NSBTS (Ψ, τ_1, τ_2). It is known that, τ_i-$NS_{int}(X) = \cup\{Y$: Y is an NSOS in (Ψ,τ_i) and $Y \subseteq X\}$. Since Y is an NSOS in (Ψ,τ_i), so by second part of Theorem 3.3, Y is a Pairwise-NSOS in (Ψ, τ_1, τ_2). Therefore, τ_i-$NS_{int}(X) = \cup\{Y$: Y is an NSOS in (Ψ, τ_i) and $Y \subseteq X\} = \cup\{Y$: Y is a Pairwise-NSOS in (Ψ, τ_1, τ_2) and $Y \subseteq X\}$ = Pairwise-$NS_{int}(X)$. Hence, τ_i-$NS_{int}(X)$ = Pairwise-$NS_{int}(X)$, for any NS X.

Theorem 3.12. In an NSBTS (Ψ, τ_1, τ_2), τ_i-$NS_{cl}(X) \subseteq$ Pairwise-$NS_{cl}(X)$.

Proof. Let X be an neutrosophic subset of an NSBTS (Ψ, τ_1, τ_2). Now, τ_i-$NS_{cl}(X)=\cap\{Y$: Y is an NSCS in (Ψ, τ_i) and $X \subseteq Y\}$. Since Y is an NSCS in (Ψ, τ_i), so by third part of Theorem 3.3, Y is a Pairwise-NSCS in (Ψ, τ_1, τ_2). Therefore, τ_i-$NS_{cl}(X) = \cap\{Y$: Y is an NSCS in (Ψ, τ_i) and $X \subseteq Y\} = \cap\{Y$: Y is a Pairwise-NSCS in (Ψ, τ_1, τ_2) and $X \subseteq Y\}$ = Pairwise-$NS_{cl}(X)$. Hence, τ_i-$NS_{cl}(X)$ = Pairwise-$NS_{cl}(X)$, for any neutrosophic set X in (Ψ, τ_1, τ_2).

Definition 3.11. Let (Ψ, τ_1, τ_2) and $(\Omega, \delta_1, \delta_2)$ be two NSBTSs. A one to one and onto mapping $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is called as:

(i) pairwise-NSSC-mapping if $\xi^{-1}(K)$ is an NS-bi-S-O-Set in Ψ, whenever K is a pairwise-NSOS in Ω.

(ii) pairwise-NSPC-mapping if $\xi^{-1}(L)$ is a NS-bi-P-O-Set in Ψ, whenever L is a pairwise-NSOS in Ω.

(iii) pairwise-NSC-mapping if $\xi^{-1}(K)$ is a NS-bi-O-Set in Ψ, whenever K is a pairwise-NSOS in Ω.

(iv) pairwise-NS-b-C-mapping if $\xi^{-1}(L)$ is a NS-bi-b-O-set in Ψ, whenever L is a pairwise-NSOS in Ω.

Theorem 3.13. Let (Ψ, τ_1, τ_2) and $(\Omega, \delta_1, \delta_2)$ be two NSBTSs. Then, every pairwise-NSC-mapping from (Ψ, τ_1, τ_2) to $(\Omega, \delta_1, \delta_2)$ is also a pairwise-NSSC-mapping.

Proof. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a pairwise-NSC-mapping. Let K be a pairwise-NSOS in Ω. Since ξ is a pairwise-NSC-mapping from (Ψ, τ_1, τ_2) to $(\Omega, \delta_1, \delta_2)$, so $\xi^{-1}(K)$ is an NS-bi-O-Set in Ψ. It is known that every NS-bi-O-Set is an NS-bi-S-O-Set. This implies, $\xi^{-1}(K)$ is an NS-bi-S-O-Set in (Ψ, τ_1, τ_2). Hence, $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a pairwise-NSSC-mapping.

Theorem 3.14. Let (Ψ, τ_1, τ_2) and $(\Omega, \delta_1, \delta_2)$ be two NSBTSs. Then, every pairwise-NSC-mapping from (Ψ, τ_1, τ_2) to $(\Omega, \delta_1, \delta_2)$ is also a pairwise-NSPC-mapping.

Proof. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a pairwise-NSC-mapping. Let K be a pairwise-NSOS in Ω. Since ξ is a pairwise-NSC-mapping from (Ψ, τ_1, τ_2) to $(\Omega, \delta_1, \delta_2)$, so $\xi^{-1}(K)$ is an NS-bi-O-Set in Ψ. It is known that every NS-bi-O-Set is also an NS-bi-P-O-Set. Therefore, $\xi^{-1}(K)$ is an NS-bi-P-O-Set in (Ψ, τ_1, τ_2). Hence, $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ is a pairwise-NSPC-mapping.

Theorem 3.15. Let (Ψ, τ_1, τ_2) and $(\Omega, \delta_1, \delta_2)$ be two NSBTSs. Then, every pairwise-NSSC-mapping from (Ψ, τ_1, τ_2) to $(\Omega, \delta_1, \delta_2)$ is a pairwise-NS-b-C-mapping.

Proof. Let $\xi:(\Psi, \tau_1, \tau_2)\rightarrow(\Omega, \delta_1, \delta_2)$ be a pairwise-NSSC-mapping. Let K be a pairwise-NSOS in Ω. Since ξ is a pairwise-NSSC-mapping from (Ψ, τ_1, τ_2) to $(\Omega, \delta_1, \delta_2)$, so $\xi^{-1}(K)$ is an NS-bi-S-O-Set in Ψ. It is known that every NS-bi-S-O-Set is an NS-bi-b-O-Set. Therefore, $\xi^{-1}(K)$ is an NS-bi-b-O-Set in Ψ. Hence, ξ:(Ψ, τ_1, τ_2)→(Ω, δ_1, δ_2) is a pairwise-NS-b-C-mapping.

Theorem 3.16. Let (Ψ, τ_1, τ_2) and (Ω, δ_1, δ_2) be two NSBTSs. Then, every pairwise-NSPC-mapping from (Ψ, τ_1, τ_2) to (Ω, δ_1, δ_2) is a pairwise-NS-b-C-mapping.

Proof. Let ξ:(Ψ, τ_1, τ_2)→(Ω, δ_1, δ_2) be a pairwise-NSPC-mapping. Let K be a pairwise-NSOS in Ω. Since ξ is a pairwise-NSPC-mapping from (Ψ, τ_1, τ_2) to (Ω, δ_1, δ_2), so $\xi^{-1}(K)$ is an NS-bi-P-O-Set in Ψ. It is known that every NS-bi-P-O-Set is an NS-bi-b-O-Set. Therefore, $\xi^{-1}(K)$ is an NS-bi-b-O-Set in Ψ. Hence, ξ:(Ψ, τ_1, τ_2)→(Ω, δ_1, δ_2) is a pairwise-NS-b-C-mapping.

Remark 3.9. Every pairwise-NSC-mapping is also a pairwise-NS-b-C-mapping.

From the above results, we draw the following figure Fig. 3:

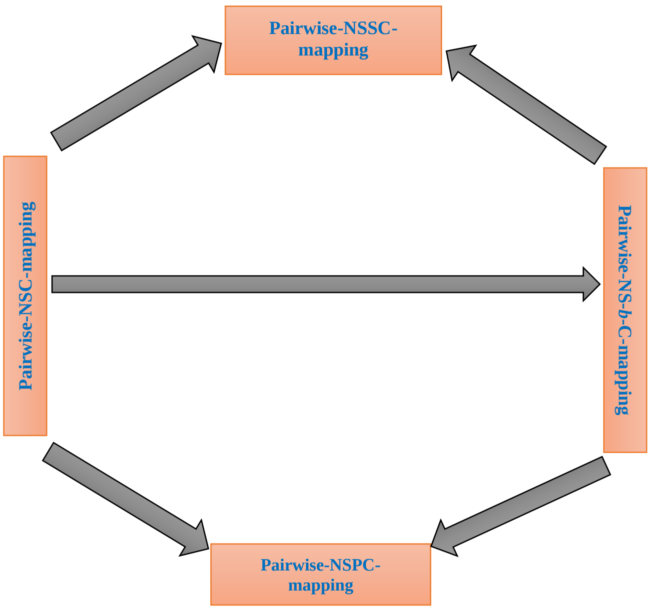

Fig. 3. Relationship between Different types of Neutrosophic Closed Mapping.

Theorem 3.17. If ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) and χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) be two pairwise-NSC-mapping, then the composition mapping χ∘ξ:(X, τ_1, τ_2)→(Z, θ_1, θ_2) is also a pairwise-NSC-mapping.

Proof. Let ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) and χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) be two pairwise-NSC-mapping. Let L be a pairwise-NSOS in (Z, θ_1, θ_2). Since χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) is a pairwise-NSC-mapping, so $\chi^{-1}(L)$ is an NS-bi-O-set in Y. Again, since every NS-bi-O-set is a pairwise-NSOS, so $\chi^{-1}(L)$ is a pairwise-NSOS in (Y, δ_1, δ_2). Since ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) is a pairwise-NSC-mapping, so $\xi^{-1}(\chi^{-1}(L))$ = $(\chi\circ\xi)^{-1}(L)$ is an NS-bi-O-Set in X. Hence, χ∘ξ:(X, τ_1, τ_2)→(Z, θ_1, θ_2) is also a pairwise-NSC-mapping.

Theorem 3.18. If ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) be a pairwise-NSPC-mapping and χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) be a pairwise-NSC-mapping, then the composition mapping χ∘ξ:(X, τ_1, τ_2)→(Z, θ_1, θ_2) is also a pairwise-NSPC-mapping.

Proof. Let ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) be a pairwise-NSPC-mapping and χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) be a pairwise-NSC-mapping. Let L be a pairwise-NSOS in (Z, θ_1, θ_2). Since χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) is a pairwise-NSC-mapping, so $\chi^{-1}(L)$ is an NS-bi-O-Set in Y. Again, since every NS-bi-O-Set is a pairwise-NSOS, so $\chi^{-1}(L)$ is a pairwise-NSOS in (Y, δ_1, δ_2). Since, ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) is a pairwise-NSPC-mapping, so $\xi^{-1}(\chi^{-1}(L))=(\chi\circ\xi)^{-1}(L)$ is a NS-bi-P-O-set in X. Therefore, $(\chi\circ\xi)^{-1}(L)$ is an NS-bi-P-O-Set in X, whenever L is a pairwise-NSOS in (Z, θ_1, θ_2). Hence, χ∘ξ:(X, τ_1, τ_2)→(Z, θ_1, θ_2) is also a pairwise-NSPC-mapping.

Theorem 3.19. If ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) be a pairwise-NSSC-mapping and χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) be a pairwise-NSC-mapping, then the composition mapping χ∘ξ:(X, τ_1, τ_2)→(Z, θ_1, θ_2) is also a pairwise-NSSC-mapping.

Proof. Let ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) be a pairwise-NSSC-mapping and χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) be a pairwise-NSC-mapping. Let L be a pairwise-NSOS in (Z, θ_1, θ_2). Since χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) is a pairwise-NSC-mapping, so $\chi^{-1}(L)$ is an NS-bi-O-Set in Y. Again, since every NS-bi-O-Set is a pairwise-NSOS, so $\chi^{-1}(L)$ is a pairwise-NSOS in (Y, δ_1, δ_2). Since, ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) is a pairwise-NSSC-mapping, so $\xi^{-1}(\chi^{-1}(L))= (\chi\circ\xi)^{-1}(L)$ is an NS-bi-S-O-Set in X. Therefore, $(\chi\circ\xi)^{-1}(L)$ is an NS-bi-S-O-Set in X, whenever L is a pairwise-NSOS in (Z, θ_1, θ_2). Hence, χ∘ξ:(X, τ_1, τ_2)→(Z, θ_1, θ_2) is also a pairwise-NSSC-mapping.

Theorem 3.20. If ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) be a pairwise-NS-b-C-mapping and χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) be a pairwise-NSC-mapping, then the composition mapping χ∘ξ:(X, τ_1, τ_2)→(Z, θ_1, θ_2) is also a pairwise-NS-b-C-mapping.

Proof. Let ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) be a pairwise-NS-b-C-mapping and χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) be a pairwise-NSC-mapping. Let L be a pairwise-NSOS in (Z, θ_1, θ_2). Since χ:(Y, δ_1, δ_2)→(Z, θ_1, θ_2) is a pairwise-NSC-mapping, so $\chi^{-1}(L)$ is an NS-bi-O-Set in Y. Again, since every NS-bi-O-Set is a pairwise-NSOS, so $\chi^{-1}(L)$ is a pairwise-NSOS in (Y, δ_1, δ_2). Since, ξ:(X, τ_1, τ_2)→(Y, δ_1, δ_2) is a pairwise-NS-b-C-mapping, so $\xi^{-1}(\chi^{-1}(L))=(\chi\circ\xi)^{-1}(L)$ is an NS-bi-b-O-Set in X. Therefore, $(\chi\circ\xi)^{-1}(L)$ is an NS-bi-b-O-Set in X, whenever L is a pairwise-NSOS in (Z, θ_1, θ_2). Hence, χ∘ξ:(X, τ_1, τ_2)→(Z, θ_1, θ_2) is also a pairwise-NS-b-C-mapping.

IV. Conclusion

In this study, we have established the notions of neutrosophic supra bi-topological space by extending the concept of neutrosophic supra topological space and bi-topological space. By defining neutrosophic supra bi-topological space, we have formulated some interesting results in the form of theorems, remarks, propositions, etc in the context of neutrosophic supra bi-topological spaces. Furthermore, we provided several well-described examples to justify our results.

Our work suggests that the concept of neutrosophic supra bi-topological space holds promise for future developments, particularly in its potential extension to pentapartitioned neutrosophic sets, bipolar pentapartitioned neutrosophic sets, and beyond, offering exciting avenues for further research and exploration.

REFERENCES

[1] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, On some new notations and functions in neutrosophic topological spaces, *Neutrosophic Sets and Systems*, **16**, 16-19, (2017).

[2] S. Das, Neutrosophic Supra Simply Open Set and Neutrosophic Supra Simply Compact Space, *Neutrosophic Sets and Systems*, **43**, 105-113, (2021).

[3] S. Das, R. Das and C. Granados, Topology on Quadripartitioned Neutrosophic Sets, *Neutrosophic Sets and Systems*, **45**, 54-61, (2021).

[4] S. Das, R. Das and S. Pramanik, Neutrosophic-Separation-Axioms, *Neutrosophic Sets and Systems*, **49**, 103-110, (2022).

[5] S. Das, R. Das and B. C. Tripathy, Multi-criteria group decision making model using single-valued neutrosophic set, *Log Forum*, **16(3)**, 421-429, (2020).

[6] S. Das, R. Das and B. C. Tripathy, Neutrosophic pre-I-open set in neutrosophic ideal bitopological space, *Soft Computing*, (2022).

[7] S. Das and S. Pramanik, Generalized neutrosophic *b*-open sets in neutrosophic topological space, *Neutrosophic Sets and Systems*, **35**, 522-530, (2020).

[8] S. Das and S. Pramanik, Neutrosophic Φ-open sets and neutrosophic Φ-continuous functions, *Neutrosophic Sets and Systems*, **38**, 355-367, (2020).

[9] S. Das and S. Pramanik, Neutrosophic simply soft open set in neutrosophic soft topological space, *Neutrosophic Sets and Systems*, **38**, 235-243, (2020).

[10] S. Das and S. Pramanik, Neutrosophic Tri-Topological Space, *Neutrosophic Sets and Systems*, **45**, 366-377, (2021).

[11] S. Das, B. Shil and S. Pramanik, SVPNS-MADM strategy based on GRA in SVPNS Environment, *Neutrosophic Sets and Systems*, **47**, 50-65, (2021).

[12] S. Das, B. Shil and B. C. Tripathy, Tangent Similarity Measure Based MADM-Strategy under SVPNS-Environment, *Neutrosophic Sets and Systems*, **43**, 93-104, (2021).

[13] R. Das, F. Smarandache and B. C. Tripathy, Neutrosophic fuzzy matrices and some algebraic operation, *Neutrosophic Sets and Systems*, **32**, 401-409, (2020).

[14] R. Das and B. C. Tripathy, Neutrosophic multiset topological space, *Neutrosophic Sets and Systems*, **35**, 142-152, (2020).

[15] S. Das and B. C. Tripathy, Pairwise neutrosophic-*b*-open set in neutrosophic bitopological spaces, *Neutrosophic Sets and Systems*, **38**, 135-144, (2020).

[16] S. Das and B. C. Tripathy, Neutrosophic simply *b*-open set in neutrosophic topological spaces, *Iraqi Journal of Science*, **62(12)**, 4830-4838, (2021).

[17] S. Das and B. C. Tripathy, Pentapartitioned Neutrosophic Topological Space, *Neutrosophic Sets and Systems*, **45**, 121-132, (2021).

[18] R. Dhavaseelan, M. Ganster, S. Jafari and M. Parimala, On neutrosophic α-supra open sets and neutrosophic α-supra continuous functions, *New Trends in Neutrosophic Theory and Applications*, **2**, 289-297, (2017).

[19] R. Dhavanseelan, M. Parimala, S. Jafari and F. Smarandache, On neutrosophic semi-supra open set and neutrosophic semi-supra continuous functions, *Neutrosophic Sets and Systems*, **16**, 39-43, (2017).

[20] E. Ebenanjar, J. Immaculate and C.B. Wilfred, On neutrosophic *b*-open sets in neutrosophic topological space, *Journal of Physics Conference Series*, **1139(1)**, 012062, (2018).

[21] P. Iswarya and K. Bageerathi, On neutrosophic semi-open sets in neutrosophic topological spaces, *International Journal of Mathematical Trends and Technology*, **37(3)**, 214-223, (2016).

[22] G. Jayaparthasarathy, V. F. Little Flower and M. Arockia Dasan, Neutrosophic supra topological applications in data mining process, *Neutrosophic Sets and Systems*, **27**, 80-97, (2019).

[23] J. C. Kelly, Bitopological spaces, *Proceedings of the London Mathematical Society*, **3(1)**, 71-89, (1963).

[24] C. Maheswari, M. Sathyabama and S. Chandrasekar, Neutrosophic generalized *b*-closed sets in neutrosophic topological spaces, *Journal of Physics Conference Series*, **1139(1)**, 012065, (2018).

[25] P. Majumder, S. Das, R. Das and B. C. Tripathy, Identification of the Most Significant Risk Factor of COVID-19 in Economy Using Cosine Similarity Measure under SVPNS-Environment, *Neutrosophic Sets and Systems*, **46**, 112-127, (2021).

[26] I. Mohammed Ali Jaffer and K. Ramesh, Neutrosophic generalized pre regular closed sets, *Neutrosophic Sets and System*, **30**, 171-181, (2019).

[27] T. Y. Ozturk and A. Ozkan, Neutrosophic bitopological spaces, *Neutrosophic Sets and Systems*, **30**, 88-97, (2019).

[28] A. Pushpalatha and T. Nandhini, Generalized closed sets via neutrosophic topological spaces, *Malaya Journal of Matematik*, **7(1)**, 50-54, (2019).

[29] K. Ramesh, Ngpr homomorphism in neutrosophic topological spaces, *Neutrosophic Sets and Systems*, **32**, 25-37, (2020).

[30] V. V. Rao and R. Srinivasa, Neutrosophic pre-open sets and pre-closed sets in neutrosophic topology, *International Journal of ChemTech Research*, **10(10)**, 449-458, (2017).

[31] P. Saha, P. Majumder, S. Das, P. K. Das and B. C. Tripathy, Single-Valued Pentapartitioned Neutrosophic Dice Similarity Measure and Its Application in the Selection of Suitable Metal Oxide Nano-Additive for Biodiesel Blend on Environmental Aspect, *Neutrosophic Sets and Systems*, **48**, 154-171, (2022).

[32] A. A. Salama and S. A. Alblowi, Neutrosophic set and neutrosophic topological space, *ISOR Journal of Mathematics*, **3(4)**, 31-35, (2012).

[33] A. A. Salama and S. A. Alblowi, Generalized neutrosophic set and generalized neutrosophic topological space, *Computer Science and Engineering*, **2(7)**, 129-132, (2012).

[34] A. A. Salama, F. Smarandache and S. A. Alblowi, New neutrosophic crisp topological concepts, *Neutrosophic Sets and Systems*, **4**, 50-54, (2014).

[35] V. K. Shanthi, S. Chandrasekar and K. Safina Begam, Neutrosophic generalized semi closed sets in neutrosophic topological spaces, *International Journal of Research in Advent Technology*, **67**, 1739-1743, (2018).

[36] F. Smarandache, Neutrosophic set: a generalization of the intuitionistic fuzzy sets, *International Journal of Pure and Applied Mathematics*, **24**, 287-297, (2005).

[37] R. Suresh and S. Palaniammal, Neutrosophic weakly generalized open and closed sets, *Neutrosophic Sets and Systems*, **33**, 67-77, (2020).

[38] B. C. Tripathy and S. Das, Pairwise Neutrosophic *b*-Continuous Function in Neutrosophic Bitopological Spaces, *Neutrosophic Sets and Systems*, **43**, 82-92, (2021).

Suman Das received his M.Sc. from Tripura University (A Central University) and his B.Sc. from Iswar Chandra Vidyasagar College, Belonia, Tripura. He is a Ph.D. candidate at Tripura University's Department of Mathematics. In 2018, he qualified UGC-NET JRF, NE-SLET and GATE. His research interests include Topology, Neutrosophy and Algebra.

Rakhal Das received M.Sc. from Tripura University (A Central University) and B.Sc. from Iswar Chandra Vidyasagar College, Belonia, Tripura. He obtained his Ph.D. from the Department of Mathematics, Tripura University (A Central University), India. He has qualified CSIR-NET JRF and NE-SLET. He is currently an Assistant Professor at ICFAI University's Department of Mathematics in Kamalghat, Agartala, Tripura. His research interests include Spatial Topology, Fuzzy Set, Neutrosophy and Algebra.

Prasanna Poojary, received M.Sc. degrees from Manipal Academy Higher Education. He completed Ph.D. degree with the Department of Mathematics, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal. He is currently an Assistant Professor Department of Mathematics, Manipal Institute of Technology Bengaluru, Manipal Academy of Higher Education, Manipal, India. His research interests include Boolean functions, Number theory, Cryptography, Neutrosophy and Coding theory.

Vadiraja Bhatta G. R., received the M.Sc. degree from Mangalore University. He completed Ph.D. degree with the Department of Mathematical and Computational Sciences (MACS), National Institute of Technology Karnataka, Surathkal. He is currently an Associate Professor with the Department of Mathematics, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal. His research interests include Latin squares, Permutation polynomial, Boolean functions, Neutrosophy and Cryptography.

Varsha Jarali, received the M.Sc. degree from Rani Channamma University, Belagavi. She is a Ph.D candidate in department of mathematics, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal. She has qualified CSIR-UGC-NET JRF, GATE and K-SET. Her research interests include Latin squares, Permutation polynomial, Boolean functions and Neutrosophy.