Performance of Two Numerical Methods in Option Price Calculation

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***Abstract*—This study employed two different numerical methods in calculating American option prices. Implied volatility was calculated using the Newton-Raphson method, while American option pricing was determined using the Monte Carlo method. American options necessitate a numerical solution to parabolic differential equations; thus, this decision is made by pursuing such a solution. A numerical solution is also necessary for the volatility value calculated from the market's implications for option prices. The Nasdaq share market served as the source of information in this study. This research examined the effectiveness of the Monte Carlo method coupled with Newton-Raphson implied volatility in American option pricing across three case studies. According to the findings, the Newton-Raphson method possessed a small error and a fast convergence rate for estimating volatility. However, the Monte Carlo option pricing was preferable to other methods since it resulted in a smaller MAPE value. The MAPE value calculated using the Monte Carlo method with Newton-Raphson implied volatility was lower than that calculated using the Monte Carlo method with historical volatility. The American option pricing generated by the Monte Carlo method with Newton-Raphson implied volatility was more in line with market option prices. The Newton-Raphson method yielded volatility values that shifted in response to market conditions. Because of this fluctuation in the volatility values, the Newton-Raphson method lent its support to the Monte Carlo method for estimating American option prices. Newton-Raphson and Monte Carlo have become two popular numerical methods for option pricing, and both delivered satisfactory results.**

***Index Terms*—American options, Newton-Raphson, Monte Carlo, Volatility**

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I. Introduction

OPTIONS are divided into two sorts based on the expiry period: European and American Options. European option contracts can only be exercised at maturity, while American option contracts can be executed anytime, starting from the contract date until the expiration time. Hence, the American options are more adaptable than the European ones. Investors can decide when to exercise their option contracts. This argument motivates many investors who trade American options so that more are traded on the share exchange [1].

Numerous methods exist for computing option pricing. In 1974, Black-Scholes invented the option price calculation method. Numerous scholars utilized this method as the basis for creating option pricing. The Black-Scholes model is a partial differential equation with a parabolic function. It is extremely challenging to determine the analytical solution to the partial differential equation of the Black-Scholes model for the American options [2], [3]. Therefore, a numerical solution to the partial differential equation is required. This study employed the Monte Carlo method. This study uses the Monte Carlo method to estimate a parameter by creating random variables from a normal distribution. Moon [4] suggests that Monte Carlo is a numerical method that quickly converges and approaches the value of the right choice. In addition, the Monte Carlo method is easy, simple, and suitable for calculating option prices [5]–[7]. This method can produce option prices close to the actual price [8] and predict option prices accurately [9].

In addition to the underlying stock price; the contract price, expiration period, risk-free interest rate, and volatility impact option prices. Volatility is a variable that describes arbitrary stock price fluctuations and cannot be detected immediately; therefore, it must be estimated in advance. There are various methods to assess volatility, one of which is historical volatility, which estimates volatility based on historical data. However, this method cannot accurately predict future volatility due to the selected period. For example, the selected time is three months ago, when the asset price volatility did not fluctuate excessively; thus, it cannot be ensured that volatility in the following period will not fluctuate excessively [10]. Moreover, Yan and Jianhui [11] suggested that implied volatility is superior to historical volatility in determining stock option prices.

Another estimation method utilizes the option prices gained from the market or implied volatility. Implied volatility is the predicted market volatility based on selecting an option contract with the same expiration date

[12]. Rahayuni et al. [12] and Mahrudinda et al. [13] investigated the Newton-Raphson, the Secant, and the Bisection methods for measuring the implied volatility of shares. Moreover, Amri et al. [14] contrasted the Newton-Raphson and Steepest Descent methods for calculating implied volatility. According to the three studies, the Newton-Raphson method is more efficient and converges faster when evaluating the implied volatility of equity.

Previous research analyzed American option pricing using the Monte Carlo method with varying volatility assumptions and created an option pricing simulation with specified volatility [4], [15]–[20]. The Monte Carlo method employs stochastic volatility to determine option prices [3], [21]–[23]. Pucci di Benisichi and Pozzi [24] applied varying volatility across the option's lifetime. Other studies assessed share volatility based on historical volatility [8], [25]–[27]. This study utilized the numerical Newton-Raphson method to calculate implied volatility.

Based on previous research conducted by Mahrudinda et al. [13], Rahayuni et al. [12], and Yan and Jianhui [11], who compared several numerical methods to determine implied volatility in European options, this study uses the Newton-Raphson method to determine implied volatility because this method converges faster and has less error. Meanwhile, American option pricing uses the Monte Carlo method. Based on research [4], [8], [9], the Monte Carlo method was chosen because it converges faster and can approach the correct choice value. The novelty in this research is the combination of two numerical methods in calculating American option prices. Consequently, the performance of the Monte Carlo method for generating option prices and the Newton-Raphson method for determining implied volatility are evaluated in this study.

II. Material and methods

A. *Newton-Raphson Implied Volatility*

The Newton-Raphson formula for determining volatility is as follows [28].

$$\sigma_{n+1} = \sigma_n - \frac{f(\sigma_n)}{f'(\sigma_n)} \tag{1}$$

Determining the implied volatility required the volatility function of $f(\sigma)$ and the first derivative of the volatility function of $f'(\sigma)$.

The price formula for the Black-Scholes Call model is as follows [29].

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{2}$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

S_0 refers to the current share price, K denotes the option contract price, T implies the expiration time (in years), r signifies the risk-free interest rate, and σ is volatility. $N(x)$ indicates the cumulative value of the standard normal distribution.

Based on equation (2), the volatility function of $f(\sigma)$ was formed as follows.

$$f(\sigma) = S_0 N(d_1) - Ke^{-rT} N(d_2) - C_M \tag{3}$$

The derivative result of the volatility function of $f'(\sigma)$ was obtained using equation (4).

$$f'(\sigma) = S_0 \frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \sigma} - Ke^{-rT}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial \sigma}$$

$$f'(\sigma) = S_0 \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \sqrt{T} \tag{4}$$

After obtaining $f(\sigma)$ and $f'(\sigma)$ the initial guess value was determined, guaranteeing that the volatility value obtained converged to a certain value. The function of equation (2) must have the first and second derivatives to ensure that the Newton-Raphson method converges. $f'(\sigma)$ reached a maximum in the interval of [0, ∞) when $e^{-0.5d_1^2} = 1$ and when $d_1 = 0$. $f'(\sigma)$ reached a maximum when

$$\hat{\sigma} = \sqrt{2\left|\frac{\ln(S_0/K) + rT}{T}\right|} \tag{5}$$

The second derivative of $f(\sigma)$ is as follows.

$$f''(\sigma) = f'(\sigma)\frac{T}{4\sigma^3}\left(\hat{\sigma}^4 - \sigma^4\right) \tag{6}$$

The upper and lower limits of the American call options were other conditions required in determining the initial implied volatility value of the Newton-Raphson method to make it converges. The upper and lower limits of the American call options were max $(S_0 - Ke^{-rT}, 0) \le C \le S_0$. The K contract value determined for the simulation must meet max $(S_0 - Ke^{-rT}, 0) \le C \le S_0$.

Thus, convergent volatility was obtained using the initial guess of $\hat{\sigma} = \sqrt{2\left|\frac{\ln(S_0/K) + rT}{T}\right|}$ and the contract value meeting inequality max $(S_0 - Ke^{-rT}, 0) \le C \le S_0$. In the Newton-Raphson method, the error of the resulting volatility was obtained from the following equation.

$$|e_r| = \left|\frac{\sigma_{n+1} - \sigma_n}{\sigma_{n+1}}\right| \tag{7}$$

B. The Monte Carlo Method of Determining American Option Prices

The Monte Carlo method involves random number trial sampling and computer-based samples. This method is also a form of probabilistic where the solution to a problem is given based on a random process [30]. The following was obtained based on the law of large numbers.

$$\frac{1}{M}\sum_{j=1}^{M}\left(S_T^{(j)}-K\right)^+ \xrightarrow{N\to\infty} \mathrm{E}^Q\left(S_T-K\right)^+$$

j = 1, 2, 3, …, M, $S_T^{(j)}$ refers to the independent sample index of the S_T probability distribution.

The stochastic differential equation built from the sample approximated the distribution of share prices using the Euler Maruyama scheme of

$$\log S_{t+\Delta t}^{(j)}=\log S_T^{(l)}+\left(\mu-\frac{1}{2}\sigma^2\right)\Delta t+\sigma\left(W_{t+\Delta t}^{(j)}-W_t^{(j)}\right) \quad (8)$$

for $\Delta t\to 0$ and $W_{t+\Delta t}^{(j)}-W_t^{(j)}\sim iid\ N\left(0,\Delta t\right)$.

This scheme is a discrete backward Riemann sum from which the Ito integral is derived. At the limit, the Monte Carlo average will be o from the true expectation.

$$\lim_{M\to\infty}\frac{1}{M}\sum_{j=1}^{M}\left(S_T^{(j)}-K\right)^+=\mathrm{E}^Q\left(S_T-K\right)^+ +o\left(\Delta t\right)$$

The option price calculation in the Monte Carlo method began by determining the share prices based on the geometric Brownian motion using the following equation.

$$S_t=S_{t_0}e^{\left[\left(r-\frac{1}{2}\sigma^2\right)\Delta t+z\sigma\sqrt{\Delta t}\right]} \quad (9)$$

r is the risk-free interest rate, σ denotes volatility, Δt refers to the time interval from one path to the next, and z represents a number of random numbers with standard normal distribution. Δt was obtained from $\Delta t = T/n$ where T denotes the maturity time (in years), and n implies the number of time partitions.

Suppose $V=\left(V_t\right)_{0\le t\le T}$ is the value of an American option. Gain based on assumed execution time in the set $P=\left\{t_0,t_1,t_2,\ldots,t_n\right\}$ with $l_i=i\Delta$ and $iN = T$. Assuming that time $t_0=0$. For example, τ is stopping time. Defined the initial value of the American option is

$$V_0^*=\sup_{0\le\tau\le T}E\left|Z_\tau\right|$$

where $Z_\tau=\exp\left(-\int_0^t r_s\,ds\right)\tilde{Z}_t$ is the option's discounted execution value.

The equation used in determining the price of American options using a Monte Carlo simulation

$$V_0^*=\inf_{M\in H_0^1}E\left[\sup_{0\le t\le T}\left(Z_t-M_t\right)\right]$$

H_0^1 is the space of martingales M where $\sup_{0\le\tau\le T}\left|Z_\tau\right|\in L^P$ and $M_0=0$. The greatest lower bound (infimum) is reached by taking $M=M^*$.

In calculating the American options, it is necessary to calculate the payoff value of the American options using the formula: $\max\left(S_T-K,0\right)$ for call option and $\max\left(K-S_T,0\right)$ for put option.

The option value is the discounted value of the option's expected payoff. The option value for t to t – 1 period is $C=e^{-rT}E\left[\max\left(S_T-K,0\right)\right]$ for call option and $P=e^{-rT}E\left[\max\left(K-S_T,0\right)\right]$ for put option. The discounted value of the option's expected payoff is calculated starting from t, t – 1, until t = 0 period.

In calculating of the American option price, the steps for generating the stock price are using Brownian geometric motion, the calculation of the option payoff, and the calculation of the expected payoff of the option until the option value is repeated n times. American option value is the average of the resulting option values in each simulation.

C. Methods

This study underwent three steps: literature studies, data collection, application and simulation in case studies, and conclusion drawing. The American option price simulation employed three case studies. Each case study's share and option information were gathered from https://finance.yahoo.com/. The data used were daily closing share price data, current share price data (S_0), option contract price (K), expiry time (T), and market option prices (Call options, C_M and Put options, P_M). Share data were obtained on August 1, 2022, October 10, 2022, and October 11, 2022, for the first, second, and third case studies. After calculating American option prices with the Monte Carlo method and Newton-Raphson implied volatility, the resulting option prices were compared to market option prices.

III. Results and Discussion

This research utilizes stock data from the Nasdaq market. Three case studies are simulated using the Monte Carlo method, using the Newton-Raphson implied volatility calculation of American option prices. The first case study uses different stocks with the same maturity date, while the second case study simulates the pricing of American options with varying contract prices but the same maturity date. The third case study applies the same contract price with different maturity date. These various case studies are applied to demonstrate how the Monte Carlo method with Newton-Raphson implied volatility and historical volatility in pricing options in various situations.

The selection of contract prices (K) in the three case studies considered the minimum and maximum limit criteria for options, $\max\left(S_0-Ke^{-rT},0\right)\le C\le S_0$. The selection of K ensured that the Newton-Raphson method converged to a certain value.

A. First Case Study

The first case study included data from ten equities with the same maturity, comprising MSFT, INTC, IBM, QCOM, NVDA, META, NFLX, AMZN, TSLA, and GOOG. The gathered data consisted of daily closing share price data, current share price data (S_0), option contract prices (K), option expiration time (T), market option prices (Call options, C_M, Put options, P_M), and interest rates of the United States Central Bank, The Fed (r). The prices of ten shares were recorded on August 1, 2022. The option expiration date was October 21, 2022; therefore, T = 81 days. The interest rate of the American central bank was 2.5%.

Tables I and II exhibit the results of calculating American option prices using the Monte Carlo method with Newton-Raphson implied volatility and historical volatility. C NR represents the call option prices with assumed Newton-Raphson implied volatility, and C His implies the call option prices with historical volatility. P NR depicts the put option prices with Newton-Raphson implied volatility, and P His signifies the put option prices with historical volatility. The values of $\frac{|C_M - C\,NR|}{C_M}$, $\frac{|C_M - C\,His|}{C_M}$, $\frac{|P_M - P\,NR|}{P_M}$, and $\frac{|P_M - P\,His|}{P_M}$ each indicate error.

The results of the calculations presented in Table I show that the Newton-Raphson method converges quite quickly. Determining the value of volatility involves a limited number of iterations using the method. It requires a maximum of five iterations, with certain sections only requiring two iterations based on the ten-section data used. These computations disclosed the high effectiveness of the Newton-Raphson method for estimating implied volatility.

The findings of option prices derived from implied volatility and historical volatility were then compared to market option prices. The Mean Absolute Percentage Error (MAPE) values were examined to determine whether the resulting option prices were close to the market option prices.

A comparison of the option price error for each stock between Newton-Raphson implied volatility and historical volatility can be seen in Fig. 1 (a) and 1 (b). Based on Table I, the MAPE value of call options with implied volatility in case studies of the same maturity and various equities was 12.6457%. However, the MAPE value of call options with historical volatility reached 18.6515% (Table II). The put options in the case study with the same maturity and various equities computed using the Monte Carlo method acquired a MAPE value of 11.5233% with Newton-Raphson implied volatility in Table I and 12.3724% with historical volatility in Table II. The MAPE values derived from implied volatility and historical volatility were less than 20% for both call and put options. According to Moreno et al. [31], a MAPE value of less than 20% suggests that the prediction method employed falls into a good category. The Monte Carlo method with implied volatility and historical volatility effectively calculated option prices.

Nevertheless, the MAPE values derived using the Monte Carlo method and implied volatility were less than historical volatility. These findings suggest that American options calculated by the Monte Carlo method and Newton-Raphson implied volatility were closer to market option prices due to their minimal inaccuracy. Thus, the Monte Carlo method with assumed Newton-Raphson volatility yielded accurate results for estimating option prices for equities with the same maturity variable but different underlying assets.

B. Second Case Study

The second case study utilized data from one share, INTC, with different contract prices but the same maturity. Data collection for INTC shares with different contract prices (K) was conducted on October 10, 2022. The selected options had two expiration dates: November 11, 2022 (T = 32 days) and April 21, 2023 (T = 193 days). At each selected maturity date, various contract prices were determined. On October 10, 2022, INTC's share price was S_0 = 25.2, and the US central bank's (The Fed) interest rate was r = 3.25%.

Tables III and IV display the results of calculating American option prices using the Monte Carlo method with Newton-Raphson implied volatility and historical volatility. These results were then compared with market option prices. This comparison aimed to discover which option prices were closer to market option prices.

The Newton-Raphson method swiftly converged when calculating the implied volatility of INTC shares maturing on November 11, 2022. This method took only three to eight iterations to settle on a certain number. The MAPE values of the American option price calculation utilizing the Monte Carlo method with Newton-Raphson implied volatility and historical volatility for the same maturity case study (November 11, 2022), and various contract prices were less than 20%. In terms of computing option prices, the Monte Carlo method, employing both Newton-Raphson implied volatility and historical volatility, was exemplary. The MAPE value for American call option prices calculated with the Monte Carlo method and Newton-Raphson implied volatility in Table III was 12.0327%. In contrast, the MAPE value calculated with historical volatility in Table IV obtained 16.953%. The comparison of call option errors between Newton-Raphson implied volatility and historical volatility at each contract price can be seen in Fig. 2 (a). The implied volatility calculated by Newton-Raphson was less than the historical volatility. The data in Table III and Table IV indicate that the MAPE value of American put option prices calculated using Newton-Raphson implied volatility was less than the value calculated using historical volatility, i.e., 10.9614% < 11.1893%. This result is also confirmed by the put option price error for each contract price presented in Fig. 2 (b). It signifies that while both Newton-Raphson implied volatility and historical volatility were appropriate methods for calculating option prices, Newton-Raphson implied volatility was superior for predicting American call and put option prices due to its reduced error rates. The computation of American option prices using the Monte Carlo method with Newton-Raphson implied volatility approximated market option prices more closely.

In this second case study, in addition to utilizing the maturity date of November 11, 2022, the computation of the American option prices for INTC shares also considered the maturity date of April 21, 2023. Tables V and VI describe the calculation results for the American option prices using the Monte Carlo method with Newton-Raphson implied volatility and historical volatility for the April 21, 2023 expiration date. The resulting option prices were then compared to the market option prices. The MAPE value was utilized to determine if the resulting option prices were close to the market option prices. The lesser the MAPE value, the smaller the resulting error. In other words, the resulting option prices approached the market option prices.

In the second case study involving INTC shares with a maturity date of April 21, 2023, the Newton-Raphson method required fewer than five iterations to converge. In short, the Newton-Raphson method quickly converged while calculating volatility. The MAPE value derived from the pricing of American call options using the Monte Carlo method with Newton-Raphson implied volatility and a maturity date of April 21, 2023, was 11.8824% in Table V. Meanwhile, the American call option prices were calculated using the Monte Carlo method, historical volatility, and a maturity date of April 21, 2023, yielding a MAPE value of 13.8298% in Table VI. The MAPE value obtained by calculating the call option prices using Newton-Raphson implied volatility was less than that acquired using historical volatility. The MAPE value derived from American put options using Newton-Raphson implied volatility was less than that derived using historical volatility. Using Newton-Raphson implied volatility and historical volatility, the MAPE values of American put options in Table V and Table VI were 8.6816% and 13.516%, respectively. Fig. 3 (a) and 3 (b) compare the errors at each contract price, where the Newton-Raphson implied volatility option price error is less than the historical volatility for almost all contract prices.

All MAPE values computed for call and put options using Newton-Raphson implied volatility and historical volatility in the same maturity case study (April 21, 2023) and varied contract values resulted in a MAPE value of less than 20%, falling within the good category. The MAPE value of American put option prices using Newton-Raphson implied volatility was less than 10%, making it a highly accurate method for determining the put option prices. It indicates that the computation of American option prices using the Monte Carlo method with Newton-Raphson implied volatility and historical volatility was reliable due to the comparatively minimal error created. In contrast, estimating American option pricing for both call and put options using Newton-Raphson implied volatility was superior to historical volatility since the resulting MAPE value was lower.

In the second case study, the Newton-Raphson method rapidly converged on the implied volatility of INTC shares. The Monte Carlo method yielded a MAPE value of less than 20%, placing it in a good category for predicting the value of American options in a case study involving the same contract price with variable expiry durations. Based on the calculation of option prices with maturities of November 11, 2022, and April 21, 2023, it showed that the calculation of American option prices using the Monte Carlo method with Newton-Raphson implied volatility produced a MAPE value less than historical volatility. This lower MAPE value suggests that the option prices derived using Newton-Raphson implied volatility were closer to the market prices. Consequently, the computation of American option prices utilizing the Monte Carlo method with Newton-Raphson implied volatility at the same maturity variable, but different contract prices provided good result.

C. Third Case Study

The final case study included data on three shares: INTC, AMZN, and NVDA. This third case study selected a certain contract price with varying maturities. Data were collected on October 11, 2022. The price of INTC shares on October 11, 2022, was $25.04. K = 26 was the contract price for the INTC share options used to simulate option prices. The simulation data for AMZN share was S_0 = 112.21 and K = 116. On October 11, 2022, the NVDA share price was S_0 = 115.86 and K = 120. The Fed rate for October 11, 2022, was 3.25%. In this third case study, the maturity dates were October 21, 2022 (T = 10 days), October 28, 2022 (T = 17 days), November 4, 2022 (T = 24 days), November 11, 2022 (T = 31 days), November 18, 2022 (T = 38 days), November 25, 2022 (T = 45 days), December 16, 2022 (T = 66 days), and January 20, 2022 (T = 101 days).

Table VII displays the results of estimating American option prices for INTC shares using the Monte Carlo method with Newton-Raphson implied volatility. Meanwhile, Table VIII exhibits the results of computing American option prices for INTC shares using the Monte Carlo method and historical volatility.

The Newton-Raphson method required only a few iterations to converge on the implied volatility of an INTC share with a contract price of K = 26. There were only two to five needed iterations. The market option prices were then compared to the American option prices produced using the Monte Carlo method with Newton-Raphson implied volatility and historical volatility. The MAPE was applied to make comparisons to determine which option price was closest to the market option price. In the third case study, the Monte Carlo method, implied volatility, and historical volatility all provided less than 20% MAPE values. The MAPE value derived from the computation of the American call option prices for INTC shares with Newton-Raphson implied volatility in a case study with the same contract price and different maturity dates was 7.6704% (Table VII). Calculating the American call option prices for INTC using historical volatility yielded a MAPE value of 12.0003% (Table VIII). The MAPE value in Table VII was 11.425% while computing the American put option prices using the Monte Carlo method with Newton-Raphson implied volatility in a case study with the same contract price and different expiry timeframes. The MAPE value for American put option prices using the Monte Carlo method and historical volatility in Table VIII was 16.8671%. The MAPE values derived from put and call option price computations using the Monte Carlo method and Newton-Raphson implied volatility were less than using the Monte Carlo method and historical volatility. Likewise, the option price error is shown at each maturity time, as in Fig. 4 (a) and 4 (b). In other words, calculating American option prices using the Monte Carlo method with Newton-Raphson implied volatility more closely approximated market option prices.

The third case study utilized AMZN shares. Table IX displays the calculation results of the American option prices for AMZN shares using the Monte Carlo method with Newton-Raphson implied volatility at the same contract price and various maturities. Meanwhile, Table X presents the calculation of American option prices with historical volatility.

The Newton-Raphson method required three to four iterations to determine the volatility of AMZN shares with a K = 116 contract price. Based on Table IX and Table X, the Monte Carlo method could accurately predict American option prices when calculating AMZN share option prices since it generated a MAPE value of less than 20% when utilizing Newton-Raphson implied volatility and historical volatility. The calculation of the American call option prices in Table IX for AMZN shares unveiled that the MAPE value using Newton-Raphson implied volatility acquired 7.4524%. In contrast, the calculation for the American put option prices for AMZN shares yielded a MAPE value of 2.77565%. Subsequently, in Table X calculating the American call option prices for AMZN shares with historical volatility produced a MAPE value of 13.6953%. Meanwhile, calculating the American put option prices on AMZN shares using historical volatility generated a MAPE value of 6.0689%. The MAPE value derived from the computation of the American option prices for AMZN shares using Newton-Raphson implied volatility in a case study with the same contract price and various maturities was less than historical volatility. This little option price error in Newton-Raphson implied volatility can also be seen in Fig. 5 (a) and 5 (b), where at most maturity times, the option price error with Newton-Raphson implied volatility was less than historical volatility. Hence, the Monte Carlo method with Newton-Raphson implied volatility produced option prices closer to market option prices because of its lower inaccuracy.

NVDA is the third share of identical contract prices and varying maturities in the case study. K = 120 was the price utilized for the contract. The calculation results of NVDA share options using the Monte Carlo method with Newton-Raphson implied volatility and historical volatility are summarized in Tables XI and XII, respectively.

In the third case study of NVDA shares, the Newton-Raphson method requires several iterations to settle on a certain volatility number. Newton-Raphson utilized three to four iterations to converge. The Monte Carlo method performed well in calculating the American option prices for NVDA. It can be seen from Table XI and Table XII that the Monte Carlo method's MAPE value being less than 20%. However, the MAPE value of the Monte Carlo method with Newton-Raphson implied volatility must be compared with historical volatility to determine whether the method yielded the option price closest to the current market price. Using Newton-Raphson implied volatility, calculating the American option prices for NVDA shares in a case study with the same contract price and varied maturity durations yielded a MAPE value of 5.6378% for call options and 6.7788% for put options as shown in Table XI. The MAPE values derived from historical volatility in Table XII were 12.71% for call options and 7.26% for put options. These MAPE values disclosed that the computation of American option prices using the Monte Carlo method with Newton-Raphson implied volatility produced a MAPE value less than historical volatility. The error in option prices at each maturity date also indicates the same thing, namely that most option prices with Newton-Raphson implied volatility have errors that were less than historical volatility, as in Fig. 6 (a) and 6 (b). This lower MAPE value suggests that American option prices derived using Newton-Raphson implied volatility were more comparable to market option prices.

The Newton-Raphson method converged fairly rapidly in estimating all implied volatility values in case studies with the same contract price but various maturity durations for INTC, AMZN, and NVDA shares. In this third case study, the MAPE values for both call and put options with implied Newton-Raphson volatility and historical volatility were less than 20%. Even some option price computations provided MAPE values below 10%. A less than 20% MAPE value indicates that the Monte Carlo method is suitable for option price prediction. A MAPE value of less than 10% suggests that the Monte Carlo method for estimating option prices is excellent or highly accurate. This method is effective for determining option prices due to its minimal error rate. In the third case study for INTC, AMZN, and NVDA shares, the option prices calculated using the Monte Carlo method with Newton-Raphson implied volatility had a smaller inaccuracy than historical volatility. Thus, the Monte Carlo method with Newton-Raphson implied volatility generated American option prices closer to market option prices. Therefore, the Monte Carlo method with Newton-Raphson implied volatility was superior for estimating option prices for the same contract price and different expiry timeframes.

Based on the simulation findings in the three case studies, all MAPE values were less than 20%, with some falling below 10%. A MAPE value of less than 20% denotes competent predicting, whereas a MAPE value of less than 10% indicates excellent predicting [31]. In short, the Monte Carlo method has become an effective way of determining option prices. By generating normally distributed random values, the Monte Carlo method provided many alternative share prices under the assumption of Brownian Geometric motion to produce several payout possibilities for each time interval. Due to the multitude of possibilities, the Monte Carlo method generated option prices with a modest error value. According to previous researchers, the Monte Carlo method under the assumption of Brownian Geometric motion in the Black-Scholes model was effective [15], [32] and demonstrated excellent performance [33], [34] in the calculation of option prices. The Monte Carlo method is a precise and accurate numerical method for calculating option prices [17], [24]. It generates option prices that closely approximate their true prices [8].

The Newton-Raphson method converged rapidly and determined volatility with a small error. Few iterations were required for the Newton-Raphson method to converge on a certain value. The initial guess value using the equation (5) assured that the Newton-Raphson method had a unique volatility solution and helped the Newton-Raphson method converge more rapidly. The conditions for the lower and upper limits of the options, namely max $(S_0 - Ke^{-rT}, 0) \leq C \leq S_0$, used to determine the contract value, also contributed to the rapid convergence of the Newton-Raphson method. These findings are comparable to those of Amri et al. [14], Mahrudinda et al. [13], and Rahayuni et al. [12], who determined that Newton-Raphson implied volatility was a numerical method that converged rapidly and had a minimal error in calculating volatility. The Newton-Raphson method for determining implied volatility was stable and convergent [35].

The MAPE values obtained in the three case studies do not form a special pattern. The size of the MAPE is not influenced by the type of stock, the size of the contract price, or the length of maturity. However, if analyzed based on the size of the MAPE in call options and put options, the largest MAPE difference between Newton-Raphson implied volatility and historical volatility occurs in the call option. In particular, the largest MAPE difference between Newton-Raphson's implied volatility and historical volatility occurred in the call study option in the third case of NVDA shares with a contract price of K = 120. Meanwhile, the smallest difference happened in the put option in the second case study of INTC shares with an expiry date of 11 November 2022. In other words, the Monte Carlo method with Newton-Raphson's implied volatility generally performs well on call options. This condition can happen because the options' lower limit and upper limit rules are used so that the Newton-Raphson method converges quickly $\text{Max}(S_0 - Ke^{-rT}) \leq C \leq S_0$. This rule indicates that the contract price K must satisfy $K \geq \dfrac{S_0 - C}{e^{-rT}}$, where the value of K is around S_0 or $K \geq S_0$. This K value is suitable for call options where the buyer expects the stock price to rise at time T to gain a profit. The call option will be exercised if $S_T > K$. By choosing K around S_0 or $K \geq S_0$, it is hoped that the stock price at T will rise and be more than K.

Employing the Monte Carlo method with Newton-Raphson implied volatility to calculate option prices resulted in a lower MAPE value than historical volatility, as presented in Fig. 7. This minimal MAPE value suggests that the ensuing inaccuracy on the actual option price was equally modest. The volatility value determined by Newton-Raphson implied volatility for a share fluctuated based on the share’s prices, contract prices, interest rates, and expiry dates. This shift in volatility value benefited the Newton-Raphson method, making it effective in determining implied volatility. It enabled the generated option prices to react closely to market movements to resemble the market option prices. In historical volatility, the volatility was derived based on the prior share price over a specific period. The volatility remained constant regardless of share prices, contract prices, interest rates, and maturity changes.

Although the variables utilized in determining the prices of these American options differed for each case study, the variable maturity was the same for all shares, the maturity time was the same for all contract prices, and the contract price was the same for all maturity times, the Monte Carlo method with Newton-Raphson implied volatility consistently generated option prices closer to market option prices. Volatility has become an essential element in option price estimation. Therefore, the accurate volatility could facilitate the option price computation. Jia [36] claims that volatility is the sole share characteristic that impacts the option’s price. For the predicted option prices to be near the market option prices, it is crucial to establish an appropriate volatility value. The performance of Newton-Raphson implied volatility, whose value changed according to market conditions, helped the Monte Carlo method calculate option prices. The Newton-Raphson method could determine implied volatility when option prices were numerically computed [37].

This study describes a suitable numerical method applied to calculating American option prices. Previous research used Monte Carlo and Newton Raphson's numerical methods separately. In this study, two numerical methods that separately perform well are combined in calculating option prices. Previous research has not explained how Monte Carlo and Newton Raphson's methods perform when combined to calculate option prices. The results of all three case studies in this study showed consistent results. In various case studies, the performance of the two numerical methods utilized in the option price computation procedure was deemed satisfactory. This research is an alternative solution for calculating American option prices that are difficult to determine the analytical solutions.

IV. Conclusion

The results of this study have demonstrated that the Newton-Raphson method for estimating implied volatility converged relatively rapidly. The Monte Carlo method yielded a MAPE value of less than 20% and, in certain case studies, even less than 10%. A MAPE value of less than 10% implies that the Monte Carlo method was very accurate in calculating option prices. So, in general the Monte Carlo method was a good method for determining American option prices. Consequently, the Monte Carlo method with Newton-Raphson implied volatility and historical volatility could calculate the American option prices. Although both methods were adequate for estimating option prices, the Monte Carlo method with Newton-Raphson implied volatility was superior to historical volatility. This conclusion was drawn based on the MAPE value of the Monte Carlo method and Newton-Raphson implied volatility, being less than historical volatility. In three case studies, the Monte Carlo method with Newton-Raphson implied volatility disclosed favorable results. Monte Carlo with Newton-Raphson implied volatility was closest to the market option prices in the three case studies. The Newton-Raphson implied volatility value changed according to market fluctuations. These value changes supported the Monte Carlo method in calculating American option prices. Two numerical methods in the American option price calculation provided option prices close to market option prices, resulting in an excellent performance.

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TABLE I
THE RESULTS OF CALCULATING AMERICAN OPTION PRICES FOR TEN SHARES USING THE MONTE CARLO METHOD WITH NEWTON-RAPHSON IMPLIED VOLATILITY

No	Share	*So*	*K*	C_M	P_M	σ	Error	Iteration	*C* NR	*P* NR	$\frac{\lvert C_M - C\,NR\rvert}{C_M}$	$\frac{\lvert P_M - P\,NR\rvert}{P_M}$
1	MSFT	278.01	300	5.24	25.45	0.25752	4.5069E-06	5	4.3146	22.0672	0.176603	0.132919
2	INTC	36.96	40	0.96	4.15	0.30383	8.0338E-07	5	0.80081	3.0333	0.165823	0.269084
3	IBM	132.04	140	2.24	10.9	0.21075	2.4701E-06	5	1.3606	8.0315	0.392589	0.263165
4	QCOM	147.43	150	9.45	11.86	0.37050	5.6998E-06	3	8.7068	10.3844	0.078646	0.124418
5	NVDA	184.41	200	12.1	26.76	0.52293	2.6715E-06	2	11.0139	26.0685	0.08976	0.025841
6	META	159.93	165	11.29	15.45	0.43808	5.82E-06	3	10.5968	15.1544	0.061399	0.019133
7	NFLX	224.9	250	13.09	39.75	0.53523	4.3398E-06	3	13.0363	35.1583	0.004102	0.115514
8	AMZN	134.95	145	5.27	14.1	0.36061	6.4182E-07	4	4.8483	13.0796	0.080019	0.072369
9	TSLA	891.45	920	87	107.5	0.58211	7.1157E-08	4	84.115	98.7572	0.033161	0.081328
10	GOOG	116.64	120	4.9	8.8	0.27986	1.8751E-07	3	4.0059	8.3727	0.182469	0.048557
										MAPE	0.126457	0.115233

TABLE II
THE RESULTS OF CALCULATING AMERICAN OPTION PRICES FOR TEN SHARES USING THE MONTE CARLO METHOD WITH HISTORICAL VOLATILITY

No	Share	*So*	*K*	C_M	P_M	σ	*C* His	*P* His	$\frac{\lvert C_M - C\,His\rvert}{C_M}$	$\frac{\lvert P_M - P\,His\rvert}{P_M}$
1	MSFT	278.01	300	5.24	25.45	0.32170	7.1136	22.0877	0.357557	0.132114
2	INTC	36.96	40	0.96	4.15	0.39899	1.2705	3.0314	0.323438	0.269542
3	IBM	132.04	140	2.24	10.9	0.29764	3.3324	8.0002	0.487679	0.266037
4	QCOM	147.43	150	9.45	11.86	0.42697	8.6269	9.7759	0.087101	0.175725
5	NVDA	184.41	200	12.1	26.76	0.51885	10.662	25.5543	0.118843	0.045056
6	META	159.93	165	11.29	15.45	0.43513	10.2634	15.2137	0.09093	0.015294
7	NFLX	224.9	250	13.09	39.75	0.49988	12.3093	35.154	0.059641	0.115623
8	AMZN	134.95	145	5.27	14.1	0.35210	4.5751	13.0785	0.13186	0.072447
9	TSLA	891.45	920	87	107.5	0.68742	80.3236	96.3019	0.07674	0.104168
10	GOOG	116.64	120	4.9	8.8	0.31259	4.2563	8.4371	0.131367	0.041239
								MAPE	0.186515	0.123724

TABLE III
CALCULATION RESULTS FOR THE AMERICAN OPTION PRICES FOR INTC SHARES USING NEWTON-RAPHSON IMPLIED VOLATILITY FOR THE MATURITY DATE OF NOVEMBER 11, 2022

No	*K*	C_M	P_M	*Implied Vol* (σ)	*Error*	*Itera-tion*	*C NR*	*P NR*	$\frac{\lvert C_M - C\,NR\rvert}{C_M}$	$\frac{\lvert P_M - P\,NR\rvert}{P_M}$
1	25	1.75	1.63	0.54487	0.000004	3	1.4375	1.39079	0.178571	0.146755
2	26	1.26	2.09	0.53277	0.0000001	3	1.2367	1.82512	0.018492	0.126737
3	27	0.86	2.77	0.52928	2.9831E-06	4	0.66389	2.4922	0.228035	0.100289
4	28	0.56	3.6	0.54023	8.5269E-06	5	0.53721	2.7802	0.040696	0.227722
5	29	0.37	3.85	0.57569	9.7057E-06	6	0.38741	3.7993	0.047054	0.013169
6	30	0.23	4.6	0.61985	8.3551E-06	7	0.28492	4.8131	0.238783	0.046326
7	31	0.15	5.46	0.68226	4.2476E-06	8	0.16973	6.326	0.131533	0.158608
8	32	0.11	6.42	0.75813	6.3323E-06	8	0.11874	6.7879	0.079455	0.057305
								MAPE	0.120327	0.109614

TABLE IV
CALCULATION RESULTS FOR THE AMERICAN OPTION PRICES FOR INTC SHARES USING HISTORICAL VOLATILITY FOR THE MATURITY DATE OF NOVEMBER 11, 2022

No	K	C_M	P_M	*His Vol (σ)*	*C His*	*P His*	$\frac{\lvert C_M - C\,His \rvert}{C_M}$	$\frac{\lvert P_M - P\,His \rvert}{P_M}$
1	25	1.75	1.63	0.396638	1.3968	1.21671	0.201829	0.253552
2	26	1.26	2.09	0.396638	1.1587	1.79609	0.080397	0.140627
3	27	0.86	2.77	0.396638	0.60493	2.4942	0.296593	0.099567
4	28	0.56	3.6	0.396638	0.52733	2.7976	0.058339	0.222889
5	29	0.37	3.85	0.396638	0.3342	3.8005	0.096757	0.012857
6	30	0.23	4.6	0.396638	0.306924	4.8083	0.334452	0.045283
7	31	0.15	5.46	0.396638	0.11464	5.8035	0.235733	0.062912
8	32	0.11	6.42	0.396638	0.104265	6.7889	0.05214	0.057461
						MAPE	0.16953	0.111893

TABLE V
CALCULATION RESULTS FOR THE AMERICAN OPTION PRICES FOR INTC SHARES USING NEWTON-RAPHSON IMPLIED VOLATILITY FOR THE MATURITY DATE OF APRIL 21, 2023

No	K	C_M	P_M	*Implied Vol* (σ)	*Error*	*Itera-tion*	*C NR*	*P NR*	$\frac{\lvert C_M - C\,NR \rvert}{C_M}$	$\frac{\lvert P_M - P\,NR \rvert}{P_M}$
1	25	3.27	3	0.40969	0.000005	3	2.8686	2.1675	0.122752	0.2775
2	26	2.77	3.53	0.40219	5.4782E-07	4	2.4776	3.2551	0.10556	0.077875
3	27.5	2.19	4.39	0.40569	3.7343E-07	4	2.018	4.3014	0.078539	0.020182
4	29	1.66	5.35	0.40641	1.1145E-07	2	1.6655	4.7793	0.003313	0.106673
5	30	1.36	6.08	0.40935	3.2898E-07	4	1.4388	5.8064	0.057941	0.045
6	31	1.14	6.65	0.42008	4.7815E-06	4	1.3028	7.084	0.142807	0.065263
7	32.5	0.82	8.2	0.43218	3.537E-06	5	1.0831	8.3248	0.320854	0.01522
								MAPE	0.118824	0.086816

TABLE VI
CALCULATION RESULTS OF THE AMERICAN OPTION PRICES FOR INTC SHARES USING HISTORICAL VOLATILITY FOR THE MATURITY DATE OF APRIL 21, 2023

No	K	C_M	P_M	HisVol (σ)	*C His*	*P His*	$\frac{\lvert C_M - C\,His \rvert}{C_M}$	$\frac{\lvert P_M - P\,His \rvert}{P_M}$
1	25	3.27	3	0.396638	2.6221	2.0626	0.198135	0.312467
2	26	2.77	3.53	0.396638	2.3105	3.11228	0.165884	0.118334
3	27.5	2.19	4.39	0.396638	1.8591	3.2939	0.151096	0.249681
4	29	1.66	5.35	0.396638	1.4669	5.8043	0.116325	0.084916
5	30	1.36	6.08	0.396638	1.266	5.7939	0.069118	0.047056
6	31	1.14	6.65	0.396638	1.3009	6.8111	0.14114	0.024226
7	32.5	0.82	8.2	0.396638	0.92364	7.3024	0.12639	0.109463
						MAPE	0.138298	0.135163

TABLE VII
THE RESULTS OF THE CALCULATION OF THE AMERICAN OPTION PRICES FOR INTC SHARES WITH NEWTON-RAPHSON IMPLIED VOLATILITY AT A CONTRACT PRICE OF K = 26

No	T	C_M	P_M	*Implied Vol* (σ)	*Error*	*Itera-tion*	*C NR*	*P NR*	$\frac{\lvert C_M - C\,NR \rvert}{C_M}$	$\frac{\lvert P_M - P\,NR \rvert}{P_M}$
1	10	0.44	1.41	0.50742	7.1074E-07	5	0.42575	1.55584	0.032386	0.103433
2	17	0.93	1.89	0.61903	6.9404e-06	3	0.80785	1.95941	0.131344	0.036725
3	24	1.03	2.26	0.55792	1.5047e-06	3	1.1215	1.96129	0.088835	0.132173
4	31	1.13	2.42	0.52326	1.2995e-07	3	1.26625	1.97272	0.120575	0.184826
5	38	1.23	2.5	0.50175	3.5015e-07	2	1.33743	1.9895	0.087341	0.2042
6	45	1.28	2.58	0.47354	3.7569e-08	3	1.33989	2.13466	0.046789	0.172612
7	66	1.52	2.78	0.44291	2.2954e-06	3	1.60161	2.94784	0.053691	0.060374
8	101	1.95	3.05	0.4334	1.0691e-07	4	1.8473	2.99005	0.052667	0.019656
								MAPE	0.076704	0.11425

TABLE VIII
THE RESULTS OF THE CALCULATION OF THE AMERICAN OPTION PRICES FOR INTC SHARES WITH HISTORICAL VOLATILITY AT A CONTRACT PRICE OF K = 26

No	T	C_M	P_M	*His Vol* (σ)	*C His*	*P His*	$\frac{\lvert C_M - C\,His \rvert}{C_M}$	$\frac{\lvert P_M - P\,His \rvert}{P_M}$
1	10	0.44	1.41	0.396638	0.459932	1.559932	0.0453	0.106335
2	17	0.93	1.89	0.396638	0.81968	1.95958	0.118624	0.036815
3	24	1.03	2.26	0.396638	1.24905	1.96526	0.21267	0.130416
4	31	1.13	2.42	0.396638	1.30892	1.96954	0.158336	0.18614
5	38	1.23	2.5	0.396638	1.41582	1.98304	0.151073	0.206784
6	45	1.28	2.58	0.396638	1.44249	1.13844	0.126945	0.558744
7	66	1.52	2.78	0.396638	1.6487	2.5124	0.084671	0.096259
8	101	1.95	3.05	0.396638	1.828	2.96498	0.062564	0.027875
						MAPE	0.120023	0.168671

TABLE IX
THE CALCULATION RESULTS OF THE AMERICAN OPTION PRICES FOR AMZN SHARES WITH NEWTON-RAPHSON IMPLIED VOLATILITY AT A CONTRACT PRICE OF K = 116

No	T	C_M	P_M	*Implied Vol* (σ)	*Error*	*Itera-tion*	*C NR*	*P NR*	$\frac{\lvert C_M - C\,NR \rvert}{C_M}$	$\frac{\lvert P_M - P\,NR \rvert}{P_M}$
1	10	2.22	5.91	0.51529	6.3064e-06	4	1.49275	5.7787	0.32759	0.022217
2	17	4.15	7.9	0.59556	3.4248e-06	3	4.1653	7.779	0.003687	0.015316
3	24	4.75	8.48	0.55131	2.5506e-07	3	4.3563	7.7932	0.082884	0.080991
4	31	5.9	9.05	0.5711	9.2487e-08	3	5.6961	8.8632	0.034559	0.020641
5	38	6	9.82	0.52076	2.4848e-07	3	5.9602	9.8129	0.006633	0.000723
6	45	6.5	8.7	0.50846	1.0403e-06	3	6.4333	8.7459	0.010262	0.005276
7	66	7.75	11	0.48073	5.6016e-06	3	7.2651	10.5774	0.062568	0.038418
8	101	9.45	12.25	0.45432	1.2064e-07	4	8.8073	11.7788	0.068011	0.038465
								MAPE	0.074524	0.027756

TABLE X
THE CALCULATION RESULTS OF THE AMERICAN OPTION PRICES FOR AMZN SHARES WITH HISTORICAL VOLATILITY AT A CONTRACT PRICE OF $K = 116$

No	T	C_M	P_M	*His Vol* (σ)	*C His*	*P His*	$\frac{\lvert C_M - C\,His \rvert}{C_M}$	$\frac{\lvert P_M - P\,His \rvert}{P_M}$
1	10	2.22	5.91	0.353558	1.16635	5.7822	0.474617	0.021624
2	17	4.15	7.9	0.353558	4.74877	7.7811	0.144282	0.015051
3	24	4.75	8.48	0.353558	4.94277	7.7919	0.040583	0.081144
4	31	5.9	9.05	0.353558	5.0533	8.8224	0.143508	0.025149
5	38	6	9.82	0.353558	5.3063	8.7434	0.115617	0.109633
6	45	6.5	8.7	0.353558	6.5133	8.7594	0.002046	0.006828
7	66	7.75	11	0.353558	7.0407	9.7813	0.091523	0.110791
8	101	9.45	12.25	0.353558	8.6614	10.8377	0.08345	0.11529
						MAPE	0.136953	0.060689

TABLE XI
CALCULATION RESULTS OF THE AMERICAN OPTION PRICES OF NVDA SHARES WITH NEWTON-RAPHSON IMPLIED VOLATILITY AT A CONTRACT PRICE OF $K = 120$

No	T	C_M	P_M	*Implied Vol* (σ)	*Error*	*Itera-tion*	*C NR*	*P NR*	$\frac{\lvert C_M - C\,NR \rvert}{C_M}$	$\frac{\lvert P_M - P\,NR \rvert}{P_M}$
1	10	3.5	7.6	0.68861	3.9364e-07	4	3.6571	7.1701	0.044886	0.056566
2	17	4.68	8.97	0.64471	1.6391e-06	3	4.3553	8.1352	0.06938	0.093066
3	24	6.1	10.4	0.66017	1.5827e-08	3	6.3993	9.1804	0.049066	0.117269
4	31	6.9	10.11	0.63799	5.8237e-07	3	6.0857	10.1636	0.118014	0.005302
5	38	8.4	12.05	0.67452	3.537e-06	3	8.7218	11.1026	0.03831	0.078622
6	45	8.59	13	0.62968	4.5383e-06	3	8.0807	12.2403	0.05929	0.058438
7	66	10.73	14.7	0.6239	8.1912e-08	4	10.0184	14.0828	0.066319	0.041986
8	101	12.67	16.65	0.57784	1.4844e-07	4	12.597	15.1339	0.005762	0.091057
								MAPE	0.056378	0.067788

TABLE XII
CALCULATION RESULTS OF THE AMERICAN OPTION PRICES OF NVDA SHARES WITH HISTORICAL VOLATILITY AT A CONTRACT PRICE OF $K = 120$

No	T	C_M	P_M	*His Vol* (σ)	*C His*	*P His*	$\frac{\lvert C_M - C\,His \rvert}{C_M}$	$\frac{\lvert P_M - P\,His \rvert}{P_M}$
1	10	3.5	7.6	0.523299	2.86319	7.1628	0.181946	0.057526
2	17	4.68	8.97	0.523299	3.7935	8.1358	0.189423	0.092999
3	24	6.1	10.4	0.523299	5.0271	9.1718	0.175885	0.118096
4	31	6.9	10.11	0.523299	6.0277	10.138	0.12642	0.00277
5	38	8.4	12.05	0.523299	7.9065	11.1176	0.05875	0.077378
6	45	8.59	13	0.523299	7.8951	12.1572	0.080896	0.064831
7	66	10.73	14.7	0.523299	9.457	14.1481	0.118639	0.037544
8	101	12.67	16.65	0.523299	11.5951	14.4871	0.084838	0.129904
						MAPE	0.1271	0.072631

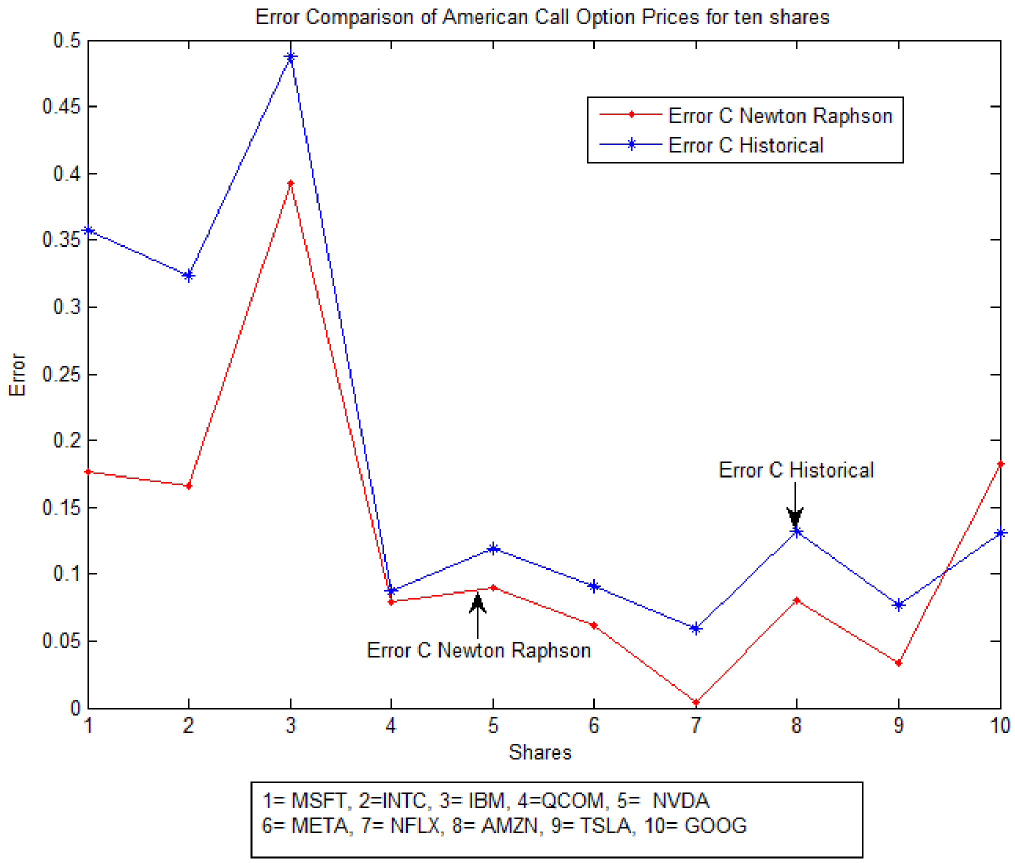

Fig. 1 (a) Error Comparison of American Call Option Prices for Ten Shares

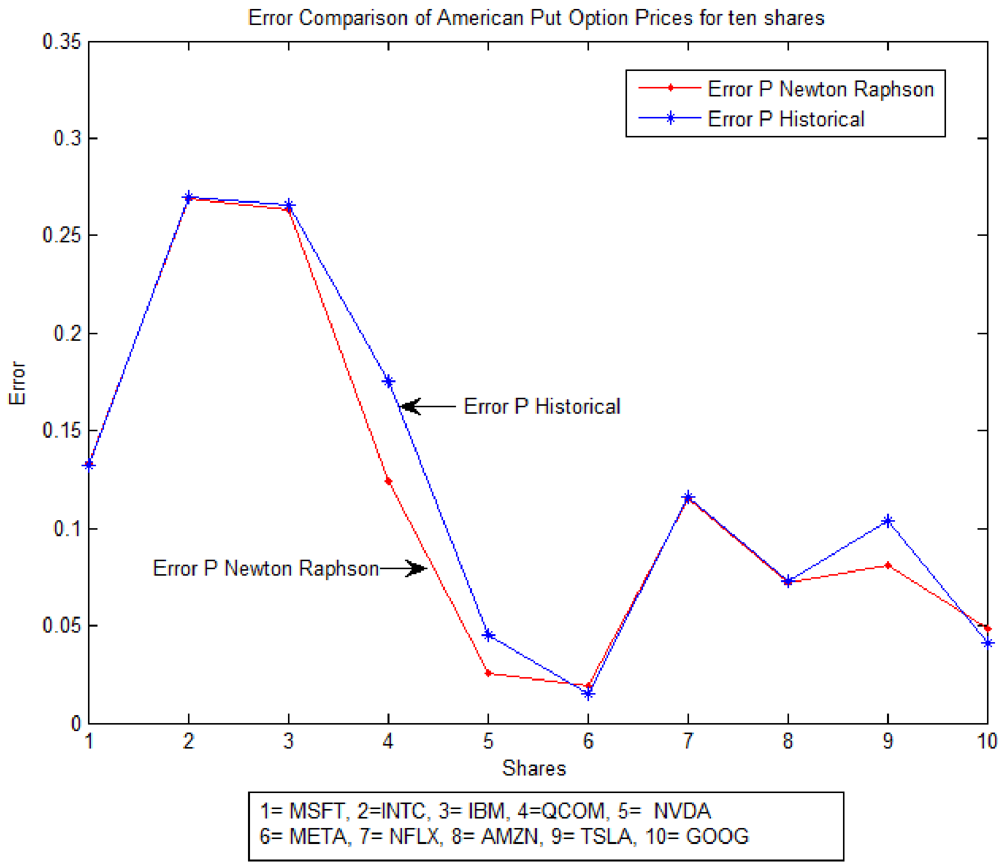

Fig. 1 (b) Error Comparison of American Put Option Prices for Ten Shares

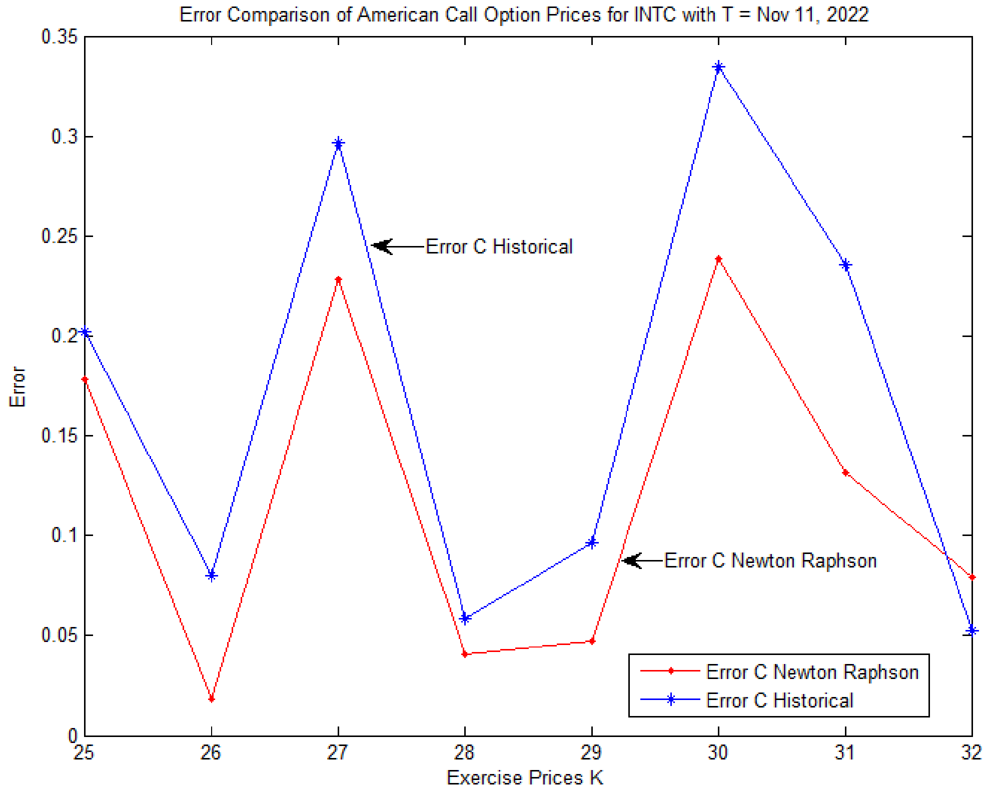

Fig. 2 (a) Error Comparison of American Call Option Prices for INTC with Maturity Date Nov, 11 2022

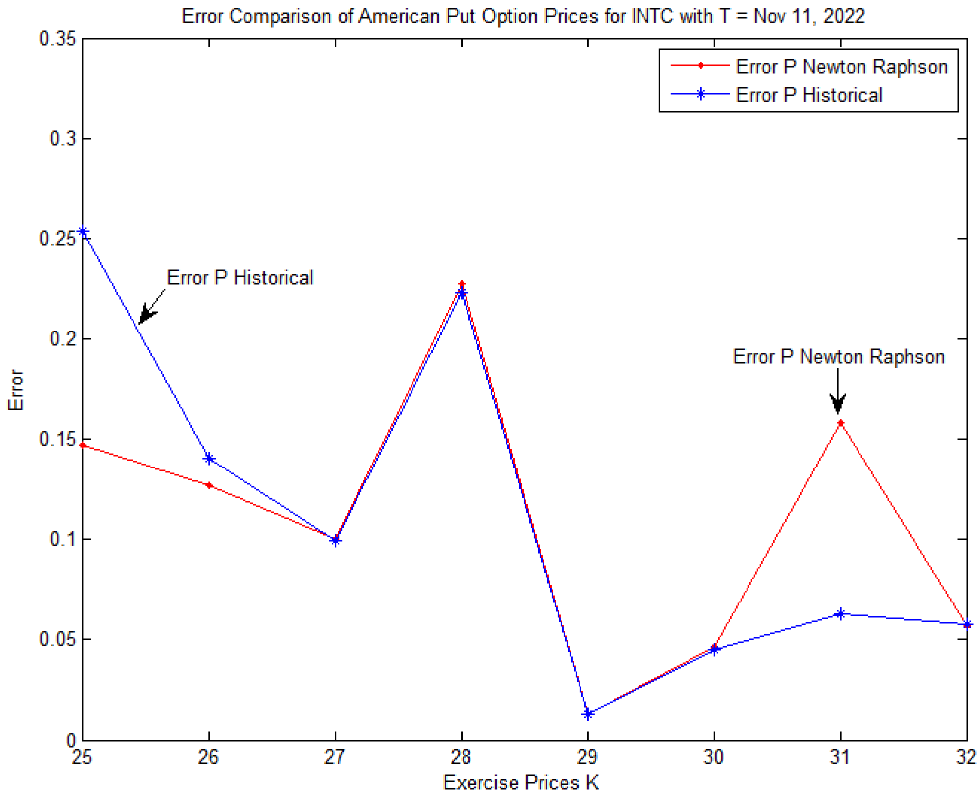

Fig. 2 (b) Error Comparison of American Put Option Prices for INTC with Maturity Date Nov, 11 2022

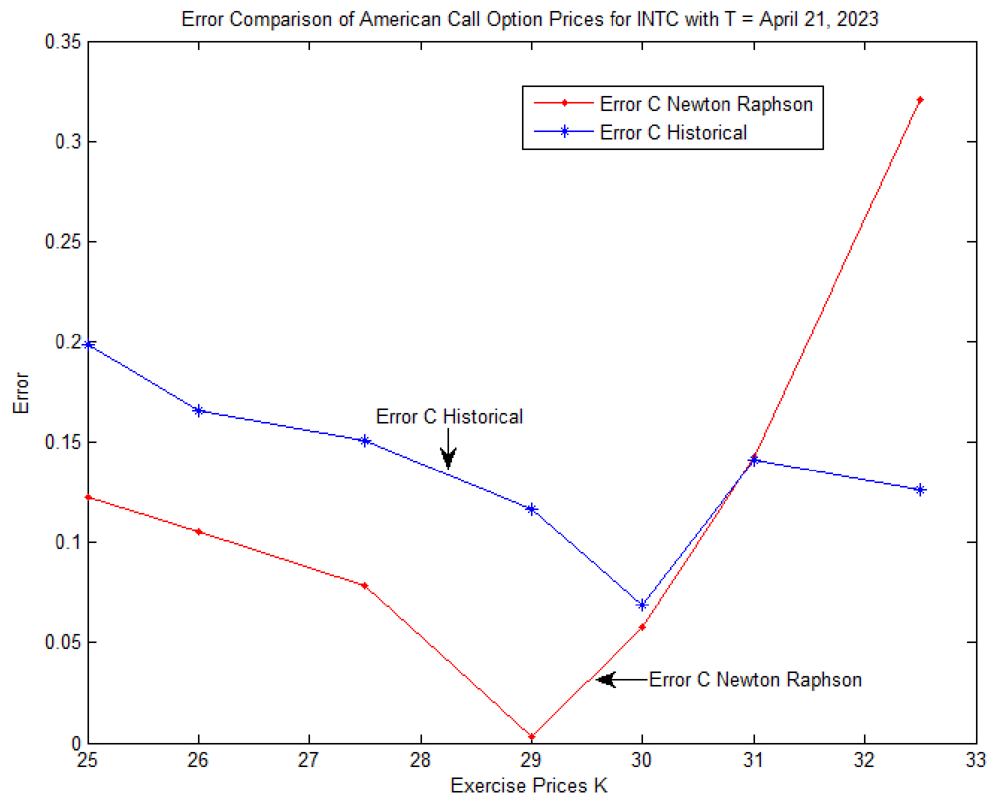

Fig. 3 (a) Error Comparison of American Call Option Prices for INTC with Maturity Date April, 21 2023

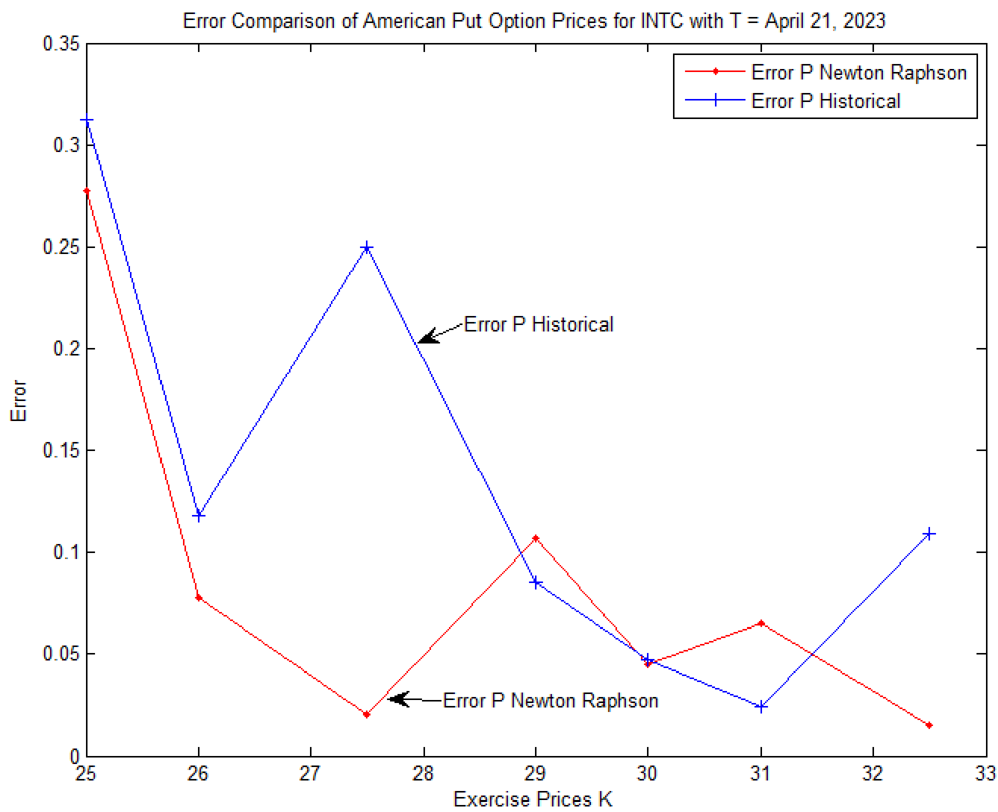

Fig. 3 (b) Error Comparison of American Put Option Prices for INTC with Maturity Date April, 21 2023

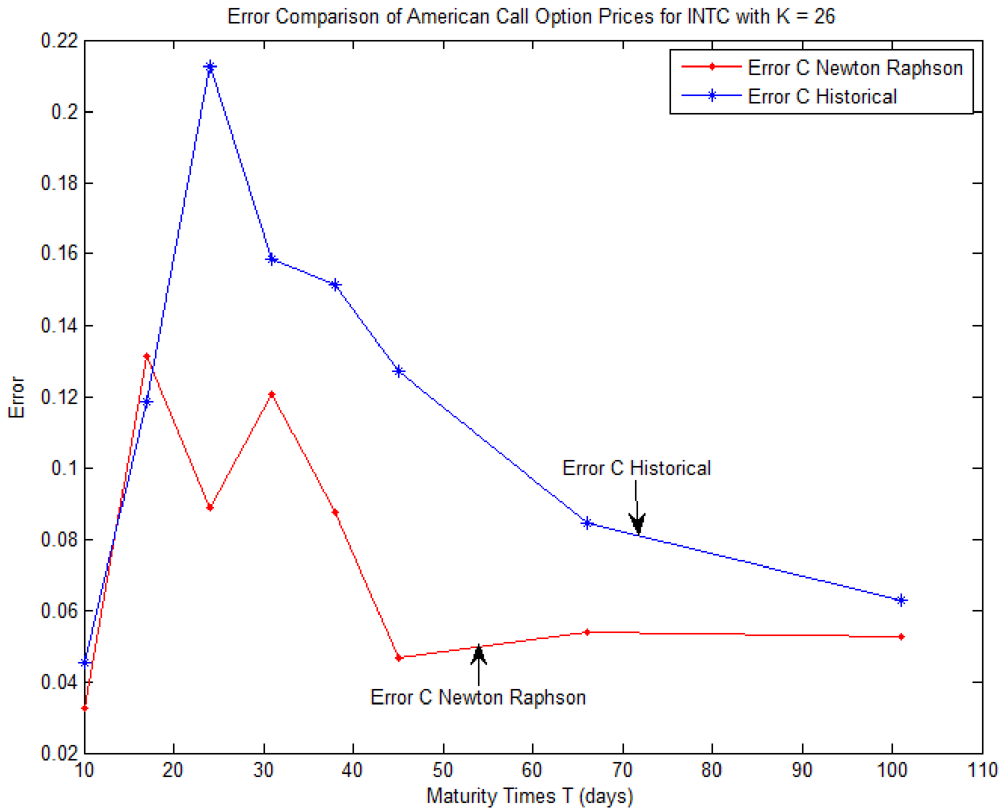

Fig. 4 (a) Error Comparison of American Call Option Prices for INTC with Exercise Price $K = 26$

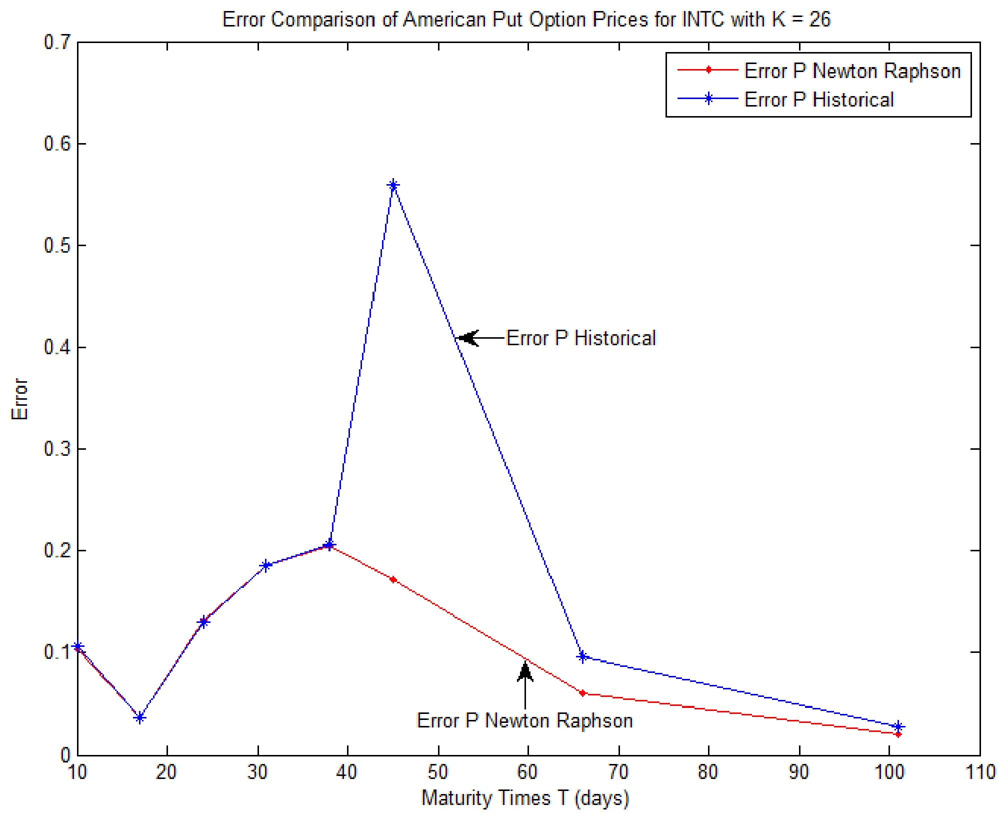

Fig. 4 (b) Error Comparison of American Put Option Prices for INTC with Exercise Price $K = 26$

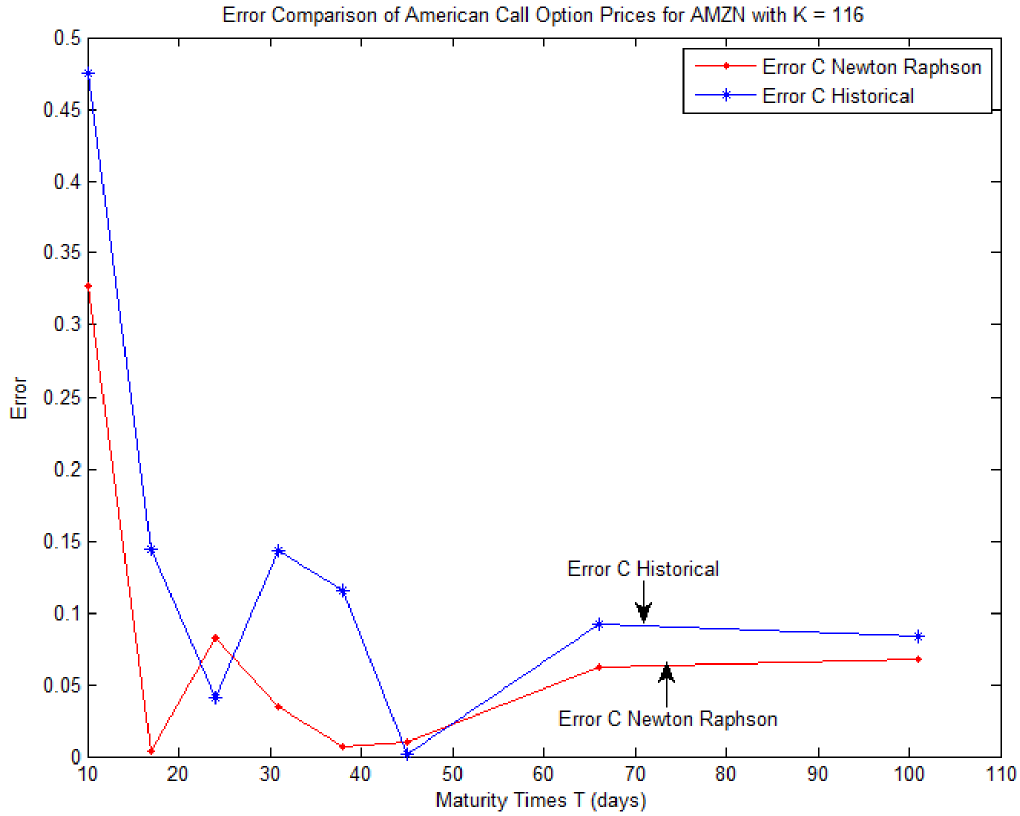

Fig. 5 (a) Error Comparison of American Call Option Prices for AMZN with Exercise Price $K = 116$

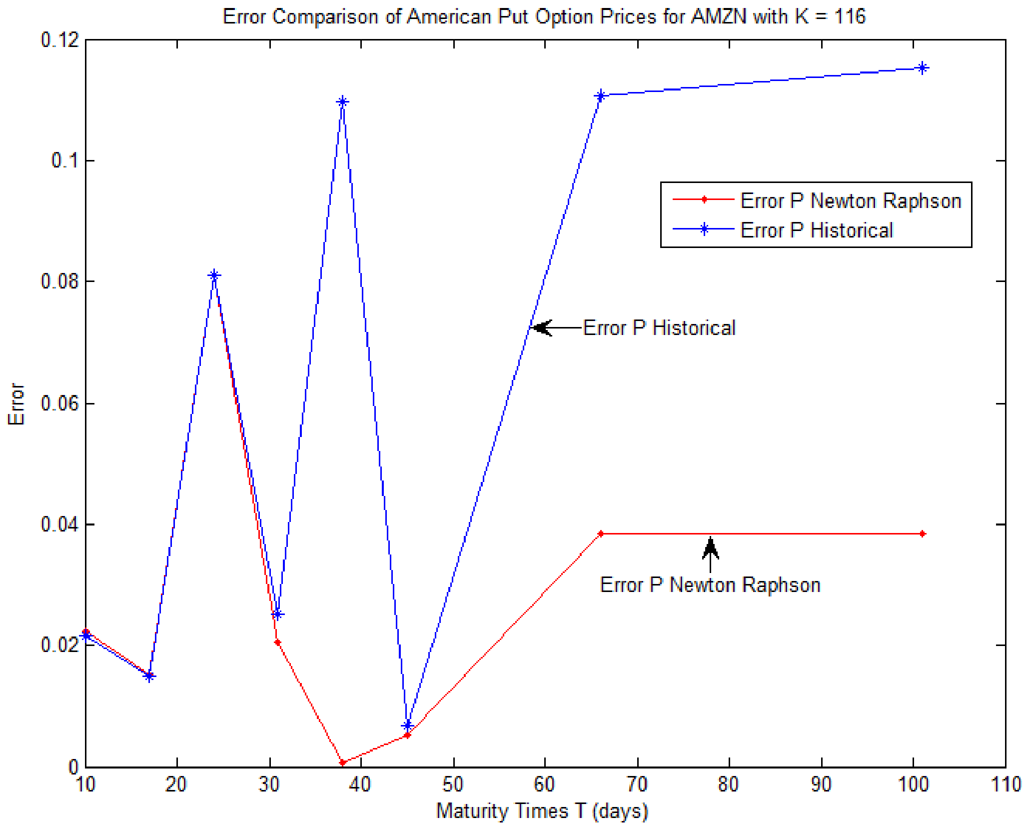

Fig. 5 (b) Error Comparison of American Put Option Prices for AMZN with Exercise Price $K = 116$

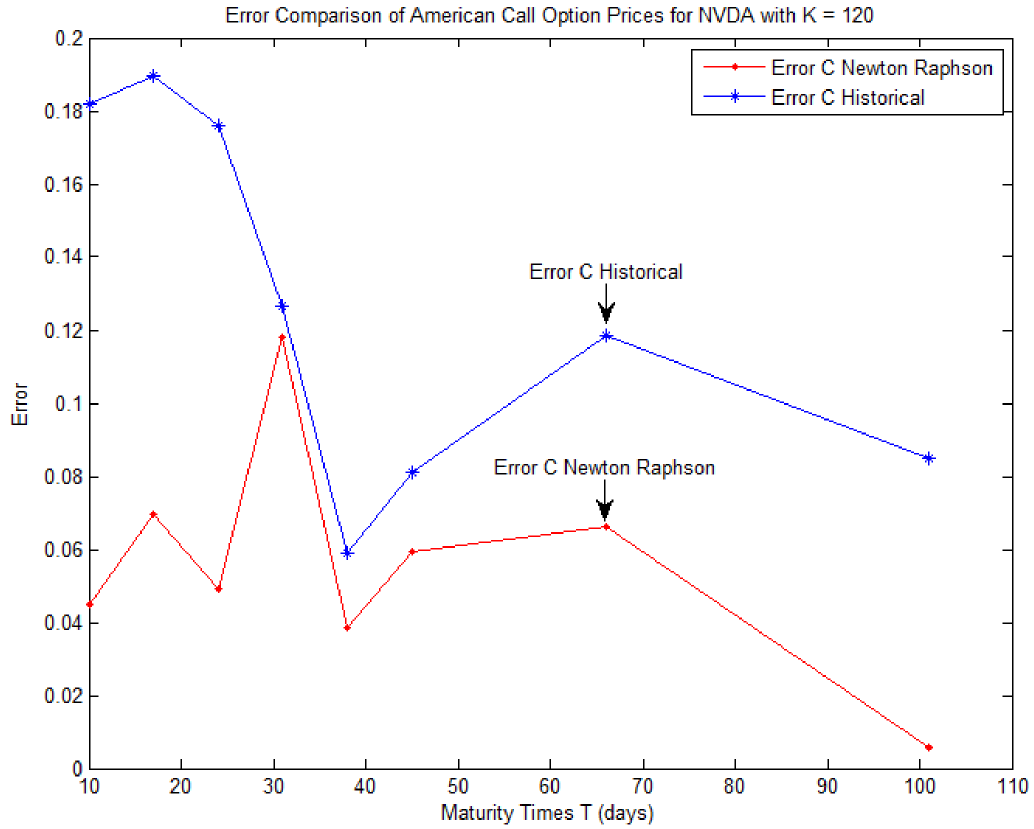

Fig. 6 (a) Error Comparison of American Call Option Prices for NVDA with Exercise Price K = 120

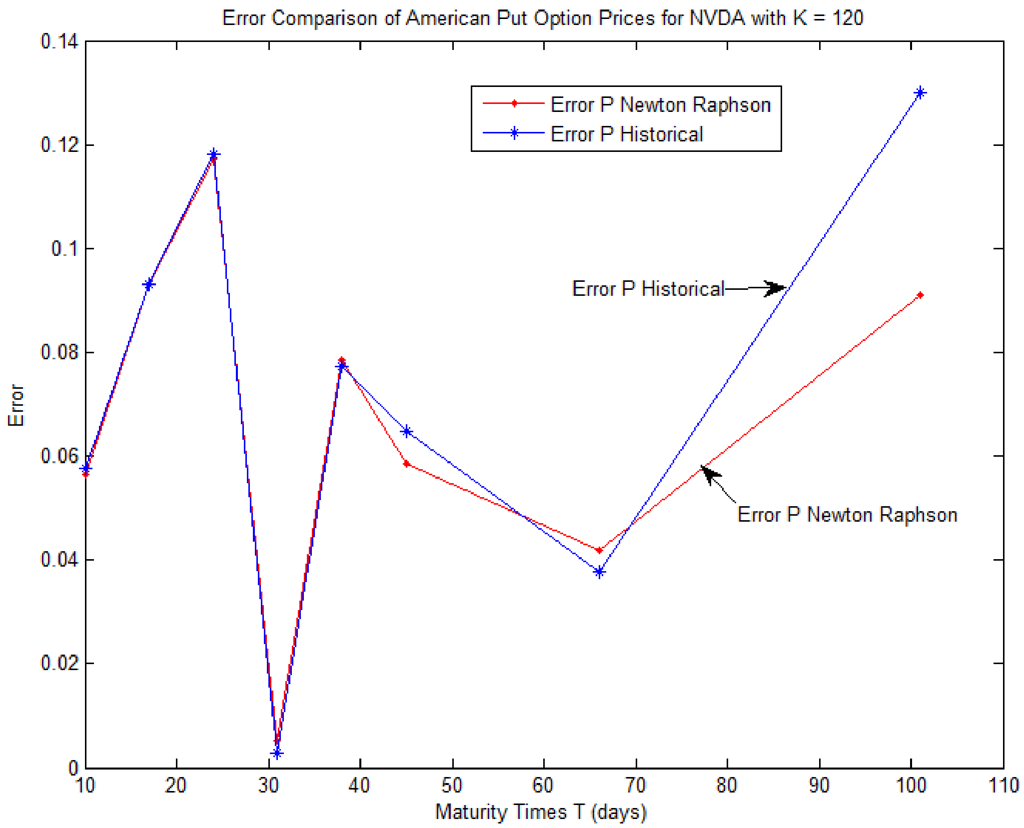

Fig. 6 (b) Error Comparison of American Put Option Prices for NVDA with Exercise Price K = 120

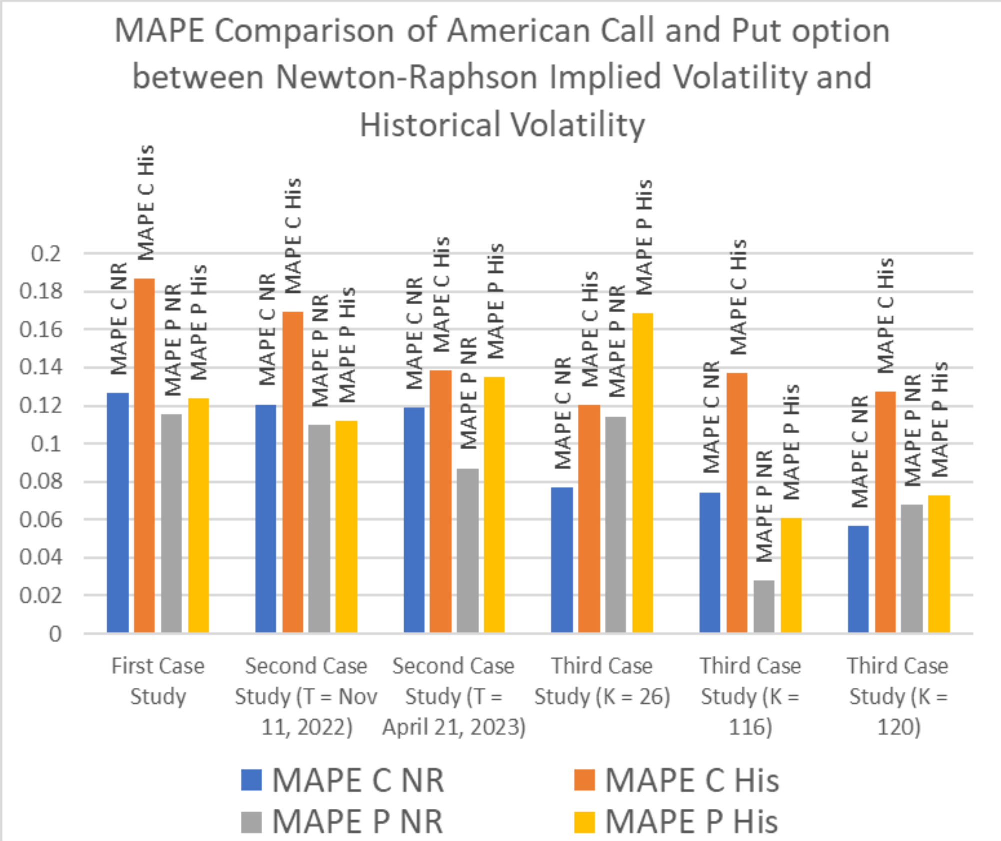

Fig. 7. MAPE Comparison of American Call and Put option between Newton Raphson's Implied Volatility and Historical Volatility

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