A Suitable Scheme for Influential Load Nodes Detection Considering Network Structural Risk Vulnerability Analysis

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***Abstract*— Swift identification of a set of influential load nodes within power systems is an important task for effective modern power grid operation and security. This paper suggests a suitable approach of Node Strength (NS) or Bus Strength (BS) scheme for effective and swift identification of sets of influential load nodes within power systems. This effective algorithm relies on the Network Structural Characteristics Theory (NSCT) of power systems. The NS scheme uses circuit theory laws to present the linear relationship among the network structure, the voltage and current in the network. The traditional Complex Network Theory (CNT) or Centrality-Based (Degree Centrality (DC), Betweeness Centrality (BC), Closeness Centrality (CC), PageRank Centrality (PRC) and Eigenvector Centrality (EC)) methods for identifying influential load nodes within power systems are also presented in the paper. Thereafter, the numerical values of the NS and each of the CNT-based methods associated with the load nodes, are determined. The relative importance of each load node is then ascertained through ranking (in the order of magnitude or priority) of the values obtained for NS and each of the CNT-based methods. Based on the order of priority, the sets of influential load nodes in the network are identified. Furthermore, performance analyses, using statistical correlation coefficient as well as structural risk assessment, are used to establish the best method suitable for quick identification of a set of influential load nodes in the system. The results obtained are corroborated with the network robustness assessment to further validate the proposed approach. The study uses IEEE 5-bus, IEEE 14-bus and Nigerian 28-bus networks as case studies. The comparison of results obtained show that the NS or BS scheme suggested is more suitable for quick identification of sets of influential load nodes in a relatively large-sized power network.**

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***Index Terms*— Bus Strength, node degree, eigenvector, centrality, voltage collapse**

I. INTRODUCTION

NEVER in history has power systems experienced the revolution and transformation that are on-going around the world recently [1]. Consequently, this transformation of the traditional configuration of power systems has led to the emergence of an electric network operation which is basically dependent on a two-way digital communication technology called smart grids [2]. Although, electric power networks are generally complex due to increase in the network interconnections, the new revolution has increased the complexity of the network structure. This, in turn, has led to various complications in solving various power system problems such as nonlinearity in the problem formulations. One of such power system problems is finding a quick solution for detecting various influential network nodes within the system. This will help the system operators to quickly identify the parts of the system that require urgent attention. Such attention could be the enhancement of voltage profile, reduction in power loss as well as the enhancement in power system transfer capability [3], [4]. Therefore, it is very imperative to identify the influential nodes so that the system's reliability, as well as its efficiency, can be improved upon. In the context of this study, critical nodes are referred to as those network nodes where there are needs for injection of reactive power. At these nodes, the voltage drop is extremely high and as such, there is a large amount of power loss associated with such nodes. However, there are some nodes within power networks whose outage, due to disturbance, could cause a detrimental effect (such as voltage instability) on the operation of the system. That is, when a disturbance occurs, removal of these nodes could lead to total blackout of the system. The importance of these nodes has been critically investigated and documented in the literature. Many existing studies have proved the importance of the location of nodes in real-time power networks [5]. The optimum operation of power systems could be greatly affected if these nodes are suddenly disconnected during disturbance. Such nodes are usually referred to as the key nodes or influential nodes.

Several approaches to predicting the distance to the location of collapse within the existing grids have been proposed in the open literature [2], [6]. The contributions offered, to the active stream of research, by these approaches have been documented. Various indices have been proposed based on the static load-flow analysis using Newton-Raphson iterative method [7]–[9]. These approaches are not without their challenges. For instance, obtaining the required solution would require several iterations with complex computation in each iterative step as a result of Jacobian computations. Another challenge that could limit the practicability of the existing approaches is the selection of a suitable slack bus among the generator buses. This is a serious bottleneck to efficient power system operations as choosing wrong slack bus could lead to wrong results and hence wrong conclusion. The detail of various approaches for analyzing vulnerability has been reviewed and presented in the literature [7]. Vulnerability analysis of power systems could be referred to as the evaluation of the influence which a local failure has on the power network before the failure occurs so that proper measures could be put in place to avert the occurrence of such failure. These challenges are tackled, in this present study, by considering the network topologies, the interconnections of the network elements as well as the impedance between the network nodes. The main merit offered by this approach is that it eliminates some of the mathematical complexity observed in the traditional methods based on power-flow analysis. Also, it requires no slack bus selection. It also avoids repetitive operational steps in the result computations and hence, it is less time-consuming. In other words, it results in a reduced computational time, and provides results for a real-time update of the network when there are changes in the practical structural characteristics of the network.

Various techniques, that were proposed in the early studies by most of the authors as regards the evaluation of performance of influential nodes in power systems, are power-flow-dependent [6]. Such methods include continuation power flow method, modal analysis, voltage stability indices-based method, P–V curve, sensitivity analysis-based approach and Q–V curve. These methods are basically iterative-based since the problem formulations are nonlinear equations [10]. As such, the solutions can only be obtained through various iterative processes. Although, these methods provide valuable insights as to how the influential nodes in a power system can be detected, they failed to account for the network topology as well as the interconnectivity among the network generators, loads and transmission lines. The main advantage of the network topology, in solving this problem, is that it formulates the problem as a linear relationship between the nodal voltages and the current flowing through the links [1], [11]. Hence, the divergence of solution is completely avoided. In recent times, the theory of complex networks has attracted considerable attention in the field of natural and social sciences, computer science, biological science and economics [4], [12]. This is gaining vast attention more recently by power system engineers and researchers across the globe in solving various power system problems. Centrality-based methods have been proposed by various authors. Some of the centralities that are commonly used in complex network theory include Eigenvector, Betweenness, LeaderRank, Closeness, PageRank and Degree centralities [13]. The solution to the problem is usually formulated by decomposing the system admittance matrix using eigenvalue analysis. Although, the significance of many of these methods has been demonstrated in the literature, there are several bottlenecks hindering the holistic application of these methods for solving various power system problems. One of such challenges is the fact that some of these Complex Network Theory (CNT)-based methods are n usually precise while those that are precise are iterative in nature and hence, computationally complex with high time consumption. For instance, in identifying important nodes in a network, Degree Centrality (DC) is a straight forward approach but it does not take into consideration the full information of the network under consideration. As such, its application is limited and not suitable in most cases. For a relatively large system, application of Betweeness Centrality (BC) and Closeness Centrality (CC) is not usually suitable because they are computationally expensive with respect to time complexity. Although, application of PageRank Centrality (PRC) is most suitable for a relatively large network, it does not work well when applied to an undirected network. Its application can only be effective on a directed network because it uses global information of the network [26]. Consequently, the traditional CNT-based methods are less accurate in some networks and therefore less reliable in identifying sets of nodes whose outage could be detrimental to the operation of the system. This greatly limits the application of the traditional CNT-based approaches to real-time power networks.

Based on the foregoing, it is of great importance to develop a faster and reliable alternative framework, which employs network local and global information in its formulation for influential nodes' detection. In this paper, the method of Network Structural Characteristics Theory (NSCT) is suggested. The following are parts of the contributions offered by the study presented in this paper: First, an effective framework, based on circuit theory laws, which completely avoids iteration in order to minimize the time complexity of the solution, is developed in this study. Second, the connectivity index for risk vulnerability assessment is suggested and implemented to determine the network strength during critical outage conditions. Third, the results obtained from the comparative analysis of all the methods based on the risk vulnerability assessment show that the suggested NSCT-based approach is more accurate and can be easily applied on large real time practical networks.

The remainder of the paper is organized as follows: Section II presents the theoretical background and the mathematical formulations of both the traditional CNT-based and the suggested NSCT-based methods. The results and discussion of results obtained are presented in section III while the paper is concluded in section IV.

II. Theoretical Background and Mathematical Formulations

This section presents the theoretical framework as well as the mathematical formulations for the traditional centrality measures suitable for identifying important nodes within a complex infrastructure such as power systems. There are several centrality measures in existence but the five suitable centrality measures for identifying important nodes are presented in this study. These include: degree centrality, closeness centrality, eigenvector centrality, PageRank and betweenness centrality [14], [15]. These are presented in the sub-sections that follow.

A. Traditional Complex Network Theory-Based Techniques

Generally, the topological structure of any given power network is usually captured in the network bus admittance through the interconnections of the network elements as well as electrical parameters of the network. As such, the network bus admittance can easily be determined using complex-weighted Laplacian method. Based on the foregoing, we modelled electric power network from the perspective of graph theory. Consider an electric power network modelled as a weighted graph $G=(V,E,W)$ whose vertex set is defined by $V(G)=\{v_1,v_2,v_3,\ldots,v_n\}$ and the network edge set defined by $E(G)=\{e_1,e_2,e_3,\ldots,e_n\}$ with each edge k between any two vertices i and j defined by $e_k=(v_i,v_j)$. The weight attached to any edge k of the network is w_{ij}. If no edge exists between vertices i and j, then $w_{ij}=0$. For any undirected weighted graph without any loop, $w_{ij}=w_{ji}$ and $w_{ii}=w_{jj}=0$ and the weight matrix is symmetrical about its diagonal. The matrix for the weight matrix of the network graph can therefore be formulated as

$$W(G)=\begin{pmatrix} 0 & w_{12} & w_{13} & . & . & . & w_{1n} \\ w_{21} & 0 & w_{23} & . & . & . & w_{2n} \\ w_{31} & w_{32} & 0 & . & . & . & w_{3n} \\ . & . & . & . & . & . & . \\ w_{n1} & w_{n2} & w_{n3} & . & . & . & 0 \end{pmatrix} \quad (1)$$

Also,

$$X(G)=\begin{pmatrix} x_1 & 0 & 0 & . & . & . & 0 \\ 0 & x_2 & 0 & . & . & . & 0 \\ 0 & 0 & x_3 & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & . & . & . & x_n \end{pmatrix} \quad (2)$$

where $x_i=\sum_{\substack{j=1 \\ j\neq i}}^{n} w_{ij}$ and the Laplacian for the network can be determined from

$$L(G)=X(G)-W(G) \quad (3)$$

1) Degree Centrality (DC) Method

This centrality measure shows the extent or degree of a vertex connection to the remaining parts of the system. Generally, the individual vertices that have more links attached to them than others are seen to be more connected compared to others within the system. This could be traced to the fact that more sources may be connected to such vertex thereby having access to information than any other vertex in the system.

In general, the Laplacian matrix formulated in equation (3) can be used to determine the electrical degree centrality of any given node v as

$$DC(v)=\frac{\deg(v)}{n-1}=\frac{L(v,v)}{n-1} \quad (4)$$

$$=\sum_{\substack{i,j\in V \\ j\neq i}}^{n}\frac{\left\| Y(i,j)\right\|}{n-1} \quad (5)$$

where n is the number of vertices.

2) Closeness Centrality (CC) Method

In determining the closeness centrality of a vertex, the shortest path between such vertex and all the other network vertices, which are connected to it via transmission line, are first determined. This centrality identifies the key node or vertex that requires a shorter electrical distance to communicate with all other connected vertices. It, therefore, ranks the closest vertex higher than the farthest. Mathematically, the closeness centrality of a node i in a network can be defined by the expression

$$CC(v)=\frac{n-1}{\sum_{i,j\in V} dy(i,j)} \quad (6)$$

where $dy(i,j)$ indicates the shortest electrical path length between the network vertices i and j.

3) Eigenvector Centrality (EC) Method

In this method, the weight associated with the first eigenvector is given to each network vertex as a centrality value. This is usually related to the adjacency matrix A in determining how significant a vertex is in a network. The electrical adjacency matrix of a network can be expressed as

$$A_t=-Y+Dg(Y) \quad (7)$$

where Y represents the network admittance and $Dg(.)$ denotes the diagonal matrix, which is extracted from the original network matrix.

The admittance matrix of the network Y can easily be obtained using

$$Y=A^{T}diag(y)A \quad (8)$$

where y is the admittance vector of the network transmission links. The elements of the n-by-n bus admittance matrix Y are determined from

$$\begin{cases} Y(i,j)=-y(i,j);\ \text{if } i\neq j \\ Y(i,j)=\sum_{\substack{i,j\in V \\ i\neq j}} y(i,j);\ \text{if } i=j \\ Y(i,j)=0;\ \text{otherwise} \end{cases} \quad (9)$$

The eigenvector centrality of a vertex i can therefore be determined from the entry v of the eigenvector η which corresponds to the largest eigenvalue μ_{max}. Consequently, the weighted eigenvector centrality can be expressed as

$$EC(v)=\left\|\frac{1}{\mu_{max}}\sum_{k=1}^{n}A(v,k)\eta_k\right\| \quad (10)$$

where $\mu\eta=A\eta$ (11)

4) Betweeness Centrality (BC) Method

In this method of approach, the weight associated with the first eigenvector

$$BC(v) = \frac{\sum_{i \neq v \neq t \in V} \sigma_{ij}(v) \Big/ \sigma_{ij}}{n(n-1) \Big/ 2} \qquad (12)$$

where σ_{ij} represents the shortest electrical path from vertex i to vertex j while $\sigma_{ij}(v)$ represents the sum total of all shortest electrical paths from vertex i to vertex j which pass through v.

5) PageRank Centrality (PRC) Method

PageRank Centrality (PRC) approach is usually used to rank webpages in order to evaluate how important the webpage is via the structure of the hyperlink system. The application of PRC in identifying important nodes in a directed network has been extensively reported in the open literature. From the graph theoretical perspective, the webpage is usually modelled as a directed graph where the vertices correspond to the webpages and the hyperlinks between any two webpages correspond to the edges of the graph. Mathematically, a PRC for any given webpage, P_w can be expressed as

$$PRC(P_w) = \sum_{P_k \in P(P_w)} \frac{PRC(P_k)}{N(P_k)} \qquad (13)$$

where $P(P_w)$ denotes those pages that point towards P_w, $N(P_k)$ denotes total out-link pages P_k. The solution to equation (13) can only be obtained through iterative procedures where

$$PRC^{j+1}(P_w) = \sum_{P_k \in P(P_w)} \frac{PRC^{j}(P_k)}{N(P_k)} \qquad (14)$$

B. Proposed Network Structural Characteristics Theory-Based Approach

From the basic circuit theory standpoint, the network equations are generally formulated based on the fundamental Kirchhoff's Law as expressed in equation (15). However, in a complex infrastructure system such as power networks, very few nodes are connected and as such equation (15) is used to capture the sparsity characteristics of power systems.

$$I = YV \qquad (15)$$

where I represents the generator and load injected current vector, Y represents the network admittance matrix, and V represents the generator and load voltage vectors for the system.

The structural topology of the interconnections between network elements such as transmission lines and nodes is usually captured through the admittance matrix of the network given in equation (15). Intuitively, Equation (15) shows that the branch currents are directly proportional to the node voltages where the constant of proportionality is the network admittance matrix, which represents the structural topology of the network. However, equation (15) becomes non-linear due to re-formulation during power-flow studies. Consequently, there is no known solution to this non-linear problem except through iterative processes, which are associated with various challenges such as divergence of the solution, local optimal solution, refactorization of Jacobian matrices, etc. In this study, the sparsity property of power network is explored to overcome some of the afore-mentioned challenges associated with power-flow-based methods.

By partitioning the equation (15) for a power network having n buses from which there are G generator buses and L load buses, we can write [1]

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix} \qquad (16)$$

where $[I_G]$ and $[I_L]$ are the elements of $[I]$, $[V_G]$ and $[V_L]$ are the elements of vectors $[V]$, $[Y_{LL}]$ represents the bus admittance of the branches that connect load bus to load bus, $[Y_{GG}]$ represents the bus admittance of the branches that connect generator bus to generator bus, $[Y_{LG}]$ represents the bus admittance of the branches that connect load bus to generator bus, $[Y_{GL}]$ represents the bus admittance of the branches that connect generator bus to load bus and $[Y_{LG}]$ represents the bus admittance of the branches that connect load bus to generator bus.

Equation (16) can be manipulated algebraically to give

$$\begin{bmatrix} V_G \\ I_L \end{bmatrix} = \begin{bmatrix} Z_{GG} & K_{GL} \\ W_{LG} & C_{LL} \end{bmatrix} \begin{bmatrix} I_G \\ V_L \end{bmatrix} \qquad (17)$$

From Equations (17), we can define the following sub-matrices:

The bus impedance of the transmission lines that connect load generator to generator bus as

$$Z_{GG} = Y_{GG}^{-1} \qquad (18)$$

The matrix of the transmission lines that connect generator bus to load bus as

$$H_{GL} = -Y_{GG}^{-1} Y_{GL} \qquad (19)$$

The matrix of the transmission lines that connect generator bus to load bus as

$$W_{LG} = Y_{LG} Y_{GG}^{-1} \qquad (20)$$

The bus impedance of the transmission lines that connect load bus to load bus, when the effect of the load buses has been completely eliminated, as

$$C_{LL} = Y_{LL} - Y_{LG} Y_{GG}^{-1} Y_{GL} \qquad (21)$$

The topological characteristics of power networks are embedded in the equations (18) to (21). Although, some of these expressions have been used to solve various operational problems combating modern power systems, there are still many problems that are yet unsolved. This problem is tackled in this paper by exploring the advantages associated with the square matrix given in equation (21). This therefore provides more insight into how power system problems could be solved faster without the need for an iterative process which is tedious and time-consuming.

An attempt is made in this paper to explore the associated benefits of the atomic theory to estimate the attractive force existing between two buses. This force is used to measure the strength and weakness associated with the transmission link between any two buses in a network.

According to Coulomb's Law, a push-pull force exists between two buses having charges Q_1 and Q_2. Also, according to Gravitational Law, a push-pull force exists between two bodies whose masses are M_1 and M_2 respectively. Based on the foregoing, it can easily be inferred that in any given power network, the strength of influence between two nodes, connected by a transmission line whose impedance is Z, is given by [1]

$$I_{s,12} = \frac{C\,(V_1 \;\; V_2)}{Z^2} \tag{22}$$

where the C in (15) is a constant. The nodal voltages at nodes 1 and 2 are V_1 and V_2 respectively. The impedance of the transmission line connecting nodes 1 and 2 is Z and it is referred to as the relative electrical distance between nodes 1 and 2.

It can be seen that the expression given in equation (22) is analogous to the maximum power flowing from node 1 whose voltage is V_1 to node 2 whose voltage is V_2 as given by

$$P_{\max} = \frac{|V_1||V_2|}{\sqrt{|Z|^2}} \tag{23}$$

where

Z represents the equivalent relative electrical distance between nodes 1 and 2.

Equation (22) can be easily manipulated to give

$$\left[NSM_{ij}\right] = K\,[V_i]\left[V_j\right]\left[\left[RED_{ij}\right]^2\right]^{-1} \tag{24}$$

where NSM is Network Strength Matrix, RED is the network equivalent Relative Electrical Distance between any two network nodes i and j while K is a constant.

Therefore, the structural characteristics of an n-bus power system can easily be captured in NSM whose diagonal element $NSM\,(i,i)$ represents the Node Strength $NS\,(i)$ or Bus Strength $BS\,(i)$ associated with node i. This implies that

$$NS\,(i) = diag\left(NSM\,(i,j)\right) \tag{25}$$

C. Performance Evaluation of the Approaches

1) Using Pearson's correlation coefficient

In this section, we present the performance evaluation for both the existing CNT-based methods and that of the NSCT-based method (Node Strength or Bus Strength) suggested in this paper. This is necessary in order to ascertain the effectiveness of each method in identifying the set of influential nodes in a power network. This evaluation helps in comparing the results obtained using the NSCT-based method with those obtained using the traditional CNT-based approaches. The Pearson's rank correlation coefficient is explored to validate the performance characteristics of the existing methods with reference to the structural-based method suggested in this paper.

Generally, Pearson's correlation coefficient between any two variables X and Y can be defined by

$$r_p = \frac{\sum\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sqrt{\left(\sum\left(X_i - \overline{X}\right)\right)} \times \sqrt{\left(\sum\left(Y_i - \overline{Y}\right)\right)}} \tag{26}$$

where X_i $(i = 1, 2, \ldots, N)$ represents the X-variable values in any given sample, N is the number of samples, $\overline{X}$ represents the mean of X-variable values, Y_i $(i = 1, 2, \ldots, N)$ represents the Y-variable values in the given sample and $\overline{Y}$ represents the mean of Y-variable values.

The strength of the association that exists between X and Y is said to be very weak if $r_p < 0.2$, weak if $0.2 < r_p < 0.39$, moderate if $0.4 < r_p < 0.59$, strong if $0.6 < r_p < 0.79$ and very strong if $r_p > 0.79$.

2) Using Structural Risk Vulnerability against Targeted Attacks

Generally, a targeted attack on a power network involves intentional criminal acts that could undermine the integrity of such power network. In order to present a holistic model of these threats, it is therefore usually assumed that the power system layout is fully known. The main aim is to evaluate the performance of the network while it is being subjected to maximum disruption while the number of attacked nodes is kept to barest minimum. In solving this vulnerability problem in a power system towards the intentional attacks, there is a need to identify the sets of influential nodes and their influence on the overall network performance. This identification, in advance, assists the system operators on how to continuously monitor and protect these influential nodes to ensure improvement in the system robustness.

In this paper, we evaluate the impact which the outage of the identified influential nodes has on the network performance. This is implemented by determining the connectivity strength of the network when it is operating under various contingency conditions. In this paper, we define Connectivity Index (CI) for any given N-bus power network as

$$CI = \frac{N - N_{outage}}{N} \times 100 \tag{27}$$

where N_{outage} represents the number of outaged nodes due to removal of the identified influential nodes from the original network. The CI is an index which lies between 0 and 1. The base case network scenario corresponds to a case when the CI is 100% since all the nodes are intact and no contingency is considered for the base case condition. Although, the case for CI corresponding to a situation where the number of outaged nodes is equal to the total number of network nodes is considered a trivial scenario in this paper.

It is often desirable to know the level of network robustness, most especially, when there is failure of one or more nodes which could lead to a total blackout of the system. In most social networks, the level of network robustness considering node failure is usually measured using the network statistical properties such as characteristic path length, global efficiency, local efficiency, and the clustering coefficient of the graph as effective indicators. Traditionally, based on the statistical properties of network topology, the average characteristic path length of a graph G is defined by

$$L_c(G) = \left(\frac{1}{n^2 - n}\right)\sum_{i \neq j} d(i,j) \tag{28}$$

where $d(i,j)$ represents the number of shortest distances between nodes i and j within the graph. The shortest distance $d(i,j)$ increases as the path connecting the two nodes i and j increases. In order words, $d(i,j) = \infty$ as the path between nodes i and j becomes increasingly large. However, in order to account for the electrical characteristic of the network, relative electrical distance is used which modifies (28) to give

$$L_c(G) = \left(\frac{1}{n^2 - n}\right)\sum_{i \neq j} RED(i,j) \tag{29}$$

The network graph global efficiency, in terms of the relative electrical distance, can be defined as

$$E_g(G) = \left(\frac{1}{n^2 - n}\right)\sum_{i \neq j} \frac{1}{RED(i,j)} \tag{30}$$

The local efficiency of the network graph, in terms of the global efficiency, can be expressed as

$$E_l(G) = \frac{1}{n}\sum_{i \in G} E_g(G_i) \tag{31}$$

The clustering coefficient of the graph is given by

$$C_c(G) = \frac{1}{n}\sum_{i \in G} \frac{2E_i}{k_i(k_i - 1)} \tag{32}$$

where E_i is the i th number of edges and k_i represents the degree of the node i.

The clustering coefficient in a graph is used to determine the extent of closeness of a node with its neighbour in forming a clique. Its values range between the value of 0, which corresponds to no cluster and 1 which corresponds to maximum or strong cluster.

III. RESULTS AND DISCUSSIONS

The effectiveness of the existing CNT-based methods and the NS method suggested in this paper is tested using two different standard networks of IEEE (IEEE 5-bus and IEEE 14-bus networks) and a practical network of Nigerian 28-bus system. The standard IEEE 5-bus system has two generator nodes and three load nodes which are interconnected by seven transmission links while the standard IEEE-14 system has five generator buses interconnected with the network nine load buses by twelve transmission lines. The Nigerian 28-bus system comprises ten generator nodes with 18 load nodes. In this study, all simulations are carried out using MATLAB 2019a software. For the sake of convenience, the generation nodes are numbered first before the load nodes. The results obtained are presented and discussed in the subsections that follow:

A. Case I: The Standard IEEE 5-bus System

Using the standard IEEE 5-bus network as the case study, the results obtained using five different CNT-based methods (Degree Centrality (DC), Eigenvector Centrality (EC), Pagerank Centrality (PRC), Closeness Centrality (CC) and Betweeness Centrality (BC)) as well as NSCT-based method (Node Strength (NS) otherwise referred to as Bus Strength (BS)) are presented in Table I. Based on these results, the load nodes are then ranked, using each method, in the order of magnitudes. For example, as presented in Table I, it is obvious that the load bus 4 has the maximum values of 9.4743, 8.3566, 3.5434, 8.6684, 1.5376 and 13.5315 using Degree Centrality (DC) method, Eigenvector Centrality (EC) method, PageRank Centrality (PRC) method, Closeness Centrality (CC) method, Betweeness Centrality (BC) method and Node Strength (NS) method respectively. This implies that load node 4 will be ranked number 1 for all the methods. This is repeated for all the load nodes in the network. The ranking results are presented as shown in Table II. Since load node 4 is ranked number 1 by all the methods, it implies that all the methods identified node 4 as the most important load node within the standard IEEE 5-bus system.

TABLE I. RESULTS FOR DIFFERENT APPROACHES USING A 5-BUS SYSTEM

Load node no	Methods					
	DC	EC	PRC	CC	BC	NS
3	7.1479	4.3520	2.5453	7.5675	1.3535	12.1211
4	9.4743	8.3566	3.5434	8.6684	1.5376	13.5315
5	1.3546	6.5474	1.6356	4.5875	0.4759	3.8637

TABLE II. INFLUENTIAL NODE IDENTIFICATION IN THE STANDARD IEEE 5-BUS SYSTEM

Node Ranking	Node number					
	DC	EC	PRC	CC	BC	NS
1	**4**	**4**	**4**	**4**	**4**	**4**
2	3	5	3	3	3	3
3	5	3	5	5	5	5

The implication of this is that removal of load node 4 has the highest influence on the network. Such a node requires special attention as regards protection against its removal as its outage could lead to total collapse of the network. For the network under consideration, load node 4 is identified by all the methods as the most influential load node in the network.

B. Case II: The Modified IEEE 14-bus System

The numerical results obtained for influential node identification within the IEEE 14-bus system are presented in Table III. Base on the order of magnitudes, the results obtained for all the methods are ranked as presented in Table IV to show the relative importance of each node within the system under consideration. As presented in Table III, for the CNT-based centrality methods, it can be seen that DC method has a maximum value of 5.6772, EC has a maximum value of 0.6768, PRC has a maximum value of 2.5765, CC has corresponds to a maximum value of 0.1457, BC corresponds to a maximum value of 26.4747 while the maximum value of NS is 3.5468.

The node corresponding to all these maximum values is load node 8 and hence, it is ranked number 1 for each of the methods. The nodes' ranking results based on the order of priority for all the methods are presented in Table IV. It can be seen that all the methods ranked load node 8 as number 1 in the standard IEEE 14-bus network. It is, therefore, obvious that all the methods identified load node 8 as the most influential node in the standard 14-bus network.

TABLE III. RESULTS FOR DIFFERENT APPROACHES USING THE MODIFIED IEEE-14 BUS SYSTEM

Load node no	Methods					
	DC	EC	PRC	CC	BC	NS
6	4.5463	0.5787	2.3431	0.0635	22.4444	2.3645
7	3.3635	0.2456	2.0675	0.0465	15.6864	0.9867
8	5.6772	0.6768	2.5765	0.1457	26.4747	3.5468
9	3.8657	0.4256	1.3553	0.1036	0.0056	1.6723
10	2.8365	0.1456	2.2635	0.0724	3.6690	0.6785
11	2.0434	0.0189	1.5460	0.0523	5.6894	0.0296
12	2.5658	0.0945	1.7588	0.0934	0.0057	0.3766
13	2.9346	0.2003	1.3553	0.1256	6.3565	0.4656
14	2.3576	0.0356	1.2466	0.1345	4.6367	0.0924

TABLE IV. INFLUENTIAL NODES IDENTIFICATION IN THE MODIFIED IEEE 14-BUS

Node Ranking	Node number					
	DC	EC	PRC	CC	BC	NS
1	**8**	**8**	**8**	**8**	**8**	**8**
2	**6**	**6**	**6**	**14**	**6**	**6**
3	9	9	10	13	7	9
4	7	7	7	9	13	7
5	13	13	12	12	11	10
6	10	10	11	10	14	13
7	12	12	9	6	10	12
8	14	14	13	11	12	14
9	11	11	14	7	9	11

C. Case III: The Nigerian 28-bus System

The structural topology of the Nigerian 28-bus system, with all nodes intact, is as shown in Fig. 1. The results of analysis obtained for a set of influential nodes identified by using the CNT-based methods and the method of NS suggested in this paper, are presented in Table V. These results are ranked based on the order of magnitudes and the top 10 influential nodes identified are presented in Table VI.

Based on the node ranking presented in Table VI for identifying a set of influential nodes in the Nigerian 28-bus system, it can be seen that DC, EC, PRC, CC, BC and NS methods ranked load nodes 14, 15, 14, 19, 19 and 19 as number 1 respectively while nodes 19, 14, 21, 14, 24 and 14 are ranked number 2 respectively. As such, nodes 14 is identified by DC and PRC methods as the most influential node. Also, load node 19 is identified by CC, BC and NS methods as the most influential load node while the EC method identified load node 15 as the most influential node.

In order to ascertain the reliability of these results, further analysis is highly desirable. This is because three different nodes are identified as the most influential load nodes by the different methods. That is, nodes 14 is identified by DC and PRC methods, node 19 is identified by CC, BC and NS and node 15 is identified by EC method. We therefore carried out the performance analysis based on Pearson's rank correlation coefficient to determine the relationship strength between the CNT-based methods and the suggested NS method. Thereafter, we implement the structural risk analysis assessment by investigating the impact of removing the identified sets on influential nodes on the entire network using N-1 and N-2 contingency criteria. The results obtained from these analyses are presented in the subsections that follow:

TABLE V. INFLUENTIAL NODE IDENTIFICATION IN THE NIGERIAN 28-BUS SYSTEM

Bus name	Bus number	Methods					
		DC	EC	PRC	CC	BC	NS
Ajah	11	0.0370	0.0071	0.0213	0.0372	20	0.2119
Akangba	12	0.0741	0.0228	0.0375	0.0438	26	0.3491
Ikeja-West	13	0.0370	0.0206	0.0194	0.0430	10	0.1464
Ajaokuta	14	0.1481	0.0661	0.0661	0.0466	97	2.3109
Aladja	15	0.0741	0.0726	0.0295	0.0395	21	0.2143
Benin	16	0.0370	0.0040	0.0209	0.0350	23	0.0370
Jebba	17	0.0370	0.0159	0.0198	0.0278	18	0.0188
Ayede	18	0.0741	0.0007	0.0406	0.0165	26	0.4743
Jos	19	0.1111	0.0377	0.0476	0.0473	176	2.4478
Kaduna	20	0.0741	0.0020	0.0376	0.0209	50	0.1080
Osogbo	21	0.1111	0.0059	0.0530	0.0220	95	0.1484
Kano	22	0.0741	0.0130	0.0366	0.0364	26	0.4417
Alaoji	23	0.0370	0.0018	0.0204	0.0182	20	0.0169
New Haven	24	0.1111	0.0150	0.0506	0.0365	131	0.0592
Onitsha	25	0.0741	0.0573	0.0305	0.0366	10	0.1715
Katampe	26	0.0370	0.0047	0.0197	0.0325	12	0.0033
Birnin Kebbi	27	0.0370	0.0206	0.0194	0.0340	14	1.1789
Gombe	28	0.0370	0.0002	0.0226	0.0142	18	2.1951

TABLE VI. THE TOP 10 INFLUENTIAL NODES IN THE NIGERIAN 28-BUS SYSTEM USING DIFFERENT METHODS

Node Ranking	Node numbering					
	DC	EC	PRC	CC	BC	NS
1	**14**	**15**	**14**	**19**	**19**	**19**
2	**19**	**14**	**21**	**14**	**24**	**14**
3	21	25	24	12	14	28
4	24	19	19	13	21	27
5	12	12	18	15	20	18
6	15	13	20	11	12	22
7	18	27	12	25	18	12
8	20	17	22	24	22	15
9	22	24	25	22	16	11
10	25	22	15	16	15	25

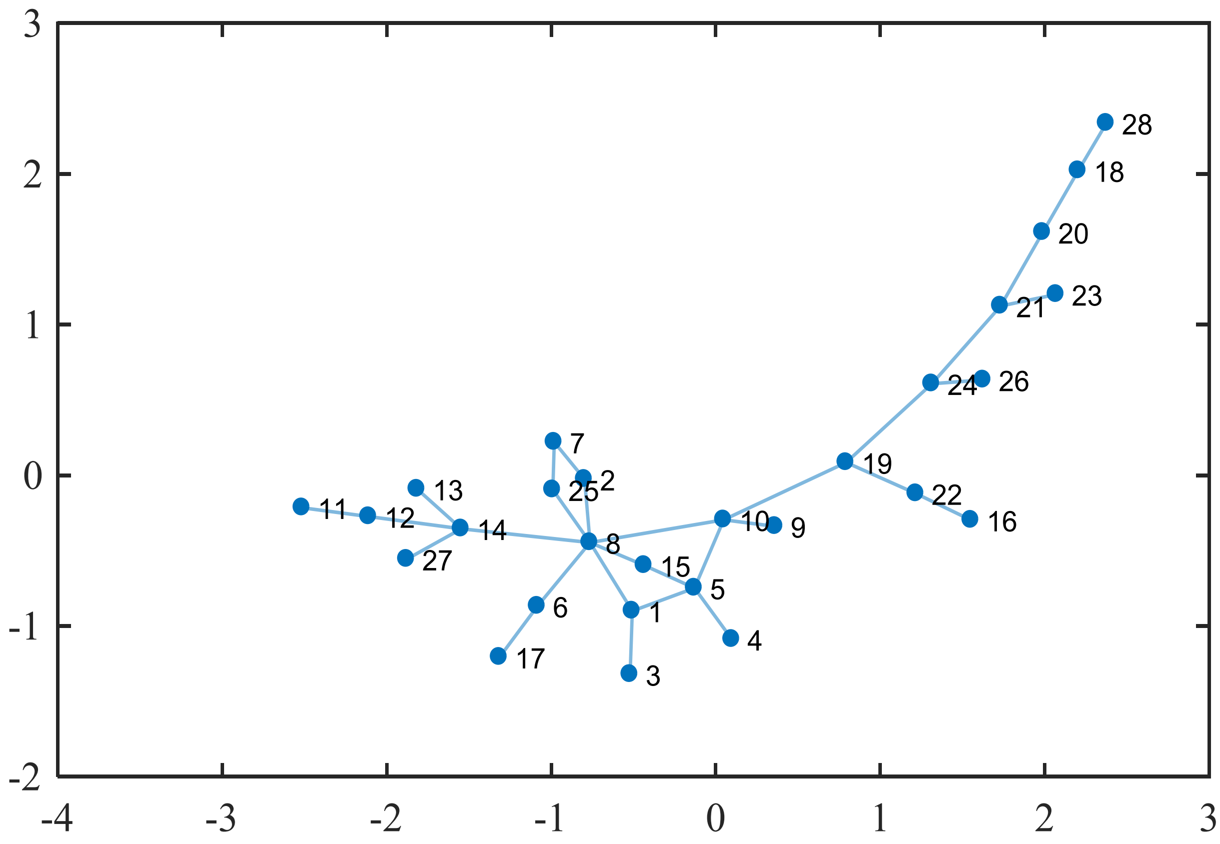

Fig. 1. Structural topology of the Nigerian 28-bus system with all the nodes intact

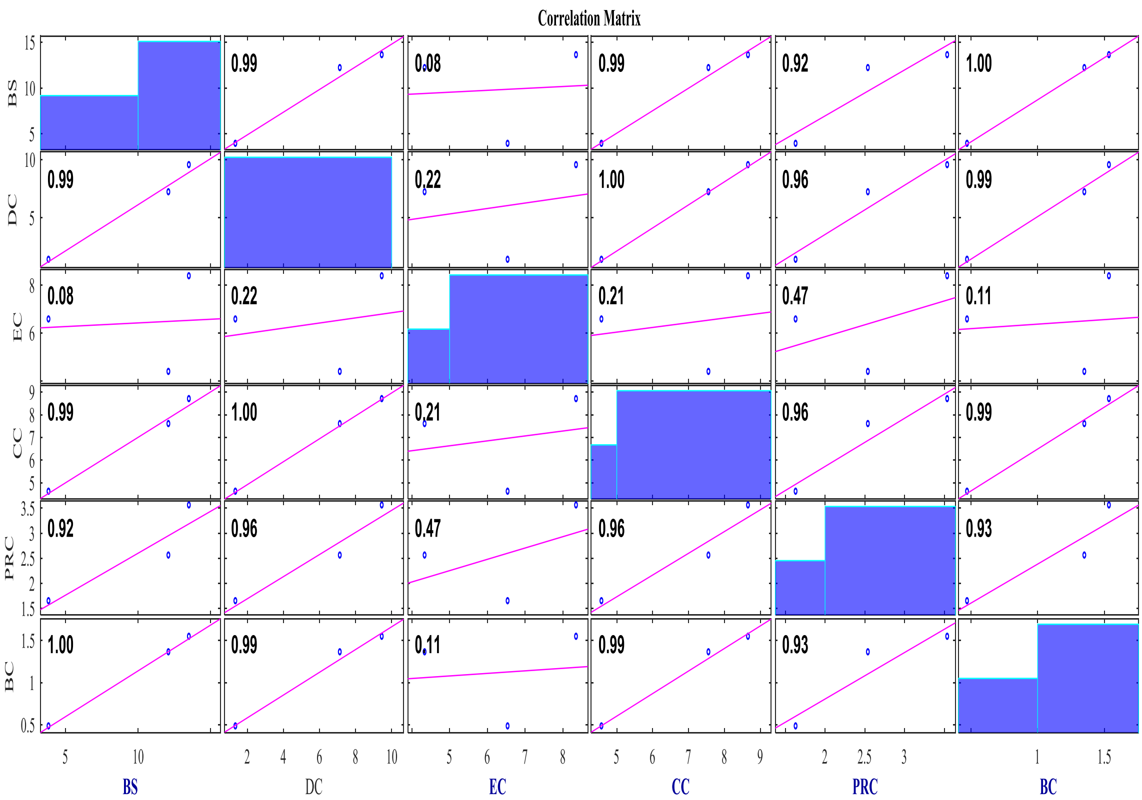

Fig. 2. Correlation matrix for the 5-bus system

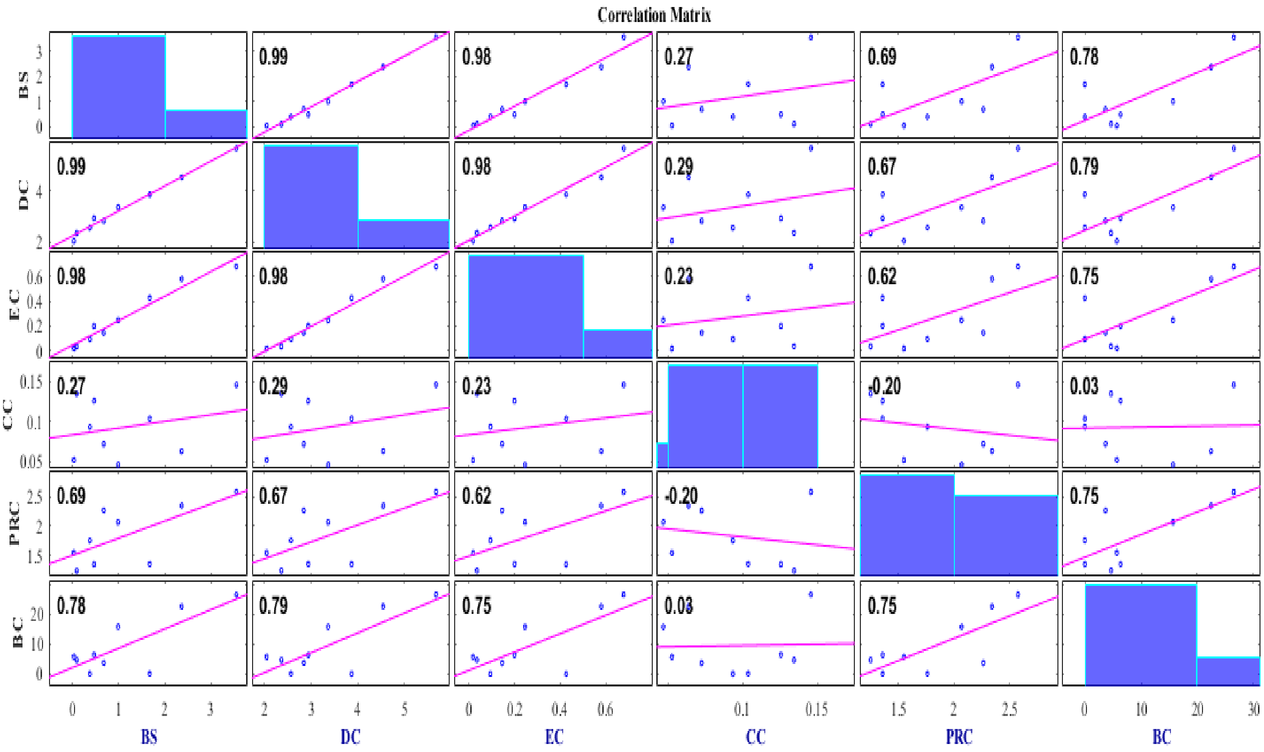

Fig. 3. Correlation matrix for the 14-bus system

1) Performance Evaluation based on Pearson Correlation Coefficient

In order to measure the strength of linear relationship that exists among all the CNT-based (DC, EC, CC, PRC and BC) methods and the NSCT-based method (BS), for effective detection of the set of influential nodes in a power system, the statistical-based approach of Pearson correlation coefficient is used. This has the capability to effectively measure the linear relationship between these methods (CNT and NSCT). The correlation matrices for the three networks considered in this work (5-bus, 14-bus and 28-bus) are presented in Figs. 2-4.

As can be seen from Fig. 2, it can be seen that the correlation coefficients between the NSCT-based and CNT-based methods are very high except with EC method whose correlation coefficient with BS method is very low (0.0797). This shows that a strong relationship exists between the node strength (BS) method and all CNT-based methods except EC method whose relationship is very weak in the 5-bus network. The correlation matrix for the 14-bus network shown in Fig. 3 indicates a high value of correlation coefficient between the NSCT-based (BS) and DC, EC, PRC and BC methods. However, it can be seen that a low correlation coefficient of 0.2661 exists between the BS and CC methods. This implies that in the standard IEEE 14-bus system, a strong linear relationship exists between the NSCT-based method and DC, EC, PRC and BC methods while weak relationship exists between the BS and CC methods in the standard 14-bus network.

The correlation matrix for the Nigerian 28-bus network which shows the type of relationship that exists between the NSCT-based method and CNT-based methods is shown in Fig. 4. As can be seen, the correlation coefficients between the BS and all the CNT-based methods are low. The implication of this is that there exists a weak correlation between these methods.

Consequently, it can easily be inferred that the capability of the CNT-based methods in identifying the important nodes in the power system decreases with the system complexity. In other words, as the system size is becoming more complex, the CNT-based methods may not be suitable for identifying the critical nodes whose removal could be detrimental to the operation of the network. As such, the solution provided using CNT-based methods may not be sufficient in detecting influential nodes in a relatively large power system. The numerical values for the correlation coefficients for the three networks considered in this paper are presented in Table VII. As presented in Table VII, another bottleneck in identifying the sets of influential nodes in large power network is that no unique solution exists with the deployment of CNT-based methods and if it exists, it could be misleading.

Based on the foregoing, there is the need to further investigate the capability strength of each method for identifying the influential nodes in a power system. This is carried out by investigating the impact which the removal of influential nodes, identified by each method, has on the entire network. The overall impact of influential node removal on the entire system is measured based on the connectivity index. The results obtained for this critical contingency assessment are presented and discussed in the section that follows:

TABLE VII. PEARSON'S RANK CORRELATION COEFFICIENT OF

Network	NS with respect to CBMs				
	DC	EC	CC	PRC	BC
5-bus	0.9893	**0.0797**	0.9918	0.9152	0.9996
14-bus	0.9935	0.9786	**0.2661**	0.6948	0.7840
28-bus	**0.3909**	**0.3061**	**0.1918**	**0.3862**	**0.4659**

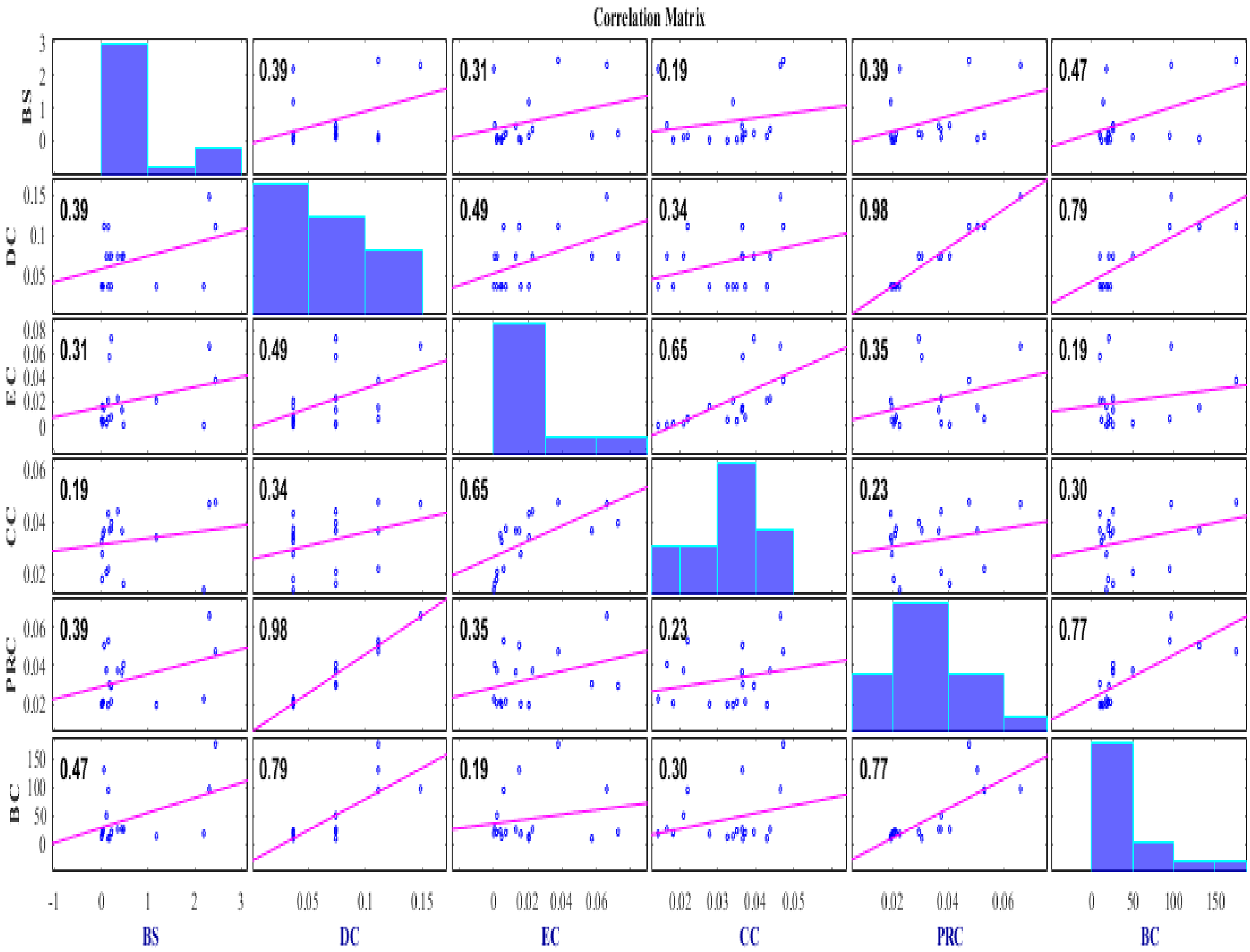

Fig. 4. Correlation matrix for the Nigerian 28-bus system

2) Performance Evaluation by Structural Risk Vulnerability Analysis

We investigate the impact of the influential node removal on the entire Nigerian 28-bus system shown in Fig. 1. In this analysis, N-1 and N-2 contingency criteria are explored. The first two influential nodes identified by each of the NSCT-based (NS) and CNT-based (CC, DC, BC, PRC and EC) methods are removed and the impact on the strength of the network is investigated using the percentage Connectivity Index (CI). The numerical results for the first two influential nodes identified by each method are presented in Table VIII. The affected load nodes when the identified influential nodes are removed as well as their associated CI are also presented in Table VIII.

Considering the contingency analysis results presented in Table VIII, it can be seen that ten load buses are affected when the most influential node identified by NS method (node 19) is outaged and the estimated CI is 64.2%. The topological structure of the network under this contingency condition is presented in Fig. 5. When the second most influential node identified by NS method (14) is outaged, it can be seen that five load buses are affected and the estimated value of CI during this condition is 82.1%. The structure of the network under this contingency condition is shown in Fig. 6. The topological structure for the network when both load buses 14 and 19 are outaged is shown in Fig. 7. The number of affected buses is fifteen and the CI drops to 46.4%. The lower value of CI associated with the removal of bus 19, as compared with that obtained when bus 14 is removed, suggests that removal of bus 19 from the network is more critical than that of bus 14. Also, based on the low percentage CI associated with the removal of buses 19 and 14 (46.4%), it can be inferred that removal of these two buses is very detrimental to the operation of the network as it could lead to island formation that could result in total network blackout.

Although, same risk vulnerability analysis is established when the CC method is used as presented in Table VIII, the CC method is associated with a high simulation time compared with the NS method. This is so because the NS method, which is solely dependent on the interconnectivity of the network and independent of the network loading conditions. As such, while the CC method is an iterative-based method, the solution provided by the NS method does not involve iteration but provides the solution in just one computational time. For the sake of comparison, the time using CC method is 4.21 seconds while that of NS method is 0.11 second. This shows the superiority of the NS method over the CC method.

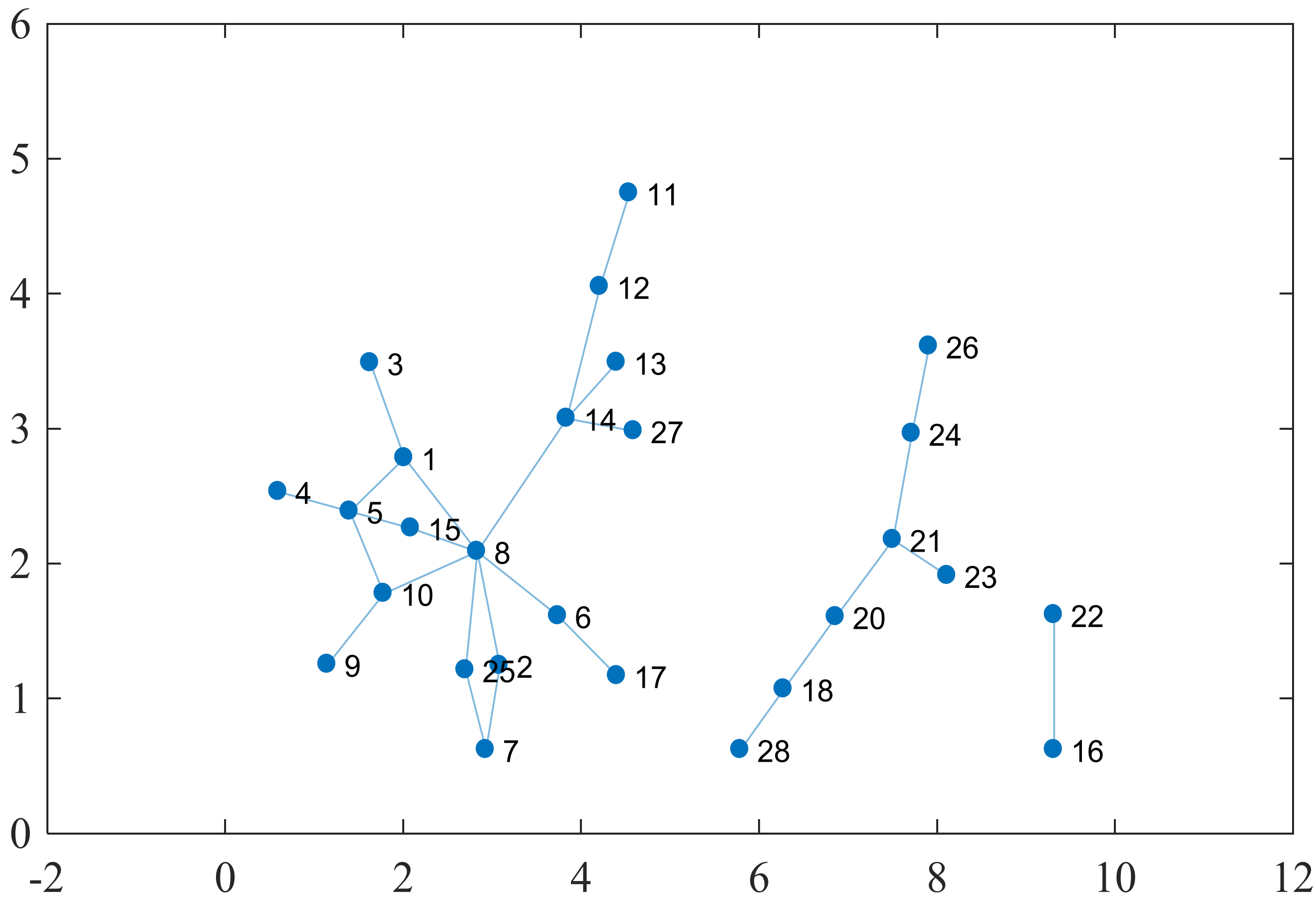

Fig. 5. Structural topology of Nigerian 28-bus system when bus 19 is outaged as identified by NS or CC method

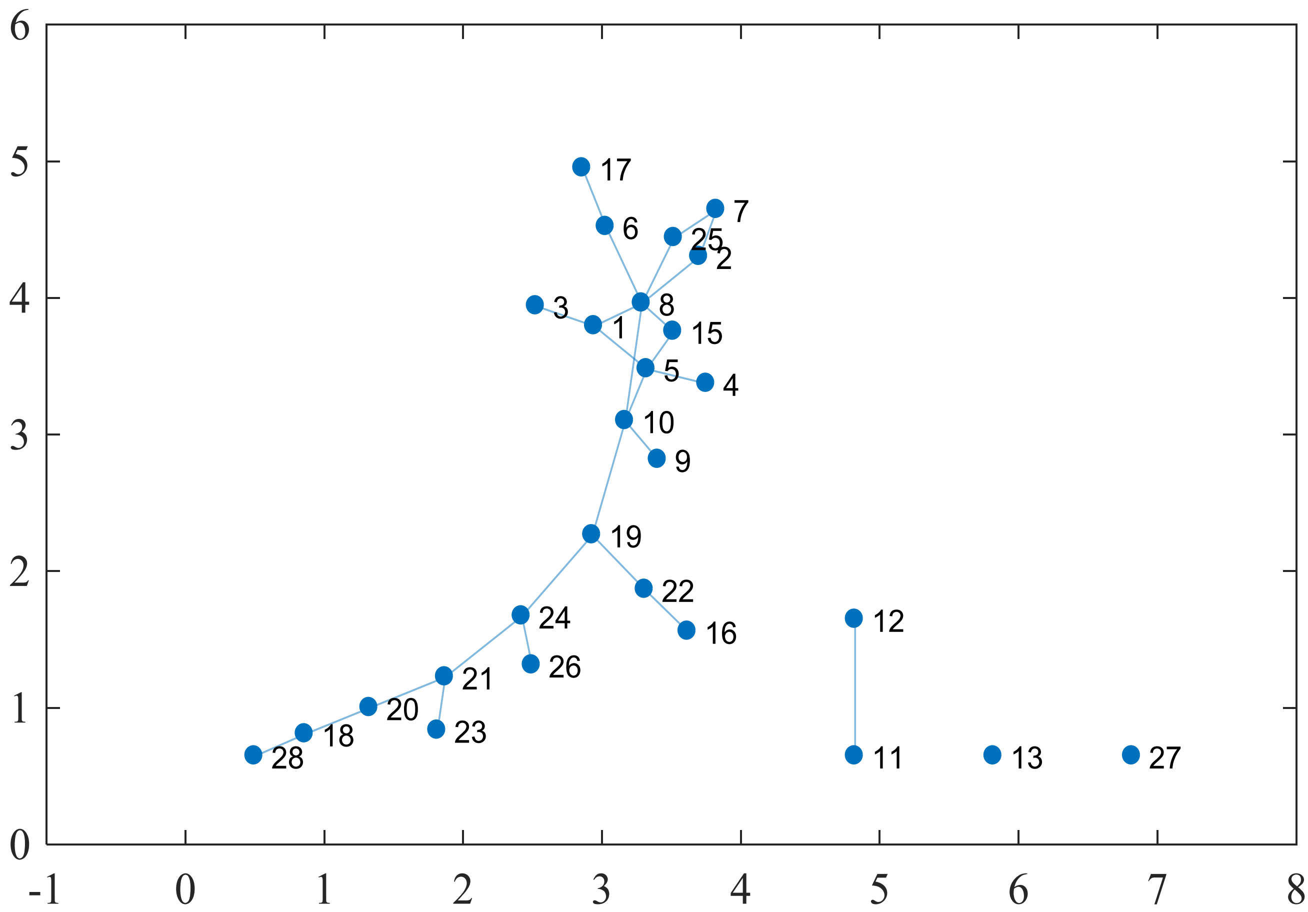

Fig. 6. Structural topology of Nigerian 28-bus system when bus 14 is outaged as identified by NS or CC method

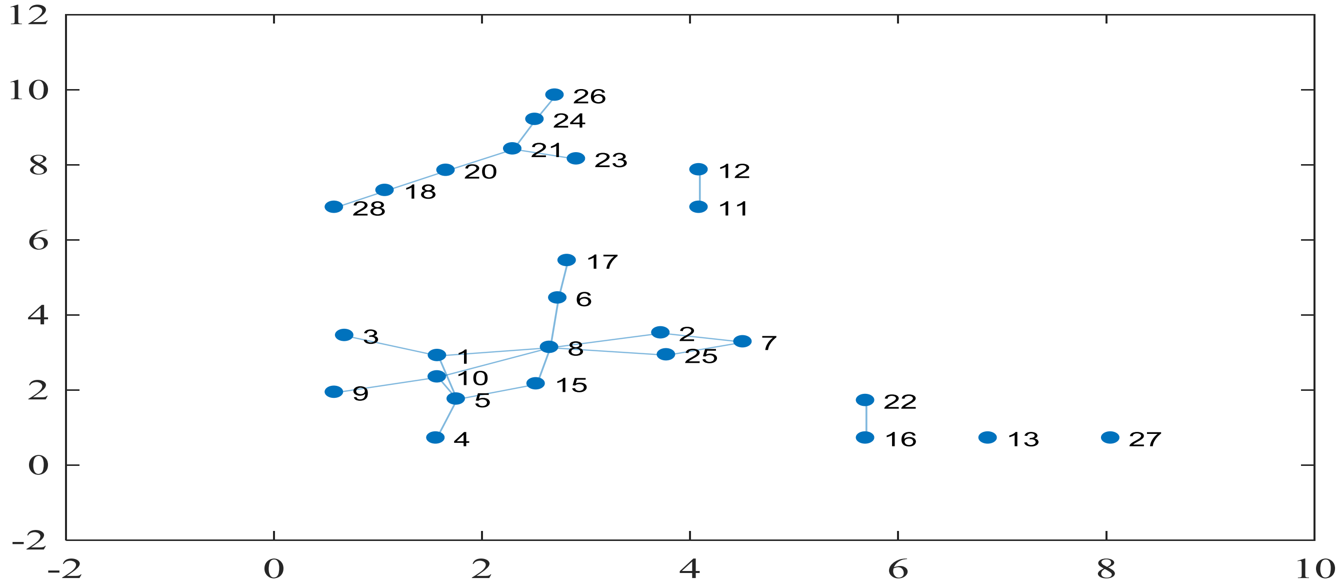

Fig. 7. Structural topology of Nigerian 28-bus system when both buses 19 and 14 are outaged as identified by NS or CC method

When the most influential node identified by the DC method (bus 14) is outaged, five other load buses are completely removed from the network (similar to Fig. 6). The associated percentage of CI for this scenario is 82.1 as presented in Table VIII. The structural interconnections of the network when the two load nodes 14 and 19 are removed from the network simultaneously are shown in Fig 8 (similar to Fig. 7). Although, with an outage of two load nodes (14 and 19), the percentage CI for the network reduces to 46.4, which is the same as that obtained for both NS and CC methods, the CI associated with the identified most influential load node is very high (82.1%) with the use of NS and CC methods. The implication of this is that with the outage of bus 14, the network integrity is far better and the impact of its outage is less critical to the network compared to the load node (node 19) identified by NS and CC methods.

TABLE VIII. INFLUENCE OF INFLUENTIAL NODE DETECTION BASED ON CONNECTIVITY INDEX IN THE NIGERIAN 28-BUS NETWORK

Method	Outaged node	Affected nodes	N_{outage}	CI (%)
NS	19	16 18, 19 20, 21, 22 23, 24, 26, 28	10	64.2
	14	11, 12 13, 14, 27	5	82.1
	19 & 14	11, 12 13, 14, 16 18, 19, 20, 21, 22, 23, 24, 26, 27, 28	**15**	**46.4**
CC	19	16 18, 19 20, 21, 22 23, 24, 26, 28	10	64.2
	14	11, 12 13, 14, 27	5	82.1
	19 & 14	11, 12 13, 14, 16 18, 19, 20, 21, 22, 23, 24, 26, 27, 28	**15**	**46.4**
DC	14	11, 14 12, 13, 27	5	82.1
	19	16 18, 19 20, 21, 22 23, 24, 26, 28	10	64.2
	14 & 19	11, 12 13, 14, 16 18, 19, 20, 21, 22, 23, 24, 26, 27, 28	**15**	**46.4**
BC	19	16 18, 19 20, 21, 22 23, 24, 26, 28	10	64.2
	24	18 20, 21 24, 23, 26, 28	7	75.0
	19 & 24	16 18, 19, 20, 21, 22, 23, 24, 26, 28	**10**	**64.2**
PRC	14	11, 14 12, 13, 27	5	82.1
	21	18, 20 21, 23, 28	5	82.1
	14 & 21	11 12, 13 14, 18, 20 21, 23, 27, 28	**10**	**64.2**
EC	15	15	1	96.4
	14	11, 14 12, 13, 27	5	82.1
	15 & 14	11, 14 12, 13, 15, 27	**6**	**78.6**

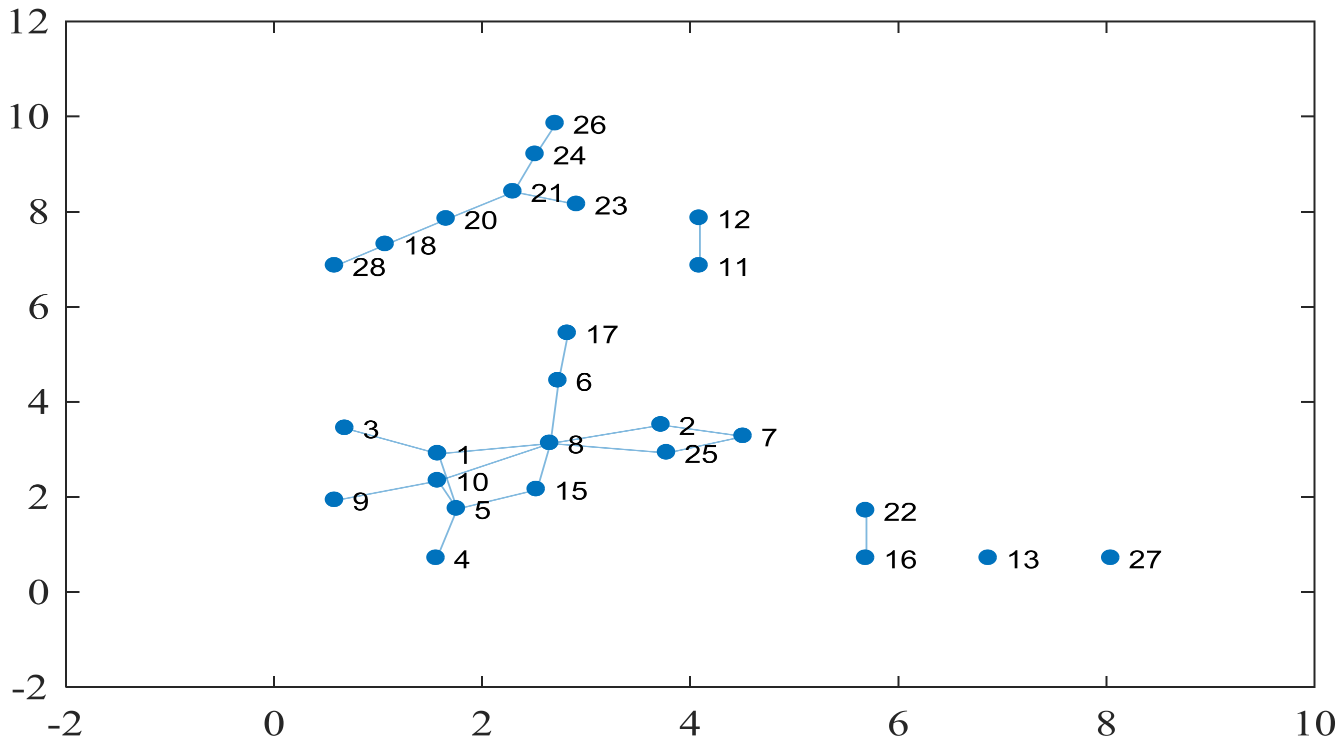

Fig. 8. Structural topology of Nigerian 28-bus system when both buses 14 and 19 are outaged as identified by DC method

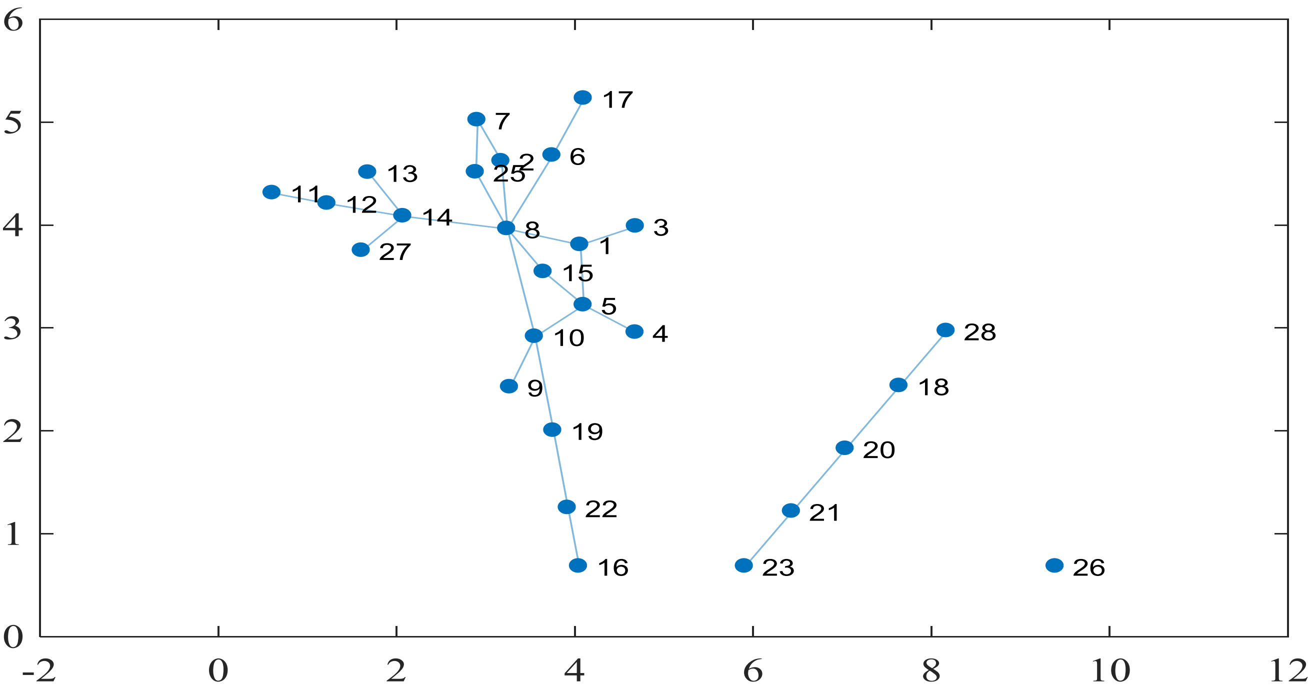

Fig. 9. Structural topology of Nigerian 28-bus system when bus 24 is outaged as identified by BC method

Although, with the BC method, bus 19 is identified as the most influential node, the next node to the most influential node identified by this method is node 24. In order to verify the effectiveness of this method compared with the results obtained using the NS method, the topological structure for the network when the identified most influential node (node 19) and the next node to it (node 24) simultaneously removed from the network, is analyzed. Using N-1 criterion, removal of load node 19 from the network leads to an outage of ten other nodes and the structural network for this scenario has been presented in Fig. 5. Also, Fig. 9 shows the structural network obtained when load node 24 is outaged from the original network. It can be seen that removal of load node 24 caused seven other load nodes to be removed. Furthermore, it can be seen, as presented in Table VIII, that when the two load nodes (19 and 24) are removed from the network, ten load nodes are affected which is the same as the number of load nodes affected when the most influential node (node 19) is removed as shown in Fig. 10. As such, the impact of removing load node 24 from the network is insignificant and the total estimated CI for this method remains 64.2%

which is less than 46.6% obtained for both NS and CC methods.
When the PRC method is applied, the structural topology of the network with node 21 removed and both nodes 14 and 21 removed are shown in Fig. 11 and Fig. 12 respectively. By going through a similar analogy, it can be seen that in PRC method, node 14 is identified as the most influential node whose outage affects only five load nodes with the CI of 82.1%. The second load node to this node 14 is identified as node 21 whose outage also affected five load nodes with the CI of 82.1. The overall network CI when these two load nodes are removed from the network is 64.2% which is much greater than that obtained with the NS and CC method.

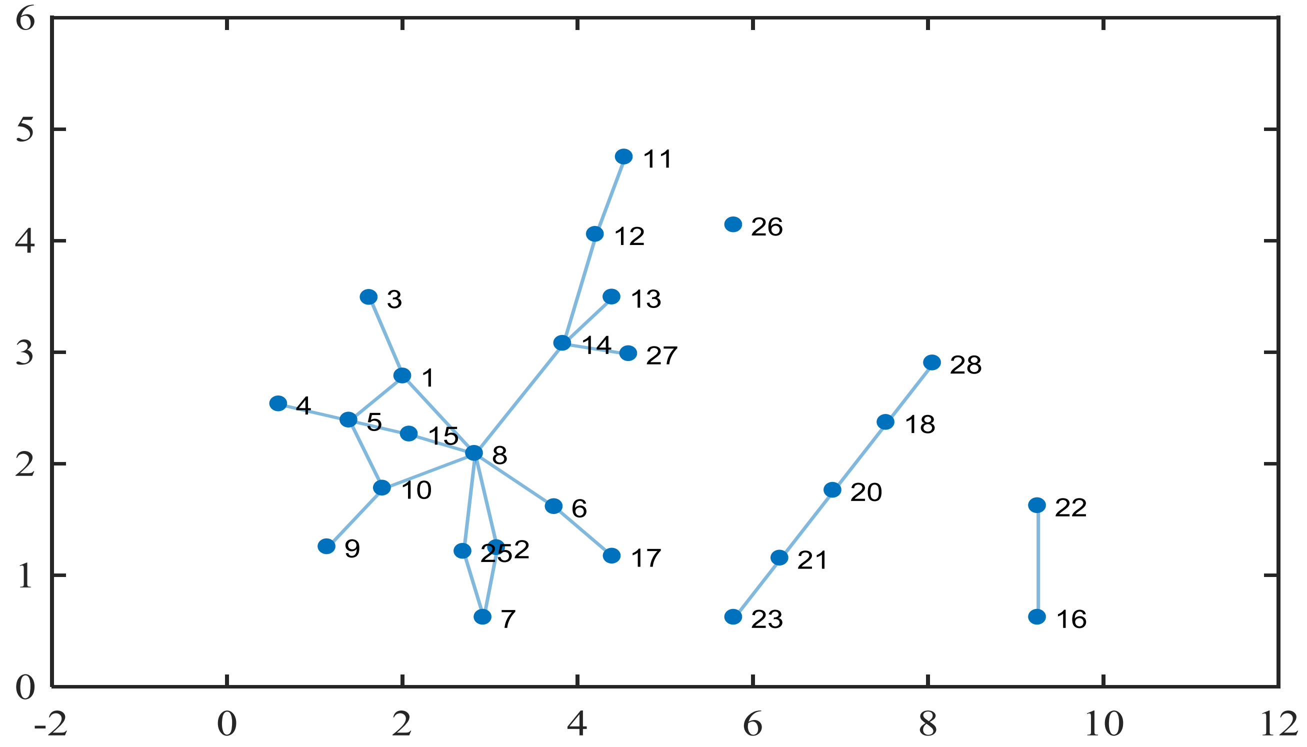

Fig. 10. Structural topology of Nigerian 28-bus system when both buses 19 and 24 are outaged as identified by BC method

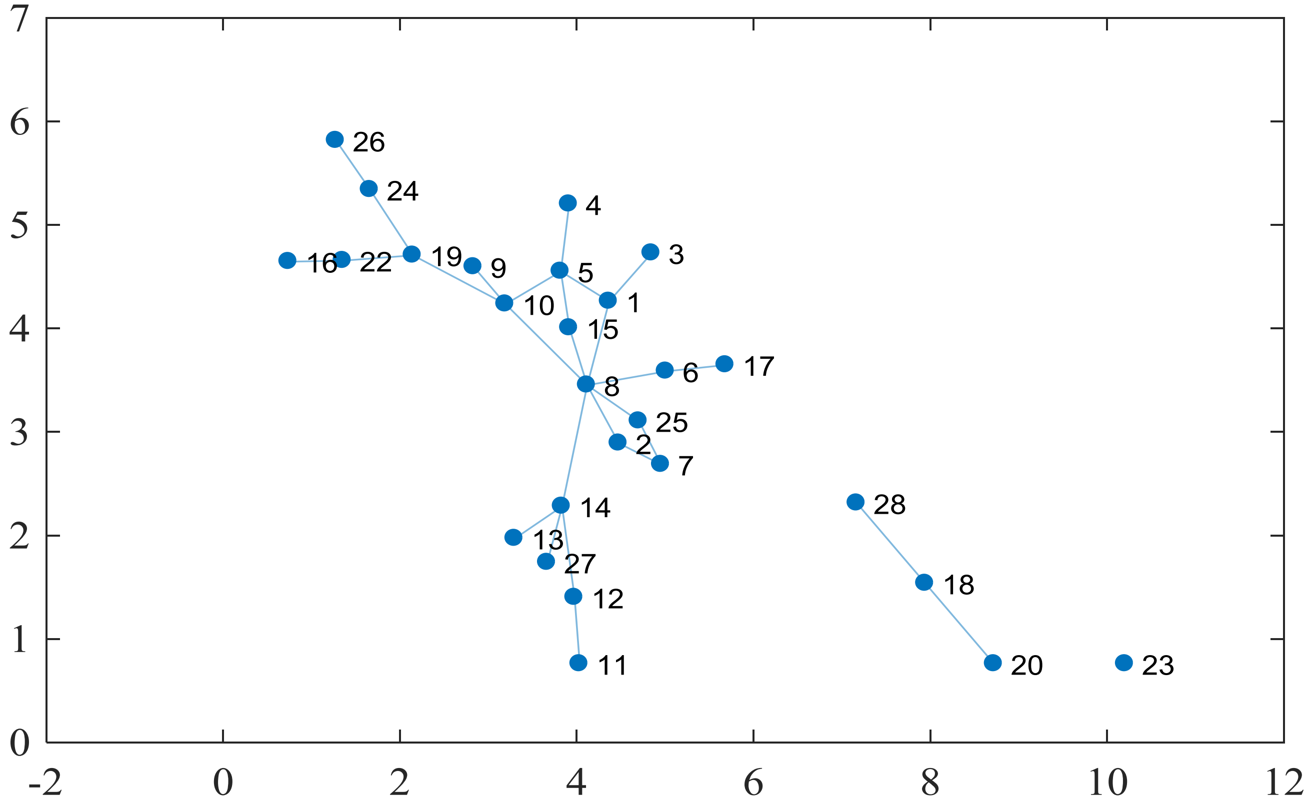

Fig. 11. Structural topology of Nigerian 28-bus system when bus 21 is outaged as identified by PRC method

With the deployment of EC method, the most influential load node identified is node 15 whose removal only affects itself with the associated CI of 96.4%. This implies that when load node 15 is outaged, the effect on the remaining network interconnections is highly insignificant and the stability of the integrity of the network is maintained. The structural topology for such network is shown in Fig. 13. As shown in Fig. 14, the removal of both the first two sets of influential load nodes (15 and 14 as identified by the EC method) in the network affected six load nodes to be outaged from the network with the CI value of 78.6% as presented in Table VIII.

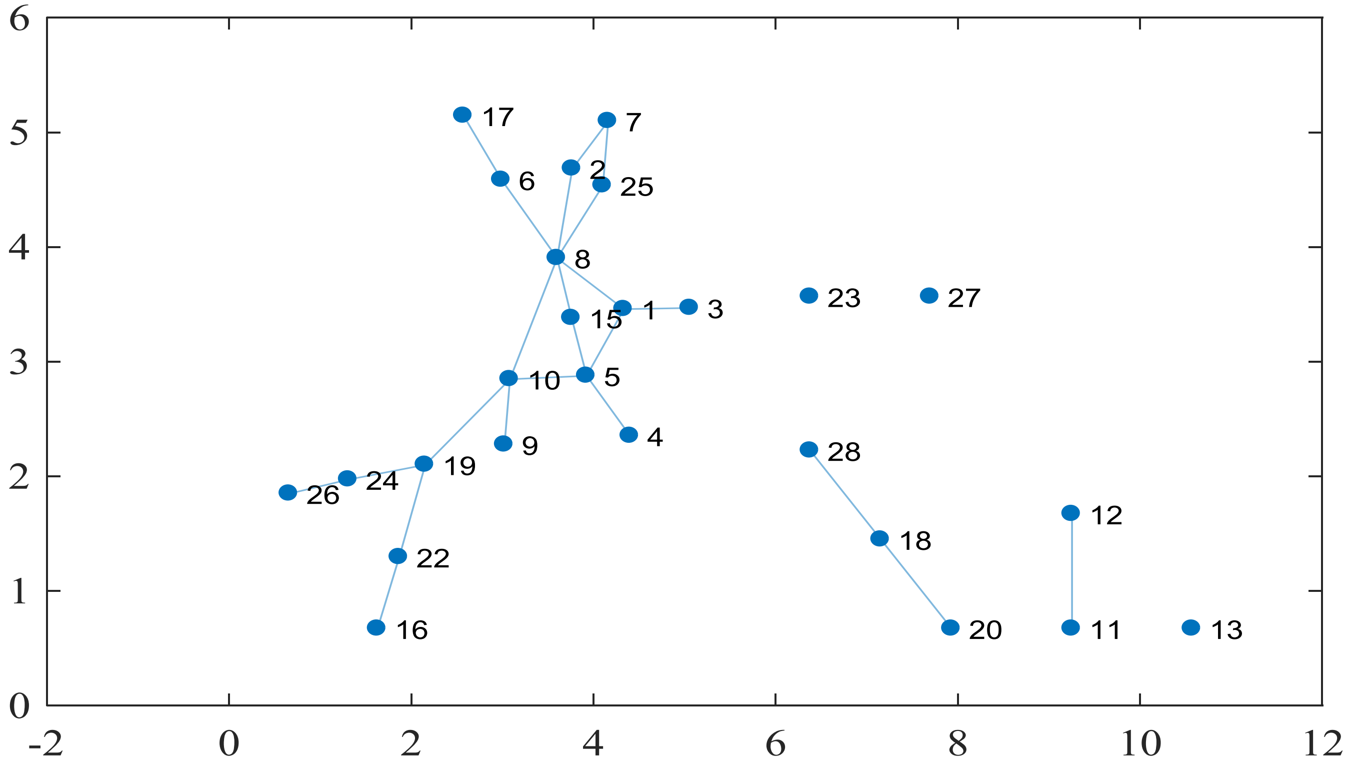

Fig. 12. Structural topology of Nigerian 28-bus system when both buses 14 and 21 are outaged as identified by PRC method

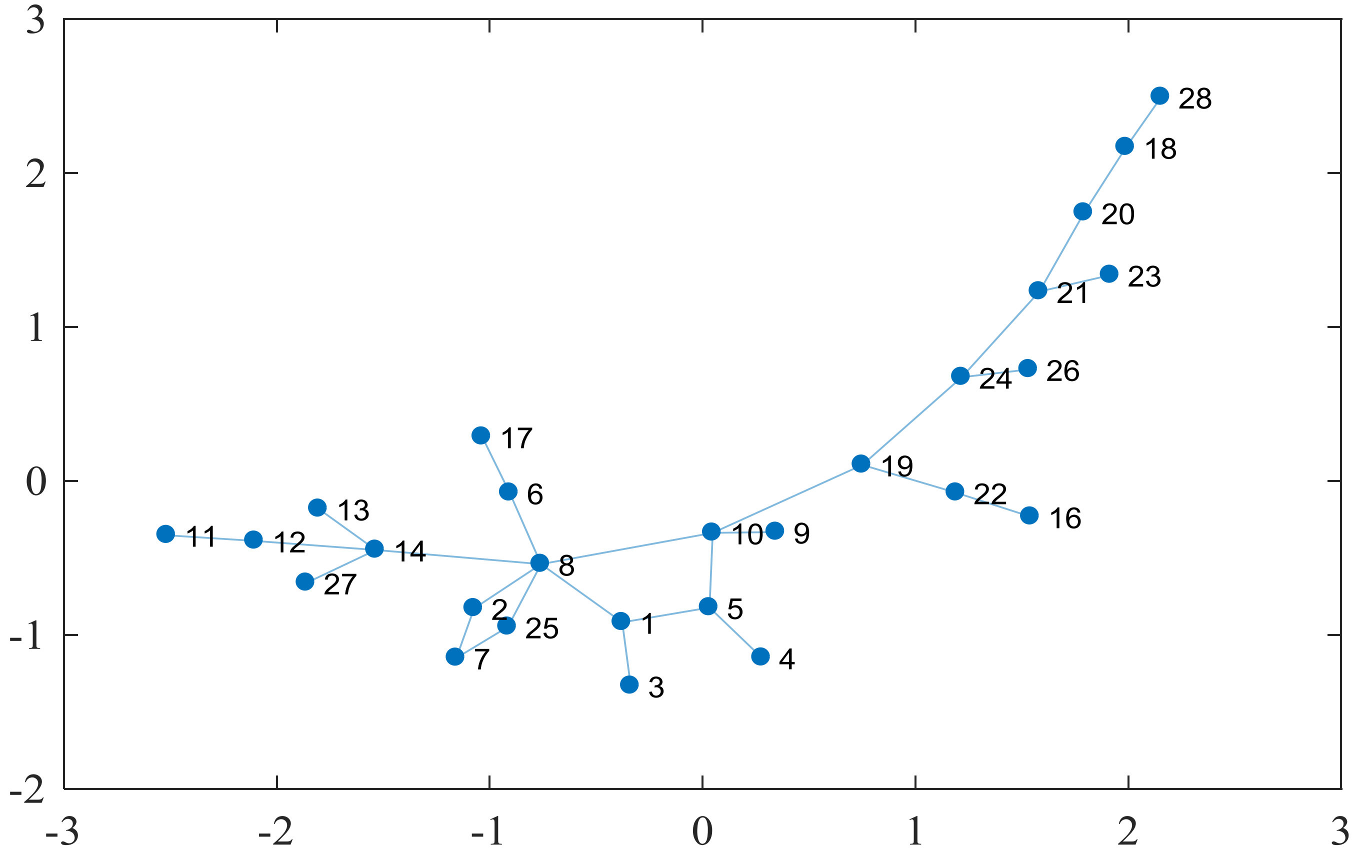

Fig. 13. Structural topology of Nigerian 28-bus system when bus 15 is outaged as identified by EC method

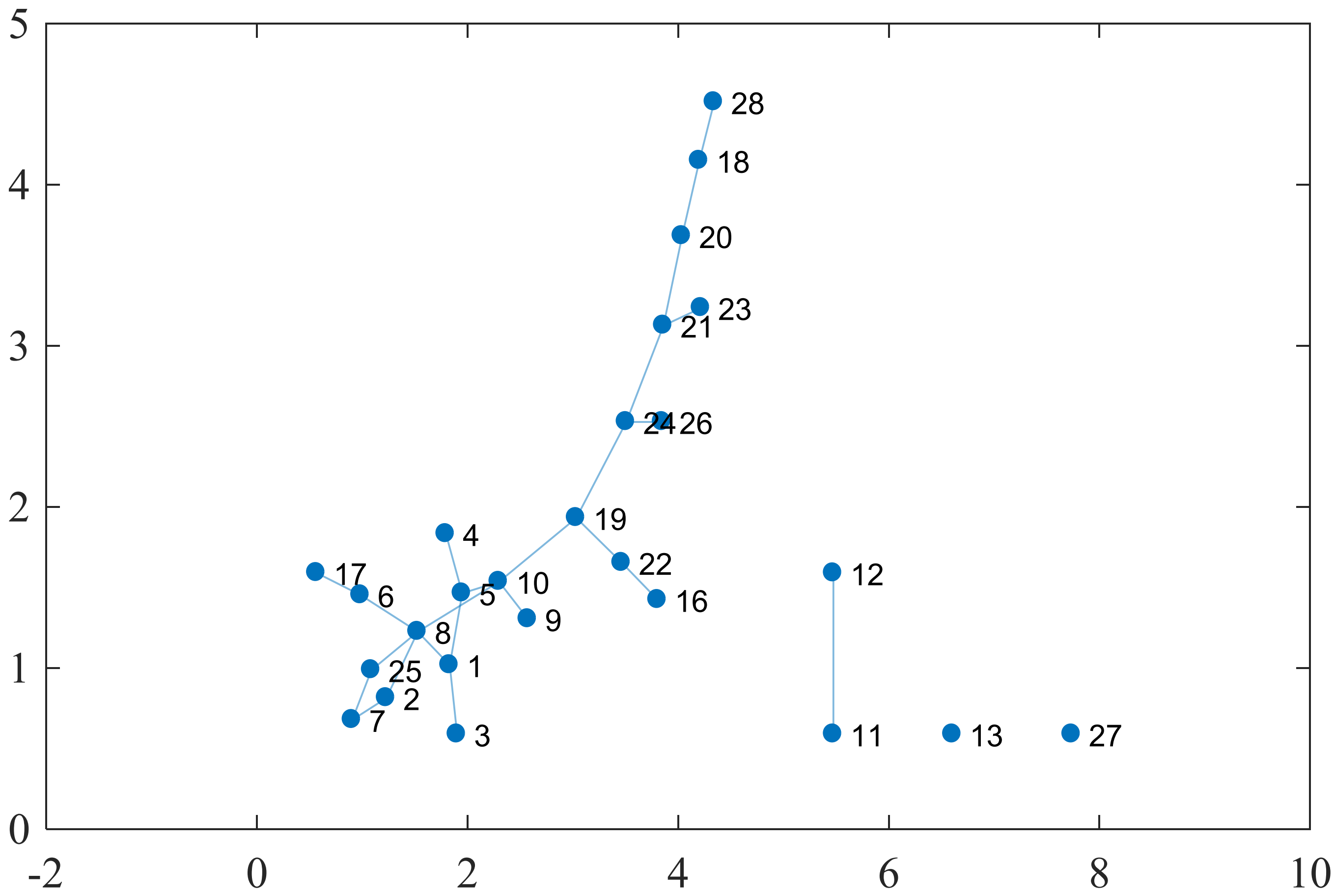

Fig. 14. Structural topology of Nigerian 28-bus system when both buses 15 and 14 are outaged as identified by EC method

It can therefore be inferred from the structural risk analysis results obtained that the most influential node which could also be referred to as the most critical node in the network is load node 19 as identified by NS, CC, DC and BC methods. This is because the impact of removing this bus 19 from the network is very high as seen from its associated CI value presented in Table VIII. It can also be seen that the load node whose outage presents the highest CI value is node 15 as identified by the EC method. This implies that removal of load node 15 from the network does not have any significant influence on the stability of the network. Also, based on this analysis, it can be seen that both PRC and EC methods are not suitable for identifying the most influential load nodes within a large undirected power system network. As a result of high degree of importance of the identified load node 19, there is therefore the need for this node to be provided with adequate protection in order to protect its outage which could be detrimental to the entire network. This will ensure maximum stability of the network and hence avoidance of frequent blackout of the system.

2) Statistical evaluation of network robustness considering single-node and multi-node outages

In this section we evaluated the basic statistical topological properties of the network in order to analyse the robustness of the network. This assessment is highly important so as to further validate the superiority of the results obtained using the proposed node strength method over other existing graph-theoretical-based methods. In this analysis, three scenarios are considered; base case (no outage) scenario, single-node case and multi-node outage scenario. We evaluated the global efficiency for the three cases and obtained the following results: 0.3298 was obtained for the no-outage case, 0.2557 was obtained when node 14 failed, 0.3264 was obtained when node 15 failed, 0.2802 was obtained when node 21 failed, and 0.2154 was obtained when node 19 failed. It can be inferred from these results that the least average global efficiency of 0.2154 is associated with the removal of node 19 from the network. This significant reduction in the network average global efficiency from 0.3264 when all the nodes are intact to 0.2154 when node 19 failed implies that the network is characterized by a low robustness when node 19 failed.

For the sake of comparison, the results for both the base case and multi-node contingency are presented in Table IX. Table IX presents the results obtained for the base case scenario when all the nodes within the network are intact as well as the multi-node scenario when two nodes are simultaneously removed from the network. It can be seen from the results that for the base case scenario, the network characteristic path length is 4.1164, the global and local efficiency are 0.3298 and 0.8520 respectively while the clustering coefficient, which represents the extent to which the network nodes are clustered, gives 0.7304.

These results show that with no contingency situation within the network, the average path length or distance between the network nodes is 4.1164 while it is found to be significantly increased when two nodes are outaged from the network. For example, when nodes 14 and 15 failed, the path length increased from 4.1164 to 12.1357, when nodes 14 and 21 failed, it increased to 14.0471, when nodes 19 and 24 failed, it turned out to be 12.3796 and when nodes 14 and 19 failed, the average characteristic path length for the network increased to 14.5675.

TABLE IX. MULTI-NODE CONTINGENCY SCENARIO

	Base Case	Outaged nodes			
		14, 15	14, 21	19, 24	14, 19
L_c	4.1164	12.1357	14.0471	12.3796	14.5675
E_g	0.3298	0.1504	0.1059	0.1272	0.0618
C_c	0.7304	0.3445	0.2377	0.3625	0.1397
E_l	0.8520	0.4761	0.4689	0.4786	0.1712

Based on these results, it can be seen that the network has the highest average characteristic path length when nodes 14 and 19 are simultaneously removed from the network. The implication of this is that the rate at which power flows through the network is significantly reduced as the path length increased. As such, the network robustness decreased with an increase in the path length. Hence, the network will experience a relatively low robustness when nodes 14 and 19 failed. Since the clustering coefficient of a network ranges between 1 and 0, it can therefore be said that the network with a clustering coefficient of 0.7304, with no contingency as shown in Table IX, is relatively high. However, it can be seen that when two network nodes failed simultaneously, the clustering coefficient of the network dropped drastically. It can be seen that the outage nodes 14 and 19 whose average clustering coefficient is 0.1397 presented the least average clustering coefficient. This significant decrease from the base case to the contingency case means that there is a significant decrease in the network robustness which presents its worst case when nodes 14 and 19 simultaneously failed. Considering the local and global efficiencies of the network, a significant difference can be seen by comparing the results when all the nodes are intact and the results when two nodes are removed from the network simultaneously. It can be seen from Table IX that the least local and global efficiency of 0.1712 and 0.0618 respectively are obtained when nodes 14 and 19 failed. Since these topological properties measure how efficient and how robust a network is, it can be said that the network is less efficient with the least robustness when nodes 14 and 19 are simultaneously removed from the network.

Based on the foregoing, it can be seen that the most critical node whose failure could lead to the disintegration of the network is node 19 as revealed by the network average global efficiency analyses. Furthermore, it can be shown that the worst-case contingency scenario which could lead to total collapse of the network is experienced when both buses 14 and 19 are outaged from the network simultaneously. These results corroborated that obtained in the previous sections using rank correlation as well as connectivity index approaches.

The comparison of the associated time complexities for the proposed approach and the five graph-theory-based methods presented are presented in Table X. It can be seen that the time complexity for all the methods increases as the network size increases. For example, considering the 5-bus system, it can be seen that the least time complexity of 0.04 seconds is associated with the DC method followed by he proposed NS method with a time complexity of 0.08 seconds and then followed by the BC method with 0.09 seconds while the highest time complexity of 0.37 seconds is associated with the CC method. Considering 14-bus system, it can be seen that the least time complexity of 0.05 seconds is associated with the DC method followed by the proposed NS method with the time complexity of 0.09 seconds while the EC method has the highest time complexity of 0.48 seconds. The least value of time complexity associated with the DC method could be attributed to the fact that the DC method has the simplest mathematical formulations and very easy to calculate amongst all graph-theory-based methods.

However, it can be seen that as the network size increases to 28-bus, the least time complexity of 0.11 seconds is associated with the NS method followed by the DC method with the time complexity of 0.18 seconds while the CC method has the highest time cost of 4.21 seconds. This shows that the proposed NS method has the least time cost when compared with the existing graph-theoretical-based methods. This strong capability of the NS method to swiftly identify the most critical node within the network could be traced to the fact that it fully explores the sparsity property of the network under consideration and as such saves a lot of computational time and computer memory space required for storage since only the non-zero elements are to be stored. For the three networks considered in this paper, by exploring the sparsity method, the number of non-zero elements to be stored in the computer memory in for the computation of the RED using the 5-bus, 14-bus and 28-bus systems are depicted in Figs. 15, 16 and 17 respectively.

These results are also presented in Table XI. It can be seen that the percentage of the non-zero elements to be stored decreases with the network size. For example, in a 5-bus system, only 19 non-zero elements are stored in the computer memory out of the total 25 elements which gives a sparsity index of 0.76 (76%). Also, in the 14-bus system, out of the total 196 elements, only 54 non-zero elements are stored in the computer memory with a sparsity index of 0.276 (27.6%) while for the 28-bus network, only 90 non-zero elements out of the total 784 elements are stored in the computer memory with a sparsity index of 0.76 (11.5%).

TABLE X. COMPARISON OF TIME COMPLEXITY FOR THE APPROACHES

Network Size	Methods					
	DC	EC	CC	PRC	BC	NS
5-bus	0.04	0.24	0.37	0.14	0.09	0.08
14-bus	0.05	0.48	0.36	0.10	0.11	0.09
28-bus	0.18	1.24	4.21	0.27	2.54	0.11

TABLE XI. SPACE COMPLEXITY ANALYSIS OF NS METHOD BASED ON SPARSITY CHARACTERISTICS

Network size	Number of nodes	Number of lines	Size of RED	Proportion of non-zero elements	Network Sparsity index
5-bus	5	7	5×5	19/25	0.760
14-bus	14	20	14×14	54/196	0.276
28-bus	28	31	28×28	90/784	0.115

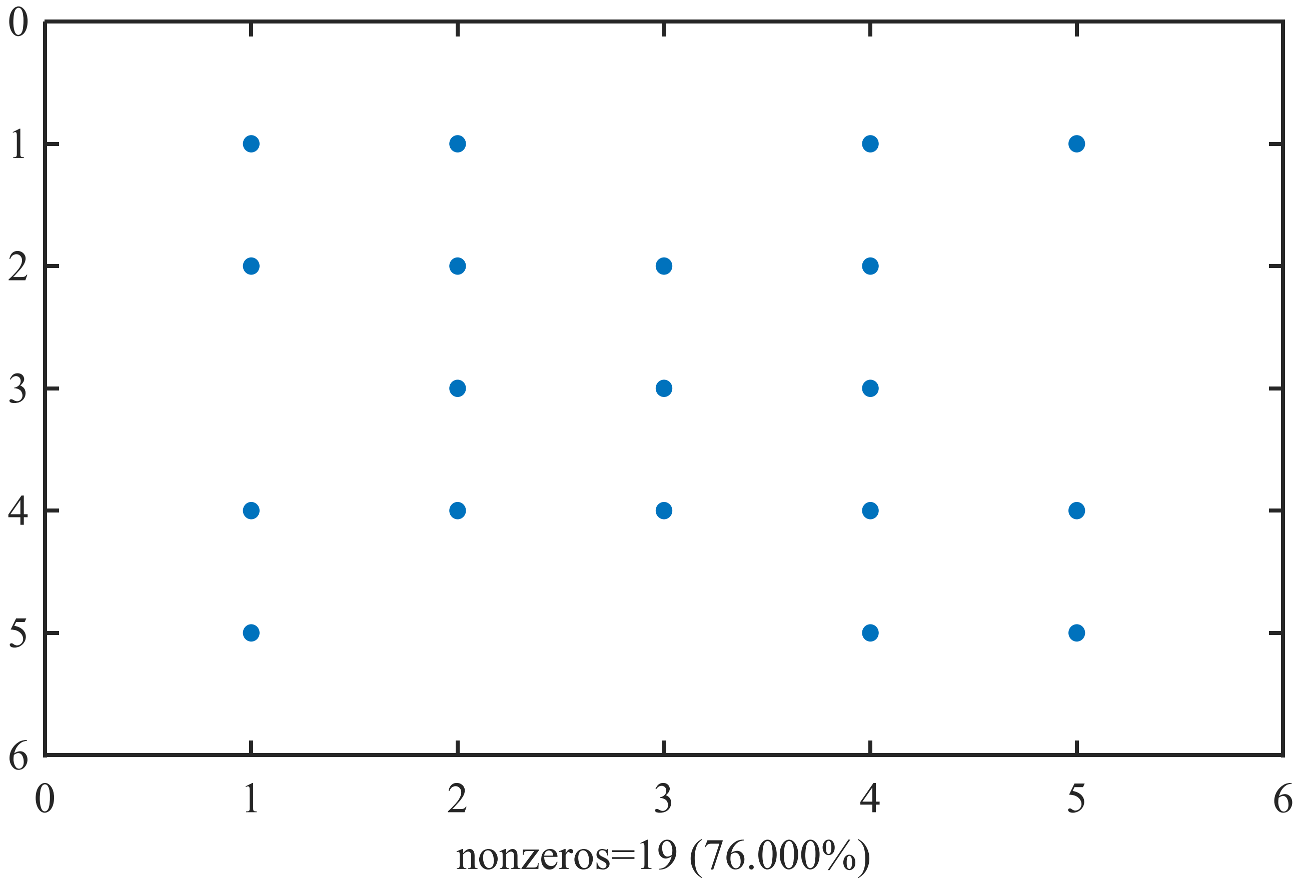

Fig. 15. Sparsity property of RED matrix in the IEEE 5-bus system

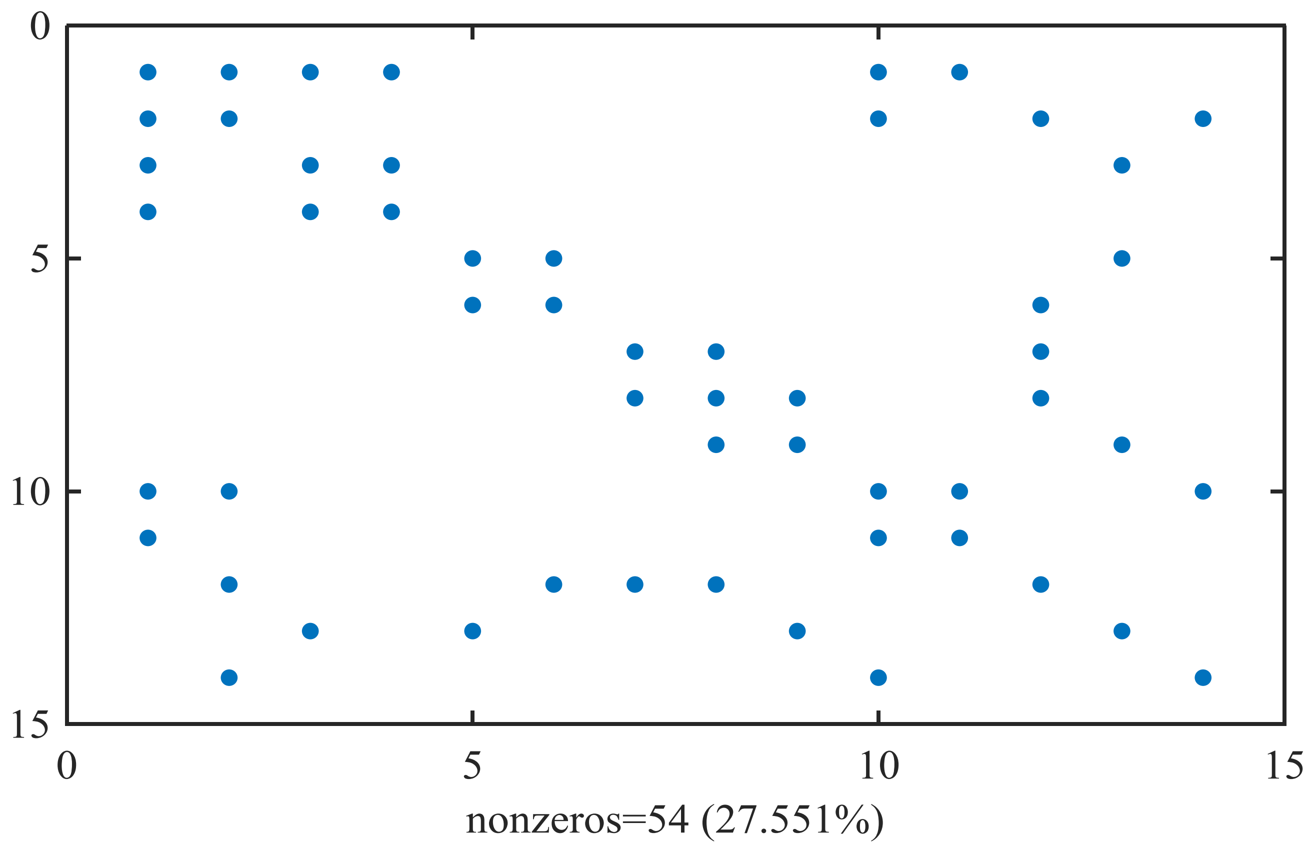

Fig. 16. Sparsity characteristic of RED matrix in the IEEE 14-bus system

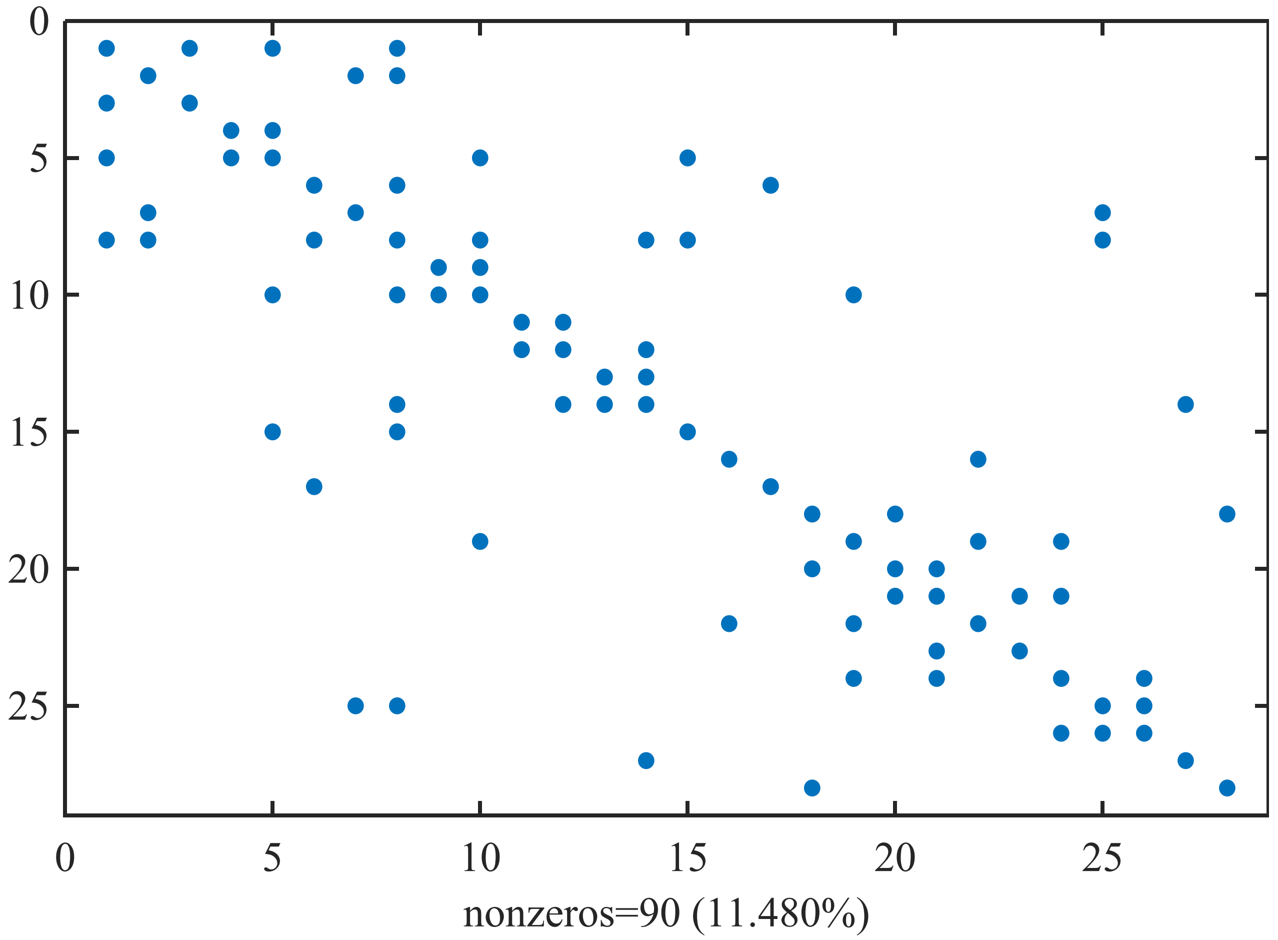

Fig. 17. Sparse symmetric matrix for the practical Nigerian 28-bus system

IV CONCLUSION

Identification of a set of identification of a set of influential as well as the most influential buses within power systems has been presented in this paper. A NSCT-based NS framework, which is suitable for swift identification of sets of influential nodes is suggested in this paper. Different traditional CNT-based methods for detecting the sets of network influential nodes are also presented for the sake of comparison. The results obtained using the NS and CNT-based methods were compared. The results showed that all the methods were resolute in quick identification of important load nodes that are susceptible to voltage instability during critical outages in a relatively small power system. However, it is found that the accuracy of the CNT-based methods is not guaranteed as the network size becomes increasingly large. Consequently, the application of CNT-based methods may not be suitable for identifying a set of influential nodes in a large-sized power practical system. It can, therefore, be inferred based on the results that the NS scheme is a better alternative approach for quick identification of most influential load nodes. This method offers some benefits over other methods, for example, it does not involve complex mathematical formulations (non-linear equations) and hence, the convergence issues faced by the iterative-based methods are completely eliminated thereby offering full advantage of space and time complexities during simulations. Consequently, this method is better positioned to serve as an efficacy, proficient and important tool for practical power system analysis, planning and operations.

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