Low-Complexity GSM Detection based on Iterative Zero Forcing and Minimum Mean Square Error

Xinhe Zhang, Siyu Zhang

Abstract—**As an extension of spatial modulation (SM) technology, generalized spatial modulation (GSM) systems can achieve higher data transmission rates with limited spectrum resources due to activate multiple transmit antennas to send data in each time slot. Simultaneously activating multiple transmit antennas increase the difficulty of signal demodulation at the receiver in GSM systems. To reduce the high computational complexity of the maximum likelihood (ML) detection algorithm in GSM systems, a low-complexity GSM detection algorithm is proposed, which combines the iterative zero forcing (IZF) algorithm and the minimum mean square error (MMSE) equalization processing detection algorithm. Firstly, the zero forcing (ZF) algorithm is used to iteratively detect all transmit antenna combinations to obtain multiple unlikely transmit antenna indices. Then the number of search antenna combinations is reduced by removing these antenna combinations. Next, MMSE equalization detection and symbol quantification on the new search set are performed. Finally, the ML algorithm is used to estimate the transmit antenna combination and transmit symbols. The effectiveness of the proposed algorithm is verified by simulation experiments. The simulation results show that the computational complexity of the proposed algorithm is much lower than that of the ML algorithm, and the bit error rate (BER) performance is close to that of the ML algorithm.**

Index Terms—**Generalized spatial modulation (GSM), maximum likelihood (ML), iterative zero forcing (IZF), transmit antenna combination (TAC), bit error rate (BER), computational complexity.**

I. INTRODUCTION

With the advent of the 5G era [1]-[2], multiple-input multiple-output (MIMO) antenna technology [3]-[4] has become one of the research focuses in 5G technology. Spatial modulation (SM) [5]-[8] is a communication technology that utilizes multiple antennas and spatial diversity, and belongs to a new type of MIMO technology. It transmits information by activated antennas, where the activation state of each antenna represents the transmitted information bits, maintaining information in the amplitude and spatial position of the signal. However, traditional SM technology only activates one antenna in each time slot for data transmission, resulting in lower transmission rate and throughput than conventional MIMO systems.

To improve the transmission rate, generalized spatial modulation (GSM) technology is proposed in [9]-[11], which is an extension of the SM technology. Its main principle is antenna diversity. It introduces the concept of multi-dimensional spatial modulation based on traditional spatial modulation, allowing information to be transmitted through multiple antennas in different combinations. GSM transmits information by selecting the combinations of activated antennas. Multiple transmit antennas can be activated in each time slot, and each antenna can carry different information bits, resulting in higher transmission rates and capacity. GSM technology can achieve higher data transmission rates under limited spectrum resources, which is crucial to meet the increasing demand for data. Since both the transmit antennas and constellation symbols carry information, antenna combination detection and symbol detection are required to estimate the information transmitted in GSM systems.

The signal demodulation algorithm at the receiver of the GSM system includes the optimal detection algorithm and the sub-optimal detection algorithm. The optimal detection algorithm is the maximum likelihood (ML) detection algorithm [12], which can achieve the lowest BER performance through exhaustive search. The high system capacity can be achieved for large-scale GSM systems, but as the system size increases, the complexity of the ML detection increases exponentially, resulting in an increased demand for computational resources. In recent years, some suboptimal detection algorithms have been proposed. The zero forcing (ZF) detection algorithm proposed in [13] is a demodulation algorithm with low computational complexity. It eliminates mutual interference between signals through pseudo-inverse matrix operations to achieve signal separation and demodulation. However, it may cause the amplification of additive Gaussian white noise at low signal-to-noise ratio, resulting in the decrease of demodulation performance. In [14], an iterative zero forcing (IZF) detection algorithm was proposed to reduce the number of detected antenna combinations through multiple iterations, which has better BER performance compared to the ZF algorithm, and also reduces the computational complexity compared to ML detection. In [15], an ordered block minimum mean square error (OB-MMSE) detection algorithm for GSM systems was proposed, which has better performance in suppressing multipath fading and noise interference. It sorts the possible antenna combinations using the pseudo-inverse operation, and then uses MMSE for each combination to achieve

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optimal symbol estimation, which can provide high demodulation performance in complex channel environments. An improved sphere decoding (SD) detection algorithm proposed in [16] reduces the exhaustive tree search for leaf nodes and limits the branches to those with valid antenna combinations, effectively reducing the detection complexity compared to the ML algorithm. A low-complexity detection algorithm by dividing transmit antennas into some groups was proposed in [17]. In this detector, the transmit antennas are divided into some groups based on the number of activated antennas at the transmitter. Only one antenna is activated in each group for transmitting modulation symbols, and the corresponding packet serial detection are performed at the receiver. In addition, detection algorithm proposed in [18] reduce the overall complexity by reordering antennas, and detection algorithm [19] reduce the computational complexity by reducing the number of transmit antenna combinations detected at the receiver.

To approximate the detection performance of the ML algorithm and reduce the complexity, a suboptimal detection algorithm IZF-MMSE was proposed in this paper. This detector reduces the computational complexity by reducing the number of antenna combinations. For the estimation of the modulation symbols, the HLML algorithm [20] for the M-QAM constellation and the LCML algorithm [21] for the M-PSK constellation are adopted. The computational complexity of the modulation symbols is independent of the modulation order, thus greatly reducing the complexity of symbol detection. The main contributions of this paper are as follows.

1) To reduce the computational complexity, a finite iteration of the zero forcing detection algorithm is carried out to shrink the search range of the antenna combinations.

2) To approach the optimal detection performance, the MMSE algorithm is used to estimate the transmitted antenna combination, and the hard-limited maximum likelihood algorithm is used to estimate the transmitted symbols.

The rest of the paper is organized as follows. The GSM system model and related algorithms are introduced in Section II. The proposed suboptimal detection algorithm is described in Section III. In Section IV, the experiment simulation results and computational complexity analysis of the proposed algorithm are presented. Finally, we summarize the paper in Section V.

Notation: Boldface uppercase letters denote matrices, boldface lowercase letters denote vectors. $(\cdot)^{-1}$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{\dagger}$ represent the inverse, transpose, Conjugate transpose and pseudo-inverse of a vector or a matrix, respectively. $\lfloor\cdot\rfloor$ denotes the floor operation. $\|\cdot\|_F$ is the Frobenius norm of a vector or a matrix. $|\cdot|$ stands for the absolute value of a complex number or the cardinality of a given set. $\Re(\cdot)$ and $\Im(\cdot)$ are the real- and imaginary-parts of a complex-value variable. $round(\cdot)$ indicates the operation of rounding a real number to the nearest integer. $\mathrm{mod}(\cdot,\cdot)$ is the modulo operation. $\min(\cdot)$ and $\max(\cdot)$ represent taking the minimum and maximum values, respectively. $C_n^k = \frac{n!}{k!(n-k)!}$ denotes the binomial coefficient. $\mathbb{C}$ represents the field of complex numbers. $\Psi = \mathrm{A}\setminus\mathrm{B}$ means that set Ψ is the complement of subset B of A if set B is a true subset of set A, i.e. $\mathrm{B}\subset\mathrm{A}$.

II. System Model

Consider a GSM system equipped with N_t transmit antennas and N_r receive antennas. The system only activates N_p transmit antennas in each time slot, and each activated antenna transmits different M-QAM or M-PSK modulation symbols. Therefore, there are a total of C_{Nt}^{Np} possible transmit antenna combinations (TACs). But among these TACs, only $N_c = 2^{\left\lfloor \log_2 C_{N_t}^{N_p} \right\rfloor}$ TACs are chosen to convey information. The binary bitstream sent in each time slot is divided into two parts. One part is used to determine which group of N_c TACs is used for data transmission, with a required information bit length of $L_1 = \left\lfloor \log_2 C_{N_t}^{N_p} \right\rfloor$, and the other part is used to map the transmit modulation symbol vector $\boldsymbol{s} = [s_1, s_2, \cdots s_{N_p}]^T$, with the information bit length of $L_2 = N_p \log_2 M$, where the modulation symbols are $s_1, s_2, \cdots, s_{N_p} \in S$. M and S denote the modulation order and the set of modulation symbols, respectively. As a result, the total number of information bits transmitted per time slot is $L = \left\lfloor \log_2 C_{N_t}^{N_p} \right\rfloor + N_p \log_2 M$. The modulation symbol vector is transmitted through the channel gain matrix $\boldsymbol{H}$ of $N_r \times N_t$ dimension, where each element $h_{i,j}$ of $\boldsymbol{H}$ represents the channel gain between the i-th transmit antenna and the j-th receive antenna, and $\boldsymbol{H}$ follows a complex Gaussian distribution with a mean of 0 and a variance of 1. The model of the received signal vector can be represented as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{C}^{N_t \times 1}$ represents the transmitted vector, $\boldsymbol{y} \in \mathbb{C}^{N_r \times 1}$ represents the received vector, and $\boldsymbol{n} \in \mathbb{C}^{N_r \times 1}$ represents the additive noise vector. Each element of the additive noise vector follows a complex Gaussian distribution with a mean of 0 and a variance of σ^2. The transmission signal $\boldsymbol{x}$ can be expressed as:

$$\boldsymbol{x} = [0, \cdots, 0, s_1, 0, \cdots, 0, s_2, 0 \cdots, 0, s_{N_p}, 0, \cdots]^T \tag{2}$$

The location of the non-zero element in (2) represents the index of the activated antenna. The non-zero elements represent the transmit symbol in each time slot.

Assuming the t-th antenna combination is used to transmit modulation symbols, the received signal vector in (1) can be equivalent to

$$\boldsymbol{y} = \sum_{t=i_1}^{i_{N_p}} \boldsymbol{h}_t \boldsymbol{s}_t + \boldsymbol{n} = \boldsymbol{H}_{\mathrm{I}} \boldsymbol{s} + \boldsymbol{n} \tag{3}$$

where $t \in \left\{ i_1, i_2, \cdots, i_{N_p} \right\}$, and $i_1, i_2, \cdots, i_{N_p}$ denotes the antenna index of the N_p activated antennas in the i-th antenna combination, $i \in \{1, 2, \cdots, N_C\}$. $\boldsymbol{h}_t$ is the t-th column of the channel matrix $\boldsymbol{H}$, $\boldsymbol{H}_I = \left(h_{i_1}, h_{i_2}, \cdots, h_{i_{N_p}} \right)$ is a submatrix of $\boldsymbol{H}$ with dimension $N_r \times N_p$, and I is the corresponding set of activated antennas. For the GSM system, the ML detector jointly detects the set of all antenna combinations and modulation symbols by exhaustively searching all possible transmit signal vectors, then it can be denoted as [22]

$$\left(\hat{I},\hat{s}\right) = \arg\min_{I\in\mathbb{Q},s\in\mathbb{S}} \| \boldsymbol{y} - \boldsymbol{H}_I \boldsymbol{s} \|_F^2 \tag{4}$$

where $\mathbb{Q} = \left\{I_1, I_2, \cdots, I_{N_c}\right\}$, I_i represents the set of N_p active antennas in the i-th antenna combination, $i \in \{1, \cdots, N_C\}$. $\mathbb{S} = S^{N_p \times 1}$ denotes the set of N_p-dimensional modulation symbol vectors. Due to the high computational complexity of the ML detection algorithm caused by jointly searching for each antenna combination and constellation point, some suboptimal low-complexity detection algorithms are proposed. The ZF detection is represented as follows:

$$\hat{\boldsymbol{x}}_{ZF} = \boldsymbol{H}^{\dagger}\boldsymbol{y} = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H\boldsymbol{y} \tag{5}$$

where $\boldsymbol{H}^{\dagger}$ is the pseudo-inverse of $\boldsymbol{H}$ and $\boldsymbol{H}^H$ is the Conjugate transpose of $\boldsymbol{H}$. In theory, the receive antennas provide diversity gain, thus the more receive antennas a GSM system has, the better its BER performance. However, as the number of receive antennas increases, the cost of the device also increases. In this case, the BER performance can be improved by reducing the number of transmit antennas. Therefore, the IZF algorithm is proposed.

The main idea of the IZF algorithm is: 1) Use the ZF algorithm to obtain the estimated transmitting symbol $\hat{\boldsymbol{x}}$ by $\hat{\boldsymbol{x}} = \boldsymbol{H}^{\dagger}\boldsymbol{y}$. If the dimension of $\hat{\boldsymbol{x}}$ is greater than N_p, proceed to the next step; 2) Find the minimum value in the estimated signal vector $\hat{\boldsymbol{x}}$, which is the least likely antenna to be activated, remove it from the antenna sequence, and then remove the antenna index from channel matrix $\boldsymbol{H}$, substitute the updated channel matrix into step 1) for iteration until the $\hat{\boldsymbol{x}}$ dimensions in step 1) are equal to N_p; 3) Implement a soft constellation decomposition algorithm for Euclidean distance through a N_p-dimensional vector $\hat{\boldsymbol{x}}$.

III. IZF-MMSE Detection Algorithm

In this section, a low-complexity detection algorithm IZF-MMSE based on IZF and MMSE algorithms is proposed. This detector adopts the iterative idea of steps 1) and 2) of the IZF algorithm, which can greatly reduce the computational complexity of the ML algorithm.

First, the ZF detection algorithm is used for iterative detection; in each iteration, a least likely transmit antenna index is obtained. Second, k least probable transmit antenna indexes are obtained by multiple iterations; in each iteration, the channel matrix is updated to compute the current least probable transmit antenna indexes, and the antenna combinations containing these antenna indexes are denoted as the set B. Then, set B is removed from all antenna combinations, i.e., set A; all combinations containing k transmit antenna indices are found, then removed from set A, and the antenna combinations left construct a new search set, denoted as set Ψ; the elements in set Ψ are then detected by MMSE equalization processing. Finally, the processed results are subjected to ML detection to find the minimum Euclidean distance $\delta_{\min}$, and the corresponding transmit antenna combination and transmission symbol for $\delta_{\min}$ are used as the final output result. The specific steps of this detection algorithm are as follows.

Step 1: According to ZF detection algorithm, the preprocess of the pseudo-inverse of the channel matrix $\boldsymbol{H}$ on the received signal vector $\boldsymbol{y}$ is to obtain $\boldsymbol{Z} = \left[z_1, z_2, \cdots, z_{N_t}\right]^T$ as follows

$$\boldsymbol{Z} = \boldsymbol{H}^{\dagger}\boldsymbol{y} = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H\boldsymbol{y} \tag{6}$$

Then select the index of the element with the minimum absolute value in $\boldsymbol{Z}$

$$(\sim, t) = \arg\min_{t\in\{1,2,\cdots,N_t\}} |\boldsymbol{Z}| \tag{7}$$

Remove the antenna index corresponding to the component obtained above from the channel matrix, that is, remove the t-th column in $\boldsymbol{H}$ and the new matrix is denoted as $\tilde{\boldsymbol{H}}$.

Step 2: To reduce computational complexity, the channel matrix $\tilde{\boldsymbol{H}}$ can be further processed on the received vector $\boldsymbol{y}$, repeating step 1

$$\tilde{\boldsymbol{Z}} = \tilde{\boldsymbol{H}}^{\dagger}\boldsymbol{y} = (\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{H}})^{-1}\tilde{\boldsymbol{H}}^H\boldsymbol{y} \tag{8}$$

Select the component with the lowest absolute value in $\left|\tilde{\boldsymbol{Z}}\right|$ and remove it from the channel matrix $\tilde{\boldsymbol{H}}$. By analogy, after k iterations, k transmit antennas with a lower likelihood will be obtained. If only one antenna needs to be removed, this step can be omitted.

Step 3: After multiple iterations, the antenna combinations containing the obtained k transmit antenna indexes, i.e., set B, is eliminated from set A containing all transmit antenna combinations to obtain a new set Ψ, $\Psi = A \setminus B$. Each antenna combination in this set is then detected sequentially using the MMSE equalization

$$\tilde{\boldsymbol{s}}_j = \left(\left(\boldsymbol{H}_{I_j}\right)^H \boldsymbol{H}_{I_j} + \sigma^2\boldsymbol{I}\right)^{-1}\left(\boldsymbol{H}_{I_j}\right)^H \boldsymbol{y} \tag{9}$$

where $\tilde{s}_j$ is the detected symbol in the j-th antenna combination in set Ψ. I is the $N_p \times N_p$ identity matrix. $\boldsymbol{H}_{I_j}$ consists of the column vectors of $\boldsymbol{H}$ corresponding to the j-th TAC in set Ψ, with $N_r' \times N_p$. N_r' is the number of elements in set Ψ.

Step 4: The following section describes modulation symbol detection for the M-QAM and M-PSK constellations for $\tilde{s}_j$. For the M-QAM constellation, if the modulation signal S with $|S| = M$ is a square or rectangular constellation, the constellation can be regarded as the Cartesian product of two sets $S_1 = N_1 - PAM$ and $S_2 = N_2 - PAM$. N_1 and N_2 are powers of 2. $N_1 - PAM$ and $N_2 - PAM$ can described as:

$N_1 - PAM = \{-N_1+1, -N_1+3, \cdots, -1, 1, \cdots, N_1-3, N_1-1\}$,

$N_2 - PAM = \{-N_2+1, -N_2+3, \cdots, -1, 1, \cdots, N_2-3, N_2-1\}$.

The quantization symbol $\boldsymbol{s}_j$ for the j-th antenna combination in set Ψ can be represented as

$$\Re\left(\boldsymbol{s}_j\right) = \min\left[\max\left(2 * round\left[\frac{\Re\left(\tilde{\boldsymbol{s}}_j\right)+1}{2}\right] - 1, -N_1+1\right), N_1-1\right] \tag{10}$$

$$\Im(\boldsymbol{s}_j) = \min\left[\max\left(2*round\left[\frac{\Im(\tilde{\boldsymbol{s}}_j)+1}{2}\right]-1, -N_2+1\right), N_2-1\right] \tag{11}$$

$$\boldsymbol{s}_j = \Re(\boldsymbol{s}_j) + j\Im(\boldsymbol{s}_j) \tag{12}$$

For the M-PSK constellation, the amplitude of its constellation point is 1. θ is the angle between $\tilde{\boldsymbol{s}}_j$ and the positive real axis of the complex plane, then the quantization symbol $\boldsymbol{s}_j$ can be shown as follows

$$\varphi_j = \frac{\theta M}{2\pi} \tag{13}$$

$$\hat{\varphi}_j = \mathrm{mod}\left(round(\varphi_j), M\right) * \frac{2\pi}{M} \tag{14}$$

$$\boldsymbol{s}_j = \cos\hat{\varphi}_j + j\sin\hat{\varphi}_j \tag{15}$$

Step 5: Next, the ML detection algorithm is used to estimate the TAC and modulation symbols. The Euclidean distance between the received signal vector $\boldsymbol{y}$ and the j-th antenna combination and its corresponding transmit symbol $\boldsymbol{s}_j$ in set Ψ is calculated. As the final detection result, the TAC and transmit symbols with the minimum of the Euclidean distance from the vector $\boldsymbol{y}$ are chosen, that is,

$$\begin{cases} \hat{j} = \arg\min_{j\in\Psi} \| \boldsymbol{y} - \boldsymbol{H}_{I_j} s_j \|^2 \\ \hat{I} = I_{\hat{j}}, \hat{s} = s_{\hat{j}} \end{cases} \tag{16}$$

where $\hat{j}$ denotes the location of the minimum Euclidean distance obtained by the ML detection algorithm between the antenna combinations and their corresponding transmit symbols in set Ψ and the vector $\boldsymbol{y}$, whose corresponding TAC and transmit symbols $(\hat{I}, \hat{s})$ are the optimal and final outputs.

The proposed IZF-MMSE detection algorithm can be described in Table I.

TABLE I
PROPOSED IZF-MMSE DETECTION ALGORITHM

1: Input: $\boldsymbol{y}$, $\boldsymbol{H}$, N_t, N_r, N_P, k, σ;

2: Calculate the pseudo-inverse of the $\boldsymbol{H}$: $\boldsymbol{Z} = \boldsymbol{H}^{\dagger}\boldsymbol{y} = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H\boldsymbol{y}$;

3: Obtain the antenna index with the lowest absolute value in $\boldsymbol{Z}$:
$(\sim, t) = \arg\min_{t\in\{1,2,\cdots,N_t\}} |\boldsymbol{Z}|$;

4: Remove the t-th column from $\boldsymbol{H}$ and change it to $\tilde{\boldsymbol{H}}$;

5: Continue with k iterations and find set B of TACs corresponding to the antennas obtained at each iteration;

6: Remove set B from TACs, set A, to obtain set Ψ;

7: perform MMSE equalization detection on set Ψ:
$\tilde{s}_j = \left((\boldsymbol{H}_{I_j})^H \boldsymbol{H}_{I_j} + \sigma^2\boldsymbol{I}\right)^{-1} (\boldsymbol{H}_{I_j})^H \boldsymbol{y}$;

8: Obtain the quantization modulation symbol s_j by using equations (10)- (12) for M-QAM constellation modulation and equations (13)-(15) for M-PSK constellation modulation;

9: Use the ML detection algorithm to estimate the TAC and modulation symbols: $\begin{cases} \hat{j} = \arg\min_{j\in\Psi} \| \boldsymbol{y} - \boldsymbol{H}_{I_j} s_j \|^2 \\ \hat{I} = I_{\hat{j}}, \hat{s} = s_{\hat{j}} \end{cases}$

10: Output the TAC and transmit symbols $(\hat{I}, \hat{s})$.

IV. SIMULATION RESULTS AND COMPUTATIONAL COMPLEXITY ANALYSIS

To verify the BER performance and computational complexity of the proposed IZF-MMSE detection algorithm under different scenarios, a series of simulation experiments is conducted in this section under the assumption of ideal channel conditions. Firstly, the BER performance of the IZF-MMSE algorithm is compared with that of the ML algorithm under the condition that whether the activated antenna indexes occur with equal probability in all TACs or not. Then, the BER performance and computational complexity of the IZF-MMSE algorithm and the ML algorithm are simulated under different iterations in the M-QAM and M-PSK constellations, respectively.

A. The Impact of Antenna Index on the Algorithm

Assuming that the channel is a quasi-static flat Rayleigh fading and Additive white Gaussian noise follows a complex Gaussian distribution with a mean of 0 and a variance of 1. Figure 1 shows the BER performance comparison of the proposed algorithm for equal or unequal occurrences of transmit antenna indexes in all TACs and ML algorithm for a GSM system in two scenarios with 1) $N_t = 6$, $N_r = 10$, $N_p = 2$, $M = 16$ and 2) $N_t = 6$, $N_r = 10$, $N_p = 3$, $M = 16$. When $N_t = 6$, $N_p = 3$, $M = 16$, the combinations of transmit antennas with antenna indexes equiprobable and non-equiprobable occurrences are shown in Table II.

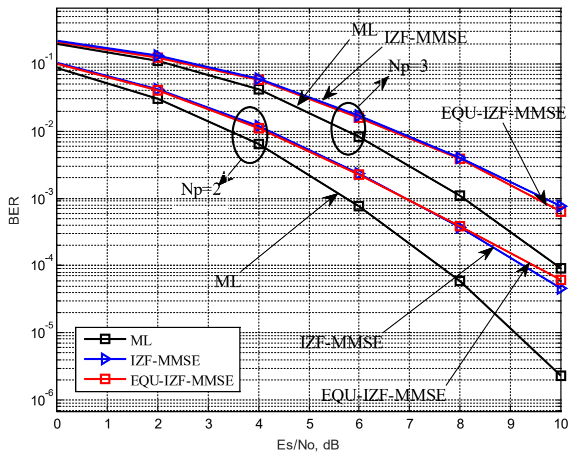

Fig. 1. The BER performance comparison between IZF-MMSE for equal or unequal occurrences of transmit antenna indexes and ML with $N_t = 6$, $N_r = 10$, $N_p = 2$ and $N_p = 3$ for 16-QAM constellation.

When each antenna index has an equal probability, the probability is 17%. When unequal probability occurs, the probability of each antenna index occurrence is 21%, 21%, 15%, 15%, and 15%, respectively.

In Figure 1, IZF-MMSE, EQU-IZF-MMSE represent the non-equal probability and equal probability, respectively. The horizontal axis of the simulation graph represents the signal-to-noise ratio (SNR), and the vertical axis represents the BER. The number of iterations k is set to 1. From Figure 1, it can be seen that the BER performance of the IZF-MMSE algorithm is close to the ML algorithm. Since the curves of the proposed algorithm for equal or unequal occurrences of transmit antenna indexes are almost the same, we can conclude that the distribution of antenna indexes in all TACs does not affect the BER performance.

TABLE II
ANTENNA COMBINATIONS WHEN ACTIVATING 3 ANTENNAS

Binary bit stream	Antenna combination (non-equal probability)	Antenna combination (equal probability)
0000	(1,2,3)	(1,2,5)
0001	(1,2,4)	(1,2,6)
0010	(1,2,5)	(1,3,4)
0011	(1,2,6)	(1,3,5)
0100	(1,3,4)	(1,3,6)
0101	(1,3,5)	(1,4,5)
0110	(1,3,6)	(1,4,6)
0111	(1,4,5)	(1,5,6)
1000	(1,4,6)	(2,3,4)
1001	(1,5,6)	(2,3,5)
1010	(2,3,4)	(2,3,6)
1011	(2,3,5)	(2,4,5)
1100	(2,3,6)	(2,4,6)
1101	(2,4,5)	(2,5,6)
1110	(2,4,6)	(3,4,6)
1111	(2,5,6)	(3,5,6)

B. BER performance with M-QAM modulation

Figure 2 shows the comparison of BER performance between the ML algorithm and the IZF-MMSE algorithm under 4-QAM modulation and $N_t = 10$, $N_r = 15$ simulation scenario for the four scenarios: the number of iterations k =1, 2, 3, and 4, respectively. It can be seen from Figure 2 that the BER performance of IZF-MMSE algorithm is close to that of the ML algorithm. The BER performance decreases as the number of iterations k increases.

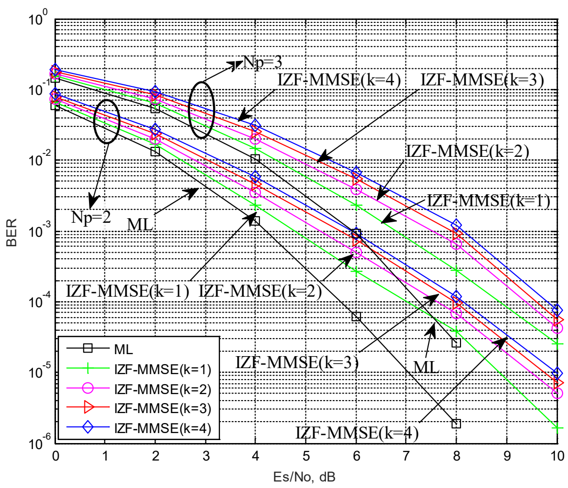

Fig. 2. The BER performance comparison between IZF-MMSE with the number of iterations k =1, 2, 3 and 4 and ML with $N_t = 10$, $N_r = 15$, $N_p = 2$ and $N_p = 3$ for 4-QAM constellation.

The IZF-MMSE algorithm can greatly reduce the number of antenna combinations detected at the receiver. Figure 3 depicts the average number of antenna combinations detected by the ML algorithm and the IZF-MMSE algorithm with the number of iterations k =1, 2, 3, and 4 at the receiver in the scenario of $N_t = 10$, $N_r = 15$, $N_p = 2$.

The average number of antenna combinations detected by the ML algorithm when $N_t = 10$, $N_r = 15$, $N_p = 2$ is 32, whereas it can be seen from Figure 3 that the IZF-MMSE algorithm detects about 25 antenna combinations at $k = 1$. The average number of antenna combinations detected decreases as the number of iterations increases. When $k = 4$, the number of detections is about 11, which accounts for 34% of the total number under different SNRs.

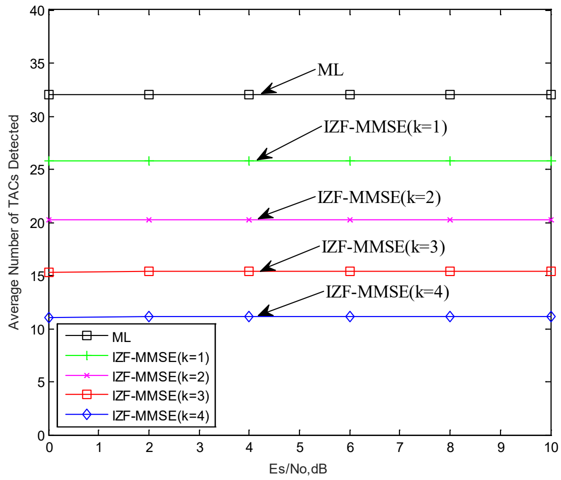

Fig. 3. Average number of TACs detected by IZF-MMSE with the number of iterations k =1, 2, 3 and 4 and ML with $N_t = 10$, $N_r = 15$, $N_p = 2$ for 4-QAM constellation.

C. BER performance with M-PSK modulation

Figure 4 depicts the BER performance of the ML algorithm and the IZF-MMSE algorithm under QPSK modulation and $N_t = 10$, $N_r = 15$ simulation scenario for the four scenarios: the number of iterations k =1, 2, 3 and 4 respectively. As shown in Figure 4, the BER performance of the proposed algorithm approaches the ML algorithm and decreases with the increasing iterations.

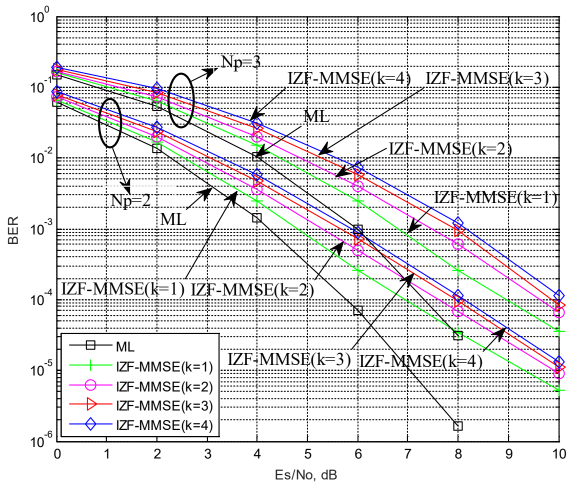

Fig. 4. The BER performance comparison between IZF-MMSE with the number of iterations k =1, 2, 3 and 4 and ML with $N_t = 10$, $N_r = 15$, $N_p = 2$ and $N_p = 3$ for QPSK constellation.

Figure 5 shows the average number of antenna combinations detected by the ML algorithm as well as the IZF-MMSE algorithm for the four different cases at the receiver in the scenario: $N_t = 10$, $N_r = 15$, $N_p = 3$. The average number of antenna combinations detected by the ML algorithm when $N_t = 10$, $N_p = 3$ is 64, while the IZF-MMSE algorithm detects about 44 antenna combinations at k =1, which accounts for 68% of the total number at different signal-to-noise ratios. As the number of iterations increases, the number of detections decreases. When k is 2, 3 and 4, the number of detections is 29, 18, and 10, respectively, which represent 45%, 28%, and 15% of all antenna combinations.

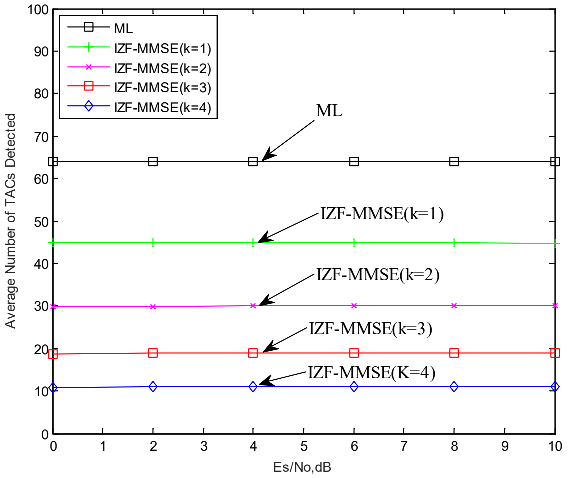

Fig. 5. Average number of TACs detected by IZF-MMSE with the number of iterations k =1, 2, 3 and 4 and ML with $N_t = 10$, $N_r = 15$, $N_p = 3$ for QPSK constellation.

D. Computational Complexity Analysis

In this subsection, we compare the computational complexity of the proposed IZF-MMSE algorithm and the ML algorithm. The computational complexity is determined by parameters N_t, N_r, N_p, M, L, and k, where L is the number of elements in set Ψ, that is, the remaining number of TACs, and k is the number of iterations. The computational complexity is defined as the total number of operations for the multiplication and division of real numbers.

The computational complexity of the IZF-MMSE algorithm consists of three components. The first part is the computation to obtain z_k during the iteration process (Lines 2-3 in TABLE I), whose complexity is $(2N_t^3 + 3N_t^2 - 5N_t)/6$ $+N_t^2 N_r + N_t N_r + N_t$ [11]. If the number of iterations is k, then repeat k times. The second part of the operation is to carry out the MMSE equalization process (Lines 7-8 in TABLE I). $\boldsymbol{H}^H\boldsymbol{H}$ requires $(2N_p^3 + 3N_p^2 - 5N_p)/6$ operations [23], multiplication with matrix $\boldsymbol{H}^H$ requires $N_p^2 N_r$ operations, and multiplication with matrix $\boldsymbol{y}$ requires $N_p N_r$ operations. The total number of computations is $(2N_p^3 + 3N_p^2 - 5N_p)/6 + N_p^2 N_r + N_p N_r$ and the total of L operations are required. The third part is the computational complexity of ML detection (Lines 9-10 in TABLE I). A total of $L(N_p N_r + N_r)$ operations are required. Thus, the computational complexity of the IZF-MMSE algorithm is shown as

$$\begin{aligned} C_{IZF-MMSE} &= k[(2N_t^3 + 3N_t^2 - 5N_t)/6 + N_t^2 N_r + N_t N_r + N_t] \\ &+ L[(2N_p^3 + 3N_p^2 - 5N_p)/6 + N_p^2 N_r + N_p N_r] \\ &+ L(N_p N_r + N_r) \end{aligned} \tag{17}$$

The computational complexity of the ML algorithm can be obtained by calculating according to (4), where the number of all TACs is denoted as $N_c = 2^{\lfloor \log_2 C_{N_t}^{N_p} \rfloor}$, and $2^{N_p \log_2 M}$ represents the number of combinations when each antenna combination carries different symbols with each requiring $N_r N_p + N_r$ operations. Therefore, the computational complexity of the ML algorithm is shown as

$$C_{ML} = 2^{N_p \log_2 M} N_r (N_p + 1) N_c \tag{18}$$

Figures 6-7 give the comparison of the computational complexity of the proposed IZF-MMSE algorithm to that of the ML algorithm for a GSM system in two scenarios with $N_t = 10$, $N_r = 15$, $N_p = 2$ and $N_t = 10$, $N_r = 15$, $N_p = 3$, respectively. It can be seen from Figure 6 that the computational complexity of the IZF-MMSE algorithm is significantly lower than that of the ML algorithm, and the complexity remains almost unchanged under different signal-to-noise ratios. As the number of iterations increases, the complexity of the algorithm also increases. As shown in Figure 6, the computational complexity of the IZF-MMSE algorithm is 5590 when only one antenna index is removed, which is about 24% of the ML algorithm, and 28%, 30%, and 32% of the ML algorithm when the number of iterations is increased to 2, 3, and 4, respectively.

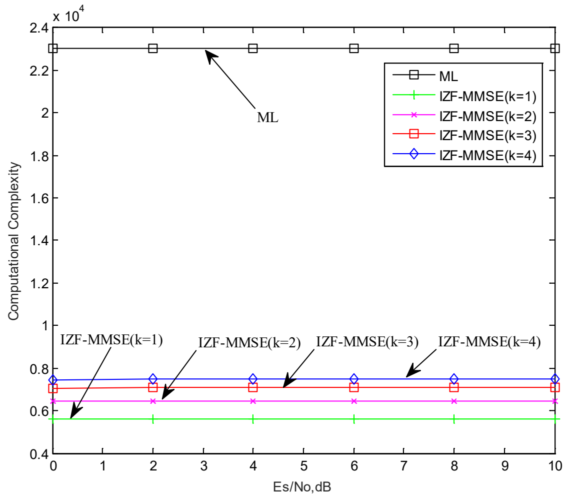

Fig. 6. The computational complexity comparison between IZF-MMSE with the number of iterations k =1, 2, 3 and and ML with $N_t = 10$, $N_r = 15$, $N_p = 2$ for QAM constellation.

In Figure 7, when N_p is 3, the computational complexity of the IZF-MMSE algorithm is 5%, 4%, 3.9%, and 3.5% of the ML algorithm when k is 1, 2, 3, and 4, respectively. The computational complexity decreases with the increase of k. The reason is as follows: The computational complexity consists of three parts. As k increases, the complexity of the first part increases, but the number of removed antennas increases, resulting in a significant decrease in the number of remaining TACs. As a result, the number of elements L in set Ψ in the second and third parts significantly decreases, which results in a reduction of the overall computational complexity.

The simulation results for PSK modulation in the above two scenarios are the same as for QAM modulation.

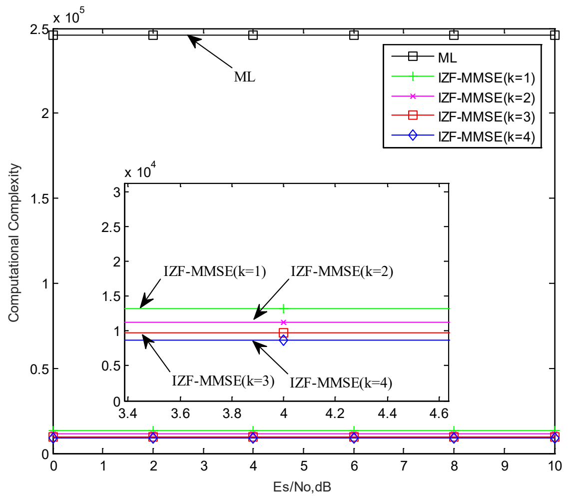

Fig. 7. The computational complexity comparison between IZF-MMSE with the number of iterations k =1, 2, 3 and 4 and ML with $N_t = 10$, $N_r = 15$, $N_p = 3$ for QAM constellation.

V. Conclusion

In this paper, an IZF-MMSE detection algorithm for GSM systems is proposed. The iterative detection of all TACs is performed to obtain multiple transmit antenna indexes with the lowest probability, then TACs containing these antenna indexes are removed from all TACs, MMSE equalization processing is performed to detect in the new search set, and finally the ML detection algorithm is utilized to estimate the transmit antenna and transmit symbols. Simulation results show that the BER performance of the proposed IZF-MMSE detector approximates that of the ML detector, and the computational complexity is significantly reduced.

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