Event-triggered Finite-time Prescribed Performance Output-feedback Control for Nonlinear Systems with Unmodeled Dynamics

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***Abstract*—A finite-time prescribed performance output-feedback adaptive control method based on event triggering is proposed for uncertain nonlinear systems with unmodeled dynamics. Firstly, dynamic signals are introduced to handle uncertain dynamic disturbances in the system, and a novel finite-time performance function is used to constrain tracking errors. In order to estimate unmeasurable states, a state observer is designed. In addition, fuzzy logic systems are introduced to approach unknown nonlinear functions in the system, greatly reducing computational complexity. Then, the event-triggered scheme is improved, which can switch between fixed threshold strategy and relative threshold strategy. On this basis, a fuzzy adaptive event-triggered controller is designed, which can guarantee that all signals of the control system are semi-globally consistent and ultimately bounded, without Zeno behavior occurring. Finally, the effectiveness of the proposed method was proven and validated.**

***Index Terms*—nonlinear systems, prescribed performance control (PPC), event-triggered, unmodeled dynamics, fuzzy logic system (FLS).**

I. Introduction

RECENTLY, the research on nonlinear system control has received increasing attention from scholars. Due to the fact that almost all practical control systems are nonlinear systems, such as aerospace, ships and vehicles, biochemistry, and other fields, the study of nonlinear systems is of great significance. Adaptive control, as an important method for nonlinear system control, is often combined with neural networks (NNs) or FLS to design controllers. In [1], [2], NNs or FLS adaptive control approaches have been proposed for time-delay nonlinear systems. For stochastic systems, FLS or NNs adaptive controllers were designed in [3], [4], [5], [6], [7]. In [8], [9], the adaptive control of the non-strict feedback systems, switching systems, and interconnected systems have also been studied respectively.

Another issue that cannot be ignored is that there are always unmodeled dynamics in actual control systems, such as ignored high-order differential terms, sensors, etc. These unmodeled dynamics can seriously affect the performance of the control system. Therefore, how to handle unmodeled dynamics is a worthwhile issue to consider. In [10], considering the unmodeled dynamics, dynamic signal was first introduced to deal with dynamic disturbances. In [11], the small gain theorem was used to prove that control systems are input-to-state stable (ISS). Based on [10], prescribed performance control scheme for time-delay nonlinear switched systems was proposed in [12], [13]. A decentralized control mechanism for a class of interconnected nonlinear systems was presented in [14]. Literature [15] proposed an output-feedback control algorithm for uncertain systems, which estimated the immeasurable states by using state observer. In addition, some system state variables cannot be fully measured, so it is also necessary to design state observer. In [16], an output-feedback tracking control method was proposed for switched stochastic pure-feedback systems, with this as a basis, literature [17] designed prescribed performance constraint. In [18], a prescribed performance output-feedback control algorithm was presented for switched nonlinear systems with nonstrict-feedback, and addressed the problem of "differential explosion".

In order to improve the performance of the control system, the prescribed performance was first proposed by Greek scholars in [19]. Since then, the PPC has been researched by a large number of scholars. In [20], a fully prescribed performance constraint scheme was proposed to constrain all errors in the control system within a bounded range. In [21], Zhao et al. proposed a prescribed performance adaptive control scheme for nonlinear systems with unknown initial conditions. However, the traditional PPC cannot satisfy the control requirements of some systems. Therefore, to further improve control accuracy, Liu et al. first presented the finite-time performance function (FTPF) in [22], where the tracking error can be converged to a bounded range within the settling time. In [23], an improved FTPF has been designed, which consists of multiple piecewise functions and allows for faster convergence of tracking errors. Literature [24] applied a tangent form of error transform, and combined the finite-time prescribed performance and command filtering to design the controller. In addition, some PPC methods have been applied to discrete-time systems [25], multi-input and multi-output (MIMO) nonlinear systems [26], practical control systems (such as robotic manipulator [27], quadrotor UAV [28]), etc.

With the increasing complexity of networked control systems, a large amount of data transmission often leads to problems such as network congestion, transmission delay, and data loss. In order to overcome the system communication overload caused by time-triggered, event-triggered scheme was proposed in [29]. Different from time-triggered, event-

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triggered is determined by predetermined event-triggered conditions rather than time. If the triggering condition can not be satisfied, the signal will remain unchanged. Therefore, communication resources can be greatly economized by event-triggered method. In the past decade, event-triggered control (ETC) has been researched by a large of scholars, e.g. [30], [31]. So far, there have been many event-triggered methods, e.g. fixed threshold strategy, relative threshold strategy, etc. In [32], [33], event-triggered controllers with fixed threshold strategy were designed. In [34], [35], [36], the relative threshold strategy was applied to design controllers. To overcome the shortcomings of fixed and relative thresholds, the switched threshold scheme was proposed in [37], and the condition of ISS was not required. The dynamic event-triggered method was first developed in [38]. In [39], static and dynamic event-triggered controllers have been designed to ensure that the control system can be adjusted more accurately. After the static event-triggered transition, an additional term can be added to the adaptive law. In [40], the intermittent event-triggered control scheme for system states was first proposed, the triggered state was replaced with continuous state, then the controller was designed. The relative threshold strategy is flexible and has been adopted by many literatures. Although the relative threshold strategy can adjust the threshold of the trigger according to the magnitude of the control signal, the measurement error of the control signal is necessarily very large when the magnitude of the control signal is too large. In this case, whenever the trigger controller works, the control signal will change abruptly, causing a large impulse to the system, it has a significant effect on the tracking control.

Inspired by the above literatures, an improved finite-time prescribed performance fuzzy adaptive output-feedback event-triggered control scheme is proposed for nonlinear systems containing unmodeled dynamics. Compared with existing results, the main contributions of this article are as follows:

(1) To deal with dynamic disturbances, a dynamic signal is introduced, the unknown continuous nonlinear functions can be approached by FLS, and the state observer is designed to estimate immeasurable states.

(2) Different from literatures [22], [23], [24], the novel finite-time performance function is designed to accelerate the convergence of tracking error to a bounded range, and the performance of the system is further improved.

(3) An improved event-triggered scheme is proposed, which can switch between relative threshold and fixed threshold strategies compared to [34], [35], [36], thus preventing system performance from being affected by excessive control signals.

II. Problem Formulation and Preliminaries

Consider a strict-feedback nonlinear system containing unmodeled dynamics as follows

$$\begin{cases} \dot{\xi} = p(\xi, x, t) \\ \dot{x}_1 = f_1(\bar{x}_1) + x_2 + \Delta_1(x, \xi, t) \\ \dot{x}_i = f_i(\bar{x}_i) + x_{i+1} + \Delta_i(x, \xi, t) \\ \dot{x}_n = f_n(\bar{x}_n) + u + \Delta_n(x, \xi, t) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in R^l, (l = 1, 2, \cdots, n)$ is the system state; $u \in R$ and $y \in R$ denote the control input and system output; $\xi \in R^{i_0}$ is the unmodeled dynamics. For $i = 1, 2, \cdots, n$, $f_i(\cdot)$ are unknown continuous nonlinear functions, $\Delta_i(\cdot)$ represent the dynamic disturbances, $\Delta_i(\cdot)$ and $p(\cdot)$ satisfy local Lipschitz condition.

Assumption 1 [1]: The desired signal y_r is a known smooth bounded function.

Assumption 2 [2]: For the dynamic disturbances $\delta_i, (i = 1, 2, \cdots, n)$, there exist unknown non-negative smooth functions φ_{i1} and φ_{i2} satisfy

$$|\Delta_i(x, \xi, t)| \le \varphi_{i1}(|\bar{x}_i|) + \varphi_{i1}(|\xi|) \tag{2}$$

Assumption 3 [10]: The unmodeled dynamics ξ are exponentially input-to-out practically stable (Exp-ISpS), consider an Exp-ISpS Lyapunov function that satisfies

$$\nu_1(|\xi|) \le V(\xi) \le \nu_2(|\xi|) \tag{3}$$

$$\frac{\partial V(\xi)}{\partial \xi}(\xi)\, p(\xi, x) \le -c_0 V(\xi) + \eta(|x_1|) + d_0 \tag{4}$$

where ν_1, ν_2, η are known K_∞-functions, c_0 and d_0 are non-negative constants.

Lemma 1 [10]: Consider the system (1), if there is a Lyapunov function V satisfying Exp-ISpS, then, when $0 < \bar{c}_0 < c_0$, any initial time $t_0 \ge 0$, any initial value $\xi_0 = \xi_0(0)$ and $r > 0$, and function $\bar{\eta}(x_1) \ge \eta(|x_1|)$, there exists a finite $T_0(\bar{c}, r_0, \xi_0) \ge 0$, the non-negative function $D(t_0, t)$ for $\forall t > t_0$ and a signal expressed as follows

$$\dot{r} = -\bar{c} + \bar{\eta}(|x_1|) + d_0, r(t_0) = r_0 \tag{5}$$

such that $D(t_0, t) = 0$ for all $t \ge T_0 + t_0$ and

$$V[\xi(t)] \le r(t) + D(t_0, t) \tag{6}$$

for $\forall t \ge t_0$, the solutions are defined. Generally, η is a smooth function and is selected as $\eta(s) = s^2 \eta_0(s^2)$. Then, (5) can be rewritten as

$$\dot{r} = -\bar{c} + x_1^2 \eta_0(x_1^2) + d_0 \tag{7}$$

where η_0 is non-negative smooth function.

Lemma 2 [41]: Consider the set $\Omega_{z_1} = \{z_1 \| z_1| < 0.8814\ell\}$ and $\ell > 0$ is constant, when $z_1 \notin \Omega_{z_1}$, the inequality $[1 - 2\tanh^2(z_1)] \le 0$ holds.

Lemma 3 [37]: For any constant $\ell > 0$ and $\bar{\kappa} \in R$, the following inequalities hold:

$$0 \le \bar{\kappa} - \bar{\kappa} \tanh(\bar{\kappa}/\ell) \le \vartheta\ell, \vartheta \le 0.2785 \tag{8}$$

Lemma 4 [15]: If $F(Z)$ is continuous function defined in the compact set Ξ, for any postive constants δ, the following inequality holds:

$$\sup_{z \in \Omega} \left| F(Z) - W^T S(Z) \right| \le \delta \tag{9}$$

where $W = [W_1, W_2, \cdots, W_N]^T$ is the ideal constant weight vector, $S(Z) = [S_1(z), S_2(z), \cdots, S_N(z)]^T$, N is the number of fuzzy rules, for $i = 1, 2, \cdots, N$, the basis functions $S_i(Z)$ are selected as follows

$$S_i(Z) = \frac{\prod_{l=1}^{n} \mu F_l^i(Z_l)}{\sum_{i=1}^{N} \prod_{l=1}^{n} \mu F_l^i(Z_l)} \tag{10}$$

Then, the fuzzy logic system $F(Z)$ can be rewritten as follows:

$$F(Z) = W^T S(Z) + \delta(Z), \delta(Z) \le \bar{\delta} \tag{11}$$

where $\bar{\delta}$ is positive constant. And W can be defined as follows

$$W := \arg\min_{W \in R^N} \left\{ \sup_{z\in\Omega} \left| F(Z) - W^T S(Z) \right| \right\} \tag{12}$$

III. CONTROLLER DESIGN

In order to proceed the subsequent controller design, define

$$\theta_i = \left\{ \|W_i\|^2 : i = 0, 1, \cdots, n \right\} \tag{13}$$

where θ_i is unknown constant, the estimation error of θ_i is expressed as $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

A. State Observer Design

For the purpose of output-feedback, the state observer is designed as follows

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + L_1(y - \hat{x}_1) \\ \dot{\hat{x}}_i &= \hat{x}_{i+1} + L_i(y - \hat{x}_1) \\ &\vdots \\ \dot{\hat{x}}_n &= u + L_n(y - \hat{x}_1) \end{aligned} \tag{14}$$

where $\hat{x}_i$ is the estimated value of x_i; L_i represents the gain of the observer, and the matrix

$$A = \begin{bmatrix} -L_1 & & \\ \vdots & I_{n-1} & \\ -L_n & \cdots & 0 \end{bmatrix} \tag{15}$$

is a strict Hurwitz matrix. Therefore, for given matrix $Q = Q^T > 0$, there exists a matrix $P = P^T$ that holds

$$A^T P + PA = -Q \tag{16}$$

Define the error of the observer as $e_i = x_i - \bar{x}_i, (i = 1, 2, \cdots, n)$. Based on (1) and (14), it yields

$$\begin{aligned} \dot{e}_1 &= e_2 - L_1 e_1 + f_1(\bar{x}_1) \\ \dot{e}_2 &= e_3 - L_2 e_1 + f_2(\bar{x}_2) \\ &\vdots \\ \dot{e}_n &= -L_n e_1 + f_n(\bar{x}_n) \end{aligned} \tag{17}$$

Equations above can be written in the compact form

$$e = Ae + F + \Delta \tag{18}$$

where $e = [e_1, e_2, \cdots, e_n]^T$; $\Delta = [\Delta_1, \Delta_2, \cdots, \Delta_n]^T$; $F = [f_1(\bar{x}_1), f_2(\bar{x}_2), \cdots, f_n(\bar{x}_n)]^T$.

Due to $f_i(\bar{x}_i)\,(i = 1, 2, \cdots, n)$ is unknown nonlinear functions, $f_i(\bar{x}_i)$ can be approximated by FLS $W_{i0}^T S_{i0}(Z)$ as follows

$$f_i(\bar{x}_i) = W_{i0}^T S_{i0}(Z) + \delta_{i0}(Z), |\delta_{i0}(Z)| \le \bar{\delta}_{i0} \tag{19}$$

Hence, there has

$$F(Z_0) = W_0^T S_0(Z) + \delta_0(Z), |\delta_0(Z)| \le \bar{\delta}_0 \tag{20}$$

where $W_0 = [W_{10}, \cdots, W_{n0}]$, $\delta_0(Z) = [\delta_{10}(Z), \cdots, \delta_{n0}(Z)]$.

For the system (1), select Lyapunov function V_0 as follows

$$V_0 = e^T P e \tag{21}$$

Taking the time-derivative of V_0 yields

$$\dot{V}_0 = e^T(A^T P + PA)e + 2e^T P(F + \Delta) \tag{22}$$

By using the Young's inequality and Assumption 2, we can get

$$2e^T PF \le \|e\|^2 + \|P\|^2 W_0^2 + \|P\|^2 \delta_0^2 \tag{23}$$

$$\begin{aligned} 2e^T P\Delta \le& 2\|e\|^2 + \|P\|^2 \left(\sum_{i=1}^n \varphi_{i1}(|\xi|) \right)^2 \\ &+ \|P\|^2 \left(\sum_{i=1}^n \varphi_{i2}(\|x\|) \right)^2 \end{aligned} \tag{24}$$

By combining (22)-(24), it can be concluded that

$$\dot{V}_0 \le -[\lambda_{\min}(Q) - 3]\|e\|^2 + B_0 \tag{25}$$

where$B_0 = \|P\|^2 W_0^2 + \|P\|^2 \left(\sum_{i=1}^n \varphi_{i1}(|\xi|) \right)^2 + \|P\|^2 \delta_0^2 + \|P\|^2 \left(\sum_{i=1}^n \varphi_{i2}(|x|) \right)^2$.

B. Prescribed Performance

The definition of error is as follows

$$\begin{cases} \zeta = x_1 - y_r \\ z_i = \hat{x}_i - \alpha_{i-1} \end{cases} \tag{26}$$

where $\alpha_i (i = 1, 2, \cdots, n)$ are the virtual control laws.

Design a finite-time performance function as follows

$$\rho(t) = \begin{cases} \left(\rho_0 - \frac{t}{t_p} \right) e^{\left(\frac{lt}{t_p - t} \right)} + \rho_\infty & 0 \le t < t_p \\ \rho_\infty & t \ge t_p \end{cases} \tag{27}$$

where the initial of $\rho(t)$ is $\rho_0 + \rho_\infty$, and ρ_0, ρ_∞, t_p are positive constants and l is smooth continuous function.

The tangent error transformation is introduced as follows

$$z_1 = \tan\left(\frac{\pi\zeta}{2\rho} \right) \tag{28}$$

Then, the derivation of z_1 is

$$\dot{z}_1 = h\left(f_1 + x_2 + \Delta_1 - \dot{y}_r - \frac{2}{\pi}\dot{\rho}\arctan z_1 \right) \tag{29}$$

where $h = \pi\left(1 + z_1^2\right)/2\rho$.

C. Virtual Controls Design

The design of the n-step backstepping controller is shown below.

Step 1 : Select a Lyapunov function as follows

$$V_1 = V_0 + \frac{1}{2}z_1^2 + \frac{1}{\lambda_0}r + \frac{1}{2\mu_1}\tilde{\theta}_1^2 \tag{30}$$

where λ_0, μ_1 are positive parameters.

The time-derivative of V_1 is

$$\begin{aligned} \dot{V}_1 \le& -[\lambda_{\min}(Q) - 3]\|e\|^2 + B_0 + z_1 h(\alpha_1 + e_2) \\ &+ z_1 h\left[f_1 + z_2 + \Delta_1 - \dot{y}_r - \frac{2}{\pi}\dot{\rho}_1 \arctan z_1 \right] \\ &+ \frac{x_1^2 \eta_0(x_1^2)}{\lambda_0} + \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0}r + \frac{1}{\mu_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \end{aligned} \tag{31}$$

Applying the Young's inequality yields

$$z_1 h\Delta_1 \leq |z_1 h|\,\varphi_{11}(|x_1|) + |z_1 h|\,\varphi_{12}(|\xi|) \tag{32}$$

$$z_1 h e_2 \leq e_2^2 + \frac{1}{4}(z_1 h)^2 \tag{33}$$

Then, let's deal with $|z_1 h|\,\varphi_{11}(|x_1|)$ and $|z_1 h|\,\varphi_{12}(|\xi|)$, respectively.

$$|z_1 h|\,\varphi_{11}(|x_1|) \leq z_1 h\bar{\varphi}_{11} + \bar{\tau}_{11} \tag{34}$$

where $\bar{\varphi}_{11} = \varphi_{11}(|x_1|)\tanh\left(\frac{z_1 h\varphi_{11}(|x_1|)}{\tau_{11}}\right)$, $\bar{\tau}_{11} = 0.2785\tau_{11}$.

$$|z_1 h|\,\varphi_{12}(|\xi|) \leq z_1 h\bar{\varphi}_{12} + \bar{\tau}_{12} + \frac{1}{4}(z_1 h)^2 + d(t_0,t) \tag{35}$$

where $\bar{\varphi}_{12} = \varphi_{12}\circ\nu_1^{-1}(2r)\tanh\left(\frac{z_1 h\varphi_{12}\circ\nu_1^{-1}(2r)}{\tau_{12}}\right)$, $d(t_0,t) = \left[\varphi_{12}\circ\nu_1^{-1}(2D(t_0,t))\right]^2$, $\bar{\tau}_{12} = 0.2785\tau_{12}$.

By using (31) - (35), it can be obtained

$$\begin{aligned}\dot{V}_1 \leq & -[\lambda_{\min}(Q) - 4]\,\|e\|^2 + B_0 + z_1 h z_2 - \frac{z_1^2}{2} \\ & + z_1(h\alpha_1 + F_1) + \bar{\tau}_{11} + \bar{\tau}_{12} + d(t_0,t) + \frac{1}{\mu_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ & + \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0}r + \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0}\end{aligned} \tag{36}$$

where $F_1(Z) = hf_1 - h\dot{y}_r - h\frac{2}{\pi}\dot{\rho}\arctan z_1 + h\bar{\varphi}_{11} + h\bar{\varphi}_{12} + \frac{1}{2}z_1 + \frac{1}{2}z_1 h^2 + \frac{2}{z_1}\tanh^2\left(\frac{z_1}{\ell}\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0}$.

Remark 1: When $z_1 = 0$, $\frac{x_1^2\eta_0(x_1^2)}{z_1\lambda_0}$ is discontinuous and can not be approximated by FLS, hence, a hyperbolic tangent function $\tanh\left(\frac{z_1}{\ell}\right)$ is introduced.

Based on Lemma 4, unknown nonlinear function F_1 can be approximated by FLS $W_1 S_1(Z_1)$ as follows

$$F_1 = W_1^T S_1(Z_1) + \delta_1(Z_1),\, |\delta_1(Z_1)| \leq \bar{\delta}_1 \tag{37}$$

where $\bar{\delta}_1$ is a positive constant.

According to the Young's inequality, it is clear that

$$\begin{aligned}z_1 F_1 = & z_1\left(W_1^T S_1(Z_1) + \delta_1(Z_1)\right) \\ \leq & \frac{1}{2a_1^2}z_1^2\|W_1\|^2 S_1^T S_1 + \frac{a_1^2}{2} + \frac{z_1^2}{2} + \frac{\bar{\delta}_1^2}{2} \\ \leq & \frac{1}{2a_1^2}z_1^2\theta_1 S_1^T S_1 + \frac{a_1^2}{2} + \frac{z_1^2}{2} + \frac{\bar{\delta}_1^2}{2}\end{aligned} \tag{38}$$

The virtual control law is set as

$$\alpha_1 = -\frac{z_1}{h}\left(k_1 + \frac{1}{2a_1^2}\hat{\theta}_1 S_1^T S_1\right) \tag{39}$$

where k_1 is positive parameter, the adaptive law is set as follows

$$\dot{\hat{\theta}}_1 = \frac{1}{2a_1^2}z_1^2 S_1^T S_1 - \gamma_1\hat{\theta}_1 \tag{40}$$

where $\gamma_1 > 0$ is constant.

Combining (36)-(40), it yields

$$\begin{aligned}\dot{V}_1 \leq & -[\lambda_{\min}(Q) - 4]\,\|e\|^2 + \sum_{l=0}^{1} B_l - k_1 z_1^2 \\ & + z_1 h z_2 + \frac{\gamma_1}{\mu_1}\tilde{\theta}_1\hat{\theta}_1 + \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0}r \\ & + \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0}\end{aligned} \tag{41}$$

where $B_1 = \frac{a_1^2}{2} + \frac{\bar{\delta}_1^2}{2} + \bar{\tau}_{11} + \bar{\tau}_{12} + d(t_0,t)$.

Step 2 : Based on $z_2 = \hat{x}_2 - \alpha_1$, the time-derivative of z_2 is expressed by

$$\begin{aligned}\dot{z}_2 &= \dot{\hat{x}}_2 - \dot{\alpha}_1 \\ &= \hat{x}_3 + L_2(y - \hat{x}_1) - \frac{\partial\alpha_1}{\partial x_1}(\Delta_1 + e_2) - \Gamma_1\end{aligned} \tag{42}$$

where $\Gamma_1 = \frac{\partial\alpha_1}{\partial x_1}(f_1 + \hat{x}_2) + \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_1}{\partial\rho}\dot{\rho} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\alpha_1}{\partial r}\dot{r}$.

Consider a Lyapunov function as follow

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2\mu_2}\tilde{\theta}_2^2 \tag{43}$$

Taking the time-derivative of V_2 yields

$$\begin{aligned}\dot{V}_2 \leq & -[\lambda_{\min}(Q) - 4]\,\|e\|^2 - k_1 z_1^2 + \frac{\gamma_1}{\mu_1}\tilde{\theta}_1\hat{\theta}_1 \\ & + \sum_{l=0}^{1} B_l + z_2(\hat{x}_3 + L_2(y - \hat{x}_1) + \Gamma_1 + z_1 h) \\ & + \left|z_2\frac{\partial\alpha_1}{\partial x_1}\right|(|e_2| + |\Delta_1|) + \frac{d_0}{\lambda_0} - \frac{1}{\mu_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \\ & - \frac{\bar{c}}{\lambda_0}r + \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0}\end{aligned} \tag{44}$$

By applying Yang's inequality, it can be obtained that

$$\left|z_2\frac{\partial\alpha_1}{\partial x_1}\right||e_2| \leq \|e\|^2 + \frac{z_2^2}{4}\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 \tag{45}$$

$$\left|z_2\frac{\partial\alpha_1}{\partial x_1}\right||\Delta_1| \leq \left|z_2\frac{\partial\alpha_1}{\partial x_1}\right|[\varphi_{11}(|x_1|) + \varphi_{12}(|\xi|)] \tag{46}$$

Then, based on Lemma 3, it can get

$$\left|z_2\frac{\partial\alpha_1}{\partial x_1}\right|\varphi_{11}(|x_1|) \leq z_2\bar{\varphi}_{21} + \bar{\tau}_{21} \tag{47}$$

where $\bar{\varphi}_{21} = \frac{\partial\alpha_1}{\partial x_1}\varphi_{11}(|x_1|)\tanh\left(\frac{z_2(\partial\alpha_1/\partial x_1)\varphi_{11}(|x_1|)}{\tau_{21}}\right)$, $\bar{\tau}_{21} = 0.2785\tau_{21}$.

$$\left|z_2\frac{\partial\alpha_1}{\partial x_1}\right|\varphi_{12}(|\xi|) \leq z_2\bar{\varphi}_{22} + \bar{\tau}_{22} + \frac{z_2^2}{4}\left|\frac{\partial\alpha_1}{\partial x_1}\right|^2 + d(t_0,t) \tag{48}$$

where $\bar{\varphi}_{22} = \frac{\partial\alpha_1}{\partial x_1}\varphi_{12}(|\xi|)\tanh\left(\frac{z_2(\partial\alpha_1/\partial x_1)\varphi_{12}(|\xi|)}{\tau_{22}}\right)$, $\bar{\tau}_{22} = 0.2785\tau_{22}$.

Combining (45)-(48) yields

$$\begin{aligned}\dot{V}_2 \leq & -[\lambda_{\min}(Q) - 5]\,\|e\|^2 - k_1 z_1^2 + \sum_{l=0}^{1} B_l \\ & + \frac{\gamma_1}{\mu_1}\tilde{\theta}_1\hat{\theta}_1 + z_2(z_3 + \alpha_2 + F_2) - \frac{z_2^2}{2} + \bar{\tau}_{21} \\ & + \bar{\tau}_{22} + d(t_0,t) + \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0}r - \frac{1}{\mu_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \\ & + \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0}\end{aligned} \tag{49}$$

where $F_2 = L_2(y - \hat{x}_1) + \Gamma_1 + z_1 h + \bar{\varphi}_{11} + \bar{\varphi}_{12} + \frac{z_2}{2}\left|\frac{\partial\alpha_1}{\partial x_1}\right|^2 + \frac{z_2}{2}$.

Then, FLS $W_2^T S_2(Z_2)$ can be used to approximate unknown nonlinear function F_2 as follows

$$F_2 = W_2^T S_2(Z_2) + \delta_2(Z_2),\, |\delta_2(Z_2)| \leq \bar{\delta}_2 \tag{50}$$

where $\bar{\delta}_2$ is psoitive constant.

According to the Young's inequality, it can get

$$\begin{aligned} z_2F_2 =& z_2\left(W_2^TS_2\left(Z_2\right)+\delta_2\left(Z_2\right)\right)\\ \leq& \frac{1}{2a_2^2}z_2^2\|W_2\|^2S_2^TS_2+\frac{a_2^2}{2}+\frac{z_2^2}{2}+\frac{\bar{\delta}_2^2}{2} \\ \leq& \frac{1}{2a_2^2}z_2^2\theta_2S_2^TS_2+\frac{a_2^2}{2}+\frac{z_2^2}{2}+\frac{\bar{\delta}_2^2}{2} \end{aligned} \quad (51)$$

The virtual control law is set as follows

$$\alpha_2=-k_2z_2-\frac{1}{2a_2^2}z_2\hat{\theta}_2S_2^TS_2 \quad (52)$$

where k_2 is a positive constant, and the adaptive law is set as

$$\dot{\hat{\theta}}_2=\frac{1}{2a_2^2}z_2^2S_2^TS_2-\gamma_2\hat{\theta}_2 \quad (53)$$

where a_2, γ_2 are positive constants.

Substituting (51)-(53) into (49) yields

$$\begin{aligned} \dot{V}_2\leq& -\left[\lambda_{\min}(Q)-5\right]\|e\|^2-\sum_{l=1}^{2}k_lz_l^2+\sum_{l=0}^{2}B_l \\ &+\sum_{l=1}^{2}\frac{\gamma_l}{\mu_l}\tilde{\theta}_l\hat{\theta}_l+z_2z_3+\frac{d_0}{\lambda_0}-\frac{\bar{c}}{\lambda_0}r \\ &+\left(1-2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0} \end{aligned} \quad (54)$$

where $B_2=\frac{a_2^2}{2}+\frac{\bar{\delta}_2^2}{2}+\bar{\tau}_{21}+\bar{\tau}_{22}+d(t_0,t)$.

Step i $(3<i<n-1)$: For $z_i=\hat{x}_i-\alpha_{i-1}$, it can obtain

$$\begin{aligned} \dot{z}_i =& \dot{\hat{x}}_i-\dot{\alpha}_{i-1} \\ =& \hat{x}_{i+1}+L_i\left(y-\hat{x}_1\right)-\frac{\partial\alpha_{i-1}}{\partial x_1}\left(e_2+\Delta_1\right)-\Gamma_{i-1} \end{aligned} \quad (55)$$

where $\Gamma_{i-1}=\sum\limits_{l=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_l}\left(\hat{x}_{l+1}+L_l\left(y-\hat{x}_1\right)\right)+\sum\limits_{l=0}^{i-1}\frac{\partial\alpha_1}{\partial\rho^{(l)}}\rho^{(l+1)}+\frac{\partial\alpha_{i-1}}{\partial x_1}\left(\hat{x}_2+f_1\right)+\sum\limits_{l=0}^{i-1}\frac{\partial\alpha_1}{\partial y_r^{(l)}}y_r^{(l+1)}+\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}+\frac{\partial\alpha_{i-1}}{\partial r}\dot{r}$.

Select a Lyapunov function as follows

$$V_i=V_{i-1}+\frac{1}{2}z_i^2+\frac{1}{2\mu_i}\tilde{\theta}_i^2 \quad (56)$$

Taking the time-derivative of V_i yields

$$\begin{aligned} \dot{V}_i\leq& -\left[\lambda_{\min}(Q)-(i+2)\right]\|e\|^2-\sum_{l=1}^{i-1}k_lz_l^2+\sum_{l=1}^{i-1}B_l \\ &+\sum_{l=1}^{i-1}\frac{\gamma_l}{\mu_l}\tilde{\theta}_l\hat{\theta}_l-\frac{1}{\mu_i}\tilde{\theta}_i\dot{\hat{\theta}}_i+\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\right|\left(|\Delta_1|+|e_2|\right) \\ &+z_i\left(\hat{x}_{i+1}+L_i\left(y-\hat{x}_1\right)+\Gamma_{i-1}+z_{i-1}\right)+\frac{d_0}{\lambda_0} \\ &-\frac{\bar{c}}{\lambda_0}r+\left(1-2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0} \end{aligned} \quad (57)$$

According to the Young's inequality, there has

$$\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\right||e_2|\leq|e|^2+\frac{z_i^2}{4}\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 \quad (58)$$

$$\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\right||\Delta_1|\leq\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\right|\left[\varphi_{11}\left(|x_1|\right)+\varphi_{12}\left(|\xi|\right)\right] \quad (59)$$

Based on Lemma 4, for $\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\right|\varphi_{11}\left(|x_1|\right)$, it yields

$$\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\right|\varphi_{11}\left(|x_1|\right)\leq z_i\bar{\varphi}_{i1}+\bar{\tau}_{i1} \quad (60)$$

where $\bar{\varphi}_{i1}=\frac{\partial\alpha_{i-1}}{\partial x_1}\varphi_{11}\left(|x_1|\right)\tanh\left(\frac{z_i(\partial\alpha_{i-1}/\partial x_1)\varphi_{11}(|x_1|)}{\tau_{i1}}\right)$, $\bar{\tau}_{i1}=0.2785\tau_{i1}$.

Similarly, for $\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\right|\varphi_{12}(|\xi|)$, one has

$$\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\right|\varphi_{12}\left(|\xi|\right)\leq z_i\bar{\varphi}_{i2}+\bar{\tau}_{i2}+\frac{z_i^2}{4}\left|\frac{\partial\alpha_{i-1}}{\partial x_1}\right|^2+d(t_0,t) \quad (61)$$

where $\bar{\varphi}_{i2}=\frac{\partial\alpha_{i-1}}{\partial x_1}\varphi_{12}\left(|\xi|\right)\tanh\left(\frac{z_i(\partial\alpha_{i-1}/\partial x_1)\varphi_{12}(|\xi|)}{\tau_{i2}}\right)$, $\bar{\tau}_{i2}=0.2785\tau_{i2}$.

Substituting (58)-(61) into (57) gets

$$\begin{aligned} \dot{V}_i\leq& -\left[\lambda_{\min}(Q)-(i+3)\right]\|e\|^2-\sum_{l=1}^{i-1}k_lz_l^2 \\ &+\sum_{l=1}^{i-1}\frac{\gamma_l}{\mu_l}\tilde{\theta}_l\hat{\theta}_l+\sum_{l=1}^{i-1}B_l+z_i\left(z_{i+1}+\alpha_i+F_i\right) \\ &-\frac{z_i^2}{2}+\bar{\tau}_{i1}+\bar{\tau}_{i2}+d(t_0,t)-\frac{1}{\mu_i}\tilde{\theta}_i\dot{\hat{\theta}}_i+\frac{d_0}{\lambda_0} \\ &-\frac{\bar{c}}{\lambda_0}r+\left(1-2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0} \end{aligned} \quad (62)$$

where $F_i=L_i\left(y-\hat{x}_1\right)+\Gamma_{i-1}+z_{i-1}+\bar{\varphi}_{i1}+\bar{\varphi}_{i2}+\frac{z_i}{2}\left|\frac{\partial\alpha_{i-1}}{\partial x_1}\right|^2+\frac{z_i}{2}$.

The unknown nonlinear function F_i can be approximated by FLS $W_i^TS_i\left(Z_i\right)$ as

$$F_i=W_i^TS_i\left(Z_i\right)+\delta_i\left(Z_i\right),\left|\delta_i\left(Z_i\right)\right|\leq\bar{\delta}_i \quad (63)$$

where $\bar{\delta}_i$ is positive constant.

By applying the Young's inequality, it can obtain

$$\begin{aligned} z_iF_i =& z_i\left(W_i^TS_i\left(Z_i\right)+\delta_i\left(Z_i\right)\right) \\ \leq& \frac{1}{2a_i^2}z_i^2\|W_i\|^2S_i^TS_i+\frac{a_i^2}{2}+\frac{z_i^2}{2}+\frac{\bar{\delta}_i^2}{2} \\ \leq& \frac{1}{2a_i^2}z_i^2\theta S_i^TS_i+\frac{a_i^2}{2}+\frac{z_i^2}{2}+\frac{\bar{\delta}_i^2}{2} \end{aligned} \quad (64)$$

The virtual control law is set as follows

$$\alpha_i=-k_iz_i-\frac{1}{2a_i^2}z_i\hat{\theta}S_i^TS_i \quad (65)$$

where k_i is positive parameter, the adaptive law is set as

$$\dot{\hat{\theta}}_i=\frac{1}{2a_i^2}z_i^2S_i^TS_i-\gamma_i\hat{\theta}_i \quad (66)$$

where a_i, γ_i are positive constants.

Combining (62)-(66) can get

$$\begin{aligned} \dot{V}_i\leq& -\left[\lambda_{\min}(Q)-(i+3)\right]\|e\|^2-\sum_{l=1}^{i}k_lz_l^2 \\ &+\sum_{l=0}^{i}B_l+\sum_{l=1}^{i-1}\frac{\gamma_l}{\mu_l}\tilde{\theta}_l\hat{\theta}_l+z_iz_{i+1}+\frac{d_0}{\lambda_0} \\ &-\frac{\bar{c}}{\lambda_0}r+\left(1-2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0} \end{aligned} \quad (67)$$

where $B_i=\frac{a_i^2}{2}+\frac{\bar{\delta}_i^2}{2}+\bar{\tau}_{i1}+\bar{\tau}_{i2}+d(t_0,t)$.

Step n: From the error $z_n = \hat{x}_n - \alpha_{n-1}$, one has

$$\begin{aligned} \dot{z}_n =& \dot{\hat{x}}_n - \dot{\alpha}_{n-1} \\ =& u + L_n\left(y - \hat{x}_1\right) - \frac{\partial \alpha_{n-1}}{\partial x_1}\left(e_2 + \Delta_1\right) - \Gamma_{n-1} \end{aligned} \tag{68}$$

where $\Gamma_{n-1} = \sum_{l=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_l}\left(\hat{x}_{l+1} + L_l\left(y - \hat{x}_1\right)\right) + \sum_{l=0}^{n-1} \frac{\partial \alpha_1}{\partial \rho^{(l)}}\rho^{(l+1)} + \frac{\partial \alpha_{n-1}}{\partial x_1}\left(\hat{x}_2 + f_1\right) + \sum_{l=0}^{n-1} \frac{\partial \alpha_1}{\partial y_r^{(l)}} y_r^{(l+1)} + \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{\partial \alpha_1}{\partial r}\dot{r}$.

Consider a Lyapunov function V_n as follows

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2\mu_n}\tilde{\theta}_n^2 \tag{69}$$

where $\mu_n > 0$ is constant.

Taking the time-derivative of the above equation yields

$$\begin{aligned} \dot{V}_n \leq& -\left[\lambda_{\min}(Q) - (n+2)\right]\|e\|^2 - \sum_{l=1}^{n-1} k_l z_l^2 + \sum_{l=0}^{n-1} B_l \\ &+ \sum_{l=1}^{n-1} \frac{\gamma_l}{\mu_l}\tilde{\theta}_l\hat{\theta}_l + \left|z_n \frac{\partial \alpha_{n-1}}{\partial x_1}\right|\left(|\Delta_1| + |e_2|\right) \\ &+ z_n\left(u + L_n\left(y - \hat{x}_1\right) + \Gamma_{n-1} + z_{n-1}\right) - \frac{1}{\mu_n}\tilde{\theta}_n\dot{\hat{\theta}}_n \\ &+ \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0}r + \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0} \end{aligned} \tag{70}$$

Applying the Young's inequality gets

$$\left|z_n \frac{\partial \alpha_{n-1}}{\partial x_1}\right||e_2| \leq \|e\|^2 + \frac{z_n^2}{4}\left(\frac{\partial \alpha_{n-1}}{\partial x_1}\right)^2 \tag{71}$$

$$\left|z_n \frac{\partial \alpha_{n-1}}{\partial x_1}\right||\Delta_1| \leq \left|z_n \frac{\partial \alpha_{n-1}}{\partial x_1}\right|\left[\varphi_{11}\left(|x_1|\right) + \varphi_{12}(|\xi|)\right] \tag{72}$$

Then, based on Lemma 4, it yields

$$\left|z_n \frac{\partial \alpha_{n-1}}{\partial x_1}\right|\varphi_{11}\left(|x_1|\right) \leq z_n\bar{\varphi}_{n1} + \bar{\tau}_{n1} \tag{73}$$

where $\bar{\varphi}_{n1} = \frac{\partial \alpha_{n-1}}{\partial x_1}\varphi_{11}\left(|x_1|\right)\tanh\left(\frac{z_n(\partial \alpha_{n-1}/\partial x_1)\varphi_{11}(|x_1|)}{\tau_{n1}}\right)$, $\bar{\tau}_{n1} = 0.2785\tau_{n1}$.

Similarly, for $\left|z_n \frac{\partial \alpha_{n-1}}{\partial x_1}\right|\varphi_{12}\left(|\xi|\right)$, one obtains

$$\left|z_n \frac{\partial \alpha_{n-1}}{\partial x_1}\right|\varphi_{12}\left(|\xi|\right) \leq z_n\bar{\varphi}_{n2} + \bar{\tau}_{n2} + \frac{z_n^2}{4}\left|\frac{\partial \alpha_{n-1}}{\partial x_1}\right|^2 + d(t_0, t) \tag{74}$$

where $\bar{\varphi}_{n2} = \frac{\partial \alpha_{n-1}}{\partial x_1}\varphi_{12}\left(|\xi|\right)\tanh\left(\frac{z_n(\partial \alpha_{n-1}/\partial x_1)\varphi_{12}(|\xi|)}{\tau_{n2}}\right)$, $\bar{\tau}_{n2} = 0.2785\tau_{n2}$.

Substituting (71)-(74) into (70) gets

$$\begin{aligned} \dot{V}_n \leq& -\left[\lambda_{\min}(Q) - (n+3)\right]\|e\|^2 - \sum_{l=1}^{n-1} k_l z_l^2 + \sum_{l=0}^{n-1} B_l \\ &+ \sum_{l=1}^{n-1} \frac{\gamma_l}{\mu_l}\tilde{\theta}_l\hat{\theta}_l + z_n\left(u - \alpha_n + \alpha_n + F_n\right) - \frac{z_n^2}{2} \\ &+ \bar{\tau}_{n1} + \bar{\tau}_{n2} + d(t_0, t) - \frac{1}{\mu_n}\tilde{\theta}_n\dot{\hat{\theta}}_n + \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0}r \\ &+ \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0} \end{aligned} \tag{75}$$

where $F_n = L_n\left(y - \hat{x}_1\right) + \Gamma_{n-1} + z_{n-1} + \bar{\varphi}_{n1} + \bar{\varphi}_{n2} + \frac{z_n}{2} + \frac{z_n}{2}\left|\frac{\partial \alpha_{n-1}}{\partial x_1}\right|^2$.

The unknown nonlinear function F_n can be approximated by FLS $W_n^T S_n\left(Z_n\right)$ as follows

$$F_n = W_n^T S_n\left(Z_n\right) + \delta_n\left(Z_n\right), \left|\delta_n\left(Z_n\right)\right| \leq \bar{\delta}_n \tag{76}$$

where $\bar{\delta}_n$ is positive constant.

According to the Young's inequality, it yields

$$\begin{aligned} z_n F_n =& z_n\left(W_n^T S_n\left(Z_n\right) + \delta_n\left(Z_n\right)\right) \\ \leq& \frac{1}{2a_n^2}z_n^2\|W_n\|^2 S_n^T S_n + \frac{a_n^2}{2} + \frac{z_n^2}{2} + \frac{\bar{\delta}_n^2}{2} \\ \leq& \frac{1}{2a_n^2}z_n^2\theta S_n^T S_n + \frac{a_n^2}{2} + \frac{z_n^2}{2} + \frac{\bar{\delta}_n^2}{2} \end{aligned} \tag{77}$$

The virtual control law is set as follows

$$\alpha_n = -k_n z_n - \frac{1}{2a_n^2}z_n\hat{\theta}S_n^T S_n \tag{78}$$

where k_n is positive parameter. Then, the adaptive law is set as

$$\dot{\hat{\theta}}_n = \frac{1}{2a_n^2}z_n^2 S_n^T S_n - \gamma_n\hat{\theta}_n \tag{79}$$

where a_n, γ_n are positive constants.

Combining (75)-(79), we can get

$$\begin{aligned} \dot{V}_n \leq& -\left[\lambda_{\min}(Q) - (n+3)\right]\|e\|^2 - \sum_{l=1}^{n} k_l z_l^2 \\ &+ \sum_{l=0}^{n} B_l + \sum_{l=1}^{n} \frac{\gamma_l}{\mu_l}\tilde{\theta}_l\hat{\theta}_l + z_n\left(u - \alpha_n\right) + \frac{d_0}{\lambda_0} \\ &- \frac{\bar{c}}{\lambda_0}r + \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right)\frac{x_1^2\eta_0(x_1^2)}{\lambda_0} \end{aligned} \tag{80}$$

where $B_n = \frac{a_n^2}{2} + \frac{\bar{\delta}_n^2}{2} + \bar{\tau}_{n2} + \bar{\tau}_{n2} + d(t_0, t)$.

D. Adaptive Event-Triggered Controller Design

In order to reduce the waste of resources, an improved event-triggered scheme is proposed in this paper. The new event-triggered controller is designed as follows

$$\begin{cases} \omega\left(t\right) = -(1+\kappa)\left[\alpha_n\tanh\left(\frac{z_n\alpha_n}{\varepsilon}\right) - \bar{m}\tanh\left(\frac{z_n\bar{m}}{\varepsilon}\right)\right] \\ u(t) = \omega(t_k), \forall t \in [t_k, t_{k+1}) \\ t_{k+1} = \inf\{t \in R|\,|E(t)| \geq \kappa\tanh(|u(t)|) + m\} \end{cases} \tag{81}$$

where ε, $\bar{m}$, m, κ are positive parameters, $t_k, k \in Z^+$ is controller update time, $E(t) = \omega(t) - \omega(t_k)$ is the measurement error. $\bar{m} > m/\left(1+\kappa\right)$. For $\forall t \in [t_k, t_{k+1})$, $|\omega(t) - \omega(t_k)| \leq \kappa\tanh(|u(t)|) + m$ is always true, and when $t \in [t_k, t_{k+1})$, $u(t) = \omega(t_k)$ is always true.

From (81), it can be seen that $|E(t)| \geq \kappa\tanh(|u(t)|) + m$, therefore From (81), it can be seen that $|E(t)| \geq \kappa\tanh(|u(t)|) + m$. There exist the time-varying parameters $|\varsigma_{1,2,3}\left(t\right)| \leq 1$ that satisfy the following

$$\begin{aligned} \omega\left(t\right) &= u(t) + \varsigma_1(t)\kappa\tanh(u(t)) + \varsigma_2(t)m \\ &= u(t) + \varsigma_3(t)\kappa u(t) + \varsigma_2(t)m \end{aligned} \tag{82}$$

Remark 2: If we directly deal with the equation containing the hyperbolic tangent function, the proof would be complicated. Here, we use the time-varying functions $\varsigma_1(t)$ and $\varsigma_3(t)$ to transform $\tanh(u(t))$ into $u(t)$, so that the proof can be done more simply.

As a reason, it follows that

$$u(t) = \omega(t) / (1 + \varsigma_3(t)\kappa) - \varsigma_2(t) m / (1 + \varsigma_3(t)\kappa) \quad (83)$$

Then, it yields

$$\begin{aligned} z_n(u - \alpha_n) =& z_n \left(\frac{\omega - \varsigma_2(t)m}{1 + \varsigma_3(t)\kappa} - \alpha_n \right) \\ \leq& \frac{z_n \omega}{1 + \kappa} - \left| \frac{z_n m}{1 - \kappa} \right| - z_n \alpha_n \\ \leq& |z_n \alpha_n| - z_n \alpha_n \tanh\left(\frac{z_n \alpha_n}{\varepsilon} \right) \\ &+ |z_n \bar{m}| - |z_n \bar{m}| \tanh\left(\frac{|z_n \bar{m}|}{\varepsilon} \right) \\ \leq& 0.557\varepsilon \end{aligned} \quad (84)$$

Finally, the time-derivative of V_n is rewritten as follows

$$\begin{aligned} \dot{V}_n \leq& - [\lambda_{\min}(Q) - (n+3)] \|e\|^2 - \sum_{l=1}^{n} k_l z_l^2 \\ &+ \sum_{l=0}^{n} B_l + \sum_{l=1}^{n} \frac{\gamma_l}{\mu_l} \tilde{\theta}_l \hat{\theta}_l + 0.557\varepsilon + \frac{d_0}{\lambda_0} \\ &- \frac{\bar{c}}{\lambda_0} r + \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right) \frac{x_1^2 \eta_0(x_1^2)}{\lambda_0} \end{aligned} \quad (85)$$

IV. STABILITY ANALYSIS

Theorem 1: Consider an uncertain nonlinear system (1) satisfying Assumption 1-3. The designed event-triggered controller (81) and virtual control laws (39), (52), (65), (78) with the adaptive laws (40), (53), (66), (79) can guarantee: (1) All signals in the closed-loop system are semi-globally uniformly ultimately bounded; (2) The tracking error can converge to a bounded range; (3) Zeno-behavior can be successfully avoided.

Proof:

(1) Due to

$$\tilde{\theta}_i \hat{\theta}_i = \tilde{\theta}_i \left(\theta_i - \tilde{\theta}_i \right) \leq -\frac{1}{2} \tilde{\theta}_i^2 + \frac{1}{2} \theta_i^2 \quad (86)$$

Substituting (86) into (85) yields

$$\begin{aligned} \dot{V}_n \leq& - [\lambda_{\min}(Q) - (n+3)] \|e\|^2 - \sum_{l=1}^{n} k_l z_l^2 \\ &- \sum_{l=1}^{n} \frac{\gamma_l}{2\mu_l} \tilde{\theta}_l^2 + \sum_{l=0}^{n} B_l + \sum_{l=1}^{n} \frac{\gamma_l}{2\mu_l} \theta_l^2 + 0.557\varepsilon \\ &+ \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0} r + \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right) \frac{x_1^2 \eta_0(x_1^2)}{\lambda_0} \end{aligned} \quad (87)$$

Let

$$c = \min \left\{ \frac{\lambda_{\min}(Q) - (n+3)}{\lambda_{\max}(P)}, 2k_i, \bar{c}, \gamma_i; i = 1, 2 \cdots n \right\} \quad (88)$$

$$b = \sum_{l=1}^{n} B_l + \sum_{l=1}^{n} \frac{\gamma_l}{2\mu_l} \theta_l^2 + 0.557\varepsilon + \frac{d_0}{\lambda_0} \quad (89)$$

Then, (87) can be rewritten as

$$\dot{V}_n \leq -cV + b + \left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right) \frac{x_1^2 \eta_0(x_1^2)}{\lambda_0} \quad (90)$$

where the sign of $\left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right) \frac{x_1^2 \eta_0(x_1^2)}{\lambda_0}$ is unknown, hence, the unknown function should be analysis in different cases.

Case 1 : For $z_1 \in \Omega_{z_1}$, according to Assumption 1, it can be seen that y_r is bounded and x_1 is bounded by construction, hence, ζ is bounded in (26), z_1 is also bounded. Due to $\eta_0(z_1^2)$ is non-negative function, so $\left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right) \frac{x_1^2 \eta_0(x_1^2)}{\lambda_0}$ is also bounded function. Let g is the bound of function. Then, (90) can be rewritten as follows

$$\dot{V}_n \leq -cV + b + g \leq -cV + h \quad (91)$$

where $h = b + g$.

Then, integrating (91) over $[0, t]$, it produces

$$0 \leq V_n(t) \leq \left(V(0) - \frac{h}{c} \right) e^{-ct} + \frac{h}{c} \quad (92)$$

Case 2 : When $z_1 \notin \Omega_{z_1}$, based on Lemma 2 and $\frac{x_1^2 \eta_0(x_1^2)}{\lambda_0} > 0$, it has

$$\left(1 - 2\tanh^2\left(\frac{z_1}{\ell}\right)\right) \frac{x_1^2 \eta_0(x_1^2)}{\lambda_0} \leq 0 \quad (93)$$

Then, (90) can be rewritten as follows

$$\dot{V}_n \leq -cV + b \quad (94)$$

Next, it can yield

$$0 \leq V_n(t) \leq \left(V(0) - \frac{b}{c} \right) e^{-ct} + \frac{b}{c} \quad (95)$$

Thus, all signals in the closed-loop control system are semi-globally uniformly ultimately bounded.

(2) According to V_1 and (92), the following equation holds:

$$\frac{1}{2} z_1^2 = \frac{1}{2} \tan^2\left(\frac{\pi\zeta}{2\rho} \right) \leq V(0) e^{-ct} + \frac{h}{c} \quad (96)$$

Transforming (96) yields

$$|\zeta| \leq \frac{2\rho}{\pi} \arctan \sqrt{2 \left(V(0) e^{-ct} + \frac{h}{c} \right)} < \rho \quad (97)$$

Therefore, the tracking error ζ can converge to a bounded range.

(3) In order to guarantee that for $\forall k \in Z^+$, the time T can satisfy $t_{k+1} - t_k > T$, it has

$$\frac{d}{dt} |E| = \frac{d}{dt} |E * E|^{\frac{1}{2}} = sign(E) \dot{E} \leq |\dot{\omega}| \quad (98)$$

According to (81), we can get

$$\dot{\omega} = \dot{\alpha}_n - \frac{\bar{m} \dot{z}_n}{\cosh^2\left(\frac{\bar{m} z_n}{\varepsilon} \right)} \quad (99)$$

Form (99), it can be concluded that $\dot{\omega}$ is bounded. Thus, there is a constant $s > 0$ satisfying $|\dot{\omega}| < s$. According to $E(t_k) = 0$ and $\lim_{t \to t_k} E(t) = -\kappa \tanh(|u(t)|) + m$, it can obtain that the lower bound of intervals time T must satisfy $T > (-\kappa \tanh(|u(t)|) + m) / s$, so the Zeno-behavior can be successfully avoided. ∎

V. SIMULATION RESULTS

Consider a nonlinear system with unmodeled dynamics as follows

$$\begin{cases} \dot{\xi} = -\xi + x_1^2 \\ \dot{x}_1 = x_1^2 e^{x_1} + x_2 + \xi x_1 x_2 \\ \dot{x}_2 = \sin(x_1 x_2) + u + \xi \sin(x_1 x_2) \\ y = x_1 \end{cases} \tag{100}$$

where $f_1(\bar{x}_1) = x_1^2 e^{x_1}$, $f_2(\bar{x}_2) = \sin(x_1 x_2)$, $\Delta_1(x, \xi, t) = \xi x_1 x_2$, $\Delta_2(x, \xi, t) = \xi \sin(x_1 x_2)$.

In order to satisfy Assumption 3 for p-subsystem in (100), consider a Lyapunov function $V(\xi) = \xi^2$, then, it yields

$$\dot{V}(\xi) \le -1.2\xi^2 + 2.5x_1^4 + 0.625 \tag{101}$$

Let $\nu_1(|\xi|) = 0.5\xi^2$, $\nu_2(|\xi|) = 2\xi^2$, $\bar{c} = 1.2, d_0 = 0.625$ and $\eta_0(|x_1|) = 2.5x_1^4$, Assumption 3 holds.

Then, define $\bar{c} = 1 \in (0, \bar{c}_0)$ and the dynamics signal r is

$$r = -\xi^2 + 2.5x_1^4 + 0.625 \tag{102}$$

Select initial conditions $[\xi(0), x_1(0), x_2(0), \hat{x}_1(0), \hat{x}_2(0), \hat{\theta}_1(0), \hat{\theta}_2(0), r(0)] = [0.5, 0.6, -0.1, 0.5, 1, 0.1, 0.1, 0]$. The design parameters $k_1 = 12, k_2 = 12, a_1 = a_2 = \mu_1 = \mu_2 = 1, \gamma_1 = \gamma_2 = 50, L_1 = 5, L_2 = 25, \bar{m} = 2, m = 0.2, \kappa = 0.5, \varepsilon = 10$. Prescribed performance function is selected as follows

$$\rho(t) = \begin{cases} \left(1 - \frac{t}{5}\right) e^{\frac{t(1+t)}{5-t}} + 0.04 & 0 \le t < 5 \\ 0.04 & t \ge 5 \end{cases} \tag{103}$$

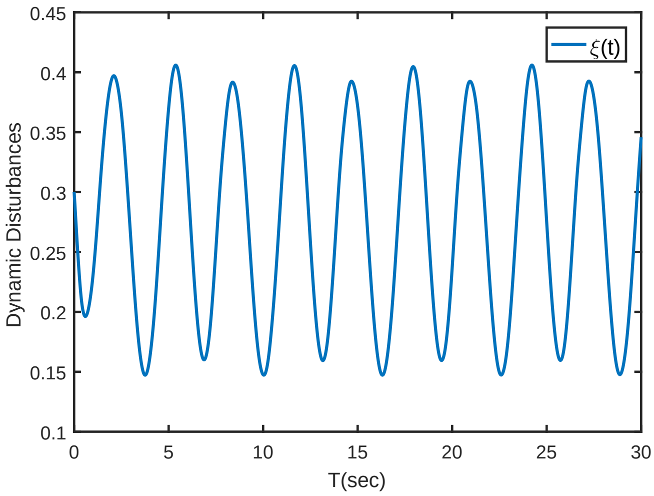

Fig. 1. Uncertain dynamic disturbances arising from unmodeled dynamics.

The dynamic disturbances is shown in Figure 1. The tracking error is shown by Figure 2, it can be seen that the proposed method (PM) can converge to a bounded range more accurately and quickly than traditional method (TM). From Figure 3, it can be seen that the system's output $y(t)$ can effectively track the reference signal $y_r(t)$. From Figure 4 and Figure 5, it can be seen that the estimation effect of the state observer can satisfy the desired requirements. The adaptive law is demonstrated by Figure 6. The event-triggered signal $u(t)$ and adaptive signal $\omega(t)$ are shown by Figure 7. Figure 8 represents the triggering instant of the three methods, in which the fixed threshold strategy triggers 328 times, the relative threshold strategy triggers 228 times,

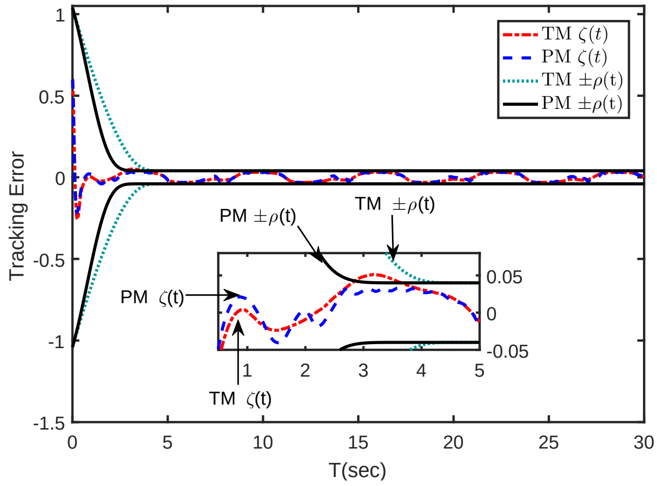

Fig. 2. Tracking errors in proposed method (PM) and traditional method (TM).

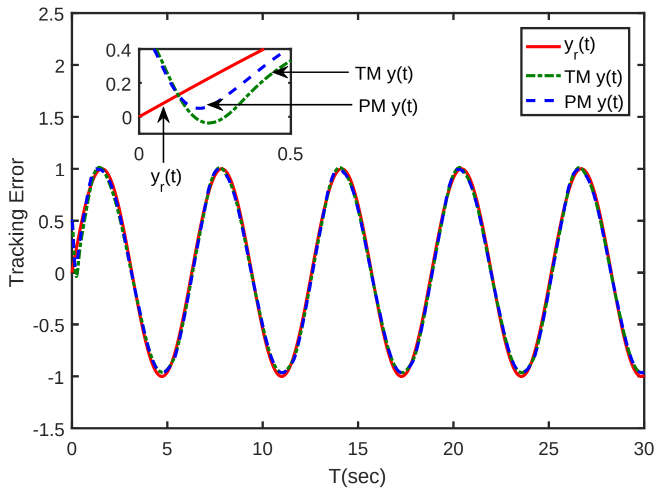

Fig. 3. System output $y(t)$ and reference signal$y_r(t)$.

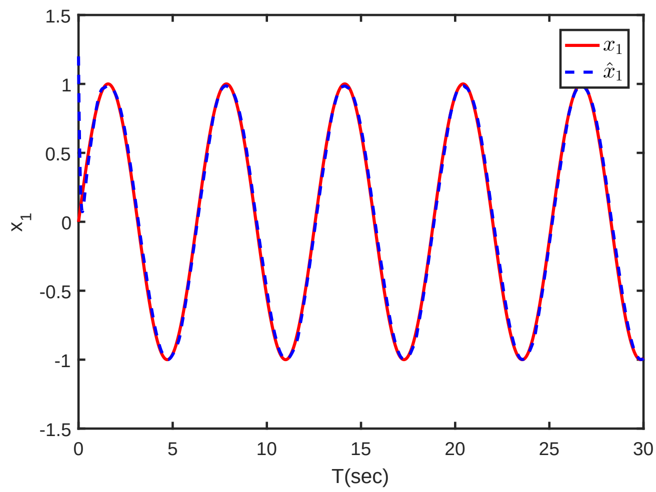

Fig. 4. The trajectories of x_1 and $\hat{x}_1$.

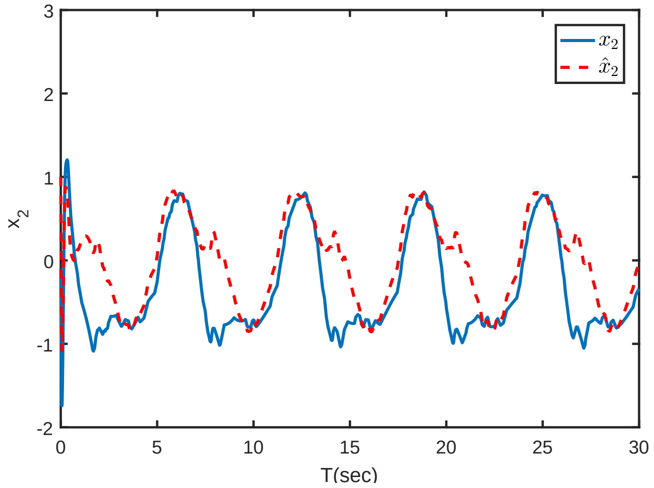

Fig. 5. The trajectories of x_2 and $\hat{x}_2$.

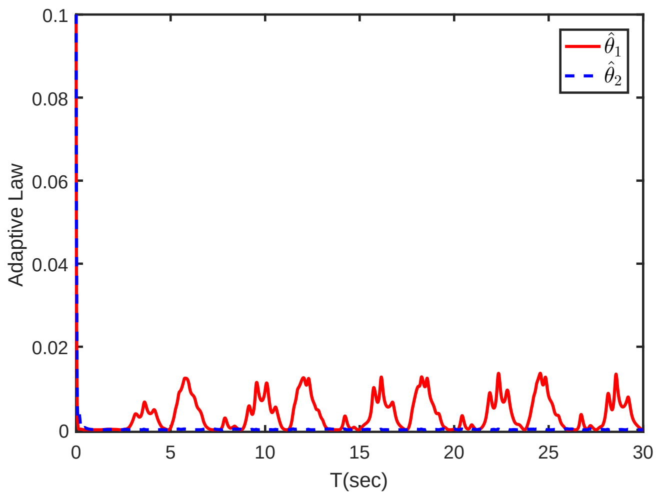

Fig. 6. Adaptive Law $\hat{\theta}_1$ and $\hat{\theta}_2$.

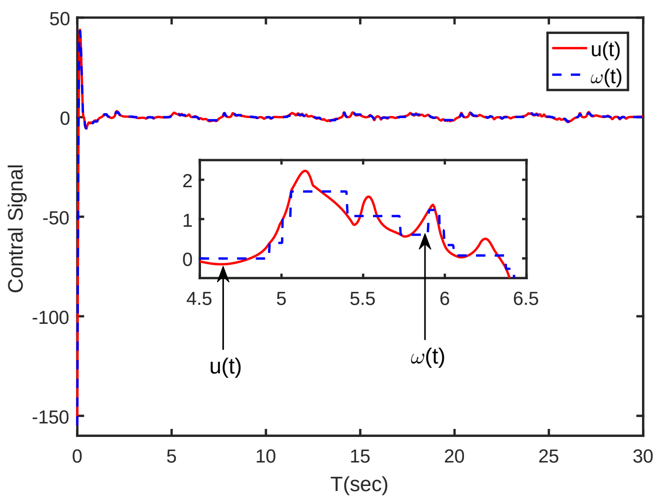

Fig. 7. Event-triggered signal $u(t)$ and adaptive signal $\omega(t)$.

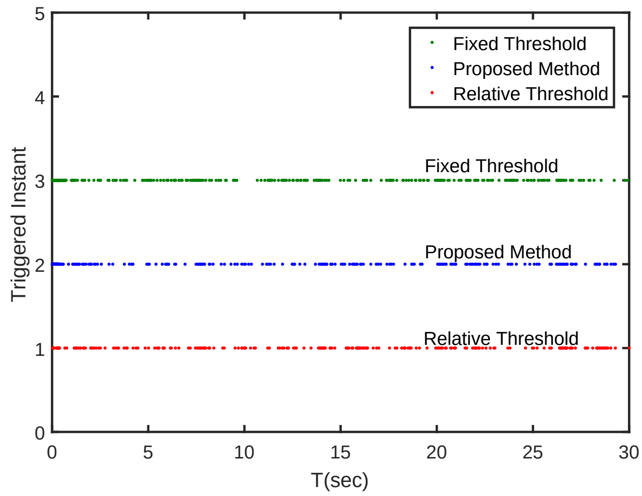

Fig. 8. Triggering instant in three methods.

and the proposed method triggers 274 times. Under the same conditions, the proposed method can save communication resources better than the fixed threshold strategy, compared with the relative threshold strategy, it will not producing a big error due to excessive control signal, so as to ensure the system performance.

VI. CONSIDERATION

In this paper, the problem of finite-time prescribed performance fuzzy adaptive event-triggered control for uncertain nonlinear systems with unmodeled dynamics has been solved. A prescribed performance fuzzy adaptive event-triggered control scheme has been designed, where prescribed performance is very important to improve the performance of system. The novel event-triggered control has accomplished the desired goal, which was to save communication resources and reduce the impact of control signal on system performance. The designed adaptive control law and event-triggered mechanism can ensure that the convergence of tracking error and all signals of the close-loop control system are bounded. This will provide better support for us to consider more complex system control in the future.

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