Research on Virtual Enterprise Partner Selection Based on Fuzzy Number Ranking Score with Distance-area Index

Junfeng Zhao, Zhiyan Chen, Xue Deng, Yechun Yu, Dayong Ye, Fengting Geng

***Abstract*—It is very crucial for virtual enterprises to select the suitable partners when facing unprecedented opportunities and challenges. Most research in this area relies on the Analytic Hierarchy Process method. However, an alternative and potentially more effective approach is proposed by using the fuzzy number ranking score model to assess virtual enterprise partners, yet there has been limited investigation in this direction. In our paper, we establish evaluation criteria for selecting partners for virtual enterprises, including capability, efficiency, cost, risk, goal congruence, and trust level. Subsequently, we develop a ranking score model based on distance-index and area-index for these six criteria in the fuzzy environment. Additionally, the score function is calculated to rank potential virtual enterprise partners by considering defuzzification value, dispersion degree, and positive-negative area indices. Furthermore, the empirical analysis verifies the effectiveness of this method to solve the problem of partner selection in virtual enterprises. The results will help decision makers to make the right selections and decisions.**

***Index Terms*—Fuzzy Numbers, Ranking Scores, Virtual Enterprises, Partners, Analytic Hierarchy Process (AHP).**

I. Introduction

Virtual enterprises integrate decentralized resources and technologies by participating in the cooperation between enterprises, so that they can quickly launch new products or services that meet the special needs of customers in the environment and the market, and achieve the purpose of sharing profits or risks. In this context, in order to better develop and innovate, the selection of virtual enterprise partners has increasingly become a key issue for enterprises. Fuzzy number ranking is an important part of the decision-making process. In the past few decades, there have been a lot of ranking methods. Fuzzy set theory is widely used in many disciplines. Because of its ability to deal with imprecise knowledge in mathematics, it is considered as a reasonable tool to deal with uncertain and fuzzy information. In the comparison of fuzzy numbers, the exploration of a general ranking method that can uniformly distinguish the size of fuzzy numbers has attracted widespread attention in the academic community. At present, the research on partner selection of virtual enterprises at home and abroad usually uses the analytic hierarchy process (AHP) for screening and sorting, and rarely uses the fuzzy number ranking score model to analyze. Therefore, the content of this study can supplement the existing research content to a certain extent, and can guide enterprises to make reasonable partner selection to a certain extent.

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A. Literature Review

Fuzzy number ranking plays an important role in the decision-making process of many applications. Jain [1] first proposed the ranking method of fuzzy numbers, and then developed many ranking methods in literatures by Chai and Das [2-3]. Chen [4] proposed a method to sort fuzzy numbers based on minimum set and maximum set. Liou and Wang [5] proposed a sorting method based on integral value index to overcome the shortcomings of Chen’ method [4]. Cheng [6] proposed a method of ranking fuzzy numbers using distance method. Chu and Tsao [7] proposed a method to sort fuzzy numbers based on the area between the centroid and the origin. However, these sorting methods have various disadvantages, such as they can not correctly sort fuzzy numbers in some cases. Ezzati et al. [8] proposed a modified sorting method based on a new symmetric fuzzy number. Unfortunately, the disadvantages related to Ezzati’s ranking method include: (1) It can't sort fuzzy numbers and their images uniformly; (2) It cannot effectively sort symmetric fuzzy numbers; (3) Non-normal fuzzy numbers cannot be sorted. Yu et al. [9] revised and improved Ezzati’s method. Chen et al. [10] proposed a fuzzy ranking method for calculating the negative side area, positive side area and centroid of generalized fuzzy numbers to evaluate the ranking scores of generalized fuzzy numbers with different left and right heights. This method provides an effective method for fuzzy risk analysis based on generalized fuzzy numbers with different left and right heights.

For the division of two kinds of ranking scoring methods, in the first category, Yager [11] and Lee [12] initially adopted the statistical center-oriented method to evaluate fuzzy numbers. Yager [11] constructed the centroid index, while Lee [12] developed the mean and standard deviation indexes. However, Cheng [13] pointed out that their limited efficiency in handling fuzzy numbers with unusually large or small data, as well as mean and divergence values. Based on the area between the centroid and the origin, Chu and Tsao [7] successfully established an area-driven ranking index. Nevertheless, due to inherent computational defects, the area index was questioned by Wang and Lee [14]. They provided some numerical examples to illustrate their counterintuitive results, and further proposed a convincing correction index to solve this problem. However, Wang and Lee's [14] area index still exhibits deficiencies in sorting correctness when encountering fuzzy numbers with the same centroid point, as indicated by Yu [15]. Nejad [18] highlighted that the area index lacks mathematical ability when the denominator value is zero. Moreover, Wang [19] pointed out that in certain special cases, the area index shows identical left, right, and total utility for fuzzy numbers. Additionally, Asady [20] noted considerable room for improvement in the area index's image ranking of fuzzy numbers.

It is worth emphasizing that the above shortcomings of the deviation-driven deviation index rekindled the development of the second category to some extent. The second category was originally proposed by Liou and Wang [5] in 1992, and they designed a ranking method that not only evaluates the fuzzy number itself, but also considers the attitude of the decision-maker to the specific purpose. In order to overcome the limitation of deviation index, Wang [17] included the decision-maker's risk attitude into the left and right areas between the fuzzy number point and the positive and negative ideal point. In order to improve the index of Liou and Wang [5], Yu and Dat [19] incorporated the decision-maker's confidence attitude into the left-right total integral value that obeys the median of fuzzy numbers. Recently, Das and Guha [20] proposed a new sorting method which aimed at solving multi-criteria decision problems by calculating the centroid of trapezoidal intuitionistic fuzzy numbers and combining expert satisfaction. However, when (a,b,c,d) is transformed into (a,a,c,d) or (a,b,c,c), or the corresponding satisfaction /dissatisfaction value is zero, then these formulae cannot work effectively.

On the surface, contrary to the multiple ranking indexes appearing in the first category, the latest ranking indexes related to the second category are still very few, which leaves wide space for further research. Based on the integration of these two categories, Nguyen [21] proposed a unified indicator, namely the product of weighted average value and weighted area, which is called centroid value and left and right area containing generalized fuzzy number, respectively. According to the comprehensive comparative study of triangular, triangular and rectangular mixed fuzzy numbers and nonlinear fuzzy numbers, the unified index shows outstanding ranking advantages in intuitive support, computational simplicity, consistency and reliability. In addition, there are some other scholars researched the application of fuzzy number sorting method. Based on the classical mean-variance (M-V) model and four kinds of ranking score formulas, Deng and Chen [22] constructed the score-variance (S-V) models. Alrefaei and Tuffaha [23] introduced an algorithm to approximate general IFN's by the n-Intuitionistic Polygonal Fuzzy Number and a new method to rank IFN. Aydin and Kahraman [24] proposed a new methodology based on fuzzy analytic hierarchy process. The proposed methodology enables multiple decision makers on evaluation.

B. Motivation

In the increasingly competitive market, customer demands are becoming more personalized and diverse. Consequently, the primary challenge for enterprises is how to effectively utilize the resources and information within and between enterprises to win the competitive advantage. Virtual enterprises offer a solution to the challenges faced by enterprises today. By employing the fuzzy number ranking score method to study the selection of virtual enterprise partners, enterprises have more perspectives and methods to measure the priority of partners. The research on the selection of virtual enterprise partners based on the fuzzy number ranking score makes it easier for people to understand the virtual enterprise, and helps more enterprises better use the management thinking of virtual enterprises to select suitable partners, so as to improve the market adaptability and competitive advantage of the enterprise itself.

C. Organization

Section I briefly describes the significance and current situation of the research, and Section II describes the basic knowledge that may be involved in this research from two aspects of fuzzy set theory and virtual enterprise partner selection model. Sections III and IV introduce the fuzzy number ranking score model based on the distance-area index and applies it to the problem of partner selection of virtual enterprises. Section V concluded the paper.

II. PRELIMINARIES

A. Fuzzy set theory

Definition 1 [25] Assuming that A is a fuzzy number, then the $\gamma-$level cut set of fuzzy number A can be expressed as $[A]^{\gamma}=\{x\in R|\mu_A(x)\geq\gamma\}=[a_1(\gamma),a_2(\gamma)]$, where $a_1(\gamma)=\min\{x\in R|\mu_A(x)\geq\gamma\}$, $a_2(\gamma)=\max\{x\in R|\mu_A(x)\geq\gamma\}$.

Definition 2 [25] If $A=(a,b;\alpha,\beta)$ is a fuzzy number, where L_A and R_A are monotonically non-increasing continuous function on $[0,1]\to[0,1]$, and, $L_A(1)=R_A(1)=0$, the membership function of A is given as

$$\mu_A=\begin{cases}L_A(\frac{a-x}{\alpha}), & a-\alpha\leq x<a,\\ 1, & a\leq x<b,\\ R_A(\frac{x-b}{\beta}), & b\leq x<b+\beta,\\ 0, & \text{others.}\end{cases}\tag{1}$$

Case 1: When L_A and R_A degenerate to linear functions, i.e., $A=(a,b;\alpha,\beta)$ is a trapezoidal fuzzy number, its membership function is

$$\mu_A=\begin{cases}1-(\frac{a-x}{\alpha}), & a-\alpha\le x<a,\\ 1, & a\le x<b,\\ 1-(\frac{x-b}{\beta}), & b\le x<b+\beta,\\ 0, & \text{others.}\end{cases} \quad (2)$$

Case 2: When L_A and R_A degenerate to linear functions and $a=b$, i.e., $A=(a;\alpha,\beta)$ is a triangular fuzzy number, its membership function is

$$\mu_A=\begin{cases}1-(\frac{a-x}{\alpha}), & a-\alpha\le x<a,\\ 1-(\frac{x-a}{\beta}), & a\le x<a+\beta,\\ 0, & \text{others.}\end{cases} \quad (3)$$

Case 3: When L_A and R_A degenerate to linear functions and $\alpha=\beta=0$, A is an interval number $A=[a,b]$.

Case 4: When L_A and R_A degenerate to linear functions and $a=b,\alpha=\beta=0$, A is a real number a.

Definition 3 [26] According to Zimmermann's definition, a fuzzy number $\tilde{A}=\left(x_1,x_2,x_3,x_4;w_{\tilde{A}}\right)$ is described as any fuzzy subset of a real straight-line R with a membership function with $\xi_{\tilde{A}}$, and

$$\xi_{\tilde{A}}(x)=\begin{cases}\xi_{\tilde{A}}^L(x), & x_1\le x\le x_2,\\ \omega, & x_2\le x\le x_3,\\ \xi_{\tilde{A}}^R(x), & x_3\le x\le x_4,\\ 0, & \text{others.}\end{cases} \quad (4)$$

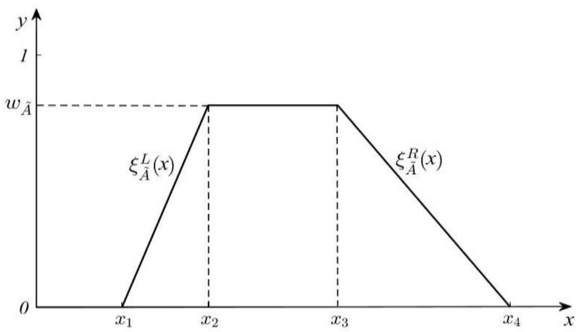

Fig.1. Fuzzy number $\tilde{A}$ and its membership function

Definition 4 [10] According to Chen's definition, the generalized trapezoidal fuzzy number defined as $\tilde{A}=\left(x_1,x_2,x_3,x_4;\mu_L,\mu_R\right)$ is shown in Fig. 2, where x_1,x_2,x_3,x_4 are real numbers, μ_L and μ_R are called the left and right height of $\tilde{A}$, and $\mu_L,\mu_R\in[0,1]$, the membership function of $\tilde{A}$ is defined as $\mu_{\tilde{A}}(x)$

$$\mu_{\tilde{A}}(x)=\begin{cases}g_1(x), & x_1\le x\le x_2,\\ g_2(x), & x_2\le x\le x_3,\\ g_3(x), & x_3\le x\le x_4,\\ 0, & \text{others.}\end{cases} \quad (5)$$

$g_1:[x_1,x_2]\to[0,1], g_2:[x_2,x_3]\to[0,1], g_3:[x_3,x_4]\to[0,1].$

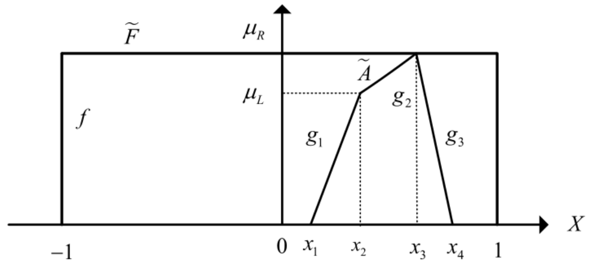

Fig. 2. Generalized fuzzy number $\tilde{A}$ and its membership function

B. Virtual Enterprise Partner Selection Model

This section focuses on a brief discussion of several aspects of virtual enterprise partners, including their selection criteria and steps.

a. Evaluation reference criteria

Participating companies access business opportunities and participate in business activities through the information infrastructure. General principles of partner selection in virtual enterprises include: the principle of complementary core competencies, the principle of rapid response, the principle of total cost effectiveness, the principle of risk minimization, the principle of consistency of partner goals, and the principle of mutual trust and loyalty. The evaluation reference criteria of the virtual partner enterprise include many factors according to the specific situation. Considering the characteristics of the virtual cooperative style, the reference criteria are: capacity- B, quality- Q, cost- C, risk- R, objective congruence- O, and trust- T.

b. Basic steps

The managers of virtual enterprises should form a reasonable comprehensive evaluation mechanism, and select suitable partners according to the market and realistic environmental changes. The basic steps of virtual enterprise partner selection are as follows: (1) initial screening of virtual enterprise industry partners; (2) selection of virtual enterprise partners; (3) combination and optimization of virtual enterprise partner selection; (4) tracking evaluation and adjustment of virtual enterprise partners.

III. Virtual Enterprise Partner Selection with Fuzzy Number Ranking Score Based on Distance-Index and Area-Index

In this section, we employ the fuzzy number ranking score function to illustrate the strengths and weaknesses of virtual business partners. Additionally, we conduct a corresponding empirical study and compare and analyze the results obtained from the model. The proposed ranking method can also be applied to different decision problems in similar settings.

A. Based on distance-index of normal fuzzy number ranking score

a. Distance index

Chen and Chen [27] proposed a generalized fuzzy number ranking method with two distance indices (defuzzification value and dispersion degree) in 2009.

The fuzzy number $\tilde{A}=\left(x_1,x_2,x_3,x_4;w_{\tilde{A}}\right)$ in Fig. 1 can be normalized as

$$\tilde{A}^*=\left(\frac{x_1}{k},\frac{x_2}{k},\frac{x_3}{k},\frac{x_4}{k};w_{\tilde{A}}\right)=\left(x_1^*,x_2^*,x_3^*,x_4^*;w_{\tilde{A}}\right), \tag{6}$$

where $k=\max_i\left(\left[\left|x_i\right|\right],1\right)$, $\left|x_i\right|$ is the absolute value of x_i, and is the upper bound of $\left|x_i\right|$. The defuzzification value and dispersion degree are defined as

$$x_{\tilde{A}^*}=\frac{x_1^*+x_2^*+x_3^*+x_4^*}{4} \tag{7}$$

and

$$STD_{\tilde{A}^*}=\sqrt{\frac{\sum_{i=1}^{4}\left(x_i^*-x_{\tilde{A}^*}\right)^2}{4-1}} \tag{8}$$

Where $STD_{\tilde{A}^*}\in[0,1.1547]$. Obviously, the dispersion degree of an accurate value is 0. When $w\in[0,1]$, the dispersion of fuzzy number $(-1,-1,1,1;w)$ is 1.1547.

b. Construction steps of score function

Step 1: Standardize by Formula (6).
Step 2: Defuzzificate by Formula (7).
Step 3: Calculate the dispersion degree by Formula (8).
Step 4: Construct score function

$$Score\left(\tilde{A}^*\right)=\frac{x_{\tilde{A}^*}\times w_{\tilde{A}}}{1+STD_{\tilde{A}^*}}, \tag{9}$$

where $Score\left(\tilde{A}^*\right)\in[-1,1]$. The bigger $Score\left(\tilde{A}^*\right)$, the better the ranking $\tilde{A}^*$.

B. Based on distance-area index of generalized fuzzy number ranking score

a. Distance index

Jiang [28] proposed to sort the generalized fuzzy numbers with different left and right heights in 2015 by two distance indexes (defuzzification and dispersion degree).

The fuzzy number $\tilde{A}_i=\left(x_{i1},x_{i2},x_{i3},x_{i4};\mu_{iL},\mu_{iR}\right)$ in Fig. 3 can be standardized as

$$\tilde{A}_i^*=\left(\frac{x_{i1}}{k},\frac{x_{i2}}{k},\frac{x_{i3}}{k},\frac{x_{i4}}{k};\mu_{iL},\mu_{iR}\right)=\left(x_{i1}^*,x_{i2}^*,x_{i3}^*,x_{i4}^*;\mu_{iL},\mu_{iR}\right), \tag{10}$$

where $k=\max_{i,j}\left(\left[\left|x_{ij}\right|\right],1\right)$, $\left|x_{ij}\right|$ is the absolute value of x_{ij}, $\left[\left|x_{ij}\right|\right]$ is the upper bound of $\left|x_{ij}\right|$, $1\le i\le n$, $1\le j\le 4$.

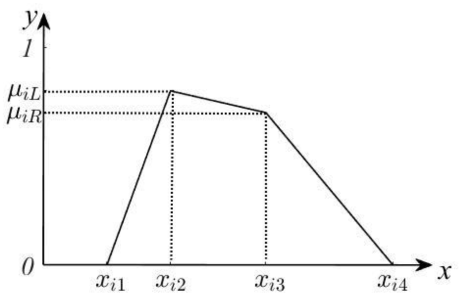

Fig. 3. The fuzzy number $\tilde{A}$

The defuzzification value and dispersion degree are as defined as

$$x_{\tilde{A}_i^*}=\frac{x_1^*+x_2^*+x_3^*+x_4^*}{4}, \tag{11}$$

and

$$STD_{\tilde{A}_i^*}=\sqrt{\frac{\sum_{i=1}^{4}\left(x_i^*-x_{\tilde{A}_i^*}\right)^2}{4-1}}, \tag{12}$$

where $STD_{\tilde{A}_i^*}\in[0,1.1547]$. Obviously, the dispersion degree of an accurate value is 0. When $\mu_{iL},\mu_{iR}\in[0,1]$, the dispersion of fuzzy number $\left(-1,-1,1,1;\mu_{iL},\mu_{iR}\right)$ is 1.1547.

b. Area index

Suppose that there are n generalized fuzzy numbers $\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n$ to be sorted, $\tilde{A}_i=\left(x_{i1},x_{i2},x_{i3},x_{i4};\mu_{iL},\mu_{iR}\right)$, $1\le i\le n$, Chen [10] proposed the following method for calculating the area.

(1) Left-negative area LN_i

Case 1: If $\mu_L\ge\mu_R$, then

$$LN_i=\int_{-1}^{x_2}f(x)dx-\int_{x_1}^{x_2}g_1(x)dx \tag{13}$$

Case 2: If $\mu_L<\mu_R$, then

$$LN_i=\int_{-1}^{x_3}f(x)dx-\left(\int_{x_1}^{x_2}g_1(x)dx+\int_{x_2}^{x_3}g_2(x)dx\right) \tag{14}$$

(2) Right-negative area RN_i

Case 1: If $\mu_L\le\mu_R$, then

$$RN_i=\int_{-1}^{x_3}f(x)dx+\int_{x_3}^{x_4}g_3(x)dx \tag{15}$$

Case 2: I f $\mu_L<\mu_R$, then

$$RN_i=\int_{-1}^{x_2}f(x)dx+\left(\int_{x_2}^{x_3}g_2(x)dx+\int_{x_3}^{x_4}g_3(x)dx\right) \tag{16}$$

(3) Left-positive area LP_i

Case 1: If $\mu_L\ge\mu_R$, then

$$LP_i=\int_{x_2}^{1}f(x)dx+\int_{x_1}^{x_2}g_1(x)dx \tag{17}$$

Case 2: If $\mu_L<\mu_R$, then

$$LP_i=\int_{x_3}^{1}f(x)dx+\left(\int_{x_1}^{x_2}g_1(x)dx+\int_{x_2}^{x_3}g_2(x)dx\right) \tag{18}$$

(4) Right-positive area RP_i

Case 1: If $\mu_L\le\mu_R$, then

$$RP_i=\int_{x_3}^{1}f(x)dx-\int_{x_3}^{x_4}g_3(x)dx \tag{19}$$

Case 2: If $\mu_L>\mu_R$, then

$$RP_i=\int_{x_2}^{1}f(x)dx-\left(\int_{x_2}^{x_3}g_1(x)dx+\int_{x_3}^{x_4}g_3(x)dx\right) \tag{20}$$

(5) Negative side area M_i

The negative side area M_i is the sum of the left and right negative side areas:

$$M_i=LN_i+RN_i \tag{21}$$

(6) Positive side area N_i

The positive side area N_i is the sum of the left and right positive side areas:

$$N_i=LP_i+RP_i \tag{22}$$

c. Model construction

Step1: Standardize by Formula (10).

Step2: Calculate left-positive, left-negative, right-positive, and right-negative areas by Formulae (13)-(20).

Step3: Calculate positive and negative areas by Formulae (21)-(22).

Step4: Introduce the reference fuzzy number. Define the reference fuzzy number $\tilde{A}_0 = (0,0,0,0;1,1)$, and calculate the negative area and positive area of the reference fuzzy number including LN_0, RN_0, LP_0 and RP_0.

Step5: Calculate the positive and negative areas of the reference fuzzy number. Define the negative area $M_0 = LN_0 + RN_0$ and positive area $N_0 = LP_0 + RP_0$.

Step6: Calculate the dispersion degree of generalized fuzzy numbers. Calculate the dispersion degree $STD_{\tilde{A}_i^*}$ of standardized generalized fuzzy numbers $\tilde{A}_i^* = (x_{i1}^*, x_{i2}^*, x_{i3}^*, x_{i4}^*; \mu_{iL}, \mu_{iR})$ by Formulae (11) and (12).

Step7: Calculate the score function value of generalized fuzzy number. Calculate the ranking score of $\tilde{A}_i^* = (x_{i1}^*, x_{i2}^*, x_{i3}^*, x_{i4}^*; \mu_{iL}, \mu_{iR})$ by

$$Score(\tilde{A}_i^*) = \frac{M_i - N_i}{M_0 + N_0 + STD_{\tilde{A}_i^*}} \tag{23}$$

Obviously, we can calculate $M_0 + N_0 = 4$, therefore,

$$Score(\tilde{A}_i^*) = \frac{M_i - N_i}{4 + STD_{\tilde{A}_i^*}} \tag{24}$$

The larger the value of $Score(\tilde{A}_i^*)$, the better the sorting order $\tilde{A}_i^*$.

IV. Empirical analysis

A. Virtual enterprise partner selection

Suppose an enterprise needs to assess and rank the performance of its virtual enterprise partners. After an initial screening process, five partners, denoted as $A_i, (i = 1,...,5)$, have been selected for sorting and scoring. The evaluation criteria consist of the following factors: ability (*B*), efficiency (*Q*), cost (*C*), risk (*R*), target consistency (*O*), and trust (*T*). Decision-makers are assumed to utilize language levels denoted as $S = \{\text{VP,P,F,G,VG}\}$, where VP, P, F, G, and VG signify "Very Poor", "Poor", "Fair", "Good" and "Very Good", respectively. The corresponding trapezoidal and triangular fuzzy numbers are detailed in Table I.

Table I The language level value of trapezoidal fuzzy number and triangular fuzzy number

Level value	Trapezoidal fuzzy number	Triangular fuzzy number
VP = Very poor	$(0,0,0.2,0.4;1,1)$	$(0,0,0.25)$
P = Poor	$(0,0.2,0.4,0.6;1,1)$	$(0,0.25,0.5)$
F = Fair	$(0.2,0.4,0.6,0.8;1,1)$	$(0.25,0.5,0.75)$
G = Good	$(0.4,0.6,0.8,1;1,1)$	$(0.5,0.75,1)$
VG = Very good	$(0.6,0.8,1,1;1,1)$	$(0.75,1,1)$

Three decision-making experts have provided their evaluations, as shown in Table II. For instance, both decision experts D_1 and D_2 assess the factor *B* of candidate A_1 as "Fair", while decision expert D_3 regards the factor *B* of candidate A_1 as "Good". The trapezoidal fuzzy number representing "Fair" is $(0.2,0.4,0.6,0.8;1,1)$, and for "Good" is $(0.4,0.6,0.8,1;1,1)$. Consequently, the trapezoidal fuzzy number for factor B of candidate A_1 is calculated as $\frac{2}{3}\times(0.2,0.4,0.6,0.8;1,1)+\frac{1}{3}\times(0.4,0.6,0.8,1;1,1)$, resulting in $(0.267,0.467,\ 0.667,0.867)$.

In Table II, the significance of the six elements is perceived differently by decision-makers. Their relative importance is assessed using language weight sets denoted as $Q = \{\text{UI,OI,I,VI,AI}\}$, where the terms UI, I, OI, VI, and AI correspond to "Unimportant", "Ordinarily Important", "Important", "Very Important" and "Absolutely Important", respectively. The corresponding trapezoidal and triangular fuzzy numbers are detailed in Table III. The assessment of element importance, as determined by experts, is presented in Table IV.

For instance, decision experts D_2 and D_3 both concur that factor *B* is deemed "Absolutely Important", while decision expert D_1 considers it "Very Important". The trapezoidal fuzzy number representing "Absolutely Important" is $(0.6,0.8,1,1;1,1)$, and for "Very Important" is $(0.4,0.6,0.8,1;1,1)$. Consequently, the trapezoidal fuzzy number for factor *B* of the candidate is calculated as $\frac{2}{3}\times(0.6,0.8,1,1;1,1)+\frac{1}{3}\times(0.4,0.6,0.8,1;1,1)$, resulting in $(0.533,0.733,0.933,1.00)$.

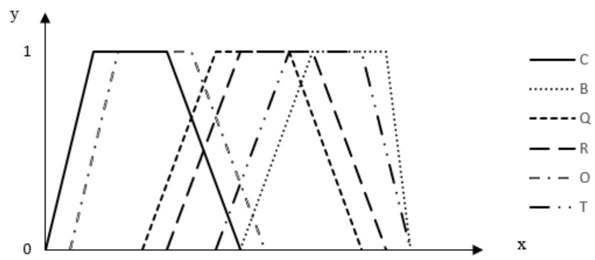

Fig.4. Trapezoidal fuzzy number results of element importance evaluation

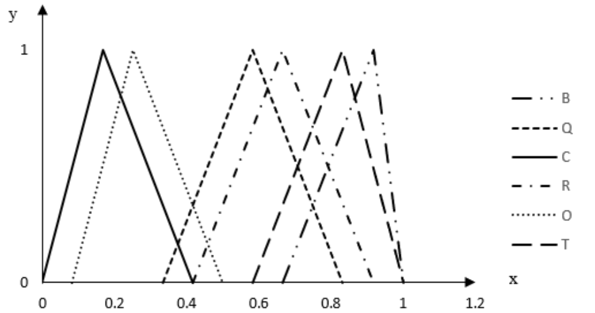

Fig. 5. Triangular fuzzy number results of element importance assessment

Table II Evaluation of decision experts

Factor	Candidate	Decision expert			r_{ij} (Trapezoid)	r_{ij} (Triangle)
		D_1	D_2	D_3		
B	A_1	F	F	G	(0.267,0.467, 0.667,0.867)	(0.333,0.583,0.833)
	A_2	VG	G	G	(0.467, 0.667,0.867,1.000)	(0.583,0.833,1.000)
	A_3	P	F	P	(0.067,0.267,0.467,0.667)	(0.083,0.333,0.583)
	A_4	VP	P	F	(0.067,0.200,0.400,0.600)	(0.083,0.250,0.500)
	A_5	G	G	F	(0.333,0.533,0.733,0.933)	(0.147,0.667,0.917)
Q	A_1	VG	VG	F	(0.467, 0.667,0.867,0.933)	(0.583,0.833,0.917)
	A_2	F	G	VG	(0.400,0.600,0.800,0.933)	(0.500,0.750,0.917)
	A_3	P	G	G	(0.267,0.467,0.667,0.867)	(0.333,.0583,0.833)
	A_4	P	VP	P	(0.000,0.133,0.333,0.533)	(0.000,0.167,0.417)
	A_5	G	P	F	(0.200,0.400,0.600,0.800)	(0.250,0.500,0.750)
C	A_1	F	G	F	(0.267,0.467, 0.667,0.867)	(0.333,0.583,0.833)
	A_2	VG	G	F	(0.400,0.600,0.800,0.933)	(0.500,0.750,0.917)
	A_3	F	F	G	(0.267,0.467, 0.667,0.867)	(0.333,0.583,0.833)
	A_4	G	G	VG	(0.467, 0.667,0.867,1.000)	(0.583,0.833,1.000)
	A_5	VG	VG	G	(0.533,0.733,0.933,1.000)	(0.667,0.917,1.000)
R	A_1	G	G	F	(0.333,0.533,0.733,0.933)	(0.147,0.667,0.917)
	A_2	F	P	P	(0.067,0.267,0.467,0.667)	(0.083,0.333,0.583)
	A_3	VG	G	VG	(0.533,0.733,0.933,1.000)	(0.667,0.917,1.000)
	A_4	F	P	VP	(0.067,0.200,0.400,0.600)	(0.083,0.250,0.500)
	A_5	P	F	P	(0.067,0.267,0.467,0.667)	(0.083,0.333,0.583)
O	A_1	G	G	G	(0.400,0.600,0.800,1.000)	(0.500,0.750,1.000)
	A_2	VG	G	F	(0.400,0.600,0.800,0.933)	(0.500,0.750,0.917)
	A_3	VG	F	VG	(0.467,0.667,0.867,0.933)	(0.583,0.833,0.917)
	A_4	F	F	F	(0.200,0.400,0.600,0.800)	(0.250,0.500,0.750)
	A_5	P	G	F	(0.067,0.267,0.467,0.667)	(0.250,0.500,0.750)
T	A_1	P	P	VP	(0.000,0.133,0.333,0.533)	(0.000,0.167,0.417)
	A_2	VG	G	G	(0.467, 0.667,0.867,1.000)	(0.583,0.833,1.000)
	A_3	G	F	VP	(0.200,0.333,0.533,0.733)	(0.250,0.417,0.667)
	A_4	P	F	VG	(0.267,0.467,0.667,0.800)	(0.333,0.583,0.750)
	A_5	VP	G	P	(0.133,0.267,0.467,0.667)	(0.167,0.333,0.583)

Table III The language weight of trapezoidal fuzzy number and triangular fuzzy number

Level value	Trapezoidal fuzzy number	Triangular fuzzy number
UI = Un-important	$(0,0,0.2,0.4;1,1)$	$(0,0,0.25)$
OI = Ordinary important	$(0,0.2,0.4,0.6;1,1)$	$(0,0.25,0.5)$
I = Important	$(0.2,0.4,0.6,0.8;1,1)$	$(0.25,0.5,0.75)$
VI = Very important	$(0.4,0.6,0.8,1;1,1)$	$(0.5,0.75,1)$
AI = Absolute important	$(0.6,0.8,1,1;1,1)$	$(0.75,1,1)$

Table IV Assessment of element importance

Factor	Decision expert			w_j (Trapezoid)	w_j (Triangle)
	D_1	D_2	D_3		
B	VI	AI	AI	(0.533,0.733,0.933,1.00)	(0.667,0.917,1.000)
Q	I	VI	I	(0.267,0.467,0.667,0.867)	(0.333,0.583,0.833)
C	UI	OI	OI	(0.000,0.133,0.333,0.533)	(0.000,0.167,0.417)
R	I	VI	VI	(0.333,0.533,0.733,0.933)	(0.417,0.667,0.917)
O	OI	UI	I	(0.067,0.200,0.400,0.600)	(0.083,0.250,0.500)
T	VI	VI	AI	(0.467,0.667,0.867,1.000)	(0.583,0.833,1.000)

The results of the evaluation of element importance using trapezoidal and triangular fuzzy numbers are depicted in Fig. 4 and Fig. 5, respectively. In Fig. 4, it is evident that the trapezoidal fuzzy number representing factor *C* is situated on the leftmost portion of the number line, indicating the lowest language level value for this factor. Conversely, the trapezoidal fuzzy number for factor *B* is positioned on the far right of the number line, suggesting the highest language level value for factor *B*. This observation remains consistent when examining the images of the triangular fuzzy numbers in Fig. 5.

B. Application model

(1) Standardized fuzzy number

In order to make the program simpler and more practical, all fuzzy numbers are defined between the interval $[0,1]$ in the examples. Therefore, normalized calculation is no longer required.

(2) Determine the weighted fuzzy decision matrix

We can compute the weighted fuzzy decision matrix using the method proposed by Chu and Lin [29], which allows us to obtain the fuzzy number ratings for partner candidates, as presented in Table V. For each cooperative partner A_i $(i=1,\ldots,5)$, the weighted rating described by a trapezoid fuzzy number is denoted as $\frac{1}{6}\sum_{j=1}^{6} r_{ij}w_j$.

For example, the weighted rating described by trapezoid fuzzy number of cooperative partner A_1 is $\frac{1}{6}\sum_{j=1}^{6} r_{1j}w_j$ = $\frac{1}{6}$ ((0.267,0.467,0.667,0.867) × (0.533,0.733,0.933,1.00) + (0.467,0.667,0.867,0.933) × (0.267,0.467,0.667,0.867) + (0.267,0.467,0.667,0.867) × (0.000,0.133,0.333,0.533) + (0.333,0.533,0.733,0.933) × (0.333,0.533,0.733,0.933) + (0.400,0.600,0.800,1.000) × (0.067,0.200,0.400,0.600) + (0.000,0.133,0.333,0.533) × (0.467,0.667,0.867,1.000)) = (0.067,0.202,0.428,0.690).

The trapezoidal fuzzy number representation of each partner's weighted rating is depicted in Fig. 6, while the triangular fuzzy number representation is shown in Fig. 7. Observing Fig. 6, we notice that the image of the trapezoidal fuzzy number for cooperative partner A4 is situated on the leftmost part of the number line, indicating the lowest weighted rating. Conversely, the image of cooperative partner A2's trapezoidal fuzzy number is positioned on the far right of the number line, indicating the highest weighted rating. This conclusion holds true when examining the triangular fuzzy number representation in Fig. 7.

Table V Weighted ratings of each partner

Cooperative partner	Weighted rating (Trapezoid)	Weighted rating (Triangle)
A_1	(0.067,0.202,0.428,0.690)	(0.087,0.315,0.617)
A_2	(0.104,0.260,0.504,0.748)	(0.162,0.405,0.690)
A_3	(0.068,0.204,0.433,0.684)	(0.106,0.318,0.611)
A_4	(0.033,0.133,0.333,0.635)	(0.051,0.207,0.475)
A_5	(0.053,0.175,0.388,0.642)	(0.057,0.280,0.575)

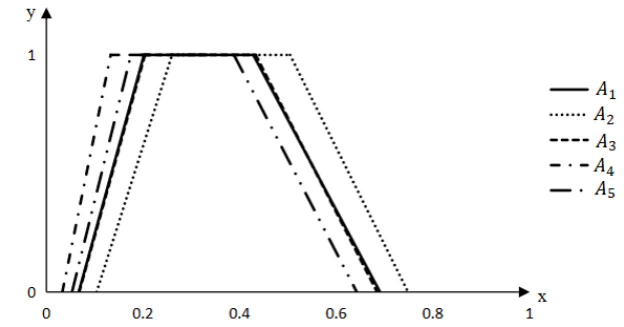

Fig. 6. Trapezoidal fuzzy number representation of each partner's weighted rating

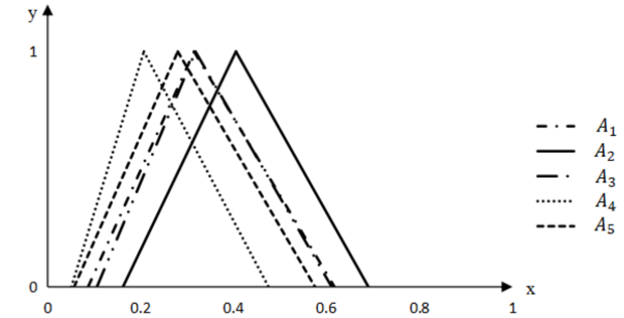

Fig. 7. Triangular fuzzy number representation of each partner's weighted rating

(3) Defuzzification

a) Fuzzy number ranking scoring model based on distance index

The final score values of the five partner candidates are shown in Table VI.

According to Table VI, the ranking order of partner candidates is $A_2 > A_3 > A_1 > A_5 > A_4$. The best candidate is A_2 with the highest score.

b) Fuzzy number ranking scoring model based on distance-area index

The final score values of the five partner candidates are shown in Table VII.

According to Table VII, the ranking order of partner candidates is $A_2 > A_3 > A_1 > A_5 > A_4$. The best candidate is A_2 with the highest score.

c) Comparative analysis

We compare the two methods, and the results are presented in Table VIII. It can be observed that the sorting results obtained from both methods are identical, indicating consistency and practicality in their results. However, for the same ranking order, the range of differences in the final scores obtained by the ranking scores of triangular fuzzy numbers is larger compared to those obtained by the trapezoidal fuzzy number index. When employing the same index, the disparity between the final scores of the fuzzy number ranking score model based on the distance-area index is greater than that of the fuzzy number ranking score model based solely on the distance index. This implies that the fuzzy number ranking score model based on the distance-area index yields superior results for trapezoidal fuzzy number ranking, particularly for similar trapezoidal fuzzy numbers, where better outcomes are achieved.

Table VI Score of each partner

Cooperative partner	Score (Trapezoid)	Ranking order	Score (Triangle)	Ranking order
A_1	0.2724	3	0.2683	3
A_2	0.3151	1	0.3314	1
A_3	0.2734	2	0.2752	2
A_4	0.2240	5	0.2012	5
A_5	0.2499	4	0.2413	4

Table VII Score of each partner

Cooperative partner	Score (Trapezoid)	Ranking order	Score (Triangle)	Ranking order
A_1	0.3246	3	0.3163	3
A_2	0.3774	1	0.3942	1
A_3	0.3253	2	0.3216	2
A_4	0.2659	5	0.2251	5
A_5	0.2954	4	0.2830	4

Table VIII Comparison of the two methods

Cooperative partner	Based on distance index				Based on distance-area index			
	Score (Trapezoid)	Ranking order	Score (Triangle)	Ranking order	Score (Trapezoid)	Ranking order	Score (Triangle)	Ranking order
A_1	0.2724	3	0.2683	3	0.3246	3	0.3163	3
A_2	0.3151	1	0.3314	1	0.3774	1	0.3942	1
A_3	0.2734	2	0.2752	2	0.3253	2	0.3216	2
A_4	0.2240	5	0.2012	5	0.2659	5	0.2251	5
A_5	0.2499	4	0.2413	4	0.2954	4	0.2830	4

V. CONCLUSION

This paper introduces two fuzzy number ranking score models based on the inherent index of fuzzy numbers, which are including one model based solely on the distance-index and the other model based on the distance-area index. We apply the two models to the research of partner selection in virtual enterprises, carry out empirical analysis, and analyze the results. In our study, we consider capability, efficiency, cost, risk, goal congruence, and trust level as the evaluation reference criteria. The fuzzy number ranking score model has proven to be an effective method for assessing virtual enterprise partners for virtual enterprises. Employing both distance-index and area-index, we develop the ranking score model and present empirical analyses to address partner selection challenges in virtual enterprises. The findings from this study can offer valuable insights for decision-makers in making selections and decisions.

REFERENCES

[1] R. Jain, "Decision making in the presence of fuzzy variables," *IEEE Transactions on Systems Man and Cybernetics*, vol. 6, no. 10, pp. 698-703, 1976.

[2] K. C. Chai, K. M. Tay and C. P. Lim, "A new method to rank fuzzy numbers using Dempster-Shafer theory with fuzzy targets," *Information Sciences*, vol. 346, pp. 302-317, 2016.

[3] S. Das and D. Guha, "A centroid-based ranking method of trapezoidal intuitionistic fuzzy numbers and its application to MCDM problems," *Fuzzy Information and Engineering*, vol. 8, no. 1, pp. 41-74, 2016.

[4] S. H. Chen, "Ranking fuzzy numbers with maximizing set and minimizing set," *Fuzzy Sets and Systems*, vol. 17, no. 2, pp. 113-129, 1985.

[5] T. S. Liou and M. J. J. Wang, "Ranking fuzzy numbers with integral value," *Fuzzy Sets and Systems*, vol. 50, no. 3, pp. 247-255, 1992.

[6] C. H. Cheng, "A new approach for ranking fuzzy numbers by distance method," *Fuzzy Sets and Systems*, vol. 95, no. 3, pp. 307-317, 1998.

[7] T. C. Chu and C. T. Tsao, "Ranking fuzzy numbers with an area between the centroid point and original point," *Computers & Mathematics with Applications*, vol. 43, no. 1-2, pp. 111-117, 2002.

[8] R. Ezzati, T. Allahviranloo, S. Khezerloo and M. Khezerloo, "An approach for ranking of fuzzy numbers," *Expert Systems with Applications*, vol. 39, no. 1, pp. 690-695, 2012.

[9] V. F. Yu, L. H. Van, L. Q. Dat, H. T. X. Chi, S. Y. Chou and T. T. T. Duong, "Analyzing the Ranking Method for Fuzzy Numbers in Fuzzy Decision Making Based on the Magnitude Concepts," *International Journal of Fuzzy Systems*, vol. 19, no. 5, pp. 1279-1289, 2017.

[10] S. M. Chen, A. Munif, G. S. Chen, H. C. Liu and B. C. Kuo, "Fuzzy risk analysis based on ranking generalized fuzzy numbers with different left heights and right heights," *Expert Systems with Applications*, vol. 39, no. 7, pp. 6320-6334, 2012.

[11] R. R. Yager, "A procedure for ordering fuzzy subsets of the unit interval," *Information Sciences*, vol. 24, no. 2, pp. 143-161, 1981.

[12] E. S. Lee and R. J. Li, "Comparison of fuzzy numbers based on the probability measure of fuzzy events," *Computers & Mathematics with Applications*, vol. 15, no. 10, pp. 887-896, 1988.

[13] C. H. Cheng, "A new approach for ranking fuzzy numbers by distance method," *Fuzzy Sets and Systems*, vol. 95, no. 3, pp. 307-317, 1998.

[14] Y. J. Wang and H. S. Lee, "The revised method of ranking fuzzy numbers with an area between the centroid and original points," *Computers and Mathematics with Applications*, vol. 55, no. 9, 2033-2042, 2008.

[15] V. F. Yu, H. T. X. Chi, L. Q. Dat, P. N. K. Phuc and C. W. Shen, "Ranking generalized fuzzy numbers in fuzzy decision making based on the left and right transfer coefficients and areas," *Applied Mathematical Modelling*, vol. 37, no. 16-17, pp. 8106-8117, 2013.

[16] A. M. Nejad and M. Mashinchi, "Ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy number," *Computers & Mathematics with Applications*, vol. 61, no. 2, pp. 431-442, 2011.

[17] Y. M. Wang and Y. Luo, "Area ranking of fuzzy numbers based on positive and negative ideal points," *Computers & Mathematics with Applications*, vol. 58, no. 9, pp. 1769-1779, 2009.

[18] B. Asady, "The revised method of ranking LR fuzzy number based on deviation degree," *Expert Systems with Applications*, vol. 37, no. 7, pp. 5056-5060, 2010.

[19] V. F. Yu and L. Q. Dat, "An improved ranking method for fuzzy numbers with integral values," *Applied Soft Computing Journal*, vol. 14, pp. 603-608, 2014, doi: https://doi.org/10.1016/j.asoc.2013.10.012

[20] S. Das and D. Guha, "A centroid-based ranking method of trapezoidal intuitionistic fuzzy numbers and its application to MCDM problems," *Fuzzy Information and Engineering*, vol. 8, no. 1, pp. 41-74, 2016.

[21] T. L. Nguyen, "Methods in ranking fuzzy numbers: a unified index and comparative reviews," *Complexity*, 2017, 3083745, doi: https://doi.org/10.1155/2017/3083745

[22] X. Deng and J. X. Chen, "Comparison and Analysis of Novel Score-Variance Portfolio Models based on Methods for Ranking Fuzzy Numbers," *IAENG International Journal of Applied Mathematics*, vol. 51, no. 3, pp. 669-679, 2021.

[23] M. H. Alrefaei and M. Z. Tuffaha, "New Operations on n-Intuitionistic Polygonal Fuzzy Numbers," *IAENG International Journal of Applied Mathematics*, vol. 51, no. 3, pp. 712-719, 2021.

[24] S. Aydin and C. Kahraman, "Evaluation of E-commerce website quality using fuzzy multi-criteria decision making approach," *IAENG International Journal of Computer Science*, vol. 39, no. 1, pp. 64-70, 2012.

[25] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no.3, pp. 338-353, 1965.

[26] H. Zimmermann, "Fuzzy Set Theory and Its Applications," Kluwer Academic Publishers, Boston, Mass, USA, 3rd edition, 2001.

[27] S. M. Chen and J. H. Chen, "Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads," *Expert Systems with Applications*, vol. 36, no. 3, pp. 6833-6842, 2009.

[28] W. Jiang, Y. Luo, X. Y. Qin and J. Zhan, "An improved method to rank generalized fuzzy numbers with different left heights and right heights," *Journal of Intelligent & Fuzzy Systems*, vol. 28, no. 5, pp. 2343-2355, 2015.

[29] T. C. Chu and Y. C. Lin, "A fuzzy TOPSIS method for robot selection," *International Journal of Advanced Manufacturing Technology*, vol. 21, no. 4, pp. 284–290, 2003.