Analysis of Magnetic Effect on UCM Fluid Flow between a Stationary and a Moving Plate

Nityanand P. Pai, Devaki B., Sampath Kumar V. S. and Pareekshith G. Bhat

Abstract—**The analysis of MHD UCM fluid flow between a pair of rectangular plates with the top plate moving either towards or apart from the stationary bottom porous plate is presented in this work. By employing similarity transformations, the governing equations of momentum are transformed into non-linear ordinary differential equations. An approximate analytical solution is achieved by adopting the homotopy perturbation technique. The aim of this work is to determine the velocity profile and coefficients of skin friction for various values of different physical parameters. It has been found that as the magnetic parameter enhances, the squeezing flow retards in injection and suction cases. It is also observed that the velocity field declines in the core region with a rise in the porosity parameter. This theoretical study is helpful in the processing of visco-elastic polymers in industry.**

Index Terms—**Upper convected Maxwell fluid, MHD flow, Squeezing flow, Homotopy Perturbation Method.**

I. Introduction

FLUID mechanics is one of the oldest disciplines of physics and a basis for comprehending numerous other branches of the applied sciences and engineering. It is the study of motion and equilibrium of fluids. Almost all engineering disciplines, as well as astrophysics, physical chemistry, plasma chemistry, geophysics, meteorology, biology, and biomedicine, are interested in the topic [1]. Fluid mechanics has become a crucial foundational subject in engineering science as a result of the progress made in mechanical, aeronautical, and chemical engineering during the past several decades, on the one hand, and the exploration of space, on the other. Franz Durst [2] explained the significance of fluid flows to science and engineering, as well as to heat and mass movement in nature during a brief survey of the history of fluid mechanics. It is stated that the fundamental equations were known by the end of the 18th century and that the early theoretical breakthroughs were explained. However, it took until the latter half of the 20th century for methods to be developed to solve these equations for engineering flows.

Studying the fluid flow between two plates is crucial due to their expanding industrial applications and significant influence on several fields of technology. The study of non-Newtonian fluid in such a medium presents challenging problems to the analyser due to the lengthy computational investigation necessary and the greater practical relevance of such types of models. The extension of these non-Newtonian fluids from viscous fluids is not straightforward because of the diverse nature present in the constitutive equation and the concurrent impacts of elasticity and viscosity. These visco-elastic effects add complexity to the ensuing differential equations.

Based on Newton's rule of viscosity, fluids can be roughly classified as Newtonian or non-Newtonian. With non-Newtonian fluids being the subject of this study, they can be roughly divided into three classes, namely rate, differential and integral. This work prioritises the rate fluid subclass, known as the Maxwell model [3]. The relaxation time effect can be clearly elucidated with the help this fluid model. K.R. Rajagopal *et al.* [4] expanded on Maxwell's methods, creating a surplus of rate type models. In addition to the typical equations of continuity and momentum in two dimensions, F. Olsson [5] studied the constitutive equation for a visco-elastic fluid for an upper-convected Maxwell (UCM) fluid, allowing for a insignificant degree of compressibility that is regarded as artificial compressibility. The UCM fluid flow above a rigid plate moving slowly in an otherwise passive fluid was theoretically examined by K. Sadeghy *et al.* [6]. Further, A.N. Kashif *et al.* [7] examined the Maxwell fluid boundary layer flow on a flat plate in the presence of pressure gradient and obtained an approximate analytical solution by employing HPM.

The Maxwell fluids gain importance because of its numerous application in the field of engineering. These fluids are ideal for hydraulic systems which require high precision control and accuracy, as a consequence of its unique properties. Hence, one of the most common use is in designing and constructing hydraulic systems. Further it has its applications in designing shock absorbers and lubrication development. As a result of its ability to flow and deform under stress, it has become one of the best choices for reducing damping and frictional vibrations. Altogether, due to its reliability and versatility, the Maxwell fluids are an essential component in various engineering applications.

The flow through porous channel/tubes draws particular interest because of their widespread use in industrial, technological and biological fields. The appearance of such flows can be observed in flow in capillaries, design of crude oil extraction, filters and underground disposal of nuclear waste material. In a uniformly porous channel, J.F. Brady [8] investigated how the inlet velocity profile developed spatially. S.M. Cox [9] considered the steady and uniform suction flow of a incompressible viscous fluid in a parallel-walled tube that is being driven through its porous walls.

Manuscript received September 25, 2023; revised January 22, 2024.

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The similarity solution to the two-dimensional Navier-Stokes equation that showed the flow of a viscous incompressible fluid in a channel with a single porous wall, was further examined by S.M. Cox [10]. Furthermore, S. W. Yuan [11] incorporated the situation of moderate to high injection or suction velocity at the walls into a steady-state laminar flow problem in channels with porous walls in two-dimensions. In order to account for visco-elasticity and inertial effects, H. Tahira *et al.* [12] conducted a theoretical examination of an incompressible Maxwell fluid flow in a channel with uniform porous walls. The steady and incompressible UCM fluid suction flow on the surface of a porous channel in two-dimensions was analysed by Choi *et al.* [13]. Later, S. Zeb *et al.* [14] analysed visco-elastic UCM fluid flow in a permeable medium.

The science that dealing with the motion of an electrically conducting fluid in the presence of magnetic field is called magneto-hydrodynamics (MHD). Electric currents produced by the conducting fluid moving through the magnetic field causes the magnetic field to change, and the influence of magnetic field on these currents results in mechanical forces that alter the fluid's movement [15]. Theoretical research on flow through the MHD channel has sparked a lot of scope due to its extensive applications in MHD seawater thrusters, pumps, accelerators and flow meters. Further, in the presence of a magnetic field, an investigation on the flow of an electrically conducting fluid through the porous medium encompasses a wide domain for scientific and technological disciplines, such as metallurgy in particular polymer processing sector, and earth science [16], [17]. A.M. Siddiqui *et al.* [18] examined the two-dimensional unsteady flow of a viscous MHD fluid between a pair of parallel plates. Further, the impact of various physical parameters on a MHD boundary layer flow of an UCM fluid in a channel with porous walls in two-dimensions has been investigated by Z. Abbas *et al.* [19]. In a rectangular porous channel with steady, incompressible fluid flow and immiscible fluids in all regions, S. Deo *et al.* [20] investigated the effects of a uniform magnetic field applied orthogonal to the direction of fluid motion on a Newtonian fluid suppressed between a pair of micro-polar fluid layers. Several other researchers have investigated the MHD flow for different geometry [21] - [22].

Homotopy perturbation method (HPM), which Ji-Huan He [23], [24] first suggested in 1998, combines homotopy in topology with conventional perturbation technique. The non-linearity in the equations makes it impossible to solve all the problems analytically using currently available methods. Nonetheless, using the assumptions, numerical techniques can produce approximations of the solution. As a result, semi-analytical approaches were created to deal with these sorts of problems [25]. Several researchers have also employed HPM to resolve various differential, integral and wave-like equations [26] - [27].

Since MHD flow and Maxwell fluids individually have a very wide scope in the technological field, the combination of the two results in one of the best options for numerous areas. Hence, the analysis of the MHD flow of UCM fluid has motivated the researchers to advance further in the regimes of the Maxwell model. Thus, the intent of this study is to analyse the steady flow of an MHD UCM fluid between a moving upper plate and a stationary lower porous plate. The problem under consideration is then solved using HPM, as it produces a rapid convergence of the series solution only after a relatively small number of iterations. This technique ensures an added advantage over solely numerical techniques for simple domains. A single computer program provides the solution for a wide range of expansion quantities. The convergence of the HPM solution is established to resolve the ensuing non-linear problem. The variations of the emerging physical parameters are emphasized on the velocity curve.

II. Mathematical Formulation

Let the two-dimensional flow of an MHD UCM fluid between two rectangular parallel plates that are placed at a distance of d apart be considered. The flow occupies the region between the plates located at $y = 0$ and $y = d$, with the plate at the top moving uniformly with velocity V_p away (or towards) from the fixed bottom porous plate, as displayed in Fig. 1.

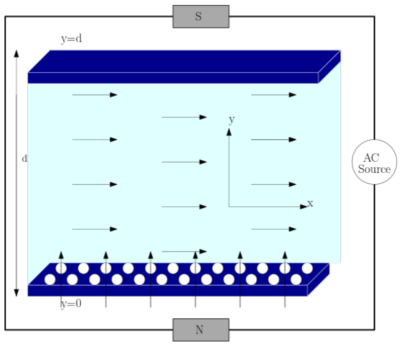

Fig. 1. Schematic diagram of the problem

The equations governing flow and momentum are

$$\nabla . V = 0, \quad (1)$$

$$\rho a = \nabla . T, \quad (2)$$

in which T, V, ρ, and a are the Cauchy stress tensor, velocity vector, density of the fluid, and the acceleration vector given by

$$\begin{aligned} a &= \frac{dV}{dt} \\ &= \frac{\partial V}{\partial t} + (V.\nabla)V. \end{aligned} \quad (3)$$

The Cauchy stress tensor for a Maxwell fluid is given by

$$T = -pI + S, \quad (4)$$

where I and p are the identity tensor and pressure. The extra stress tensor, S is given by the relation

$$(1 + \lambda \frac{D}{Dt})S = \mu A_1, \quad (5)$$

where λ and μ represent the relaxation time and the dynamic viscosity. A_1, the kinematical tensor is expressed as

$$A_1 = L + L^T, \quad (6)$$

where L is the gradient of V. The equations of continuity and momentum for a MHD UCM fluid flow in two-dimensions reduces to

$$\frac{du}{dx} + \frac{dv}{dy} = 0, \quad (7)$$

$$u\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}+\lambda[u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}] \\ =\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}(u+\lambda v\frac{\partial u}{\partial y})-\phi\frac{\nu}{k}u. \tag{8}$$

The x and y components of velocity are given by u and v respectively. B_0 is the uniform magnetic field along $y-$ axis, k, σ and ϕ are the permeability of porous medium, electrical conductivity, and porosity, respectively.

The relevant boundary conditions are as follows

$$u(x,y)=0, \quad v(x,y)=AV_p, \quad at \quad y=0, \tag{9}$$
$$u(x,y)=0, \quad v(x,y)=V_p, \quad at \quad y=d. \tag{10}$$

The similarity transformations are

$$\hat{x}=\frac{x}{d}, \quad \hat{y}=\frac{y}{d}, \quad u=-V_p\hat{x}f'(\hat{y}), \quad v=V_pf(\hat{y}). \tag{11}$$

The resulting equation from (8) is

$$f^{iv}-M[Rf''+De(f'f''+ff''')] \\ -Kf''+R[f'f''-ff'''] \\ +De[2f(f'')^2+2(f')^2f''-f^2f^{iv}]=0. \tag{12}$$

Equations (9) and (10) reduce to

$$f=A, \quad f'=0 \quad at \quad y=0, \tag{13}$$
$$f=1, \quad f'=0 \quad at \quad y=1, \tag{14}$$

in which

$$De=\frac{\lambda V_p^2}{\nu}, \quad M=\frac{\sigma B_0^2 d}{\rho V_p}, \quad K=\frac{d^2\phi}{k}, \quad R=\frac{dV_p}{\nu},$$

are called the Deborah number, magnetic number, porosity parameter, and Reynolds number, respectively. Here, $A>0$ denotes suction and $A<0$ denotes injection, whereas $R>0$ and $R<0$ denote the movement of the upper plate apart and towards the bottom plate.

III. Method of Solution

The problem considered here is solved using homotopy perturbation method. The combination of homotopy in topology with conventional perturbation technique is called Homotopy Perturbation Method. As HPM does not require any small parameters in the equations, it can overcome the drawbacks of conventional perturbation techniques. It is up to the researcher to decide on the initial approximation using potential unknown constants. The estimates produced by this approach are accurate for both large and small parameters. The homotopy perturbation approach is conveniently implemented, and it accelerates the convergence of series solutions. The first few terms of the series solutions of the flow are

$$f_0=A+3y^2-3Ay^2-2y^3+2Ay^3. \tag{15}$$

$$f_1= -\frac{1}{420}(A-1)(y-1)^2y^2(3(K(7 \\ -14y)+R(-32+7M-10y-14My \\ +12y^2-8y^3+2A(-19+5y-6y^2 \\ +4y^3)))+2De(-48-4y+40y^2 \\ +84y^3-250y^4+100y^5+3M(-10 \\ -11y-12y^2+8y^3)+2A^2(39-86y \\ +104y^2+42y^3-125y^4+50y^5) \\ -A(3M(25-11y-12y^2 \\ +8y^3+2(15-88y+124y^2 \\ +84y^3-250y^4+100y^5))))). \tag{16}$$

$$f_2= \frac{1}{8408400}(-1+A)(-1+y)^2y^2(De^2((13M^2 \\ (-5279-3240y-1201y^2+1931y^3 \\ +10885y^4+11956y^5-25088y^6 \\ -7168y^7)+8M(70560+74190y \\ +77820y^2+129498y^3-213218y^4 \\ +42807y^5-97993y^6+488362y^7 \\ -393750y^8+87500y^9)+16(24387 \\ +24576y+24765y^2+24954y^3 \\ -191073y^4+228678y^5+69279y^6 \\ +74330y^7-298997y^8+266250y^9 \\ -12100y^{10}-22000y^{11})+16A^4 \\ (-148149+500324x-818168x^2 \\ +265740x^3+939238x^4-1319050x^5+ \\ 663327x^6+7354x^7-282253x^8 \\ +266250x^9-121000x^10)...))). \tag{17}$$

$$f_3= -\frac{1}{3422050632000}(-1+A)y^4(96A^6De^3 \\ (155156376900-244107689280y \\ +12608317240y^2+258538079916y^3 \\ -586153351980y^4+503520953200y^5 \\ -150277297170y^6-80918341140y^7 \\ +130236797054y^8-183839106360y^9 \\ +237846040350y^{10}-161403707240y^{11} \\ -8453999205y^{12}+93570454782y^{13} \\ -79849549150y^{14}+33695917500y^{15} \\ -7541803500y^{16}+718267000y^{17}) \\ +6De^3y(323M^2(4083534-8079344y \\ +3117192y^2-14864850y^3+30180150y^4 \\ -18962944y^5+25272702y^6-49619115y^7 \\ +41636280y^8-15133440y^9+2017792y^{10} \\ +646M^2(-9932160-10047751y \\ +52458246y^2-57845580y^3 \\ +148383300y^4-283188906y^5...)s)...)). \tag{18}$$

IV. Results and Discussion

In the present study, the problem of an electrically conducting UCM fluid flowing between a pair of rectangular plates of which the top plate moves either towards ($R<0$) or apart ($R>0$) from the fixed bottom porous plate through which fluid is sucked ($A>0$) or injected ($A<0$) is examined. The preliminary goal of this work is to study the squeezing flow of MHD UCM fluid, considering the effects of suction and injection on the velocity field and the coefficient of skin friction. Fig. 2 – Fig. 19 and Table I - Table III demonstrate how the physical parameters (viz. R, De, M, and K) for injection and suction affect the velocity profile and skin frictions of a squeezing flow. Using HPM, a novel kind of series solution is proposed.

The velocity function of the fluid, $f'(y)$, is depicted in Fig. 2 - Fig. 19, which is found to be parabolic in nature. Fig. 2 and Fig. 3 show the velocity profiles in both injection ($A < 0$) and suction ($A > 0$) for various values of the Reynolds number. Fig. 2 shows that the velocity profile, in case of injection, increases in the range $0 \leq y \leq 0.5$ and drops in the domain $0.5 \leq y \leq 1$ as R rises when the top plate moves towards the fixed bottom plate whereas declines in the region $0 \leq y \leq 0.5$ and elevates in $0.5 \leq y \leq 1$ as R increases when $R > 0$. The same trend is seen in Fig. 3 for $A > 0$, i.e., as the top plate moves towards the fixed bottom plate, the velocity field of the MHD UCM fluid intensifies in the range of $0 \leq y \leq 0.5$ and falls in the domain $0.5 \leq y \leq 1$ as R increases whereas when $R > 0$, $f'(y)$ decreases in the range of $0 \leq y \leq 0.5$ and is found to be rising in domain $0.5 \leq y \leq 1$.

When the plates move away from each other, as the strength of the applied magnetic field increases, there develops a drag force known as Lorentz force causes the momentum boundary layer to thicken. As a result, the flow is impeded, leading to a reduction in the magnitude of the velocity profile. Thus, as the plates move away from each other, the magnitude of the velocity profile decreases with an increase in magnetic parameter.

The velocity profiles for variation in various physical parameters in case of injection are displayed in Fig. 4 – Fig. 9. In squeezing condition ($R < 0$), $f'(y)$ increases as M rises across the plane with $R = -2$, $K - 0.1$ and $De = 0.1$ is observed in Fig. 4. The velocity profile is found to be increasing as the magnetic parameter rises, peaking at $y = 0.5$. As the plates move apart from the stationary bottom plate ($R > 0$), Fig. 5 shows the opposite behaviour, where the velocity profile is found to be decreasing as the magnetic parameter rises as the top plate moves apart from the stationary bottom plate, maximum at $y = 0.55$, when $R = -2$, $K - 0.1$ and $De = 0.1$.

The velocity profile decreases in the region of $0 \leq y \leq 0.5$ and increases in $0.5 \leq y \leq 1$, with an increment in the Deborah number as the top plate moves towards the lower one as shown in Fig. 6. This is because the lower Deborah number implies the fluid is more viscous, and the relaxation time is much longer than the characteristic time scale of the flow. Hence, the velocity decreases in the first half because the fluid resists the deformation caused by the flow. However, as De increases, the fluid becomes less viscous, and the relaxation time becomes comparable with the characteristic time scale of the flow. Thus, the velocity increases in the second half as the fluid flows more easily. The same trend is found when the top plate moves apart from the fixed bottom porous plate shown in Fig. 7. As the K value rises, the velocity field increases in the region $0 \leq y \leq 0.25$, after which it begins to retard until $y = 0.7$ further, it increases in the region $0.7 \leq y \leq 1$ as the plates move towards as depicted in Fig. 8. It is evident from Fig. 9 that as the top plate moves apart from the bottom plate, the velocity profile retards in the region $0.3 \leq y \leq 0.75$ as the value of K increases.

The impact of various physical parameters on the velocity profile in the case of suction ($A > 0$) is depicted in Fig. 10 - Fig. 15. Under the squeezing condition ($R < 0$), $f'(y)$ increases as M rises throughout the plane, as observed in Fig. 10. As the magnetic parameter rises, it is demonstrated that the velocity profile increases as well, culminating at $y = 0.4$. Fig. 11, which depicts the opposite behaviour when the top plate moves apart from the bottom plate ($R > 0$), demonstrates that the velocity field profile is increasing as the magnetic parameter increases, reaching a maximum at $y = 0.6$.

The velocity profile increases in the region $0 \leq y \leq 0.5$ and decreases in $0.5 \leq y \leq 1$, with an increase in the Deborah number when the top plate moves towards the lower one as displayed in Fig. 12. When the top plate moves apart from the stationary bottom porous plate, shown in Fig. 13, the trend is seen to be opposite. As observed in Fig. 14, as the K value rises, the velocity field profile increases in the region $0 \leq y \leq 0.2$, after which it starts to retard until $y = 0.65$. It then rises again when the plates move towards, in the region $0.65 \leq y \leq 1$. It is evident from Fig. 15 that, when the top plate moves apart from the bottom plate, the velocity profile retards in the region $0.3 \leq y \leq 0.8$ as the value of K increases.

The impact of suction and injection on the velocity profile are displayed in Fig. 16 - Fig. 19. Fig. 16 and Fig. 18 represent the velocity profile graphically in the case of suction for plates moving away and towards, respectively, whereas Fig. 17 and Fig. 19 demonstrate the case of injection. It is evident from Fig. 16 and Fig. 18 that as suction increases, the velocity profile declines with an increase in suction parameter for both plates moving apart and towards. In the case of injection, it is evident from Fig. 17 and Fig. 19 that an increase in injection results in the increase of velocity profile in both cases.

Table I - Table III represent the impact of pertinent parameters on the numerical values of the skin friction coefficient. Table I and Table II illustrate how the coefficients of skin friction $f''(0)$ and $f''(1)$ vary with the magnetic number M, Reynolds number R, and Deborah number De for the conditions $A = -0.5$ and $A = 0.5$ with the porosity parameter $K = 0.1$. As R rises, skin friction intensity decreases for $A = -0.5$ and $A = 0.5$. It has been noted that skin friction rises when the plates are moving towards one another as opposed to when the top plate moves apart from the bottom stationary plate. This is true for both injection and suction cases.

Table III represents the numerical values of the skin friction coefficient for different values of the parameter A. It is evident from Table III that the magnitude of skin friction coefficient increases with increased injection at both lower and upper plates as the plates move towards each other. A similar behaviour is observed in the case of plates moving apart. It can be inferred from the table that the magnitude of skin friction coefficient decreases with increased suction at both upper and lower plates in the case of plates moving apart. A similar trend is observed in the case of plates moving towards each other.

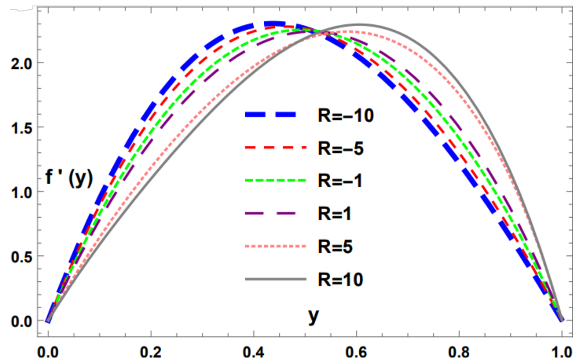

Fig. 2. Effect of variation of R on $f'(y)$ for $A = -0.5, M = 0.2, K = 0.2, De = 0.2$

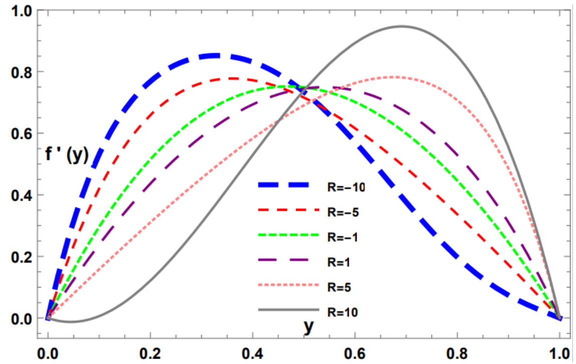

Fig. 3. Effect of variation of R on $f'(y)$ for $A = 0.5, M = 0.2, K = 0.2, De = 0.2$

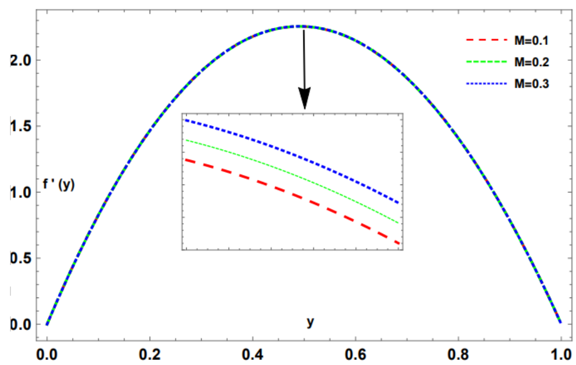

Fig. 4. Effect of variation of M on $f'(y)$ for $A = -0.5, R = -2, K = 0.1, De = 0.1$

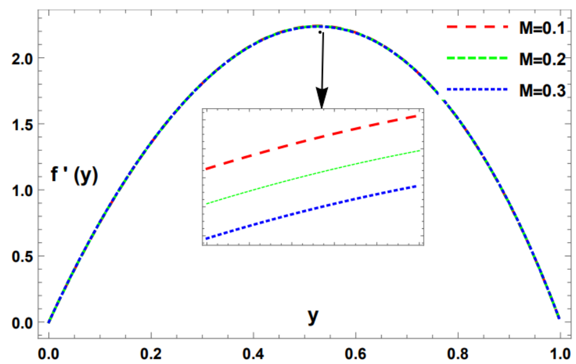

Fig. 5. Effect of variation of M on $f'(y)$ for $A = -0.5, R = 2, K = 0.1, De = 0.1$

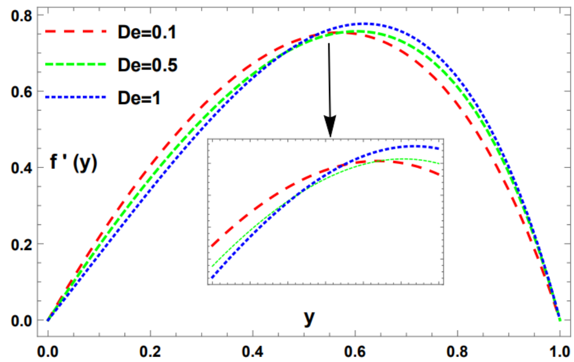

Fig. 6. Effect of variation of De on $f'(y)$ for $A = -0.5, R = -2, K = 0.1, M = 0.1$

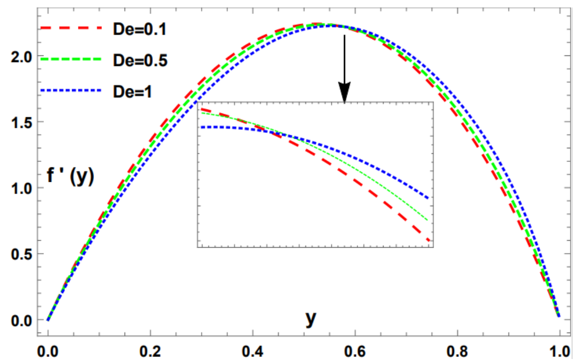

Fig. 7. Effect of variation of De on $f'(y)$ for $A = -0.5, R = 2, K = 0.1, M = 0.1$

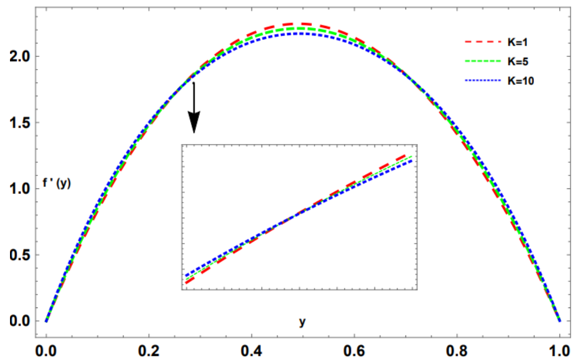

Fig. 8. Effect of variation of K on $f'(y)$ for $A = -0.5, R = -1, M = 0.2, De = 0.2$

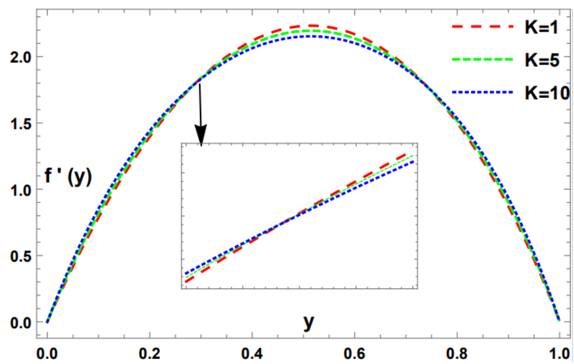

Fig. 9. Effect of variation of K on $f'(y)$ for $A = -0.5, R = 1, M = 0.2, De = 0.2$

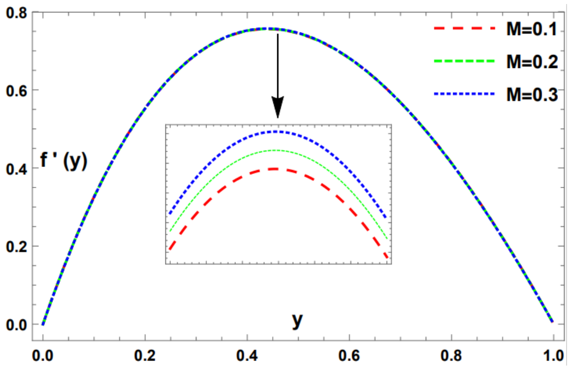

Fig. 10. Effect of variation of M on $f'(y)$ for $A = 0.5, R = -2, K = 0.1, De = 0.1$

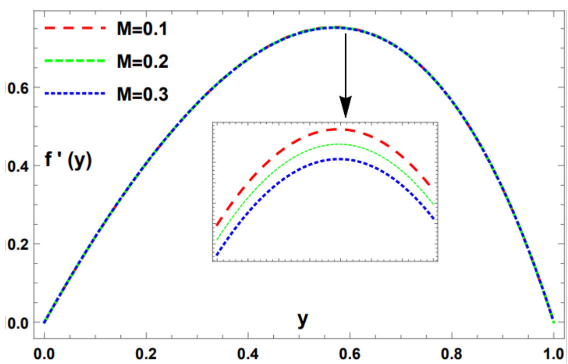

Fig. 11. Effect of variation of M on $f'(y)$ for $A = 0.5, R = 2, K = 0.1, De = 0.1$

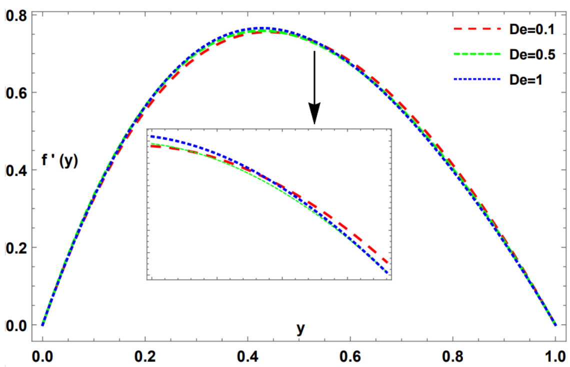

Fig. 12. Effect of variation of De on $f'(y)$ for $A = 0.5, R = -2, K = 0.1, M = 0.1$

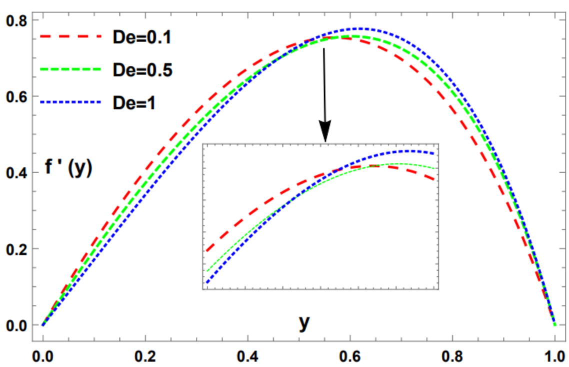

Fig. 13. Effect of variation of De on $f'(y)$ for $A = 0.5, R = 2, K = 0.1, M = 0.1$

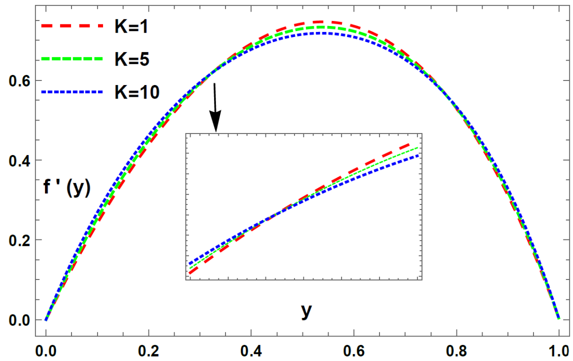

Fig. 14. Effect of variation of K on $f'(y)$ for $A = 0.5, R = -2, De = 0.2, M = 0.2$

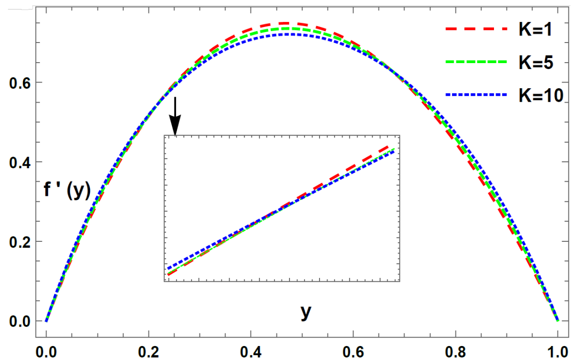

Fig. 15. Effect of variation of K on $f'(y)$ for $A = 0.5, R = 2, De = 0.2, M = 0.2$

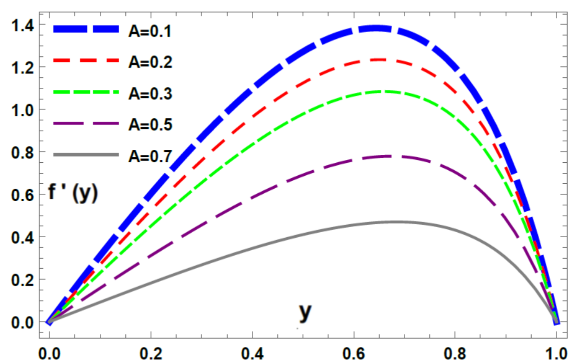

Fig. 16. Effect of suction on $f'(y)$ for $R = 5, De = 0.2, K = 0.2, M = 0.2$

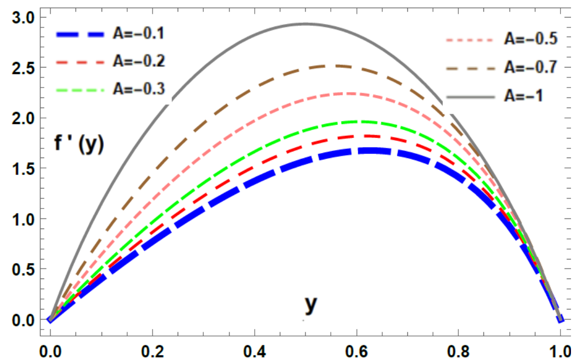

Fig. 17. Effect of injection on $f'(y)$ for $R = 5, De = 0.2, K = 0.2, M = 0.2$

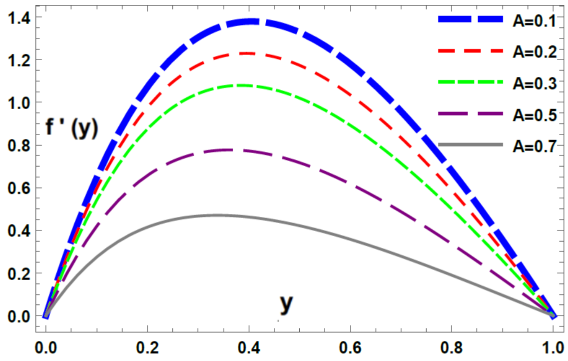

Fig. 18. Effect of suction on $f'(y)$ for $R = -5, De = 0.2, K = 0.2, M = 0.2$

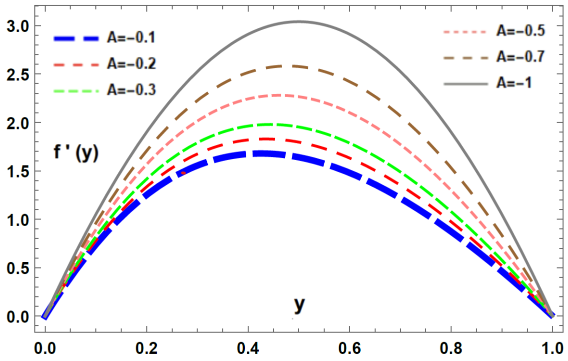

Fig. 19. Effect of injection on $f'(y)$ for $R = -5, De = 0.2, K = 0.2, M = 0.2$

TABLE I
INJECTION: SKIN FRICTION COEFFICIENTS FOR $A = -0.5, K = 0.1$

R	De	$M = 0.1$		$M = 0.3$	
		$f''(0)$	$f''(1)$	$f''(0)$	$f''(1)$
-10	0.1	10.69020	-6.63060	10.48820	-6.50273
-5		9.99593	-7.45883	9.89528	-7.33844
-1		9.25132	-8.67588	9.22571	-8.66051
1		8.70444	-9.73008	8.72977	-9.78353
5		7.00152	-13.65460	7.27806	-13.77930
10		4.55102	-18.93240	5.13253	-19.32050
-10	0.5	10.70890	-6.76840	10.27660	-7.55429
-5		10.04230	-7.74117	9.94976	-7.63663
-1		9.19635	-9.39134	9.17308	-9.46220
1		8.54346	-10.99190	8.58099	-11.20710
5		6.37442	-17.85880	6.74141	-17.72180
10		4.06584	-21.02010	4.66251	-21.67830

TABLE II
SUCTION: SKIN FRICTION COEFFICIENTS FOR $A = 0.5, K = 0.1$

R	De	$M = 0.1$		$M = 0.3$	
		$f''(0)$	$f''(1)$	$f''(0)$	$f''(1)$
-10	0.1	7.98767	-1.70023	7.46117	-0.84397
-5		5.21908	-1.80407	5.20448	-1.75374
-1		3.37469	-2.65646	3.35853	-2.65227
1		2.62497	-3.50643	2.63086	-3.52693
5		1.64018	-6.33950	1.69830	-6.33205
10		1.36324	-11.81070	1.30286	-11.11270
-10	0.5	7.66400	-0.30219	7.66213	-0.36881
-5		5.61369	-1.71582	5.56621	-1.66084
-1		3.39719	-2.69670	3.29002	-2.73229
1		2.41134	-4.02481	2.39028	-4.13546
5		1.33455	-9.39148	1.40736	-9.48284
10		0.286024	-12.48760	0.54215	-12.65720

TABLE III
SKIN FRICTION COEFFICIENTS FOR $De = 0.5, K = 0.1$

A	R	$M = 0.1$		$M = 0.3$	
		$f''(0)$	$f''(1)$	$f''(0)$	$f''(1)$
-0.5	-5	10.04230	-7.74117	9.94976	-7.63663
-0.3		9.30965	-6.28520	9.22407	-6.18976
-0.1		8.53284	-4.95065	8.45541	-4.86508
0.1		7.68571	-3.74315	7.61847	-3.66983
0.3		6.73223	-2.67239	6.67886	-2.61354
0.5		5.61369	-1.71582	5.56621	-1.66084
-0.5	5	6.37442	-17.85880	6.74141	-17.72180
-0.3		4.83638	-16.63320	5.07661	-17.01730
-0.1		3.65269	-15.48780	3.84098	-15.79500
0.1		2.72662	-13.8559	2.87233	-14.09040
0.3		1.97743	-11.74110	2.08546	-11.90700
0.5		1.33455	-9.39148	1.40736	-9.48284

V. CONCLUSION

This article focuses on the upper convected Maxwell fluid flow between two plates, one of which is stationary, and the other is moving with injection and suction. Here, the effect of the magnetic field on the UCM fluid flow is analysed with the help of HPM, leading to the following conclusions:

1) The velocity field profile rises with an increase in Reynolds number, attaining a maximum value, after which the velocity profile falls as the top plate moves towards and away from the stationary lower plate.
2) As the magnetic field increases, the velocity profile increases in case of the top plate moving towards the stationary lower, whereas the velocity profile decreases as the top plate moves away.
3) The velocity profile retards in the core region with an increase in porosity parameter as the top plate moves towards and away from the lower.
4) The velocity field retards with an increase in the Deborah number reaching the peak and further increases in both cases, the top plate moving towards and away from the stationary lower plate in case of injection.
5) In the case of suction, the velocity field enhances with an increase in Deborah number reaching the peak and further decreases as the top plate moves towards the stationary lower plate, whereas the velocity profile retards with an increase in Deborah number reaching the peak and further increases as the top plate moves away from the lower.
6) The velocity profile decreases with an increase in suction, whereas it increases with an increase in injection for both plates moving towards and apart from each other.
7) An increase in injection increases the magnitude of skin friction coefficient at upper and lower plates in both cases, that is, plates moving towards and away from each other. In contrast, the magnitude of the skin friction coefficient decreases with an increase in suction.

ACKNOWLEDGMENT

The authors would like to thank Manipal Academy of Higher Education, Manipal for their constant support.

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