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Analyzing the effects of the Footprint of Uncertainty in Type-2 Fuzzy Logic Controllers

Roberto Sepulveda, Patricia Melin, Antonio Rodriguez, Alejandra Mancilla, Oscar Montiel

Abstract— **Uncertainty is an inherent part in controllers used for real-world applications. The use of new methods for handling incomplete information is of fundamental importance in engineering applications. We simulated the effects of uncertainty produced by the instrumentation elements in type-1 and type-2 fuzzy logic controllers to perform a comparative analysis of the systems' response, in the presence of uncertainty. We are presenting a proposal to optimize interval type-2 membership functions using an average of two type-1 systems we did some experiments where we optimized the MFs under different ranges values for the Footprint Of Uncertainty (FOU) and different noise values.**

Index Terms—**Type-2 Fuzzy Logic, Intelligent Control.**

I. Introduction

Uncertainty affects decision-making and appears in a number of different forms. The concept of information is fully connected with the concept of uncertainty. The most fundamental aspect of this connection is that the uncertainty involved in any problem-solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way [1]. The general framework of fuzzy reasoning allows handling much of this uncertainty, fuzzy systems employ type-1 fuzzy sets, which represents uncertainty by numbers in the range [0, 1]. When something is uncertain, like a measurement, it is difficult to determine its exact value, and of course type-1 fuzzy sets makes more sense than using crisp sets [2]. However, it is not reasonable to use an accurate membership function for something uncertain, so in this case what we need is another type of fuzzy sets, those which are able to handle these uncertainties, the so called type-2 fuzzy sets [3]. So, the effects of uncertainty in a system can be handled in a better way by using type-2 fuzzy logic because it offers better capabilities to cope with linguistic uncertainties by modeling vagueness and unreliability of information [4, 5, 6, 7, 8].

Recently, we have seen the use of type-2 fuzzy sets in fuzzy logic systems to deal with uncertain information [9, 10, 11]. So we can find some papers emphasizing on the implementation of a type-2 Fuzzy Logic System (FLS) [12]; in others, it is explained how type-2 fuzzy sets let us model and minimize the effects of uncertainties in rule-base FLSs [13]. Some research works are devoted to solve real world applications in different areas, for example, in signal processing type-2 fuzzy logic is applied in prediction in Mackey-Glass chaotic time-series with uniform noise presence [14, 15]. In medicine, an expert system was developed for solving the problem of Umbilical Acid-Base (UAB) assessment [16]. In industry, type-2 fuzzy logic and neural networks was used in the control of non-linear dynamic plants [17, 18, 19, 20, 21, 22].

This work deals with the optimization, through a random search, of interval type-2 membership functions in a fuzzy logic controller (FLC), the behavior of the FLC after optimization of the MFs under different range values for the Foot of Uncertainty (FOU) and different noise values. It is a fact, that in the control of real systems, the instrumentation elements (instrumentation amplifier, sensors, digital to analog, analog to digital converters, etc.) introduce some sort of unpredictable values in the information that has been collected [23, 24]; so, the controllers designed under idealized conditions tend to behave in an inappropriate manner [25]. Since, uncertainty is inherent in the design of controllers for real world applications, with the purpose of analyzing the effects of uncertainty we plan a set of experiments, which are explained next:

In the first test, we are presenting how to deal with it using type-2 FLC to diminish the effects of imprecise information. We are supporting this statement with experimental results, qualitative observations, and quantitative measures of errors. For quantifying the errors, we utilized three widely used performance criteria, these are: Integral of Square Error (ISE), Integral of the Absolute value of the Error (IAE), and Integral of the Time multiplied by the Absolute value of the Error (ITAE) [26].

As a second test, we optimized the parameters of the Gaussian membership functions (MFs) using a random search

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method, ISE as the fitness function; we used, as an output, the average of two type-1 systems. We optimized the MFs of the FLC for different values of the FOU and for different noise values introduced to the system in a simulated way.

This paper is organized as follows: section II presents an introductory explanation of type-1 and type-2 FLCs and the performance criteria for evaluating the transient and steady state closed-loop response in a computer control system. Section III is devoted to the experimental results. In this section we are showing details of the implementation of the feedback control system used, we are presenting results from several experiments. The plant was tested using several signal to noise ratio, we are including a performance comparison between type-1 and type-2 fuzzy logic controllers, versus optimized type-2 FLCs, an analysis of the results of optimized MFs for different ranges of the FOU and different noise levels. Finally, in section IV, we have the conclusions.

II. Fuzzy Controllers

A. Type-1 Fuzzy Controllers

In the 40's and 50's, many researchers proved that many dynamic systems can be mathematically modeled using differential equations. These previous works represent the foundations of the Control theory, which, in addition with the Transform theory, provided an extremely powerful means of analyzing and designing control systems [27]. These theories were being developed until the 70's, when the area was called System theory to indicate its definitiveness [28]. Its principles have been used to control a very big amount of systems taking mathematics as the main tool to do it during many years. Unfortunately, in too many cases this approach could not be sustained because many systems have unknown parameters or highly complex and nonlinear characteristics that make them not to be amenable to the full force of mathematical analysis as dictated by the Control theory.

Soft computing techniques have become a research topic, which is applied in the design of controllers [29]. These techniques have tried to avoid the above-mentioned drawbacks, and they allow us to obtain efficient controllers, which utilize the human experience in a more related form than the conventional mathematical approach [30, 31, 32]. In the cases in which a mathematical representation of the controlled systems cannot be obtained, the process operator should be able to express the relationships existing in them, that is, the process behavior.

A FLS, described completely in terms of type-1 fuzzy sets is called a type-1 fuzzy logic system (type-1 FLS). It is composed by a knowledge base that comprises the information given by the process operator in form of linguistic control rules; a fuzzification interface, who has the effect of transforming crisp data into fuzzy sets; an inference system, that uses them in conjunction with the knowledge base to make inference by means of a reasoning method; and a defuzzification interface, which translates the fuzzy control action so obtained to a real control action using a defuzzification method [27].

In our paper, the implementation of the fuzzy controller in terms of type-1 fuzzy sets, has two input variables such as the error $e(t)$, the difference between the reference signal and the output of the process, as well as the error variation $\Delta e(t)$,

$$e(t) = r(t) - y(t) \quad (1)$$

$$\Delta e(t) = e(t) - e(t-1) \quad (2)$$

so the control system can be represented as in Fig. 1.

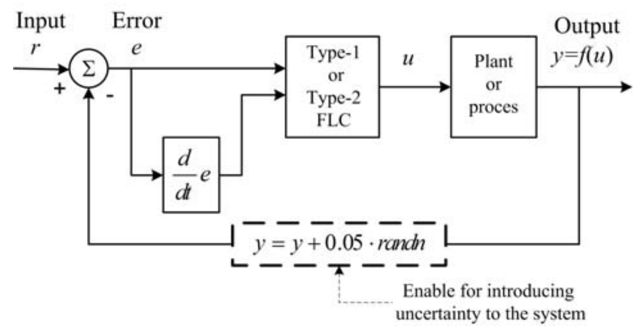

Figure 1. System used for obtaining the experimental results in this paper.

B. Type-2 Fuzzy Controllers

A FLS described using at least one type-2 fuzzy set is If we have a type-1 membership function as in Fig. 2, and we blurring it to the left and to the right then, at a specific value x', the membership function value u', takes on different values which not all be weighted the same, so we can assign an amplitude distribution to all of those points.

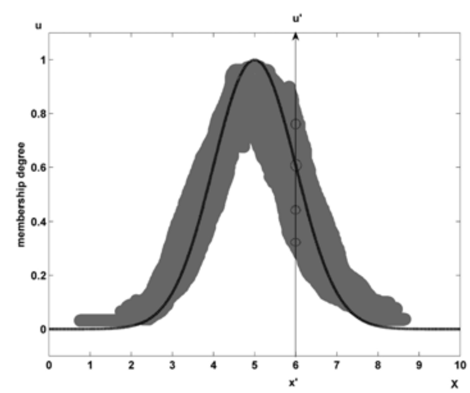

Fig. 2. Blurred type-1 membership function.

Doing this for all $x \in X$, we create a three-dimensional membership function –a type-2 membership function– that characterizes a type-2 fuzzy set [3,13]. A type-2 fuzzy set $\tilde{A}$, is characterized by the membership function:

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \quad (3)$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$. Another expression for $\tilde{A}$ is,

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \qquad J_x \subseteq [0,1] \quad (4)$$

Where $\int\int$ denote union over all admissible input variable

x and u. For discrete universes of discourse $\int$ is replaced by $\sum$ [13]. In fact $J_x \subseteq [0,1]$ represents the primary membership of x and $\mu_{\tilde{A}}(x,u)$ is a type-1 fuzzy set known as the secondary set. Hence, a type-2 membership grade can be any subset in [0,1], the primary membership, and corresponding to each primary membership, there is a secondary membership (which can also be in [0,1]) that defines the uncertainty for the primary membership.

This uncertainty is represented by a region called footprint of uncertainty (FOU). When $\mu_{\tilde{A}}(x,u) = 1, \forall u \in J_x \subseteq [0,1]$ we have an interval type-2 membership function, as shown in Fig. 3. The uniform shading for the FOU represents the entire interval type-2 fuzzy set and it can be described in terms of an upper membership function $\overline{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$.

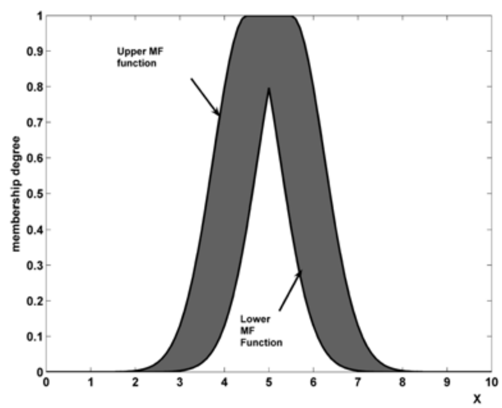

Fig. 3. Interval type-2 membership function.

A FLS described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain [33]. On the other hand, type-2 FLSs, are very useful in circumstances where it is difficult to determine an exact certainty, and measurement uncertainties [3].

It is known that type-2 fuzzy sets let us to model and to minimize the effects of uncertainties in rule-based FLS. Unfortunately, type-2 fuzzy sets are more difficult to use and understand than type-1 fuzzy sets; hence, their use is not widespread yet. In [13] were mentioned at least four sources of uncertainties in type-1 FLSs:

1. The meanings of the words that are used in the antecedents and consequents of rules can be uncertain (words mean different things to different people).
2. Consequents may have histogram of values associated with them, especially when knowledge is extracted from a group of experts who do not all agree.
3. Measurements that activate a type-1 FLS may be noisy and therefore uncertain.
4. The data used to tune the parameters of a type-1 FLS may also be noisy.

All of these uncertainties translate into uncertainties about fuzzy set membership functions. Type-1 fuzzy sets are not able to directly model such uncertainties because their membership functions are totally crisp. On the other hand, type-2 fuzzy sets are able to model such uncertainties because their membership functions are themselves fuzzy. A type-1 fuzzy set is a special case of a type-2 fuzzy set; its secondary membership function is a subset with only one element, unity.

Similar to a type-1 FLS, a type-2 FLS includes fuzzifier, rule base, fuzzy inference engine, and output processor. The output processor includes type-reducer and defuzzifier; it generates a type-1 fuzzy set output (from the type-reducer) or a crisp number (from the defuzzifier) [34,35]. A type-2 FLS is again characterized by IF-THEN rules, but its antecedent or consequent sets are now type-2. Type-2 FLSs, can be used when the circumstances are too uncertain to determine exact membership grades such as when training data is corrupted by noise. In our case, we are simulating that the instrumentation elements (instrumentation amplifier, sensors, digital to analog, analog to digital converters, etc.) are introducing some sort of unpredictable values in the collected information.

In the case of the implementation of the type-2 FLC, we have the same characteristics as in type-1 FLC, but we used type-2 fuzzy sets as membership functions for the inputs and for the output.

C. Performance criteria

For evaluating the transient closed-loop response of a computer control system we can use the same criteria that normally are used for adjusting constants in PID (Proportional Integral Derivative) controllers. These are [26]:

1. Integral of Square Error (ISE).

$$\text{ISE} = \int_0^{\infty} [e(t)]^2 \, dt \qquad (5)$$

2. Integral of the Absolute value of the Error (IAE).

$$\text{IAE} = \int_0^{\infty} |\, e(t) \,| \, dt \qquad (6)$$

3. Integral of the Time multiplied by the Absolute value of the Error (ITAE).

$$\text{ITAE} = \int_0^{\infty} t \,|\, e(t) \,| \, dt \qquad (7)$$

The selection of the criteria depends on the type of response desired, the errors will contribute different for each criterion, so we have that large errors will increase the value of ISE more heavily than to IAE. ISE will favor responses with smaller overshoot for load changes, but ISE will give longer settling time. In ITAE, time appears as a factor, and therefore, ITAE will penalize heavily errors that occurs late in time, but virtually ignores errors that occurs early in time. Designing using ITAE will give us the shortest settling time, but it will produce the largest overshoot among the three criteria considered. Designing considering IAE will give us an intermediate results, in this case, the settling time will not be so large than using ISE nor so small than using ITAE, and the same applies for the overshoot response. The selection of a particular criterion depends on the type of desired response.

III. Experimental Results

Fig. 1 shows, the feedback control system that was used for achieving the results of this paper. It was implemented in Matlab where the controller was designed to follow the input as closely as possible. The plant was modeled using equation (8)

$$y(i) = 0.2 \cdot y(i-3) \cdot 0.07 y(i-2) + 0.9 \cdot y(i-1) + 0.05 \cdot u(i-1) + 0.5 \cdot u(i-2) \quad (8)$$

The controller's output was applied directly to the plant's input. Since we are interested in comparing the performance between type-1 and type-2 FLC systems versus optimized type-2 FLC system, and type-2 FLCs versus type-2 FLCs optimized under different ranges of the FOU and different noise levels, we have the next three cases:

1. Considering the system as ideal, that is, we did not introduce in the modules of the control system any source of uncertainty. See experiments 1, and 2.
2. Simulating the effects of uncertain modules (subsystems) response introducing some uncertainty at 10 db SNR noise levels. See experiments 3 and 4.
3. Optimizing the MFs under different sizes of the FOU and noise levels. See experiments 5 and 6.

For case 1, as is shown in Fig. 1, the system's output is directly connected to the summing junction, but in the second case, the uncertainty was simulated introducing random noise with normal distribution (the dashed square in Fig. 1). We added noise to the system's output $y(i)$ using the Matlab's function "randn" which generates random numbers with Gaussian distribution. The signal and the aditive noise in turn, were obtained with the programmer's expression (9), the result $y(i)$ was introducedh to the summing junction of the controller system. Note that in (9) we are using the value 0.05, for experiments 3 and 4, but in the set of tests for experiment 5 and 6 we varied this value to obtain different SNR values.

$$y(i) = y(i) + 0.05 \cdot randn \quad (9)$$

We tested the system using as input, a unit step sequence free of noise, $r(i)$. For evaluating the system's response and compare between type-1 and type-2 fuzzy controllers, we used the performance criteria ISE, IAE, and ITAE. In Table III, we summarized the values obtained for each criterion considering 400 units of time. For calculating ITAE we considered a sampling time $T_s = 0.1$ sec.

For experiments 1, 2, 3, and 4 the reference input r is stable and noisy free. In experiments 3 and 4, although the reference appears clean, the feedback at the summing junction is noisy since we introduced deliberately noise for simulating the overall existing uncertainty in the system, in consequence, the controller's inputs e (t) (error), and $\Delta e(t)$ contains uncertainty data.

In experiment 5, we tested the systems, type-1 and type-2 FLCs, introducing diverse values of noise η, this is modifying the signal to noise ratio SNR [36,37,38], see equation (10),

$$SNR = \frac{\sum |s|^2}{\sum |\eta|^2} = \frac{P_{signal}}{P_{noise}} \quad (10)$$

Because many signals have a very wide dynamic range [37], SNRs are usually expressed in terms of the logarithmic decibel scale, SNR(db), as we can see in equation (11),

$$SNR(db) = 10 \log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) \quad (11)$$

In Table IV, we show, for different values of SNR(db), the behavior of ISE, IAE, ITAE for type-1 and type-2 FLCs. In almost all the cases the results for type-2 FLC are better than type-1 FLC.

In type-1 FLC, we selected Gaussian membership functions (Gaussian MFs) for the inputs and for the output. A Gaussian MF is specified by two parameters $\{c,\sigma\}$:

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \quad (12)$$

where c represents the MFs center and σ determines the MFs standard deviation. For each input of the type-1 FLC, e (t) and $\Delta e(t)$, we defined three type-1 fuzzy Gaussian MFs: negative, zero, positive. The universe of discourse for these membership functions is in the range [-10 10]; their centers are -10, 0 and 10 respectively, and their standard deviations are 9, 2 and 9 respectively.

For the output of the type-1 FLC, we have five type-1 fuzzy Gaussian MFs: NG, N, Z, P and PG. They are in the interval [-10 10], their centers are -10, -4.5, 0, 4, and 10 respectively; and their standard deviations are 4.5, 4, 4.5, 4 and 4.5. Table I illustrates the characteristics of the inputs and output of the type-1 FLC. For the type-2 FLC, as in type-1 FLC we also selected Gaussian MFs for the inputs and for the output,

TABLE I
CHARACTERISTICS OF THE INPUTS AND OUTPUT OF TYPE-1 FLC.

Variable	**Term**	**Center** c	**Standard deviation** σ
Input e	negative	-10	9
	zero	0	2
	positive	10	9
Input Δe	Negative	-10	9
	Zero	0	2
	positive	10	9
Output *cde*	NG	-10	4.5
	N	-4.5	4
	Z	0	4.5
	P	4	4
	PG	10	4.5

but in this case we have an interval type-2 Gaussian MFs with a fixed standard deviation, σ, and an uncertain center, ie.,

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \quad c \in [c_1, c_2] \quad (13)$$

In terms of the upper and lower membership functions, we have for $\bar{\mu}_{\tilde{A}}(x)$,

$$\bar{\mu}_{\tilde{A}}(x) = \begin{cases} N(c_1, \sigma; x) & x < c_1 \\ 1 & c_1 \le x \le c_2 \\ N(c_2, \sigma; x) & x > c_2 \end{cases} \quad (14)$$

and for the lower membership function $\underline{\mu}_{\tilde{A}}(x)$,

$$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} N(c_2, \sigma; x) & x \le \dfrac{c_1 + c_2}{2} \\ N(c_1, \sigma; x) & x > \dfrac{c_1 + c_2}{2} \end{cases} \quad (15)$$

where $N(c_1, \sigma, x) \equiv e^{-\frac{1}{2}\left(\frac{x-c_1}{\sigma}\right)^2}$, and $N(c_2, \sigma, x) \equiv e^{-\frac{1}{2}\left(\frac{x-c_2}{\sigma}\right)^2}$, [3].

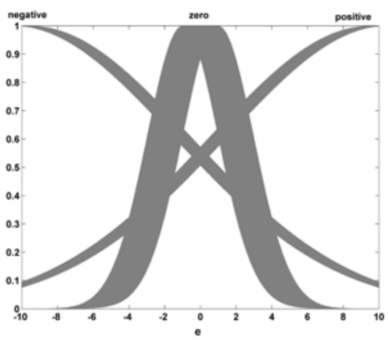

Fig.4. Input e MFs for the type-2 FLC.

Hence, in type-2 FLC, for each input we defined three interval type-2 fuzzy Gaussian MFs: negative, zero, positive in the interval [-10 10], as illustrates Figures 4 and 5; for computing the output we have five interval type-2 fuzzy Gaussian MFs NG, N, Z, P and PG, with uncertain center and fixed standard deviations in the interval [-10 10], as can be seen in Fig. 6. Table II shows the characteristics of the inputs and output of the type-2 FLC.

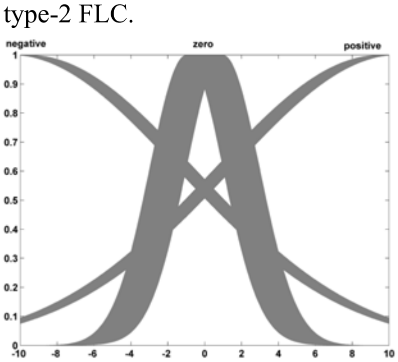

Fig.5. Input Δe MFs for the type-2 FLC.

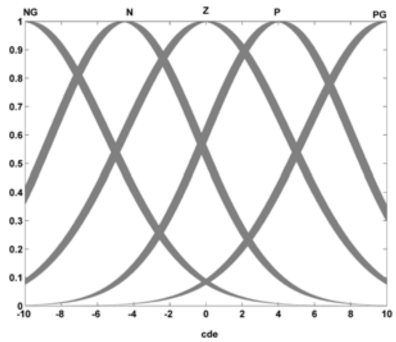

Fig.6. Output cde MFs for the type-2 FLC.

For type-2 FLCs, we used, in experiments 2, and 4, basically the free software available online [39], and for experiments 5, and 6 we obtained the output of the type 2 FLC system with the average of two type-1 FLCs.

TABLE II
CHARACTERISTICS OF THE INPUTS AND OUTPUT OF TYPE-2 FLC.

Variable	**Term**	**Center** c_1	**Center** c_2	**Standard deviation** σ
Input e	negative	-10.25	-9.75	9.2
	zero	-0.25	0.25	2.2
	positive	9.75	10.25	9.2
Input Δe	Negative	-10.25	-9.75	9.2
	Zero	-0.25	0.25	2.2
	positive	9.75	10.25	9.2
Output cde	NG	-10.25	-9.75	4.5
	N	-4.75	-4.25	4
	Z	-0.25	0.25	4.5
	P	3.75	4.25	4
	PG	9.75	10.25	4.5

Experiment 1. Ideal system using a type-1 FLC.
In this experiment, we did not add uncertainty data to the system, in Table III, we listed the obtained values of ISE, IAE, and ITAE for this experiment.

TABLE III
COMPARISON OF PERFORMANCE CRITERIA FOR TYPE-1 AND TYPE-2 FUZZY LOGIC CONTROLLERS FOR 10 db SIGNAL NOISE RATIO. VALUES OBTAINED AFTER 200 SAMPLES.

Performance Criteria	**Type-1 FLC**		**Type-2 FLC**	
	Ideal System	Syst. with uncertainty	Ideal System	Syst. with uncertainty
ISE	5.2569	205.0191	5.2572	149.3097
IAE	13.8055	155.9412	13.7959	131.77
ITAE	46.0651	1583.4	45.8123	1262.2

Experiment 2. Ideal system using a type-2 FLC.
Here, we used the same test conditions of Experiment 1, but in this case, we implemented the controller's algorithm with type-2 fuzzy logic. The corresponding performance criteria are listed in Table III.

Experiment 3. System with uncertainty using a type-1 FLC. In this case we simulated, using equation (9), the effects of uncertainty introduced to the system by transducers, amplifiers, and any other element that in real world applications affects expected values. In Table III, we can see the obtained values for ISE, IAE, and ITAE performance criteria for a simulated 10 db signal noise ratio.

Experiment 4. System with uncertainty using a type-2 FLC. In this experiment, we introduced uncertainty in the system, in the same way as in Experiment 3. In this case, we used a type-2 FLC and we improved those results obtained with a type-1 FLC (Experiment 3), see table III.

Experiment 5. Optimizing the interval type-2 MFs of the FLC for 10 db of SNR, varying the FOU.

To optimize the interval type-2 MFs of the FLC, we simulated the system using two type-1 FLCs. We maintained constant the centers of the Gaussian MFs of the inputs and we varied its standard deviations. After optimization, and taking ISE as the fitness function, we found the best values of the MFs. To see the effects of varying the size of the FOU in the optimization of the type-2 MFs, for 10 db signal to noise ratio, we established different search intervals for the shadow of the MFs. We began with a narrow interval and finished with the wider one. First we repeated the conditions of experiment 6, in the case of the inputs e and Δe for the negative and positive MFs we started with an interval between 8 and 9, and for the term zero with an interval between 1 and 2. After the optimization, we calculated ISE, IAE and ITAE for the noise levels from 6 to 30 db of SNR.

The next step was to increase the search interval as follow, an interval between 7 and 9 for the negative and positive terms of inputs e and Δe, for the term zero from 0.5 to 2 and after the optimization was calculated ISE, IAE and ITAE. Finally, the broader search interval was, between 5 and 9 for the negative and positive terms of inputs e and Δe, from .5 to 2 for the zero term. Then again we obtained the values for ISE, IAE and ITAE. In table IV, can be seen the optimized values for the standard deviations of the MFs. In table V, we can see the comparison between these results. In Fig. 7 we have that the best results were obtained using a wider search interval of the FOU.

Experiment 6. Optimizing the interval type-2 MFs of the FLC for 24 db of SNR, varying the FOU.

We repeated the steps of experiment 5, but in this case optimizing for 24 db of SNR.

Table VI shows the optimized values for the standard deviations of the MFs, and in table VII we can see the comparison between the results obtained for each variation of the FOU. Similar than in experiment 7, in Fig. 8 we obtained the best results using a wider search interval of the FOU.

In figure 9, we plotted the best results of ISE for experiment 5 and the best for experiment 6. In Fig.10, we have a zoomed view of the curves between 18 and 30 db. In these figures we are showing that for higher noise levels we obtained the better results of ISE with the MFs optimized for 10 db, and for lower noise levels with the ones optimized for 24 db of SNR.

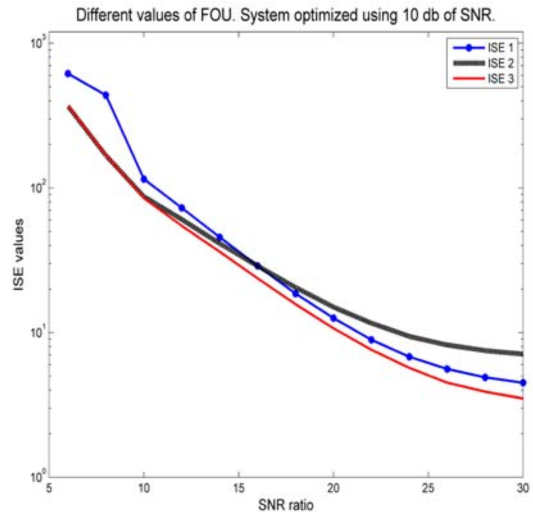

Fig. 7. System optimized for 10 db of SNR, with different search intervals for optimizing the MFs.

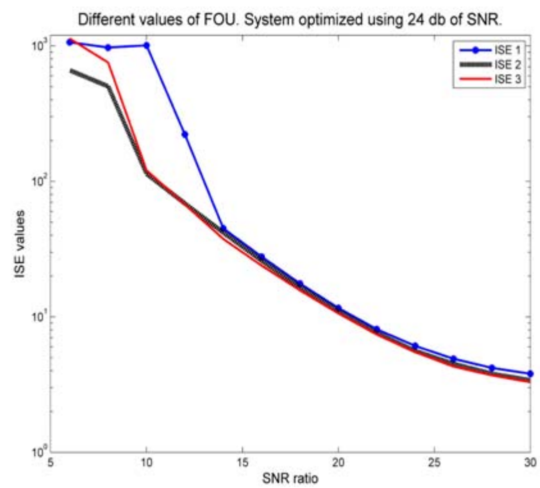

Fig. 8. System optimized for 24 db of SNR, with different search intervals for optimizing the MFs.

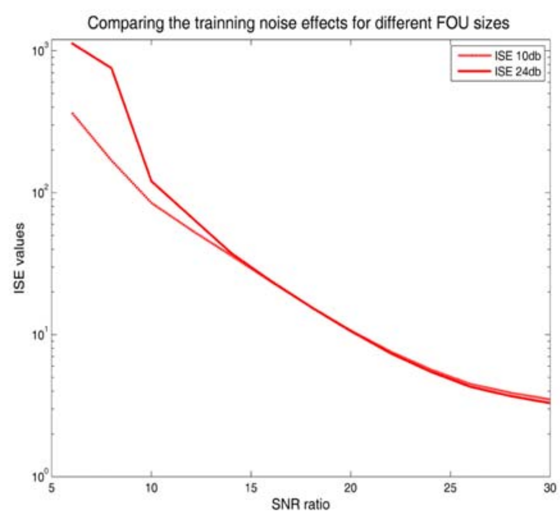

Fig. 9. A comparative ISE values for different FOU sizes and noise levels for optimizing the MFs.

TABLE IV

COMPARISON OF THE CHARACTERISTICS OF THE OPTIMIZED MFs FOR DIFFERENT INTERVALS OF THE FOU OF TYPE-2 FUZZY LOGIC CONTROLLER, FOR 10 DB SIGNAL NOISE RATIO.

Variable	**Type-2 FLC Intervals for MFs of *e* and Δe. σ between 8 to 9 for the positive and negative terms; σ between 1 to 2 for the zero term. $c_1 = c_2$.**			**Type-2 FLC Intervals for MFs of *e* and Δe. σ between 7 to 9 for the positive and negative terms; σ between 0.5 to 2 for the zero term. $c_1 = c_2$.**			**Type-2 FLC Intervals for MFs of *e* and Δe. σ between 5 to 9 for the positive and negative terms; σbetween 0.5 to 2 for the zero term. $c_1 = c_2$.**		
	Center c_1	**Standard Deviation σ_1**	**Standard Deviation σ_2**	**Center c_1**	**Standard Deviation σ_1**	**Standard Deviation σ_2**	**Center c_1**	**Standard Deviation σ_1**	**Standard Deviation σ_2**
Input e	-10	9	8.0298	-10	9	7.8717	-10	9	5.7384
	0	2	1.8911	0	2	1.3571	0	2	0.9101
	10	9	8.1167	10	9	7.6215	10	9	6.6960
Input Δe	-10	9	8.7767	-10	9	8.6036	-10	9	7.666
	0	2	1.0987	0	2	0.5008	0	2	0.6479
	10	9	8.5129	10	9	8.7118	10	9	7.5595
Output cde	-10	4.5	4.5	-10	4.5	4	-10	4.5	4
	-4.5	4	4	-4.5	4	4.5	-4.5	4	4.5
	0	4.5	4.5	0	4.5	4	0	4.5	4
	4	4	4	4	4	4.5	4	4	4.5
	10	4.5	4.5	10	4.5	4.5	10	4.5	4.5

TABLE V

COMPARISON OF PERFORMANCE CRITERIA FOR DIFFERENT INTERVALS OF THE FOU OF TYPE-2 FUZZY LOGIC CONTROLLER WITH OPTIMIZED MFS, FOR 10 DB SIGNAL NOISE RATIO. VALUES OBTAINED AFTER 200 SAMPLES.

SNR (db)	**Type-2 FLC Intervals for MFs of *e* and Δe. σ between 8 to 9 for the positive and negative terms; σ between 1 to 2 for the zero term. $c_1 = c_2$.**			**Type-2 FLC Intervals for MFs of *e* and Δe. σ between 7 to 9 for the positive and negative terms; σ between 0.5 to 2 for the zero term. $c_1 = c_2$.**			**Type-2 FLC Intervals for MFs of *e* and Δe. σ between 5 to 9 for the positive and negative terms; σbetween 0.5 to 2 for the zero term. $c_1 = c_2$.**		
	ISE	IAE	ITAE	ISE	IAE	ITAE	ISE	IAE	ITAE
6	616.4	274.7	3005	366.9	212.6	2204.3	368.7	211.4	2218.7
8	437.3	226.7	2509	167.4	142.9	1379.8	168.9	141.5	1338.2
10	115	116.6	1119.6	87.1	104.2	974.5	84.8	98.7	929.9
12	72.8	90.9	866.8	60.5	86	815.1	54.7	80.7	750.6
14	45.6	71.3	674.1	41.3	70.2	769.5	36.2	64.8	608.7
16	28.9	56.3	528.4	28.9	58	567.9	23.7	52.2	490.3
18	18.6	45.2	419.4	20.5	48.6	479.5	15.7	42	395.1
20	12.6	37	337	15	41.4	410.5	10.7	34.1	320
22	8.9	30.8	273.8	11.6	36.3	360.3	7.6	28	260.8
24	6.8	26.3	227.7	9.4	32.9	325.9	5.7	23.5	214.8
26	5.6	23.1	195.6	8.2	31.2	304.8	4.5	20.2	180.1
28	4.9	21	172.8	7.5	30.5	294.3	3.9	17.7	153.8
30	4.5	19.6	157.8	7.1	30	287.1	3.5	16.3	135.8

TABLE VI

COMPARISON OF THE CHARACTERISTICS OF THE OPTIMIZED MFs FOR DIFFERENT INTERVALS OF THE FOU OF TYPE-2 FUZZY LOGIC CONTROLLER, FOR 24 DB SIGNAL NOISE RATIO.

Variable	Type-2 FLC Intervals for MFs of *e* and Δe. σ between 8 to 9 for the positive and negative terms; σ between 1 to 2 for the zero term. $c_1 = c_2$.			Type-2 FLC Intervals for MFs of *e* and Δe. σ between 7 to 9 for the positive and negative terms; σ between 0.5 to 2 for the zero term. $c_1 = c_2$.			Type-2 FLC Intervals for MFs of *e* and Δe. σ between 5 to 9 for the positive and negative terms; σbetween 0.5 to 2 for the zero term. $c_1 = c_2$.		
	Center c_1	Standard Deviation σ_1	Standard Deviation σ_2	Center c_1	Standard Deviation σ_1	Standard Deviation σ_2	Center c_1	Standard Deviation σ_1	Standard Deviation σ_2
Input e	-10	9	8.0441	-10	9	7.7631	-10	9	6.963
	0	2	1.4188	0	2	1.4478	0	2	0.8629
	10	9	8.0045	10	9	7.2071	10	9	5.0858
Input Δe	-10	9	8.6585	-10	9	8.1797	-10	9	7.5034
	0	2	1.8657	0	2	1.1885	0	2	1.2757
	10	9	8.5921	10	9	8.1486	10	9	7.4305
Output cde	-10	4.5	4.5	-10	4.5	4	-10	4.5	4
	-4.5	4	4	-4.5	4	4.5	-4.5	4	4.5
	0	4.5	4.5	0	4.5	4	0	4.5	4
	4	4	4	4	4	4.5	4	4	4.5
	10	4.5	4.5	10	4.5	4.5	10	4.5	4.5

TABLE VII

COMPARISON OF PERFORMANCE CRITERIA FOR DIFFERENT INTERVALS IN THE FOU OF TYPE-2 FUZZY LOGIC CONTROLLER WITH OPTIMIZED MFS, FOR 24 DB SIGNAL NOISE RATIO. VALUES OBTAINED AFTER 200 SAMPLES.

SNR (db)	Type-2 FLC Intervals for MFs of *e* and Δe. σ between 8 to 9 for the positive and negative terms; σ between 1 to 2 for the zero term. $c_1 = c_2$.			Type-2 FLC Intervals for MFs of *e* and Δe. σ between 7 to 9 for the positive and negative terms; σ between 0.5 to 2 for the zero term. $c_1 = c_2$.			Type-2 FLC Intervals for MFs of *e* and Δe. σ between 5 to 9 for the positive and negative terms; σbetween 0.5 to 2 for the zero term. $c_1 = c_2$.		
	ISE	IAE	ITAE	ISE	IAE	ITAE	ISE	IAE	ITAE
6	1065.9	362.9	4640.1	661	288	3325	1132	385.5	4760.1
8	971.5	337.4	4465.3	503.7	247.1	286.2	754.8	308.2	3845.4
10	1008.8	331.8	4513.1	113.8	115.1	1102.9	120.6	116.7	1054.3
12	222.4	165.9	1985.4	68.7	87.1	820.3	67.4	85.8	800.8
14	45	71.2	679.3	41.9	67.8	633.8	37.6	64.4	602.8
16	27.8	55.1	521.5	26	53.4	496.5	23.9	51.5	481
18	17.6	43.4	407.5	16.6	42.3	390.7	15.7	41.3	383.3
20	11.6	34.5	319.7	11	33.8	307.9	10.6	33.2	305.3
22	8.1	28.1	253.8	7.6	27.5	244.1	7.4	27	243.9
24	6.1	23.2	202.1	5.6	22.8	194.9	5.5	22.3	195.8
26	4.9	19.5	162.1	4.5	19	156.5	4.3	18.6	157.6
28	4.2	16.9	131.1	3.8	16.2	126.6	3.7	15.8	128
30	3.8	15	107.3	3.4	14	103.5	3.3	13.7	105.2

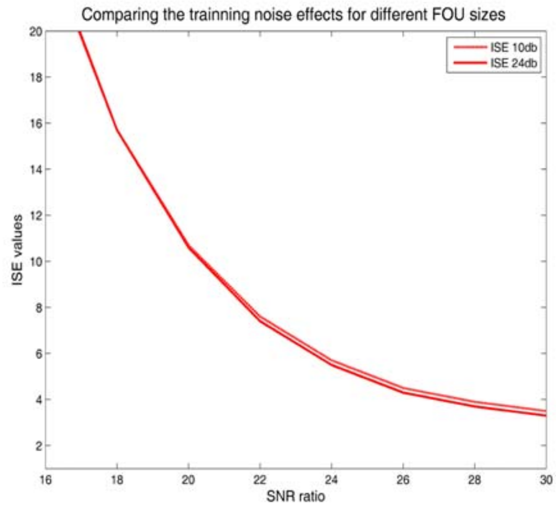

Fig. 10. An amplification of the curves showing a comparative ISE values for different FOU sizes and noise.

IV Conclusions

We observed and quantified using performance criteria such as ISE, IAE, and ITAE that in systems without uncertainties (ideal systems) is a better choice to select a type-1 FLC since it works a little better than a type-2 FLC, and it is easier to implement it. It is known that type-1 FLC can handle nonlinearities, and uncertainties up to some extent.

Unfortunately, real systems are inherently noisy and nonlinear, since any element in the system contributes with deviations of the expected measures because of thermal noise, electromagnetic interference, etc., moreover, they add nonlinearities from element to element in the system.

We are concluding that using a type-2 FLC in real world applications can be a good option since this type of system is a more suitable system to manage high levels of uncertainty, as we can see in the results shown in tables III, V and VII.

We found that the optimized membership functions (MFs) for the inputs of a type-2 system increases the performance of the system for high noise levels. Also when the search interval for optimizing the MFs is wider we obtained better results in the performance of the system as can be seen in the ISE, IAE, ITA values in tables V and VII, so these results indicate that the type-2 fuzzy system can handle in a better way the uncertainty introduced to the system.

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