Global Stabilization of Robot Control with Neural Network and Sliding Mode

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***Abstract*—This paper presents a neural network global PID-sliding mode control method for the tracking control of robot manipulators with bounded uncertainties. A certain sliding mode controller with PID sliding function is developed. In this controller, the switching gain is tuned by a single-input radial-basis-function neural network on the reachable condition of sliding mode. Thus, the effect of chattering can be alleviated. Moreover, global sliding mode is realized by designing an exponential dynamic sliding function. Mathematical proof of the stability and convergence of the control system is given. Simulation results demonstrate that the chattering and the steady state errors are eliminated and satisfactory trajectory tracking is achieved.**

***Index Terms*—Neural network, Robot, Robustness, Sliding mode control.**

I. Introduction

A well known approach to the control of uncertain systems by nonlinear feedback laws is the sliding mode control [1]-[3]. Sliding mode controller design provides a systematic approach to the problem of maintaining stability in the face of modeling imprecision and uncertainty. However, chattering problem is a major drawback of sliding mode control. The boundary layer is used to avoid chattering phenomena [4]. The cost of this technology is a reduction in the accuracy of the tracking performance [5, 6].

In general, sliding mode control has two phases in the control process. One is the reaching mode, and the other is the sliding mode. The former is the phase of initial states toward the sliding surface. When the system trajectory stays on the sliding surface, the control system can reject uncertainties and disturbances. However, robust tracking is guaranteed only after the system states reach the sliding surface, and therefore robustness is not guaranteed during the reaching phase. In order to overcome this problem, some researches have been proposed, such as global sliding mode control [7-9]. Here, global sliding mode is realized by designing an exponential dynamic sliding function. In [10]-[12], the sliding mode control with PID sliding surface for robot manipulators were presented. Simulation results demonstrated that PID sliding surface provided faster response than that of traditional PD-manifold controller.

In this paper, a robust neural network global sliding mode PID-controller is proposed to control a robot manipulator with parameter variations and external disturbances. The chattering phenomenon is eliminated by substituting a single-input radial-basis-function (RBF) neural network. Moreover, a theoretical proof of the stability and the convergence of the proposed scheme are provided.

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II. Robot Manipulator Model

A. Dynamics of the Robot Manipulator

Consider an n-link robot manipulator, which takes into account the friction forces and disturbances, with the equation of motion given by [13],

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + T_d = \tau , \tag{1}$$

where $q \in R^n$ is the joint angular position vector of the robot manipulator; $\tau \in R^n$ is the applied joint torques; $M(q) \in R^{n\times n}$ is the inertia matrix; $C(q,\dot{q}) \in R^{n\times n}$ is the effect of Coriolis and centrifugal forces; $G(q) \in R^n$ is the gravitational torques; and $T_d \in R^n$ is the vector of generalized input due to disturbances.

B. Properties of the Robot Manipulator

Property 1. The inertia matrix $M(q)$ is symmetric and positive definite and satisfies

$$m_1 I_n \le M(q) \le m_2 I_n , \ \forall q \in R^n , \tag{2}$$

where m_1 and m_2 are positive constant, and $I_n \in R^{n\times n}$ is the identity matrix.

Property 2. The Coriolis and centrifugal matrix $C(q,\dot{q})$ satisfies

$$\|C(q,\dot{q})\| \le \zeta_c \|q\| , \ \forall q, \dot{q} \in R^n , \tag{3}$$

where ζ_c is a positive constant, and $\|(\cdot)\|$ is the Euclidean norm.

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Property 3. The gravity term is bounded as

$$\|G(q)\| \leq g_b , \ \forall q \in R^n , \tag{4}$$

where g_b is a known positive function of *q*.

Property 4. Using a proper definition of the matrix $C(q,\dot{q})$, the $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric and satisfies

$$x^T \left[\dot{M}(q) - 2C(q,\dot{q})\right] x = 0 , \ \forall x \in R^n . \tag{5}$$

III. Neural Network Global Sliding Mode Controller

A. *Definition of Sliding Function*

Let the tracking error vector be

$$e = q - q_d , \ e \in R^n , \tag{6}$$

where q_d is the desired trajectory. The sliding function is defined as

$$\sigma(t) = \dot{e} + \Lambda_1 e + \Lambda_2 \int_0^t e dt - \beta(t) , \tag{7}$$

where Λ_1 and Λ_2 are constant positive definite diagonal matrices. Now we define $\beta(t)$ as $\beta(t) = \sigma(0)\exp(-\alpha t)$, where $\alpha > 0$ and $\sigma(0)$ is the initial value of sliding function. The choice of $\beta(t)$ should satisfy: (1) $\beta(0) = \dot{e}(0) + \Lambda_1 e(0)$, (2) $\beta(t) \to 0$ as $t \to \infty$, and (3) $\dot{\beta}(t)$ exits and is bounded.

Notably, the function $\beta(t)$ drives system states in any state space directly to the sliding mode without a reaching phase. In other words, the system states are initially located in the sliding mode. If system states are maintained on the surface for $t > 0$, then *e* approaches zero and $q \to q_d$.

The follow sliding condition will be used to develop the control law,

$$\frac{1}{2}\frac{d}{dt}\left[\sigma^T M(q)\sigma\right] < 0 . \tag{8}$$

Equation (8) means that the distance to the sliding surface decreases to zero eventually along with all system trajectories. Thus, the system states are driven to the sliding surface on which sliding mode takes place.

B. *Definition of Control Input*

Let the subscript "$_o$" stand for the nominal value, and symbol "Δ" stand for the uncertain value, i.e., $M = M_o + \Delta M$, $C = C_o + \Delta C$, $G = G_o + \Delta G$.

Assumption 1. The uncertainties of the *n*-link robot manipulator (1) can be lumped as Δf,

$$\Delta f = \Delta M(\Lambda_1 \dot{e} + \Lambda_2 e - \ddot{q}_d) - \Delta C(\dot{q}_d - \Lambda_1 e - \Lambda_2 \int_0^t e dt)$$
$$+ \dot{\beta} - \Delta G - T_d . \tag{9}$$

The control input of conventional sliding mode control consists of a continuous nominal control part, and a discontinuous switching control part. The switching control part causes the chattering problem. Here we propose a single-input RBF neural network to find a suitable gain matrix to replace the switching control input.

Define the sliding mode control law as follows, based on equivalent control,

$$\tau = -M_o(\Lambda_1 \dot{e} + \Lambda_2 e - \ddot{q}_d + \dot{\beta}) + C_o(\dot{q}_d - \Lambda_1 e - \Lambda_2 \int_0^t e dt)$$
$$+ G_o - A\sigma - K . \tag{10}$$

where

$$A = diag\left[a_1 \quad a_2 \quad \cdots \quad a_n\right], \ a_i \text{ is positive constant,} \tag{11}$$

$$K = \left[k_1 \quad k_2 \quad \cdots \quad k_n\right], \ k_i = W_{k_i}^T \Phi_{k_i}(\sigma_i) . \tag{12}$$

The gain matrix *K* is obtained by single-input RBF neural network. The symbol W_{k_i} is the $m \times 1$ vector of output layer weights, *m* is the number of nodes in hidden layer, and $\Phi_{k_i}(\sigma_i) = \left[\phi_{k_i}^1 \quad \phi_{k_i}^2 \quad \cdots \quad \phi_{k_i}^m\right]^T$ is the $m \times 1$ vector of outputs of hidden layer nodes. They can be chosen as Gaussian-type function,

$$\phi_{k_i}^j(\sigma_i) = \gamma_i \exp(-\left|\sigma_i - \mu_i^j\right|^2 / 2\upsilon_i^{j2}) , \tag{13}$$

where μ_i^j and υ_i^j are the center and variance of the *j*th basis function of the *i*th RBF neural network. The gain γ_i is a positive constant.

Define $W_{k_{id}}$ is the ideal value of W_{k_i}, so that $k_i = W_{k_{id}}^T \Phi_{k_i}(\sigma_i)$ is the optimal compensation for Δf_i, where Δf_i is the *i*th row of Δf. According to the property of universal approximation of RBF neural network, there exists $\delta_i > 0$ and the following condition satisfied,

$$\left|\Delta f_i - W_{k_{id}}^T \Phi_{k_i}(\sigma_i)\right| \leq \delta_i , \tag{14}$$

where δ_i is positive and can be chosen small.

C. *Stability Proof*

Theorem 1. Consider an *n*-link robot manipulator, as described in (1), which contains unknown but bounded uncertainties. If (1) is controlled applying the control input (10) to (13), then the control system (1) is globally stable.

Proof. Choose the Lyapunov function candidate as

$$V(t) = \frac{1}{2}\sigma^T(t) M(q) \sigma(t) . \tag{15}$$

Then,

$$\dot{V}(t) = \sigma^T M(q)\dot{\sigma} + \frac{1}{2}\sigma^T \dot{M}(q)\sigma . \quad (16)$$

Using Property 4 and Assumption 1 and substituting control law (10) to (12) and Gaussian-type function (13), then Eq. (16) becomes

$$\begin{aligned}\dot{V}(t) &= -\sum_{i=1}^{n} a_i\sigma_i^2 + \sum_{i=1}^{n}\sigma_i\left[\Delta f_i - W_{k_{id}}^T \Phi_{k_i}(\sigma_i)\right] \\ &\le -\sum_{i=1}^{n} a_i\sigma_i^2 + \sum_{i=1}^{n}\left|\sigma_i\right|\left|\Delta f_i - W_{k_{id}}^T \Phi_{k_i}(\sigma_i)\right| . \quad (17)\end{aligned}$$

Form the property of universal approximation of RBF neural network, assume

$$\left|\Delta f_i - W_{k_{id}}^T \Phi_{k_i}(\sigma_i)\right| \le \delta_i \le \rho_i \left|\sigma_i\right| , \quad (18)$$

where $0 < \rho_i < 1$. Then, the second term on the right side of (17) satisfies

$$\left|\sigma_i\right|\left|\Delta f_i - W_{k_{id}}^T \Phi_{k_i}(\sigma_i)\right| \le \rho_i\sigma_i^2 . \quad (19)$$

Therefore, one can get

$$\dot{V}(t) \le -\sum_{i=1}^{n} a_i\sigma_i^2 + \sum_{i=1}^{n}\rho_i\sigma_i^2 . \quad (20)$$

Since a_i is a positive constant and $a_i > \rho_i$ is chosen, it is clear that

$$\dot{V}(t) \le 0 . \quad (21)$$

Equation (21) guarantees the decay of the energy of σ as long as $\sigma \ne 0$. Thus, the overall system is stable.

■

IV. EXAMPLE AND SIMULATION RESULTS

Consider a two-link robot manipulator [14] as shown in Fig. 1. The parameter matrices are as follows:

$$M(q) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3\cos(q_2) & \theta_2 + 2\theta_3\cos(q_2) \\ \theta_2 + 2\theta_3\cos(q_2) & \theta_2 \end{bmatrix}, \quad (22)$$

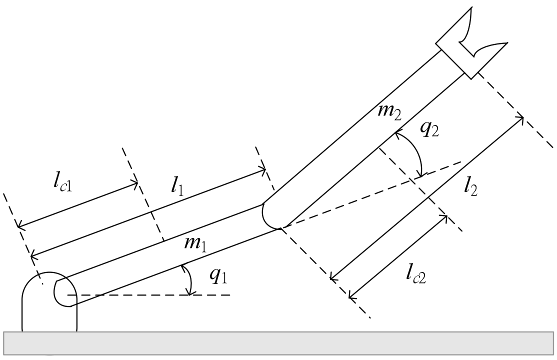

Fig. 1. Two-link robot manipulator

$$C(q,\dot{q}) = \begin{bmatrix} -\theta_3\sin(q_2)\dot{q}_2 & -\theta_3\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ \theta_3\sin(q_2)\dot{q}_1 & 0 \end{bmatrix}, \quad (23)$$

$$G(q) = \begin{bmatrix} g(\theta_4 + \theta_5)\cos(q_1) + g\theta_6\cos(q_1 + q_2) \\ g\theta_6\cos(q_1 + q_2) \end{bmatrix}, \quad (24)$$

where g is the gravitational acceleration and

$$\theta_1 = m_1 l_{c1}^2 + m_2 l_1^2 + I_1 , \quad (25)$$

$$\theta_2 = m_2 l_{c2}^2 + I_2 , \quad (26)$$

$$\theta_3 = m_2 l_1 l_{c2} , \quad (27)$$

$$\theta_4 = m_1 l_{c1} , \quad (28)$$

$$\theta_5 = m_2 l_1 , \quad (29)$$

$$\theta_6 = m_2 l_{c2} . \quad (30)$$

Assume that the parameters of the unloaded robot are given by Table 1. The desired trajectories are

$$q_d = \begin{bmatrix} q_{d1} \\ q_{d2} \end{bmatrix} = \begin{bmatrix} 1.6 - 1.6\exp(-8t) - 12.8t\exp(-8t) \\ 1.6 - 1.6\exp(-8t) - 12.8t\exp(-8t) \end{bmatrix}. \quad (31)$$

Regarding an unknown load carried by the robot as part of the second link, the parameters m_2, l_{c2} and I_2 change to $m_{2O} + \Delta m_2$, $l_{c2O} + \Delta l_{c2}$, and $I_{2O} + \Delta I_2$, respectively. Suppose that the variation of parameters lies in the intervals: $0 \le \Delta m_2 \le 3$, $0 \le \Delta l_{c2} \le 0.25$, and $0 \le \Delta I_2 \le 0.5$. The external disturbance is assumed to be

$$T_d = \begin{bmatrix} 3.2 + 2\cos(0.02t) \\ 3.5 + 1.7\sin(0.02t) \end{bmatrix}. \quad (32)$$

In order to achieve that the desired response of each joint of the manipulator being a second-order critically damped response, we choose damping ratio to be 1 and natural frequency to be 13 rad/sec. Therefore, the sliding function constants are $\Lambda_1 = \begin{bmatrix} 26 & 0 \\ 0 & 26 \end{bmatrix}$ and $\Lambda_2 = \begin{bmatrix} 169 & 0 \\ 0 & 169 \end{bmatrix}$. The control input is chosen as in (10) to (13). The matrix A is $A = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$. The gains of Gaussian-type function are $\gamma_1 = 18$ and $\gamma_2 = 12$.

The simulation results are shown in Figs. 2 to 5. Figs. 2 and 3 show that both q_1 and q_2 converge to the desired trajectories. From Figs. 4 and 5, it is obvious that chattering of the control input is eliminated by applying the proposed method.

Table 1. Parameters of the robot manipulator

m_1	m_{2o}	l_1	l_{2o}	l_{c1}	l_{c2o}	I_1	I_{2o}
10	5	1	0.5	0.5	0.5	10/12	5/12

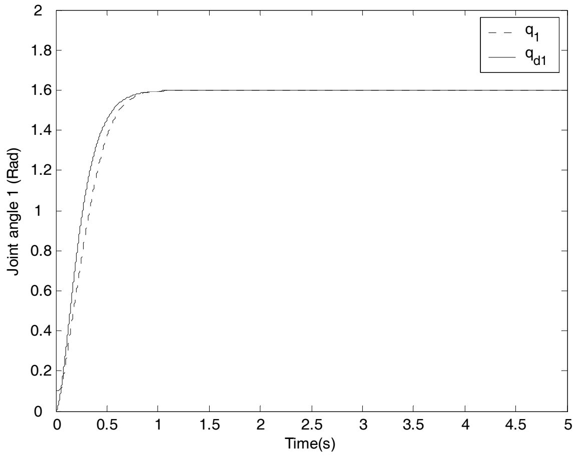

Fig. 2. The response of q_1 and desired path q_{1d}

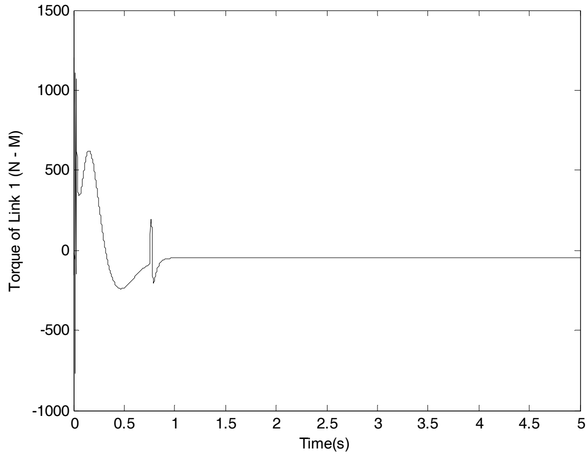

Fig.4. The control input τ_1

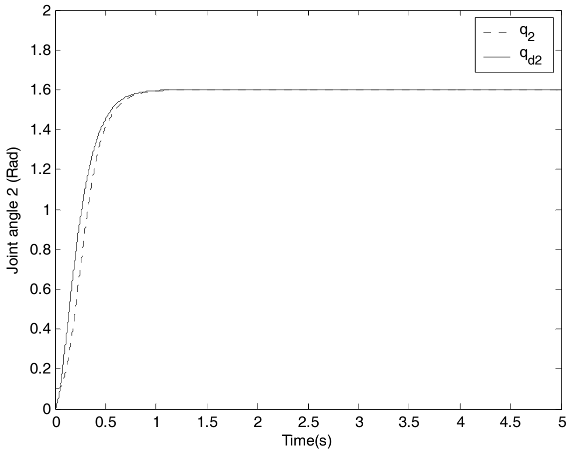

Fig. 3. The response of q_2 and desired path q_{2d}

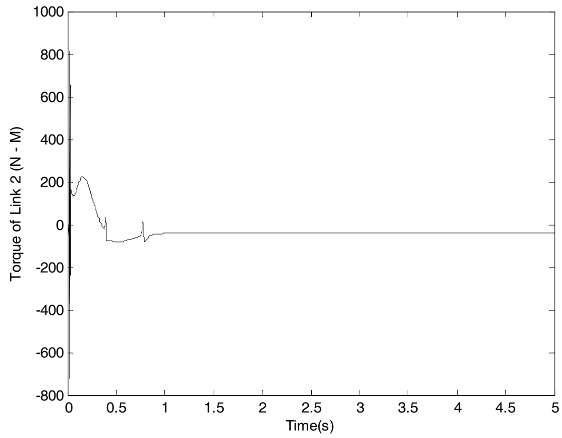

Fig.5. The control input τ_2

V. Conclusion

In this paper, a robust neural network global sliding mode PID-controller is proposed to control a robot manipulator with parameter variations and external disturbances. In classical sliding mode control, the control input gain is chosen to be lager than the bound of the uncertainties, which means the controller has to have a prior knowledge of the uncertainties. The proposed method can compensate the uncertainties. The common problem of input chattering is also eliminated and hence the control input is smooth. The other advantage of the proposed method is that it possesses sliding mode characteristics all the time without a reaching phase.

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