Engineering Letters, 17:4, EL_17_4_15

Determination of Loss of Coolant Accident (LOCA) in Nuclear Power Plants Using Signal Processing Method

R. Mahmoodi, M. Shahriari, A. Zolfaghari

***Abstract*—A major objective in reactor design is to provide the capability to withstand a wide range of postulated events without exceeding specified safety limits. Assessment of the consequence of hypothetical Loss of Coolant Accident (LOCA) in primary circuit is an essential element to address fulfillment of acceptance criteria. In addition to analysis of behavior of plants during a LOCA, finding the position of rupture help to manage accident in a right direction. In this work, the transient vibration signal from a pipe rupture is used to determine the position of LOCA. A finite element formulation (Galerkin Method) is implemented to include the effect of fluid-structure interaction (FSI). The coupled equations of fluid motion and pipe displacement are solved. The results are in good agreement with published data.**

Fast Fourier Transform (FFT) provides an alternate way of representing data: instead of representing the vibration signal amplitude as a function of time, we represent this signal by how much information is contained at different frequencies. The most of the frequencies of the structure and the fluid coupled are present in the FFT of structural response and then the dominant frequency of excitation is obtained.

In addition, the Power Spectral Density (PSD), a measurement of energy at various frequencies is worked out. To convert signals from the time domain to the frequency domain and to obtain PSD of those signals, this analysis was performed using MATLAB software.

***Index Terms*—Signal Processing, Fast Fourier Transform, Power Spectral Density, Finite element method, Loss of Coolant Accident**

I. Introduction

When in a fluid-filled piping system under stationary flow conditions at some location the fluid velocity changes, pressure surges are induced and propagate at speed of sound through the system. Spectacular pressure surges, referred to as water hammer, can be caused by e.g. rapid closing or opening of valve, or to the stopping or starting of pump. Other causes of water hammer are loud rejection of turbine, seismic excitation and pipe rupture.

Manuscript received August 15, 2009.

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Tijsseling et al. [2] presented a very detailed review of transient phenomena in liquid-filled pipe systems. He dealt with water hammer, cavitation, structural dynamics and fluid-structure interaction (FSI). The main focus was on the history of FSI research in time domain.

Wiggert et al. [3] used the MOC to study transients in pipeline systems. They identified seven wave components, coupled axial compression of liquid and pipe material, along transverse shear and bending of the pipe elements in two principal direction and torsion of the pipe wall.

Heinsbroek [4] reported an application of FSI in the nuclear industry. His analysis was based on a combination of MOC and FEM, His conclusion was that the MOC technique was superior for axial dynamics.

In this paper, the numerical resolution of the coupled problem is carried out with a finite element method, which leads to the definition of a matrix problem. Information about vibration of structure is contained in the peak amplitude and frequency content of the initial wave.

Q.S.Li et al. [5] used frequency domain analysis of FSI in liquid-filled pipe systems by transfer matrix method, along taking the Laplace Transform. The method implement both harmonic and frequency response analysis. Especially is helpful to obtain the solution at any point of system. It is also shown how the frequencies and mode shapes change with the radius and material properties of the piping system.

The main purpose of the present study is to perform a frequency analysis that focuses on fluid-structure interaction effect and their influence on the vibration signal of the pipe with signal processing method to finding the position of pipe rupture when LOCA accident is found.

II. Basic Equations

A. Finite element method

The transient behavior of fluid-filled pipeline system is governed by acoustic waves propagating in both fluid and pipe pressure wave in the fluid, referred to as water hammer coexist with the pipe wall. The behavior of the fluid is governed by extended water hammer equations. Whereas, the dynamics of the pipe is modeled by standard beam theory. For studding the FSIs in pipeline, the model proposed by Wiggert et al. [3] has been used. The set of pipe dynamic equations suggested by Wiggert et al. [3] is shown below:

$$EA_p u'' - m\ddot{u} + 2\nu AP' = 0 \qquad (1)$$

$$\rho_w \dot{V} + P' = 0 \qquad (2)$$

(Advance online publication: 19 November 2009)

$$\dot{p}+\rho_w a^{*2}V'=0 \quad (3)$$

where

$$a^{*2}=\frac{(K_f/\rho_w)}{1+K_f D/E^*t},\ E^*=\frac{E}{1-\nu^2}$$

and K_f is the fluid bulk modulus, m_p, E, ν, A_p, D, ρ_w, t, u, P and V, are the mass per unit length of pipe, Young's modulus of elasticity, Poisson's ratio, the cross-sectional area, the inner diameter, density of fluid, the thickness of the pipeline, displacement of the pipe in x-direction, pressure and velocity of flow respectively. If the derivative of (2), with respect to the axial direction and (3), with respect to time respectively is taken, one of the variables can be eliminated.

Two wave equations can then be obtained, either in terms of pressure or in terms of velocity. The wave equations obtained are elliptical in nature and suitable for solution by FEM. Since the boundary condition for the valve opening event is in terms of flow velocity, it is easier to use the wave equation in terms of flow velocity which is given by:

$$\frac{\partial^2 V}{\partial x^2}-\frac{1}{a^{*2}}\frac{\partial^2 V}{\partial t^2}-2\nu\frac{\partial^3 u}{\partial x^2\partial t}=0 \quad (4)$$

The relation between pressure and velocity given by (3), is used to obtain the pressure from velocity. With assuming the effect of pipe displacement on the fluid is negligible, one can eliminate the third term of (3), to reduce (4), to two terms. The finite element of wave (4), and pressure equation ((3)), using Galerkin technique is obtained as:

$$[G]\{\ddot{V}\}+[H]\{V\}=\{0\} \quad (5)$$

$$[A]\{P\}+[B]\{V\}=\{0\} \quad (6)$$

For modeling the pipe, 3D beam element with six degree-of-freedom per node is used. From "(1)," will get:

$$[M]\{\ddot{u}\}+[K]\{u\}-[S]\{P\}=\{0\} \quad (7)$$

where [M] and [K] are the mass matrix and stiffness matrix of the pipe and the interaction of pressure with the structure due to the Poisson coupling is [S]. [G], [H], [A], [B] and [S] are obtained through using Galerkin method by implement Hermity equation as shape functions. The Runge-Kutta fourth order integration scheme is used to evaluate the transient response for the valve opening or closure event. The fully coupled equation in state space form is given by:

$$\begin{Bmatrix}\dot{u}\\ \ddot{u}\\ \dot{V}\\ \ddot{V}\\ \dot{P}\end{Bmatrix}=\begin{bmatrix}0 & [I] & 0 & 0 & 0\\ -[M]^{-1}[K] & 0 & 0 & 0 & [M]^{-1}[S]\\ 0 & 0 & 0 & [I] & 0\\ 0 & 0 & -[G]^{-1}[H] & 0 & 0\\ 0 & 0 & -[A]^{-1}[B] & 0 & 0\end{bmatrix}\begin{Bmatrix}u\\ \dot{u}\\ V\\ \dot{V}\\ P\end{Bmatrix} \quad (8)$$

The size of matrix is so large and numerical overflow errors to occur. To overcome this difficulty, the modal reduction technique is used to reduce the size of both structural and fluid matrices (Jayaraj et al. [6]). The first few mode shapes of the structure $[\varphi_s]$ as well as the fluid $[\varphi_f]$ are used for transforming the respective variables by substituting:

$$\{u\}=[\varphi_s]\{x\} \quad (9)$$

$$\{V\}=[\varphi_f]\{V_m\} \quad (10)$$

If the frequencies of the fluid are much higher than those of The fundamental frequency of the structure, one has still to include a few mode shapes of the structure having frequencies in the range of fluid frequencies, as it can resonate. After substitution and multiplying throughout by $[\varphi_s]^T$ and $[\varphi_f]^T$ respectively, one gets:

$$[\varphi_S]^T[M][\varphi_S]\{\ddot{x}\}+[\varphi_S]^T[K][\varphi_S]\{x\}-[\varphi_S]^T[S_2]\{p\}=0 \quad (11)$$

$$[\varphi_f]^T[G][\varphi_f]\{\ddot{V}_m\}+[\varphi_f]^T[H][\varphi_f]\{V_m\}=0 \quad (12)$$

$$[A]\{\dot{P}\}+[B][\varphi_f]\{V_m\}=0 \quad (13)$$

The values of the structural variables as well as of the fluid flow velocity available in the modal coordinates are transformed to the nodal coordinates by multiplying with $[\varphi_s]$ and $[\varphi_f]$ respectively. The pressure values can be used directly without transformation.

B. *Assumption*

The formulation is based on using of linear elasticity, no buckling, negligible radial inertia, low Mach number. Furthermore, it is understood that instantaneous pressure will remain above vapor pressure so that no cavitation will occur.

III. Validation Studies

Tijsseling et al. [1] used the water hammer theory for the fluid coupled with beam theory for the pipe to model FSI problems in non-rigid pipeline systems. They used simple to model represent by the four equations. The system analyzed consists of a pipe with length of 20 m. The other parameters of the reservoir-pipe-valve system have been shown in fig. 1. In the steady state situation fluid is flowing from reservoir to valve.

The structural boundary conditions for the pipeline system are no displacements at the valve as well as at the upstream reservoir end. At t=0 ms, the valve is closed instantaneously. The pressure rise propagates with a sound speed towards the reservoir. Fig. 2 shows the pressure history near the valve due to valve closure, a comparison of the results from the four model formulations and in good agreement with Tijsseling et al. [1]. Table 1 shows the fluid and structural frequencies of system. For investigation modal reduction, the fluid frequencies are evaluated from the finite element from (5).

IV. Signal Processing Analysis

A. *Frequency domain analysis*

A software utility set of graphically driven procedures that run under the MATLAB environment, was developed to illustrate the signal processing techniques used. Validating numerical results of equations with Tijsseling et al. [1], we extended our test problem for a pipe with 2 m length. Fig. 3

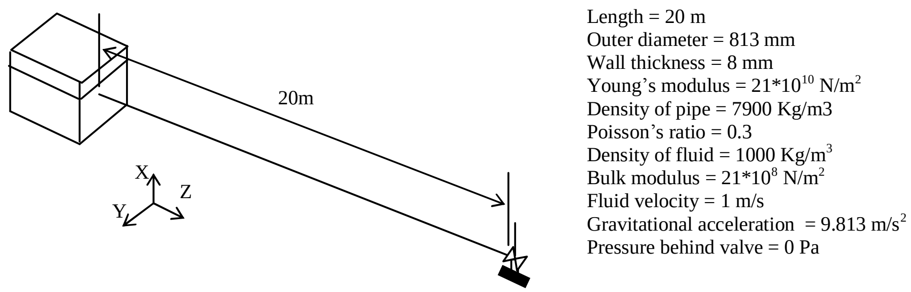

Fig. 1: Tijsseling geometry and parameters

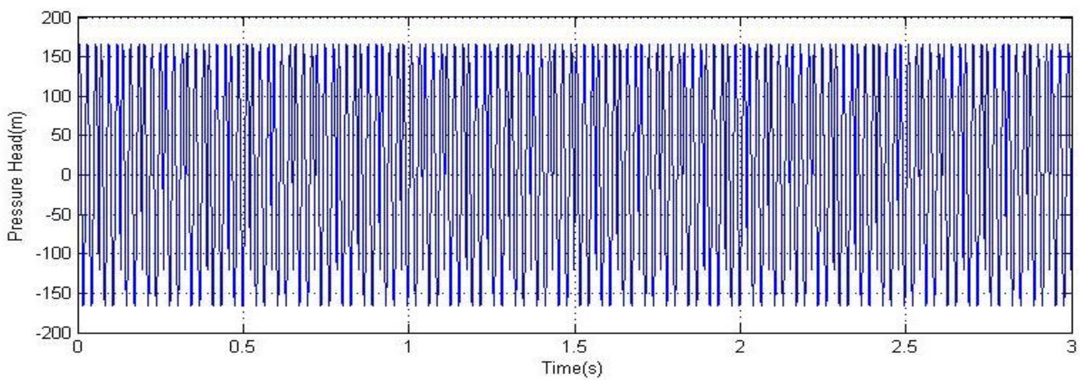

Fig. 2: Pressure history near the valve from present study

Table 1: Structural and fluid frequencies, in Hz, for Tijsseling et al [1] geometry

Serial no.	Structural frequency	Fluid frequency
1	43.96	167.28
2	113.76	334.43
3	221.67	667.8
4	365.32	833.77
5	544.8	999

illustrates time domain plots of displacement of the pipe for different values of x. clearly, as x increases (from valve), the displacement of pipe decreases. Fig. 4 shows this decrease during the time as maximum displacement in length of the pipe. FFTs and power spectrum are useful for measuring the frequency content of vibration transient signals. FFTs produce the average frequency content of signal over the entire time that signal is acquired. For this reason, to use FFT for the vibration signals analysis or in case, one needs the energy at each frequency line where it is required as a result of power spectrum computation. In this simulation the sampling frequency and sampling rate are 500 kHz and 2.5 kHz respectively.

In other to verify the major frequency component of excitation, the FFT of the structural response including the effect of vibration of the fluid on the structure is shown in Fig. 5. In this case, some of frequencies are suppressed and some are slightly deviated form original values. The dominant frequency of excitation of the system is 109 Hz. The frequencies of pipe system are depending to the length of pipe and constant versus the length of it. It is concluded from Fig. 4 that is maximum displacement of the pipe decreases with increases x with damped oscillation trend, the amplitude of vibration signals in FFT plots (Fig. 5) is decreasing. Fig.6 shows decreasing damped trend, which represent the mode shape of structure at the dominant frequency 109 Hz. The natural frequency of the structure and fluid are found separately without coupling the structure and fluid for this case, and it is shown in Table 1. The first sixth frequencies of pipe system are given in Table 2 where the amplitudes obtained at those frequencies in x=20 cm.

B. *Power Spectral Density*

It is important to find severity of the pressure surge impacts that clear intensity of LOCA. The Power Spectral Density (PSD), a measurement of the energy at various

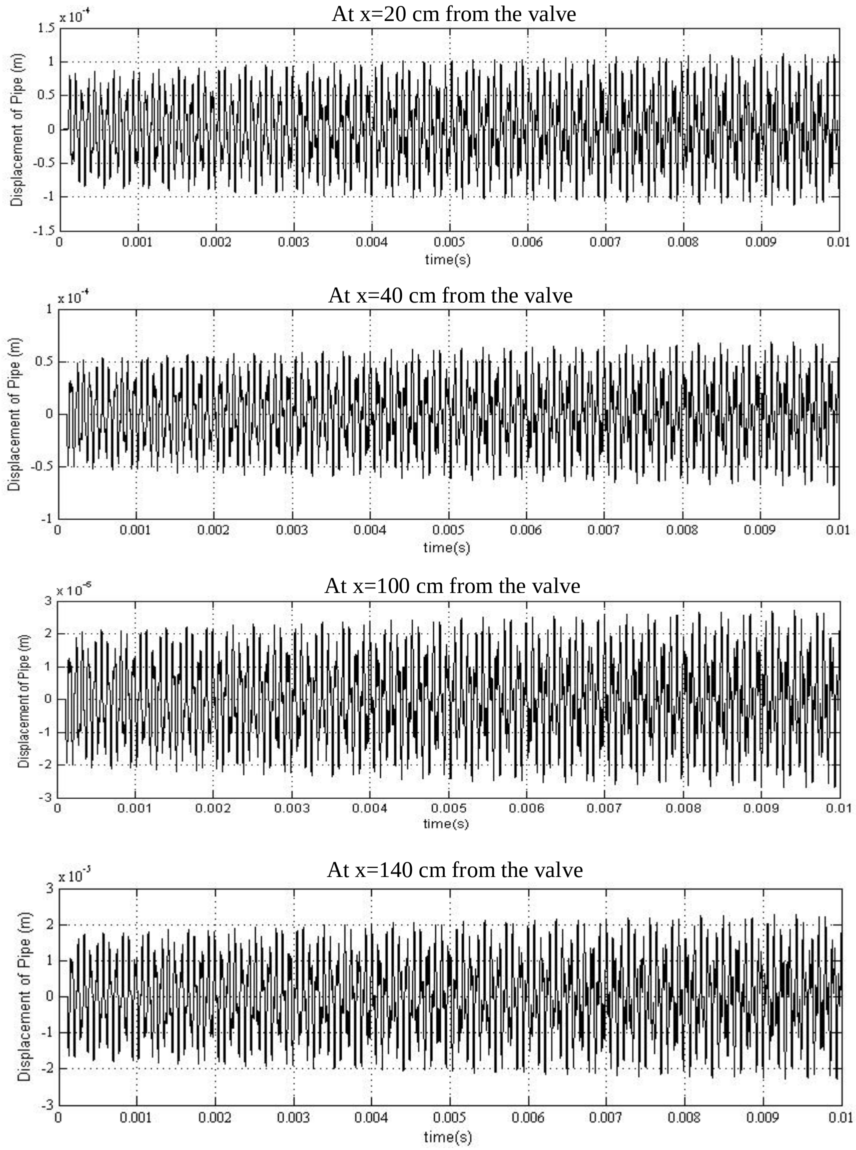

Fig. 3: Time domain plots of the displacement of pipe

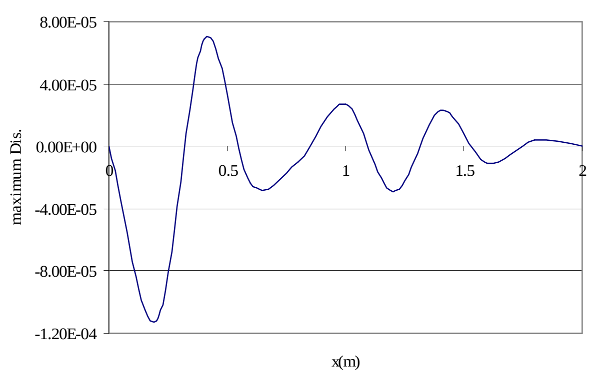

Fig. 4: The maximum displacement of pipe

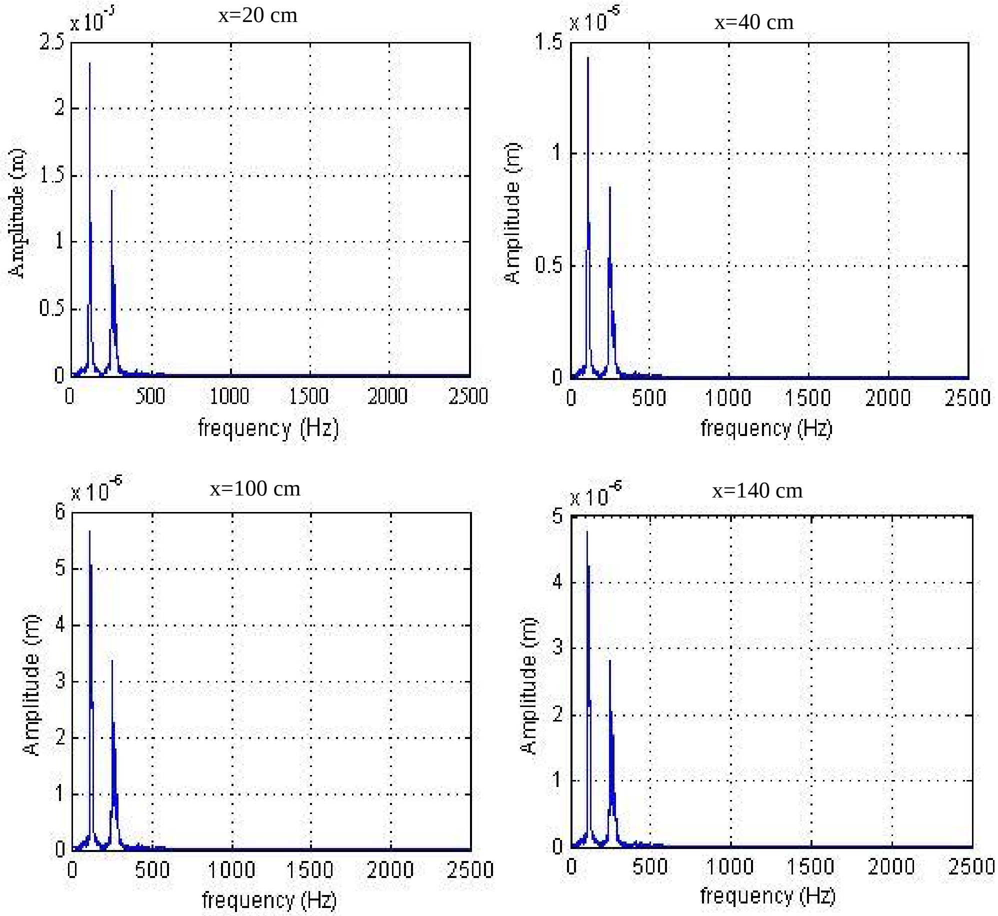

Fig 5: Frequency domain of vibration signals with FFT method

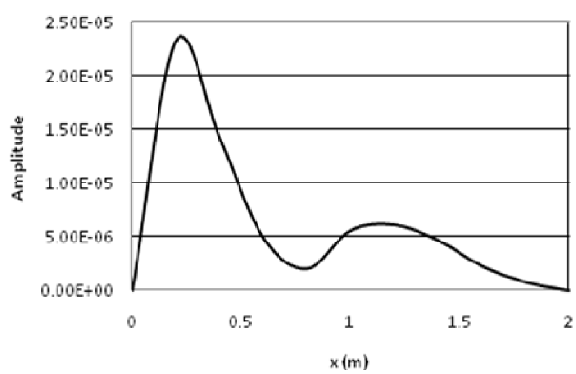

Fig. 6: Mode shape of pipe system in dominant frequency (109 Hz)

frequencies is computed. Fig. 7 shows the PSD of the vibration signals at the different position of pipe length. It is obvious that the energy of the vibration signals reduce along increasing length of pipe which has been shown in Fig. 7.

V. Conclusion

It can be proved, theoretically, that in a single straight pipe with open-closed and fixed-fixed boundaries for liquid and pipe respectively, FSI influences the impact-induced system vibration. For investigation of pipe vibration of pipe, a finite element formulation for the fully coupled pipe dynamic theory is introduced and the water hammer phenomenon which occurs due to sudden valve closure or opening is modeled using a new velocity based finite element formulation. The comparison of the results of the pressure formulation with a benchmark problem validates the present formulation. The rupture of pipe causes water hammer phenomenon and loss of coolant accident (LOCA) in nuclear power plants. With detecting vibration of pipe in abnormal conditions and considering water hammer due to rupture of pipe, loss of coolant accident can be recognized. Signal processing methods e.g. FFTs are effective procedures to process the vibration signals caused by LOCA for finding natural frequencies and mode shapes in pipeline systems. The dominant frequency of the combined pipe system, coupled

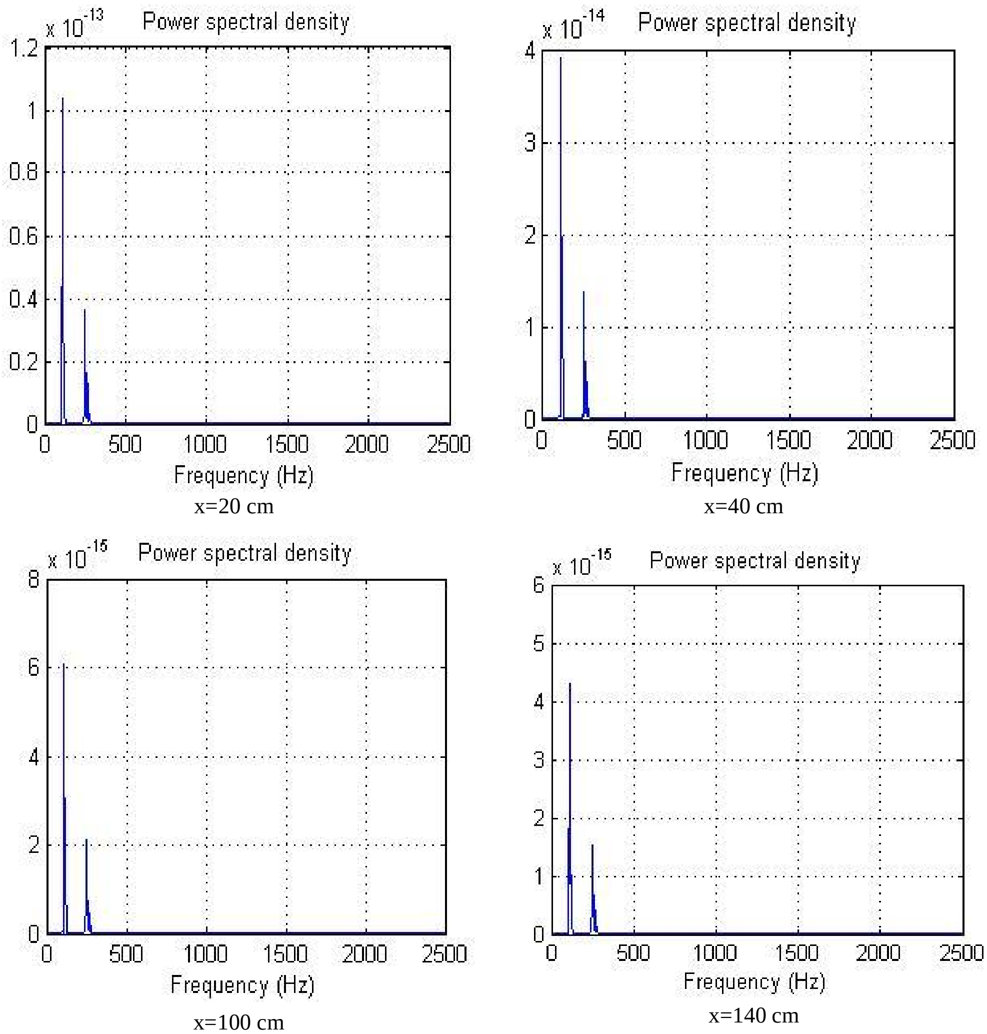

Fig. 7: The Power Spectral Density (PSD) at different values of x

Table 2: Frequencies in Hz of Pipe system and their amplitude at x=20 cm

Serial no.	Frequency	Amplitude
1	190	2.33E-5
2	112	1.532E-5
3	115	1.656E-5
4	120	6.941E-6
5	248	1.386E-6
6	252	6.19E-6

fluid and structure can be obtained by Fast Fourier Transfer. The severity of those signals in other to energy of each signal can be finding with the PSD analysis.

ACKNOWLEDGMENT

The authors also appreciate the kind help of Mahdi Aghaie and Mehdi Zangian.

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