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Sliding Mode Control of Space Robot for Unknown Target Capturing

Takuro KOBAYASHI and Shinichi TSUDA

***Abstract*—Space Robot is extensively utilized in space program, such as the Space Station Freedom. Furthermore the space robot is expected to maintain failed satellites and to capture space debris in the autonomous manner in the near future. This paper deals with the robust control of space robot in capturing operation of the target and controlling the spacecraft motion under unknown parameters, like mass and inertia tensor. The sliding mode control is applied to obtain the above robust control. Numerical simulations were conducted and the validity of our approach is demonstrated.**

***Index Terms*—Space robot, Robust Control, Sliding mode control**

I. Introduction

Space robot technology has been rapidly developed and extensively used in the space station program. Most of these space robots are a kind of remote manipulator systems controlled by astronauts from inside or outside of space station. In the space application more intelligent system is desirable to reduce the workload and hazardous risk of those astronauts. Therefore in the near future this technology will be expected to perform the wider range of operations, such as to maintain failed satellites and to capture space debris in the autonomous manner by the space robot. This capability will tremendously decrease the extravehicular operations of astronauts, which are most time consuming and terribly exhausting. In this respect the autonomy will be mandatory.

In the space robot operation there are a few features like the reactive behavior of attitude motion of the space robot by robot arm operation and the parameter change in attitude dynamic equations of motion by capturing the target and so on. Generally speaking the failed target and debris will not be accurately known a priori and freely rotating, that is, some of physical parameters are unknown. In the above respect some kind of robustness of the space robot control must be incorporated[1].

This study deals with the space robot operation, i.e., controlling the attitude of the space robot and controlling the robot arm under the changed mass property. The sliding mode control[2] is applied to the control of attitude motion and the robot arm in which the absolute supremum value method[3] was used to assure the robustness.

Takuro KOBAYASHI is with NEC Engineering Limited, Higashikanagawa 4-10-27, Shinagawa 140-0002 Tokyo, Japan phone: 0463-6713-1111

Shinichi TSUDA is with School of Engineering , Tokai University, Kitakaname 4-1-1, Hiratsuka Japan 259-1292, phone:0463-58-1211 (e-mail:stsuda@keyaki.cc.u-tokai.ac.jp)

II. Model of Space Robot

The Model of a space robot is illustrated in Fig.1. A robot arm is mounted on the body of the spacecraft. The robot arm is articulated with 3 rotary joints and the motion of the robot arm is assumed to be two dimensional. This assumption is not inconsistent with reality. The out of plane motion can be separated from the in-plane motion.

Link lengths of robot arm are given by $l_i\,(i=0,1,2,3)$. The position of the space robot in the inertia space is denoted by the coordinates X, Y and angles of joints are $\theta_i\,(i=0,1,2,3)$ in which θ_0 gives the attitude angle of space robot body.

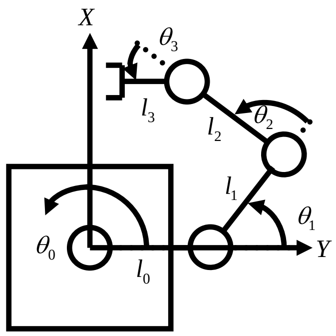

Fig.1 Model of Space Robot

III. Equations of Motion

Dynamical equations of motion for space robot are derived using Lagrange formula. It will be obtained as follows.

K is the kinetic energy and P is the potential energy, then, Lagrange equations of motion is expressed in the following:

$$Q_{ib} = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i}\,. \tag{1}$$

Both energies are given as below:

$$K = \frac{1}{2}mv^T v + \frac{1}{2}\omega^T I\omega \tag{2}$$

$$P = mgl\,. \tag{3}$$

where m, $\vec{v}$, $\vec{\omega}$ and I are mass , velocity vector, angular velocity vector and moment of inertia, respectively.

Detailed Geometry of the space robot is shown in the Appendix A.

Center of mass for the space robot and each link is given by

$s_{iX}, s_{iY}\,(i=0,1,2,3)$. And the velocity is v_i. a_i expresses

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the length between the joint and link center of mass, and b_i gives the length between the joint and link center of mass.

Then we have the following relationships:

$$
\begin{aligned}
&s_{0X}(t) = X_0(t) \\
&s_{0Y}(t) = Y_0(t) \\
&v_0(t) = \dot{s}_{0X}{}^2(t) + \dot{s}_{0Y}{}^2(t) \\
&s_{1X}(t) = X_0(t) + l_0 C_0 + a_1 C_{01} \\
&s_{1Y}(t) = Y_0(t) + l_0 S_0 + a_1 S_{01} \\
&v_1(t) = \dot{s}_{1X}{}^2(t) + \dot{s}_{1Y}{}^2(t) \\
&s_{2X}(t) = X_0(t) + l_0 C_0 + l_1 C_{01} + a_2 C_{012} \\
&s_{2Y}(t) = Y_0(t) + l_0 S_0 + l_1 S_{01} + a_2 S_{012} \\
&v_2(t) = \dot{s}_{2X}{}^2(t) + \dot{s}_{2Y}{}^2(t) \\
&s_{3X}(t) = X_0(t) + l_0 C_0 + l_1 C_{01} + l_2 C_{012} + a_3 C_{0123} \\
&s_{3Y}(t) = Y_0(t) + l_0 S_0 + l_1 S_{01} + l_2 S_{012} + a_3 S_{0123} \\
&v_3(t) = \dot{s}_{3X}{}^2(t) + \dot{s}_{3Y}{}^2(t)
\end{aligned} \tag{4}
$$

The kinetic energies are described as below:

$$
\begin{aligned}
&K_0 = \frac{1}{2} m_0 v_0^2 + \frac{1}{2} I_0 \omega_0^2 \\
&K_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_1 (\omega_0 + \omega_1)^2 \\
&K_2 = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_2 (\omega_0 + \omega_1 + \omega_2)^2 \\
&K_3 = \frac{1}{2} m_3 v_3^2 + \frac{1}{2} I_3 (\omega_0 + \omega_1 + \omega_2 + \omega_3)^2
\end{aligned} \tag{5}
$$

The potential energies for free floating bodies on the orbit are given by the following:

$$
\begin{aligned}
&P_0 = 0 \\
&P_1 = 0 \\
&P_2 = 0 \\
&P_3 = 0
\end{aligned} \tag{6}
$$

Those equations are summarized as in eq.(7) by substituting the above relations, where $M(\theta)$ is the inertia matrix and $h(\theta,\dot{\theta})$ includes centrifugal and Coriolis terms. $u(t)$ is translational control force, attitude control and joint control torque vector for space robot.

$$M(\theta)\ddot{q}(t) + h(\theta,\dot{\theta}) = u(t) \tag{7}$$

where

$$q = [X \quad Y \quad \theta_0 \quad \theta_1 \quad \theta_2 \quad \theta_3]^T$$

$$\theta = [\theta_0 \quad \theta_1 \quad \theta_2 \quad \theta_3]^T .$$

Further we assume the following relations:

$$M(\theta) = M^0(\theta) + \triangle M(\theta) \tag{8}$$

$$h(\theta,\dot{\theta}) = h^0(\theta,\dot{\theta}) + \triangle h(\theta,\dot{\theta}) \tag{9}$$

In which $M^0(\theta)$ and $h^0(\theta,\dot{\theta})$ are defined as nominal value matrix and vector, $\triangle M(\theta)$ and $\triangle h(\theta,\dot{\theta})$ are called deference from nominal values and absolute supremum values are defined as bellow;

$$\left| \triangle M_{ij}(\theta) \right| \le \hat{M}_{ij}(\theta) \tag{10}$$

$$\left| \triangle h_i(\theta,\dot{\theta}) \right| \le \hat{h}_i(\theta,\dot{\theta}) \tag{11}$$

And further, absolute supremum values of elements of time derivative $\dot{M}_{ij}(\theta)$ of matrix $M(\theta)$ was also defined in the following manner;

$$\left| \dot{M}_{ij}(\theta) \right| \le \hat{\dot{M}}_{ij}(\theta) \tag{12}$$

The absolute supremum value $\hat{v}_i(q,t)$ will be given as follows;

$$\left| \{ M(\theta)\ddot{q}_d(t) \}_i \right| \le \hat{v}_i(\theta,t) \tag{13}$$

IV. SLIDING MODE CONTROL

The sliding mode control restricts the trajectory of plant states on a hyper plane by the control and slides it to the equilibrium point in an asymptotic manner.

First let us design the switching hyper plane. The target trajectory is given by q_d and controlling errors are defined by the followings;

$$e(t) = q(t) - q_d(t) \tag{14}$$

$$\dot{e}(t) = \dot{q}(t) - \dot{q}_d(t) \tag{15}$$

And then we give the switching hyper plane as an equation (10).

$$\sigma(t) = \Lambda e(t) + \dot{e}(t) \tag{16}$$

where

$$\Lambda = diag(\lambda_1, \cdots, \lambda_n) \qquad \lambda_i > 0 .$$

If $\sigma(t) = 0$ holds, then, $e(t)$ in eq. (16) satisfies the asymptotic stable differential equation and $e(\infty) \to 0$ is assured. In order to secure the state is approaching to the hyper plane, the following Lyapunov function is introduced. And the negative definiteness of its time derivative will be proved.

$$V(\sigma) = \frac{1}{2} \sigma^T M \sigma \tag{17}$$

The time derivative of eq.(17) is given by

$$
\begin{aligned}
V &= \frac{1}{2} \sigma^T \dot{M} \sigma + \sigma^T M \dot{\sigma} \\
&= \frac{1}{2} \sigma^T \dot{M} \sigma + \sigma^T (M \Lambda \dot{e} + M \ddot{q} - M \ddot{q}_d) \\
&= \frac{1}{2} \sigma^T \dot{M} \sigma + \sigma^T (M \Lambda \dot{e} - h + u - M \ddot{q}_d)
\end{aligned} \tag{18}
$$

Let us define $u(t)$ as follows;

$$u(t) = -M^0(\theta)\Lambda \dot{e} + h^0(\theta,\dot{\theta}) - P\sigma - Q\,\mathrm{sgn}(\sigma) \tag{19}$$

where

$$P := diag(P_{11}(t), \cdots, P_{nn}(t))$$

$$Q := diag(Q_{11}(t), \cdots, Q_{nn}(t)) ,$$

then, we obtain

$$
\begin{aligned}
\dot{V} &= \frac{1}{2} \sigma^T \dot{M} \sigma + \sigma^T [M \Lambda \dot{e} - h - M \ddot{q}_d] \\
&\quad + \sigma^T \left[-M^0 \Lambda \dot{e} + h^0 - P\sigma - Q\,\mathrm{sgn}(\sigma) \right] \\
&= -\sigma^T \left[P - \frac{1}{2} \dot{M} \right] \sigma \\
&\quad + \sigma^T \left[-Q\,\mathrm{sgn}(\sigma) + \triangle M \Lambda \dot{e} + \triangle h - M \ddot{q}_d \right]
\end{aligned} \tag{20}
$$

Here we choose P and Q which satisfy $\dot{V}(s) < 0$.

In the first place elements of the diagonal matrix Q are determined as below;

$$Q_{ii}(t) = \sum_{j=1}^{n} \{ \hat{M} \Lambda \}_{ij} \left| \dot{e}_j \right| + \hat{h}_i + \hat{v}_i \tag{21}$$

Then we have

$$\sigma^T Q \operatorname{sgn}(\sigma) \geq \sigma^T \left[\triangle M \Lambda \dot{e} - \triangle h - M \ddot{q}_d \right] \quad (22),$$

And the second term of eq. (20) becomes negative semi-definite. In the next place if we define elements of diagonal matrix P as follows;

$$P_{ii}(t) = \sum_{j=1}^{n} \hat{\dot{M}}_{ij} / 2 + k_i \quad , k_i > 0 \quad (23),$$

then, the first term $P - \frac{1}{2}\dot{M}$ of eq.(20) is given by the following,

$$\frac{1}{2}\begin{bmatrix} \sum_{j=1}^{n} \hat{\dot{M}}_{1j} - \dot{M}_{11} & -\dot{M}_{12} & \cdots & -\dot{M}_{1n} \\ -\dot{M}_{21} & \sum_{j=1}^{n} \hat{\dot{M}}_{2j} - \dot{M}_{22} & & -\dot{M}_{2n} \\ \vdots & & \ddots & \vdots \\ -\dot{M}_{n1} & -\dot{M}_{n2} & \cdots & \sum_{j=1}^{n} \hat{\dot{M}}_{nj} - \dot{M}_{nn} \end{bmatrix} + K \quad (24).$$

By the Gershgorin's theorem, for an arbitrary matrix $A = [a_{ij}]$, if the following inequality is satisfied;

$$a_{ij} \geq \sum_{k=1, k \neq i}^{n} |a_{ik}| \quad (25),$$

then, the matrix A is positive semi-definite. Therefore if we apply $k_i > 0$ to the eq. (24), then, we have the negative definiteness of the first term in eq. (20). This means $\dot{V} < 0$ .The above concludes the proof of the negative definiteness of the Lyapunov function.

A typical design process for the absolute supremum will be shown in the Appendix B, where $\Delta M_{55}(\vec{\theta})$ is evaluated.

And in order to avoid the chattering phenomena, we introduce saturation function in place of sgn function.

$$sat(\sigma/\varepsilon) = \begin{cases} 1 & \sigma > \varepsilon \\ \sigma/\varepsilon & |\sigma| \leq \varepsilon \\ -1 & \sigma < -\varepsilon \end{cases} \quad (26)$$

V. Numerical Simulations

We conducted numerical simulations for the space robot model defined in Fig.1. And to perform the mission two phases are introduced.

PHASE I

To capture the target the robot arm follows the motion of the target for 10 seconds. By this operation grasping operation will be completed

In order to realize to follow the target, a goal trajectory $r_d(t)$ for the position of endeffector of the robot arm is defined and then, the joint trajectory for $q_d(t)$ is calculated. The position of the center of target is X_t and Y_t, and the distance between the center of the target and the grasping point is given by r_t. And the target has the rotational motion. Then we have the following relations;

$$r_d(t) = \begin{bmatrix} Xt + r_t \cdot \cos\left(\frac{\pi t}{360} + \frac{3\pi}{2}\right) \\ Y_t + r_t \cdot \sin\left(\frac{\pi t}{360} + \frac{3\pi}{2}\right) \\ \frac{\pi t}{360} + \frac{\pi}{2} \end{bmatrix} \quad (27)$$

$$q_d(t) = \begin{bmatrix} 0 & 0 & 0 & \theta_{1d} & \theta_{2d} & \theta_{3d} \end{bmatrix} \quad (28)$$

PHASE II

After the grasping operation the velocity of the endeffector will be controlled to be 0 [m/sec].

To realize the above operation a goal trajectory for the joint velocity is given by linear functions of time which reduce the velocity to 0 [m/sec] after the 30 [sec]. The joint velocity vector is given by eq. (29).

The supremum value is determined by Table 1.

$$\dot{q}_d(t) = \begin{bmatrix} 0 & 0 & 0 & \dot{\theta}_{1d}(t) & \dot{\theta}_{2d}(t) & \dot{\theta}_{3d}(t) \end{bmatrix} \quad (29)$$

Table 1 Parameters of the Target

	Target	Assumed Value for determining the Suremum Value
Mass $[Kg]$	500	600
Moment of Inertia $[Kgm^2]$	333.33	400
Rotational Velocity $[\deg/s]$	0.5	0.5
Size	2[m]×2[m]	2[m]×2[m]

In Table 2 parameters for the space robot are defined.

Table 2 Parameters of the Space Robot

	Body	Link 1	Link 2	Link 3
Mass $[Kg]$	1500	40	40	30
Link Length $[m]$	1.5	1.5	1.5	1.0
Moment of Inertia $[Kgm^2]$	1000	30	30	10
Initial Angle [deg]	0	45	90	-45

Other parameters are assumed as follows;

$$k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 100$$
$$\lambda_1 = \lambda_2 = 15, \lambda_3 = 10, \lambda_4 = \lambda_5 = \lambda_6 = 5$$
$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0.05 .$$

Some of the above parameters are determined by iterative manner.

Positions of the space robot and the target at 0 and 10 seconds are illustrated in Fig.2.

Results of the Phase I are shown in Figs.3-9.

The performance of tracking the target is satisfactory and the error of tracking was below 1 [mm].

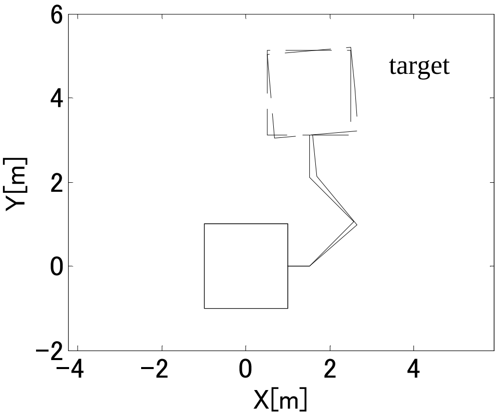


Fig.2 Target and Space Robot Position at 0 and 10 [sec]

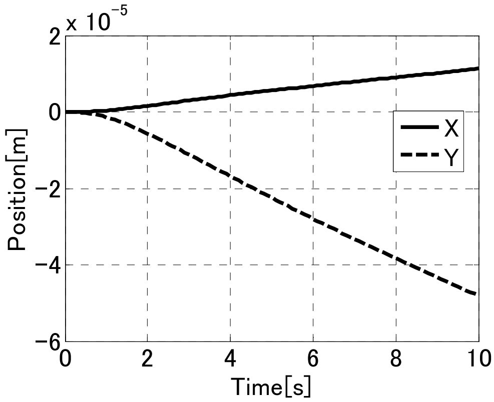


Fig.3 History of Space Robot Position

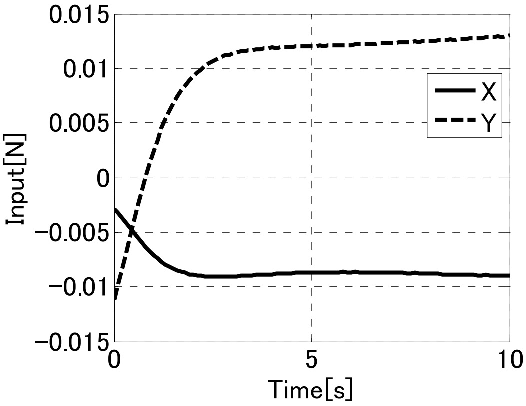


Fig.4 History of Control Force for Space Robot

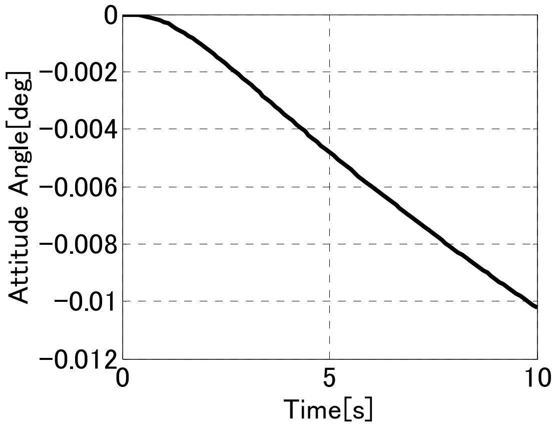


Fig.5 History of Space Robot Attitude Angle

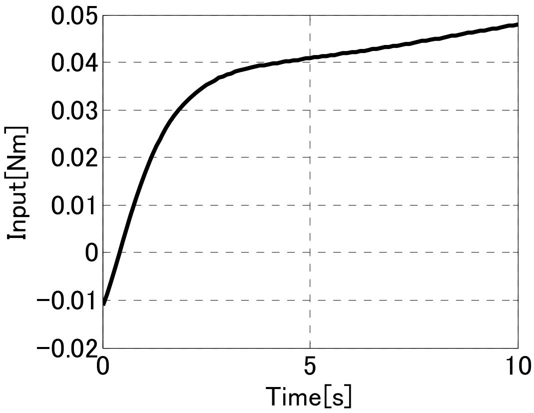


Fig. 6 History of Control Input Torque

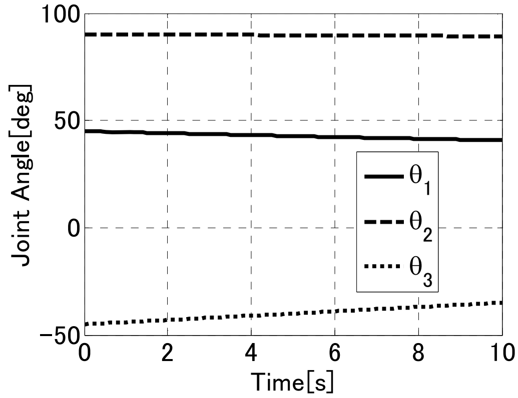


Fig.7 History of Joint Angles

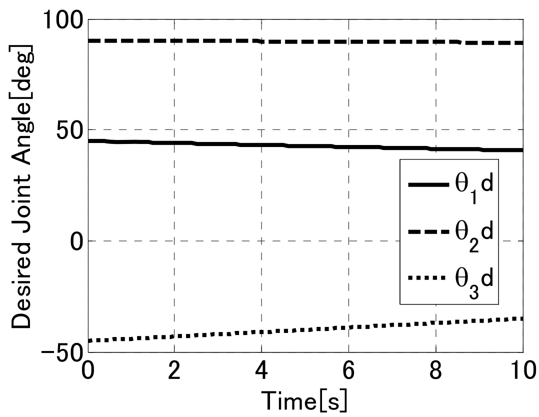


Fig.8 History of Desired Joint Angles

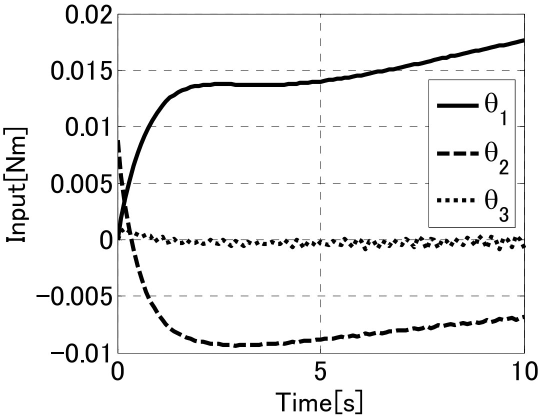


Fig.9 History of Joint Torque Input

Results of Phase II control are given by Figs.10-19.

The control of position and velocity of the space robot is satisfactory and control input for spacecraft position and joint angles is sufficiently small, for instance, the maximum torque for both the space robot attitude control and joint control is smaller than 1 [Nm]. These values are consistent with the space application.

In general mounted thruster forces are from 1N to 10N for thousand kg class spacecrafts and typical arm length for the torque will be 2m or 3m. Furthermore, typical toque capability by reaction wheel for the attitude control of spacecraft is 1Nm. These facts validate the applicability of our approach.

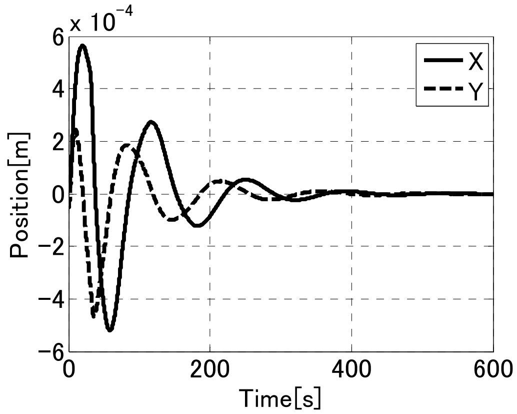


Fig.10 History of Space Robot Position

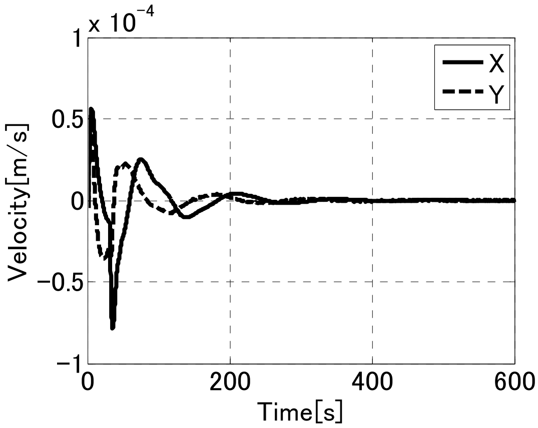


Fig.11 History of Space Robot Velocities

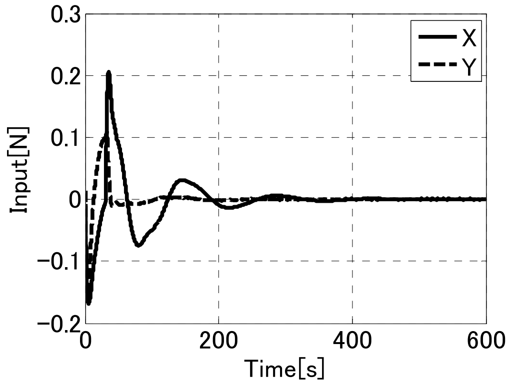


Fig.12 History of Translational Control Input

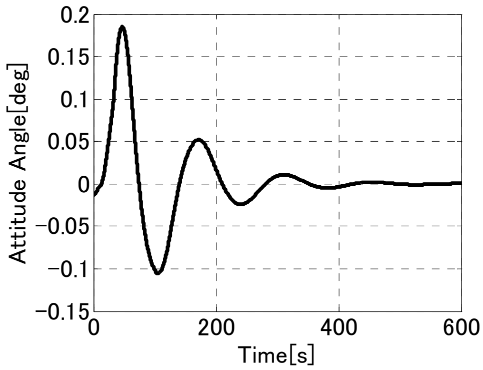


Fig.13 History of Space Robot Attitude Angle

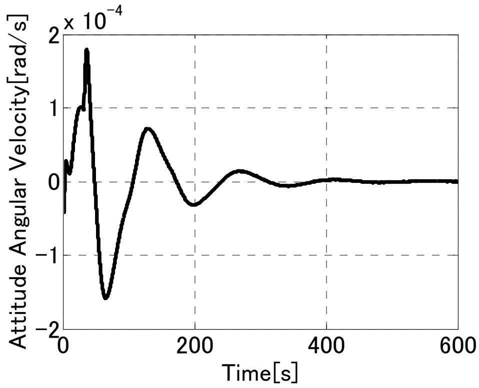


Fig.14 History of Attitude Angle Velocity

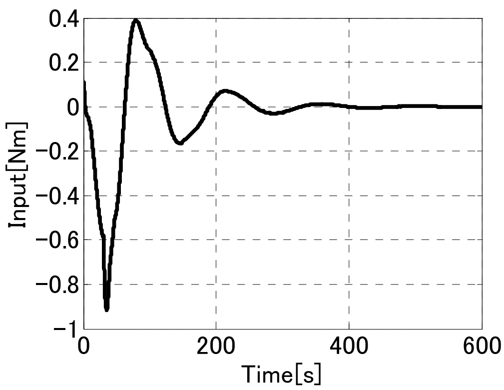


Fig.15 History of Torque Control Input for Space Robot

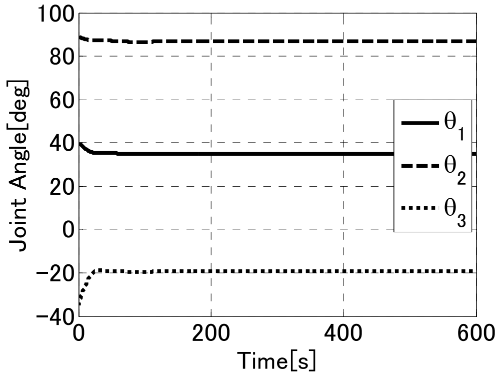


Fig.16 History of Joint Angles for Space Robot

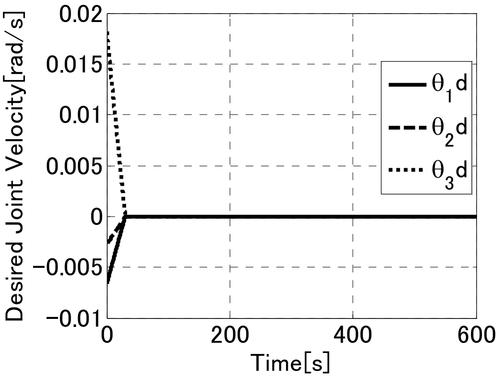


Fig.17 History of Desired Joint Velocities

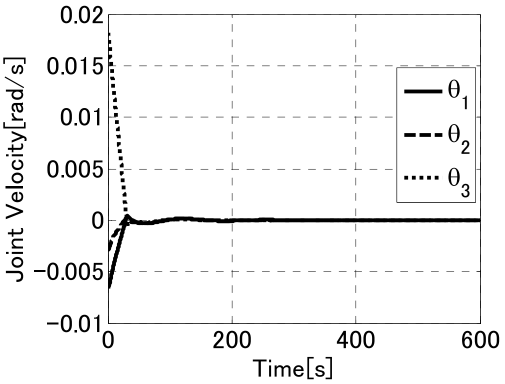


Fig.18 History of Joint Angles

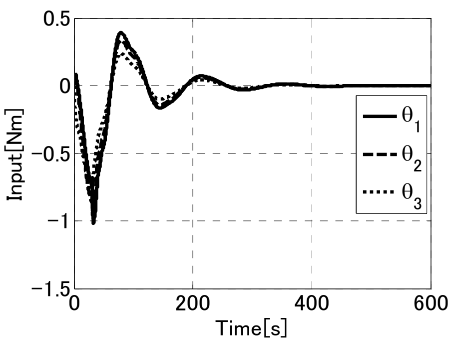
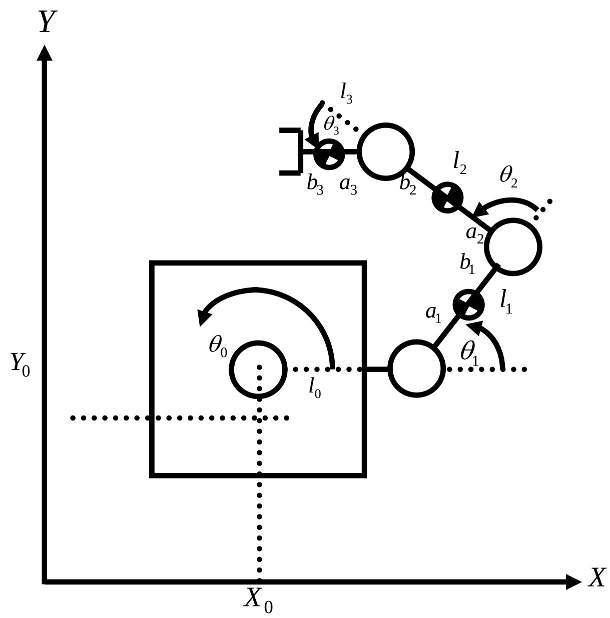


Fig.19 History of Torque Control Input

VI. CONCLUSION

In this study robust control of space robot for unknown target capturing operation was discussed. The target initially has freely rotating motion, therefore we defined two phases, in which we have operations of grasping the target and stabilizing both the space robot and the target. The sliding mode control was applied to have the robustness of control. Numerical simulations were conducted and the results show the consistency with space application requirement. This validates our approach.

Appendix A: Geometry of the Space Robot



Appendix B: Evaluation of Absolute Supremum Value

Unknown values for the equation of motion will be target moment of inertia I_{target}, mass m_{target} and location of center of mass of joint 3 and link 3 a_{target} which includes unknown target. If we assume a_3 is the distance of Link 3 and target center of mass before the connection with target and a_4 is distance after the connection, then, we obtain the following:

$$a_{target} = a_3 + a_4 . \qquad \text{(B.1)}$$

Let us take the typical value $\Delta M_{55}(\vec{\theta})$ for the evaluation of the absolute supremum value.

$$\Delta M_{55}(\vec{\theta}) = I_{target} + 2a_3 a_4 m_3 + \frac{9}{4} m_{target} + a_3^2 m_{target} + 2a_3 a_4 m_t + a_4^2 m_t + \left(3a_3 m_{target} + 3a_4 m_3 + 3a_4 m_{target}\right)\cos\theta_3 \qquad \text{(B.2)}$$

In order to design the supremum values we assume upper values of target moment of inertia I_{upper}, mass m_{upper} and distance of Joint 3 and Link 3 center of mass as follows:

$$I_{target} \le I_{upper}$$

$$m_{target} \le m_{upper} \quad \text{(B.3)}$$
$$a_{target} \le a_{upper}$$

By substituting (B.3) in to (B.2), we obtain the supremum value of $\Delta M_{55}(\vec{\theta})$ as below:

$$\left|\Delta M_{55}(\vec{\theta})\right| \le \hat{M}_{55}(\vec{\theta}) \cdot$$

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