Designing Four-bar Linkages for Path Generation with Worst Case Joint Clearances

Samer K. Mutawe, Yahia M. Al-Smadi, Rajpal S. Sodhi

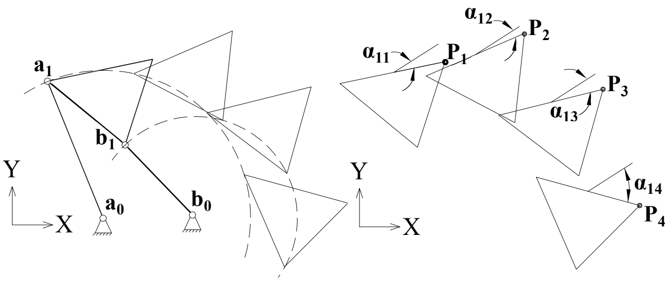
Abstract— **The calculation of mechanism parameters required to achieve or approximate a set of prescribed rigid-body path points is determined by the use of Four-bar path generation. This work will discuss the use of path generation while considering the position tolerance resulting from joint running clearance. Not only will this work discuss the use of path generation of four-bar mechanism in its familiar form, it will include position tolerance as a result of joint running clearance. The standards of American National Standard Institute (ANSI) will be applied in this study. This paper will implement the joint tolerances into the conventional planar four-bar path generation model displacement matrix as a design constraint. This synthesis will produce a planar four-bar mechanism with moving pivots and length tolerance limits from which any mechanism can be created to travel throughout the prescribed path for a coupler point with its specified tolerance. An example to demonstrate the synthesis of a four-bar mechanism with joint tolerances is included.** [1]

Index Terms—**Path generation, joint tolerances, worst case tolerance, mechanism tolerance**

I. INTRODUCTION

WHEN a rigid-body must achieve a specific displacement sequence for effective operation (e.g., specific tool paths and/or orientations for accurate fabrication operations), path generation mechanism synthesis becomes essential. The evaluation of the mechanism variables required to pass through or approach a group of prescribed rigid-body path points is the objective of four-bar path generation. In Fig. 1 four prescribed rigid-body poses are defined by the coordinates of variables **p** and α (path generation model input), and the model outputs are the calculated coordinates of $\mathbf{a}_0$ and $\mathbf{b}_0$ and moving pivot variables $\mathbf{a}_1$ and $\mathbf{c}_1$. Many contributions have been made in the field of planar mechanism synthesis with tolerances. Manufacturing processes and loading and unloading of the mechanism which increases joint clearance after service period and causes impulsive forces are examples of factors attributing to these tolerances. The effect of the required joint assembly tolerances on the four-bar mechanism synthesis is studied.

Many methods and analyses have been considered to include error caused by joint clearances and link geometry tolerances in the mechanism synthesis. Graphical and mathematical approaches to investigate the efficiency of planar mechanisms to approximate the coupler poses considering the errors/tolerances in mechanism linkages were developed by [2] and [3]. Recent contributions performed by [4], [5] and [6] modeled the joint clearance as a massless virtual link (clearance link) and investigated the joint clearance effect on the mechanism performance to achieve the prescribed coupler curves/points. A method to predict the limits of the tolerance region by choosing the clearance value was also proposed by [6].



(a) (b)

Fig. 1 Prescribed rigid-body poses (a) and calculated planar four-bar mechanism (b)

II. CONVENTIONAL PLANAR FOUR-BAR PATH GENERATION ANALYSIS

Equations (1) through (3) present a conventional planar four-bar path generation model introduced by [1]

$$\left(\left[\mathbf{D}_{1j}\right]\mathbf{a}_1-\mathbf{a}_0\right)^T\left(\left[\mathbf{D}_{1j}\right]\mathbf{a}_1-\mathbf{a}_0\right)-L_1^2=0\,, \tag{1}$$

$$\left(\left[\mathbf{D}_{1j}\right]\mathbf{b}_1-\mathbf{b}_0\right)^T\left(\left[\mathbf{D}_{1j}\right]\mathbf{b}_1-\mathbf{b}_0\right)-L_2^2=0\,, \tag{2}$$

$$\left[\mathbf{D}_{1j}\right]=\begin{bmatrix}\cos\alpha_{1j} & -\sin\alpha_{1j} & p_{jx}-p_{1x}\cos\alpha_{1j}+p_{1y}\sin\alpha_{1j}\\ \sin\alpha_{1j} & \cos\alpha_{1j} & p_{jy}-p_{1x}\sin\alpha_{1j}-p_{1y}\cos\alpha_{1j}\\ 0 & 0 & 1\end{bmatrix} \tag{3}$$

where j=1,2,3,4

These equations express the fixed length constraint of links $\mathbf{a}_0$-$\mathbf{a}_1$ and $\mathbf{b}_0$-$\mathbf{b}_1$ throughout the prescribed rigid-body

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displacements. The scalar lengths of links $\mathbf{a}_0$-$\mathbf{a}_1$ and $\mathbf{b}_0$-$\mathbf{b}_1$ in (1) and (2) are defined as variables L_1 and L_2, respectively.
The planar displacement matrix in Equation (3) is used to calculate 6 of the 10 unknown variables $\mathbf{a}_0$, $\mathbf{a}_1$, L_1, $\mathbf{b}_0$, $\mathbf{b}_1$, and L_2 with two arbitrary choices of parameters (where $\mathbf{a}_0=[a_{0x}, a_{0y}, 1]$, $\mathbf{a}_1=[a_{1x}, a_{1y}, 1]$, $\mathbf{b}_0=[b_{0x}, b_{0y}, 1]$, and $\mathbf{b}_1=[b_{1x}, b_{1y}, 1]$.

III. TOLERANCE ANALYSIS.

While considering joint assembly tolerances, this paper presents a technique of synthesizing planar four-bar mechanism. In an effort to define the linkages with a tolerance to approximate the desired coupler points trajectory with tolerable accuracy, many goal functions have been formulated. Studies carried out by [16] and [17] on RRCC mechanism and multi-phase four-bar mechanism respectively show the mechanism motion synthesis with a prescribed tolerance for one position, where work presented in [7] shows analytical solutions for the kinematic analysis of position, velocity, acceleration and transmission angle of geared linkage mechanisms. Several optimization algorithms, objective/goal functions and techniques on the shape of coupler curve and points have been presented in [11-14]. [8] developed a nonlinear optimization to investigate four-bar with structural constraints.

In this research, the consideration of the joint clearance tolerances was added to the synthesis of the four-bar mechanisms [23]. The bilateral tolerance system selected for this study is in accordance with [9] which specify bilateral tolerance for pin-hole clearance fit. The main required tolerance for operation of four-bar mechanism was selected and presented herein. For any prescribed rigid-body pose to be achieved, the plus or minus deviation from the specified value would be the allowable tolerance limits. The variation required is the running clearance which is specified when mating parts are assembled, description of clearance fits can be found in [9]. Fig. 2 Portrays the clearance adopted in this investigation, which will be medium running fits RC5 and RC6. These fits were chosen due to their suitability for high running speed or heavy journal pressures.

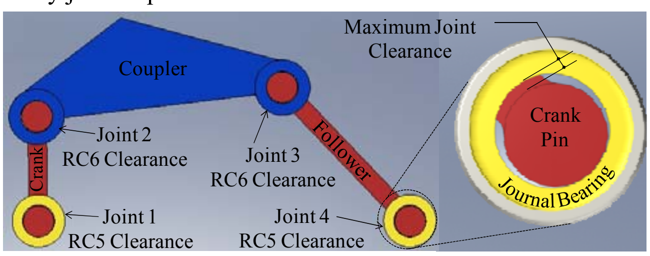

Fig. 2 Joint running clearance required for planar four-bar mechanism

IV. WORST CASE TOLERANCE SYNTHESIS

The tolerance analysis field uses Worst case tolerance model frequently. The tolerance stack-up of the mechanism is measured in a simple form by summing the absolute limits of the tolerances. The allowances for coupler path point **p** will be calculated by implementing the worst case tolerance. Based on the tolerance accumulation model (4) estimated by [19] and [20], the simplified tolerance model (5) is concluded in [21].

$$\Delta\Phi_1 = \left|\frac{\partial F_1}{\partial a}\right| tol_a + \left|\frac{\partial F_1}{\partial b}\right| tol_b + \left|\frac{\partial F_1}{\partial c}\right| tol_c \qquad (4)$$

where $\Delta\Phi_1$ is the worst case variation.
The predicted assembly tolerance is

$$\delta_{Assembly} = \sum_{i=1}^{n} |T_i| \qquad (5)$$

where n is number of parts considered in the tolerance analysis.

After converting the dimension with tolerance limits to a mean dimension with symmetrical tolerance limits, the stack-up tolerance is the total sum of tolerance limit variation to the mean dimension for each part. Section VI demonstrates and explains a worst case tolerance analysis.

V. MODIFICATION OF POSES DISPLACEMENT MATRIX CONSIDERING TOLERANCE

A tolerance region for each coupler point to be positioned within is produced by joint clearance tolerance. Studies done by [10] and [11] demonstrated that the region of the moving pivot point in four-bar mechanism takes the shape of a rectangle with curved sides. If δ_x and δ_y applied on each coupler point pose, then a tolerance region of a box shape would predict the limits with reasonable accuracy. On the other hand, the tolerance region for coupler point positions was stated by [12, 13 and 14] to be an ellipse shape. [15] constructed a reliable region S_R for RCCC Mechanism using reliability analysis in mechanism synthesis. Russell and Sodhi [16] adopted point tolerance for RRSS mechanism by considering δ_x and δ_y for one pose only. This adoption would produce a tolerance region of a box shape if covariance is not calculated; this produces a tolerance region with reasonable accuracy. Similar consideration was performed by [17] in which a square tolerance region for one coupler point pose was suggested. [18] formulated a design sensitivity of an elliptical tolerance region versus square shape tolerance region. The tolerance region presented in this paper is considered as a square or a box shape. The work of [6], [16] and [17] for choosing the clearance value has been adopted. Therefore, (5) is defined to compute the maximum tolerance value for each coupler point pose 1 through 4. The tolerance regions limited by $\pm\delta_x$ and $\pm\delta_y$ for coupler path point **p** is shown in Fig. 3.

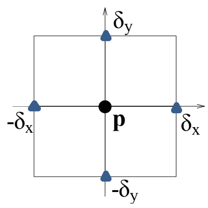

Fig. 3 Tolerance region

The tolerance calculated from (5) will be used in the poses displacement matrix of the coupler point as shown in (6). Several cases of tolerance limits (i.e. 0δ, $+\delta_x$, $-\delta_x$, $+\delta_y$, $-\delta_y$, $+\delta_x$ and $+\delta_y$, $-\delta_x$ and $-\delta_y$, $+\delta_x$ and $-\delta_y$, and $-\delta_x$ and $+\delta_y$) are investigated, moving pivots $\mathbf{a}_1$, $\mathbf{b}_1$ and Links L_1 and L_2 are synthesized for each case.

$$\left[\mathbf{D}_{1j}\right]=\begin{bmatrix} \cos\alpha_{1j} & -\sin\alpha_{1j} & (p_{jx}+\delta_x)-p_{1x}\cos\alpha_{1j}+p_{1y}\sin\alpha_{1j} \\ \sin\alpha_{1j} & \cos\alpha_{1j} & (p_{jy}+\delta_y)-p_{1x}\sin\alpha_{1j}-p_{1y}\cos\alpha_{1j} \\ 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

where j=2,3,4

VI. EXAMPLE

Dimensions used in this example are in SI units. Path generation program can be used with prescribed values of $\mathbf{a}_0$=(0, 0), $\mathbf{b}_0$=(0.5080, 0), and initial guesses of $\mathbf{a}_1$=(0.2540, 0.3048), L_1=0.3810, $\mathbf{b}_1$=(0.6096, 0.3048), and L_2=0.5080. Worst case tolerance model (δ) is calculated in Table I which later will be used in (6) to generate the area described in Section V. Table II shows the prescribed rigid body poses for planar four bar mechanism. Joint number and clearance are based on Fig. 2. Coupler path points can fall anywhere within the calculated region. Nine tolerance cases have been discussed and investigated as shown in Table III. Rigid-body poses 1 through 4 correspond to link $\mathbf{a}_0$-$\mathbf{a}_1$ rotation angles of θ_1= 45°, 70°, 120°, and 150° respectively. Therefore, the displacement angles $(\delta\theta)_{1j}$ for link $\mathbf{a}_0$-$\mathbf{a}_1$ are 25°,75° and 105° respectively.

Table III shows the calculated coordinates of the moving pivot variables $\mathbf{a}_1$ and $\mathbf{b}_1$ and link lengths L_1 and L_2. All rigid-body path points achieved by the constructed mechanisms were studied and found to be within the calculated worst case tolerance range. The highlighted case shown in Table III is taken as an example in this study; others can be done in the same manner. This case generates the longest link lengths L_1 and L_2. The calculated rigid-body path points after implementing the parameters of the synthesized mechanism (a_{1x}, a_{1y}, L_1, b_{1x}, b_{1y}, L_2) for the highlighted case is included in Table IV. Rigid-body path points 1 through 8 correspond to crank angles of θ_1= 29.5195°, 32.9152°, 30.3376°, and 24.1722° respectively.

The ranges of the achieved pivot variables for the given tolerance region are represented by the perimeter of the solid line in the plots of Fig. 4. The perimeter represents the value of these pivot variables for which, the rigid-body position tolerances will be within the prescribed limit. For the given tolerance, a least square best fit can be obtained for each of the variables. These best fit curves are represented in Fig. 4 using dashed-line format. Since only nine cases were analyzed here, the shape of the best fit curve is a nine-sided polynomial. But, a close examination of the data clearly indicates that for the entire square tolerance region (Fig. 3) the best fit curve will be a circle. The radius of this best fit curve represents the values of the pivot variable for which the given tolerances will always be met.

TABLE I
WORST CASE TOLERANCES ANALYSIS

Joint	Clerance Fit		Nominal Size	LL	UL	Centered Dimension	$\lvert\delta\rvert$
1	RC5	Hole	0.5	0	0.001	0.5005	0.0005
		Pin		-0.0019	-0.0012	0.49845	0.0016
2	RC6	Hole	0.5	0	0.0016	0.5008	0.0008
		Pin		-0.0022	-0.0012	0.4983	0.0017
3	RC6	Hole	0.5	0	0.0016	0.5008	0.0008
		Pin		-0.0022	-0.0012	0.4983	0.0017
4	RC5	Hole	0.5	0	0.001	0.5005	0.0005
		Pin		-0.0019	-0.0012	0.49845	0.0016
						$\sum\lvert\delta_i\rvert$	0.0091

TABLE II
PRESCRIBED RIGID-BODY POSES FOR PLANAR FOUR-BAR MECHANISM

	p	α
Pose 1	0.1479, 0.7330	30
Pose 2	0.0056, 0.8490	33.3966
Pose 3	-0.3981, 0.8081	30.8200
Pose 4	-0.6086, 0.6202	24.6646

TABLE III
CALCULATED COORDINATES OF THE MOVING PIVOT VARIABLES $\mathbf{a}_1$ AND $\mathbf{b}_1$ AND SCALAR LINK LENGTHS L_1 AND L_2 FOR NINE COMBINATION CASES OF WORST CASE TOLERANCE δ

	0δ	$+\delta_x$	$-\delta_x$	$+\delta_y$	$-\delta_y$	$+\delta_x$, $+\delta_y$	$-\delta_x$, $-\delta_y$	δ_x , $-\delta_y$	$-\delta_x$, δ_y
a_{1x}	0.3233	0.3230	0.3235	0.3233	0.3233	0.3231	0.3235	0.3230	0.3235
a_{1y}	0.3233	0.3232	0.3233	0.3230	0.3236	0.3229	0.3236	0.3236	0.3230
L_1	0.4572	0.4572	0.4572	0.4571	0.4573	0.4571	0.4573	0.4573	0.4571
b_{1x}	0.8466	0.8464	0.8469	0.8467	0.8466	0.8464	0.8469	0.8464	0.8469
b_{1y}	0.5068	0.5067	0.5070	0.5064	0.5072	0.5063	0.5074	0.5071	0.5066
L_2	0.6095	0.6094	0.6097	0.6094	0.6097	0.6093	0.6098	0.6096	0.6095

TABLE IV
RIGID-BODY POSES ACHIEVED BY SYNTHESIZED PLANAR FOUR-BAR MECHANISM FOR CHOSEN $\mathbf{a}_1$, $\mathbf{b}_1$ FROM TABLE III

	p
Pose 1	5.8242, 28.8577
Pose 2	0.2129, 33.4278
Pose 3	-15.6935, 31.8129
Pose 4	-23.9877, 24.4063

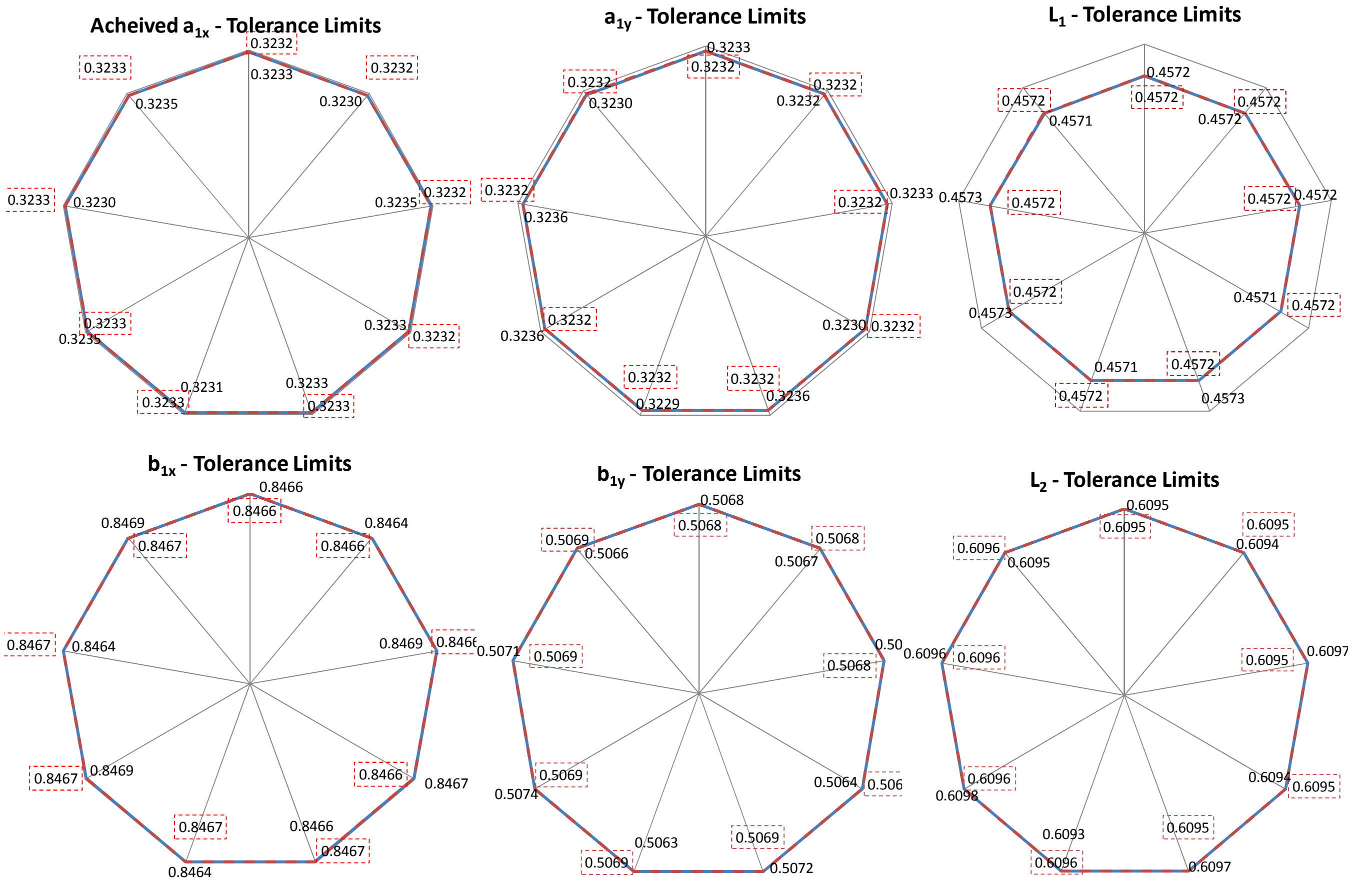

Fig. 4. Plots of synthesized moving pivot points with tolerance limits, dotted line denotes the best fit curve for the achieved moving pivots pose

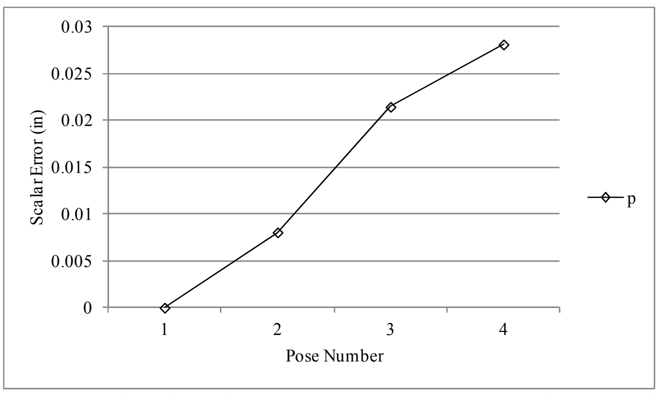

Fig. 5. Plots of synthesized moving pivot points with tolerance limits, dotted line denotes the best fit curve for the achieved moving pivots pose

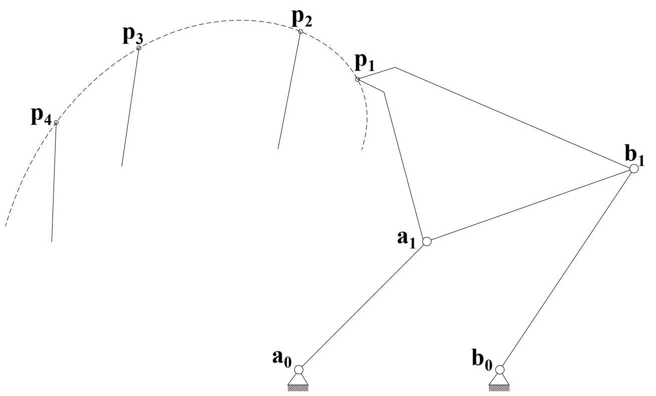

Fig. 6: Plots of synthesized path generator

VII. DISCUSSION

The mathematical analysis software MathCAD was used to codify and solve the formulated path with tolerance program. CAD software was used to model the prescribed rigid-body poses. CAD enables one to specify more reasonable initial guesses for the unknown mechanism than arbitrary guessing. One should be cautious for the mechanism not to fall into a lock-up or binding position which occurs when the pivots $\mathbf{a}_1$, $\mathbf{b}_1$, and $\mathbf{b}_0$ are collinear. The tolerance modeling technique adopted by [22] will be considered in future work.

VIII. CONCLUSIONS

Four-bar path generation is used to synthesize a mechanism which passes through or approximates prescribed rigid-body positions. This work discussed the path generation of four-bar mechanism with path points worst case tolerance which is due to joint clearance during assembly stage. ANSI standard for clearance fit tolerances was incorporated in the rigid-body displacement matrix. The synthesized mechanism approximates the prescribed rigid-body path points within the calculated path points tolerances. This study concludes that considering joint running clearance tolerance in the synthesis of four-bar path generation mechanism improves the accuracy of the synthesized mechanism.

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