On Optimizing a WWTP Design using Multi-objective Approaches

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Abstract—**In this paper, the multi-objective formulation of an optimization problem arising from an activated sludge (AS) system of a wastewater treatment plant (WWTP) design optimization is solved through a multi-objective genetic algorithm. Two multi-objective approaches are proposed. First, a solution to the WWTP design is provided, regardless of its location, date of construction or the involved unit operations. The variables that mostly influence the cost of the system define the objectives and are simultaneously optimized. Second, two crucial objectives for the correct operation of the AS system are simultaneously optimized: the investment and operation costs are minimized and the effluent quality is maximized. Since the objectives are conflicting, several trade-offs between objectives are obtained through the optimization process. The direct visualization of the trade-offs through Pareto curves assists the decision-maker in the selection of crucial design and operation variables. The numerical results show that the proposed methodology produces improved results with physical meaning when compared with previous work.**

Index Terms—**Activated sludge system, multi-objective optimization, WWTP modelling.**

I. Introduction

The high costs associated with the design of Wastewater Treatment Plants (WWTP) have motivated research in the area of process modelling and optimization of the treatment water processes. Thus, one of the main research issues in WWTP is focused on the search for the minimum cost design. Several works on this area, mainly using simulation procedures, may be found in literature [1], [2]. They choose the most economic design among a set of selected and tested designs by simulation. However, this choice does not guarantee that the found solution is optimal. More recently, an interesting optimization of an activated sludge process based on biodegradation kinetics correlation is proposed by Zhang *et al.* [3]. The procedure aims to minimize the bioreactor volume, and a sequence of computation steps follows until there is a confirmation that the solution obtained is optimal, within a certain tolerance. This is not a real optimization procedure. Due to the complexity of the involved mathematical models, as well as the high computational costs associated with the optimization, real optimization procedures related with WWTP cost minimization are rare in the literature.

A cost function based on the structure proposed by Tyteca [4] has been used in previous studies [5], [6], [7]. In the last two cited papers, an augmented Lagrangian framework is used to minimize solely the WWTP cost. In [5], a preliminary study has been carried out aiming to simultaneously minimize the cost function and the amount of pollution in the effluent of the WWTP. However, when the objective is to minimize the cost function, both investment and operation costs are considered. Both costs emerge as depending on the decision variables that mostly influence the cost of each unit operation considered in the study. The aeration tank volume, air flow, settling tank area and settling tank depth are the most important. Since the costs depend on economic data that is collected in a particular region, and at a specific time, the value of money can vary enormously. Including both investment and operation costs in the same analysis requires that the operation costs have to be updated to a present value, so that they can be added to the investment costs. This implies that a discount rate has to be predicted to the life span of the WWTP.

The new alternative methodology that is herein proposed considers the use of the variables that mostly influence the cost separately as objectives to be minimized. This way no knowledge about the local or when the WWTP is being built are required. This allows to overcome the disadvantage of using a cost function that is limited in space and time. However, this alternative formulation comprises a multi-objective approach to cost minimization via the simultaneous minimization of, for example, the aeration tank volume, the air flow, and the settling tank area and depth. This approach allows a generic and flexible solution to the design of the WWTP, in the sense that it does not depend on the location and the date of construction.

When cost minimization is used, another objective could be pursued since strict laws on effluent quality also demand an efficient plant. Thus, the amount of pollution in the effluent of the WWTP could also be minimized. Since these goals are in conflict, a bi-objective approach to the WWTP optimal design is then required [5].

These multi-objective approaches to the optimization problem with several criteria enable the decision-maker to simultaneously consider the WWTP process from different perspectives. Therefore, it is possible to optimally balance between different conflicting design objectives and select the final design. For instance, the cost is conflicting with the effluent quality. When multi-objective problems are solved, no single solution that is optimal with respect to all objectives exists. Instead, there is a set of optimal solutions, known as Pareto-optimal solutions, reflecting compromises between the objectives.

The main contributions of this study are the following. First, we address the approach that minimizes the influential variables, while maintaining the effluent quality according to the strict laws. This is mostly appropriate when a WWTP

Manuscript received May 14, 2013; revised September 12, 2013. This work was financed by FEDER funds through COMPETE–Operational Programme Thematic Factors of Competitiveness and by portuguese funds through FCT–Foundation for Science and Technology, within the project FCOMP-01-0124-FEDER-022674.

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draft project is required. No specific knowledge about the location and date of construction are required for this analysis. Then, when a final decision is made to build a WWTP in a specific region, a detailed optimal design process is to be required. Our proposal relies on the previously referred bi-objective approach to the WWTP optimization. In this case, the cost function itself is minimized and the effluent quality is maximized via the minimization of the amount of pollution. We note that the cost function is non-convex, and the other equations and inequalities arising from the biological process modelling of the unit operation considered in this study, defining the constraints of the problem, are highly non-linear, and some are even non-differentiable. The set of feasible solutions is also very small. Solving such a mathematical programming problem is a challenging issue for the continuous non-linear optimization area. Since the bi-objective approach is a complex and difficult to solve problem, the output of the first approach aims to define tighter lower and upper bounds for the most influential variables (aeration tank volume, air flow, settling area and settler depth) and to generate an initial approximation closer to feasibility, when solving the bi-objective problem.

To get the equality and inequality constraints of the problem, we show the modelling process of an activated sludge system using: i) the ASM1 model [8] for the aeration tank; ii) a combination of the ATV design procedure [9] with the double exponential model [10], that has proven to be more robust than each one of the two models used separately [11] for the secondary settler. We remark that this is a general procedure that can be easily extended to other models, operation modes or even to other units in the WWTP.

Addressing the WWTP design problem through multiple objectives and solving it afterwards by considering an *a posteriori* perspective is not a very common approach in this area.

Briefly, in [12], an interactive design tool using the GPS-X simulator [13] and the IND-NIMBUS [14] multi-objective optimizer is implemented. A multi-objective approach based on genetic algorithms is used in [15]. However, the goal is to select treatment trains where extent and reliability of the treatment are high, whereas the capital, operation, maintenance and land area requirements are low. The bi-objective problem is then solved with NSGA [16], by minimizing both the capital cost and overall environmental impact. In [17], single-, two- and three-objective functions problems are solved using NSGA-II [18]. This is an evolutionary algorithm for multi-objective programming originally developed by Srinivas and Deb [16]. The problems are based on a simple model for the aeration tank and the secondary settler is assumed a perfect clarification tank. The authors in [19] present support tools based on multicriteria decision analysis to support the conceptual design of an activated sludge system. In [20], a surrogate based method, ParEGO, combined with an integrated urban wastewater model, is used. The therein presented real time control problems are solved and the obtained results are compared with those of NSGA-II. In [21], a methodology that combines the multi-objective genetic algorithm, NSGA-II [18], with WWTP existing models and simulation software is used to provide a framework for the evaluation, optimization and comparison of WWTP control laws. The methodology is applied to the BSM1 model [22]. In [23], an optimal control problem concerning the management of a wastewater treatment system comprising several plants is addressed through a multi-objective approach using the weighting method to look for the Pareto-optimal solutions. In [24], scalarization methods and the NSGA-II are applied to the optimal design of a biochemical reactor with respect to two conflicting economic objectives. Other efficient multi-objective scalarization strategies are available in [25], [26].

These various multi-objective approaches to WWTP designs differ from the herein presented paradigm where we use an evolutionary algorithm to produce a representative set of approximations to the Pareto-optimal solutions. Therefore, the search for optimality is performed without any *a priori* preference information of the decision-maker. After the optimization process, the decision-maker selects one solution from the set of trade-off solutions according to his/her preferences.

Genetic algorithms [27] are population-based algorithms and, therefore, particularly suitable to tackle multi-objective problems. They can, in principle, find multiple widely different Pareto-optimal solutions in a single run [28]. Furthermore, they do not require any differentiability or convexity assumptions and can deal with complex search spaces, as well as non-convex Pareto fronts. In this paper, we aim at solving the highly non-linear constrained multi-objective optimization problems, by using an efficient version of a genetic algorithm, known as Multi-objective Elitist Genetic Algorithm (MEGA for short). This variant of the genetic algorithm (GA) relies on an elitist concept based on a secondary population that prevents the loss of non-dominated solutions found during the search and also improves the convergence rate and the distribution of the solutions in objective space [29], [30].

The remainder of the paper is organized as follows. Sections II and III present a detailed description of the mathematical model that contains the equality, inequality and simple bound constraints of the proposed WWTP formulation. Section IV introduces the objective functions developed for both multi-objective approaches, and describes the multi-objective elitist genetic algorithm. Section V is devoted to the numerical results and we conclude and present some ideas for future work in Section VI.

II. The activated sludge system

A typical WWTP has four units. The first unit is a primary treatment, which is a physical process and aims to eliminate the gross solids and grease, so avoiding the blocking up of the secondary treatment. Although the dimensioning of such a unit is usually empirical and based on the wastewater to be treated, its cost is not affected by the (biological, chemical and biochemical) characteristics of the wastewater. The cost just corresponds to the civil engineering construction work of a tank. This is the reason why this process is not included in the optimization procedure. The next two units define the secondary treatment of the wastewater. It is the most important treatment in the plant because it eliminates the soluble pollutants. This is a biological process which, in the case herein studied, comprises an aeration tank and a clarifier that aims to separate the biological sludge from the treated water. There are other biological treatments but this is, by far, the most widely used. Finally, the last unit is used to treat the biological sludge that is wasted by the secondary settler. When the wastewater is very polluted and the secondary treatment does not provide the demanded quality, a tertiary treatment, usually a chemical process, can be included. There are many other possible WWTP layouts, but most of them have the above described treatments.

This paper aims to address optimization approaches that are based on computing optimal values for the decision variables that mostly influence the WWTP cost. The values of the variables are to be minimized simultaneously as separate objectives. Furthermore, the operation and design optimization of the plant, in terms of minimum total cost function (investment and operation costs) and minimum of daily pollution defined through a quality index function [22], is then explored.

The work herein presented focus solely on the secondary treatment, in particular on an activated sludge system that consists of an aeration tank and a secondary settler. The mathematical models used to describe the aeration tank and the settling tank are the ASM1 model [8] and the ATV model [9], combined with the double exponential model (DE) [10], respectively.

III. The mathematical model

A procedure to provide an efficient WWTP design optimization that can be easily adapted to any WWTP, regardless of its location or the involved unit operations, is hereupon presented. The mathematical model can be subdivided into seven types of equations, as it will be shown. The system under study consists of an aeration tank, where the biological reactions take place, and a secondary settler for the sedimentation of the sludge and clarification of the effluent.

To describe the aeration tank we chose the activated sludge model n.1 (ASM1), described by Henze *et al.* [8], which considers both the elimination of the carbonaceous matter and the removal of the nitrogen compounds. This model is widely accepted by the scientific community, as it produces good predictive values by simulation [22]. This means that all state variables keep their biological interpretation. The tank is considered a completely stirred tank reactor (CSTR) in steady state.

For the settling tank, a combination of the ATV design procedure [9] with the DE model [10] is used. The ATV model is usually used as a design procedure to new WWTP. It is based on empirical equations that were obtained by experiments and does not contain any solid balances, although it contemplates peak wet weather flow (PWWF) events. The DE model is the most widely used in simulations and it produces results very close to reality. However, since it does not provide extra sedimentation area needed during PWWF events, the resulting design has to consider the use of security factors that yield an over-dimensioned and expensive unit. Previous work [11] shows that this combined model is prepared to overcome PWWF events without over dimensioning and provided the most equilibrated WWTP design when compared with the other two used separately. When these three designs were introduced in the GPS-X simulator [13] and a stress condition of a PWWF value of five times the normal flow was imposed, only the combined model was able to support this adverse condition maintaining the quality of the effluent under the values imposed by the portuguese law.

A. Mass balances around the aeration tank

The first set of equations come from the mass balances around the aeration tank. The Peterson matrix of the ASM1 model [8] is used to define the model for the mass balances. For a CSTR, it is assumed that the mass of a given component entering the tank minus the mass of the same compound in the tank, plus a reaction term (positive or negative) equals the accumulation in the tank of the same compound:

$$\frac{Q}{V_a}(\xi_{in}-\xi)+\sum_j \nu_{\xi j}\rho_j=\frac{d\xi}{dt}, \tag{1}$$

where Q is the volumetric flow of the effluent, V_a is the aeration tank volume and ξ represents the concentration of the compound (in g COD/m^3). It is convenient to refer that in a CSTR the concentration of a compound is the same at any point inside the reactor and at the effluent of that reactor. The second term on the left hand side of equation (1) defines the reaction term for the compound in question, and is the sum of the product of the stoichiometric coefficients, $\nu_{\xi j}$, with the expression of the process reaction rate, ρ_j, of the ASM1 Peterson matrix [8].

In steady state, the accumulation term given by $\frac{d\xi}{dt}$ is zero, because the concentration is constant in time. The ASM1 model involves 8 processes incorporating 13 different components. The mass balances for the inert materials, S_I and X_I, are not considered because they are transport-only components.The process rates are the following:

- Aerobic growth of heterotrophs

$$\rho_1=\mu_H\left(\frac{S_S}{K_S+S_S}\right)\left(\frac{S_O}{K_{OH}+S_O}\right)X_{BH}; \tag{2}$$

- Anoxic growth of heterotrophs

$$\rho_2=\mu_H\left(\frac{S_S}{K_S+S_S}\right)\left(\frac{K_{OH}}{K_{OH}+S_O}\right)\left(\frac{S_{NO}}{K_{NO}+S_{NO}}\right)\eta_g X_{BH}; \tag{3}$$

- Aerobic growth of autotrophs

$$\rho_3=\mu_A\left(\frac{S_{NH}}{K_{NH}+S_{NH}}\right)\left(\frac{S_O}{K_{OA}+S_O}\right)X_{BA}; \tag{4}$$

- Decay of heterotrophs

$$\rho_4=b_H X_{BH}; \tag{5}$$

- Decay of autotrophs

$$\rho_5=b_A X_{BA}; \tag{6}$$

- Ammonification of soluble organic nitrogen

$$\rho_6=k_a S_{ND} X_{BH}; \tag{7}$$

- Hydrolysis of entrapped organics

$$\rho_7=k_h\frac{\frac{X_S}{X_{BH}}}{K_X+\frac{X_S}{X_{BH}}}\left[\left(\frac{S_O}{K_{OH}+S_O}\right)+\eta_h\left(\frac{K_{OH}}{K_{OH}+S_O}\right)\left(\frac{S_{NO}}{K_{NO}+S_{NO}}\right)\right]X_{BH}; \tag{8}$$

- Hydrolysis of entrapped organic nitrogen

$$\rho_8=\rho_7\frac{X_{ND}}{X_S}; \tag{9}$$

where μ_H, η_g, μ_A, b_H, b_A, k_a, k_h and η_h are kinetic parameters.

Using the process rates and equation (1), the mass balances for the state variables can be obtained. As an example, the mass balance for the soluble substrate (S_S) is

$$\frac{Q}{V_a}\left(S_{S_{in}} - S_S\right) - \frac{1}{Y_H}\rho_1 - \frac{1}{Y_H}\rho_2 + \rho_7 = 0, \qquad (10)$$

where Y_H is one of the stoichiometric parameters. Similar equations can be obtained for the other state variables: slowly biodegradable substrate (X_S), heterotrophic active biomass (X_{BH}), autotrophic active biomass (X_{BA}), particulate products arising from biomass decay (X_P), nitrate and nitrite nitrogen (S_{NO}), $NH_4^+ + NH_3$ nitrogen (S_{NH}), soluble biodegradable organic nitrogen (S_{ND}), particulate biodegradable organic nitrogen (X_{ND}), alkalinity (S_{alk}), and oxygen (S_O).

B. Composite variables

In a real system, some state variables are, most of the time, not available from direct measurements. Thus, readily measured composite variables are used instead. They are defined as follows:

- Particulate chemical oxygen demand: $X = X_I + X_S + X_{BH} + X_{BA} + X_P$;
- Soluble chemical oxygen demand: $S = S_I + S_S$;
- Chemical oxygen demand: $COD = X + S$;
- Volatile suspended solids: $VSS = \frac{X}{icv}$;
- Total suspended solids: $TSS = VSS + ISS$;
- Biochemical oxygen demand: $BOD = f_{BOD}\left(S_S + X_S + X_{BH} + X_{BA}\right)$;
- Total nitrogen of Kjeldahl: $TKN = S_{NH} + S_{ND} + X_{ND} + i_{X_B}\left(X_{BH} + X_{BA}\right) + i_{X_P}\left(X_P + X_I\right)$;
- Total nitrogen: $N = TKN + S_{NO}$;

where icv and f_{BOD} define ratios to convert units.

C. Quality constraints

Quality constraints are usually derived from environmental law restrictions. The most used are related with limits in COD, N, and TSS at the effluent. In mathematical terms, these constraints are defined as $COD_{ef} \leq COD_{law}$, $N_{ef} \leq N_{law}$, $TSS_{ef} \leq TSS_{law}$, where the subscript "ef" means effluent.

D. Constraints of the secondary settler

Traditionally, the importance of the secondary settler is underestimated when compared with the aeration tank. However, it plays a crucial role in the activated sludge system. For example, the clarification efficiency of the settling tank has great influence on the treatment plant efficiency because the particulate fraction arising from biomass contributes to the major portion of effluent COD. Further, it has been observed that the investment cost of a typical settling tank in a WWTP context could reach 25% of the total [31]. Thus, when trying to reduce both investment and operation costs, the importance of the secondary settler is by far emphasized.

A good settling tank has to accomplish three different functions. As a thickener, it aims to produce a continuous underflow of thickened sludge to return to the aeration tank; as a clarifier, it produces a good quality final effluent; and as a storage tank it allows the conservation of the sludge in peak flow events. None of these functions could fail. If that happens the effluent will be of poor quality and the overall behavior of the system can be compromised. The behavior of a settling tank depends on its design and operation, namely the hydraulic features, as the flow rate, the physical features, as inlet and sludge collection arrangements, site conditions, as temperature and wind, and sludge characteristics. The factors that most influence the size of the tank are the wastewater flow and the characteristics of the sludge. As the influent flow is known, the optimization of the sedimentation area and depth must rely on the sludge characteristics, which in turn are related with the performance of the aeration tank.

The ATV design procedure contemplates the PWWF events, in which the sludge mass transferred from the biological reactor is $\Delta X\, V_a$, where ΔX is the change in the sludge concentration within the aeration tank. A reduction of 30% on the sludge concentration for a PWWF event is considered. A higher reduction of the sludge concentration into the biological reactor may compromise the entire process.

A way of turning around this problem is to allocate a certain depth, defined by $h_3 = (\Delta X V_a DSVI)/(480 A_s)$, where A_s is the sedimentation area and $DVSI$ is the diluted sludge volume index, to support the fluctuation of solids during these events. This sludge storage depth depends on the mass that needs to be stored during a PWWF.

When this zone is considered, a reduction in the sedimentation area is allowed. The transferred sludge causes the biological sludge concentration in the reactor at PWWF to decline, which allows a higher overflow rate and therefore a smaller surface area. However, the greater the decrease in reactor concentration is, the greater is the mass of sludge to be stored in the settler tank, so the deeper the tank needs to be. The ATV procedure allows a trade-off between surface area and depth and one may select the area/depth combination that suites the particular site under consideration.

The compaction zone where the sludge is thickened in order to achieve the convenient concentration to return to the biological reactor, depends only on the characteristics of the sludge, and is given by $h_4 = (X_p DSVI)/1000$, where X_p is the sludge concentration in the biological reactor during a PWWF event.

The clear water zone, h_1, and the separation zone, h_2, are set empirically, in our case to 1 m. The depth of the settling tank, h, is the sum of these four zones.

The sedimentation area is still related to the peak flow, Q_p, by the expression $Q_p/A_s \leq 2400\,(X_p DSVI)^{-1.34}$.

On the other hand, the double exponential model assumes a one dimensional settler, in which the tank is divided into ten layers of equal thickness.Some simplifications are considered. No biological reactions take place in this tank, meaning that the dissolved matter concentration is maintained across all the layers. Only vertical flux is considered and the solids are uniformly distributed across the entire cross-sectional area of the feed layer ($j = 7$, in our case). This model is based on a traditional solids flux analysis but the flux in a particular layer is limited by what can be handled by the adjacent layer. The settling function, described by Takács *et al.* in [10], which represents the settling velocity, is given by $v_{s,j} = \max\left(0, \min\left(v_0', w_0\right)\right)$, where $v_{s,j}$ is the settling velocity in layer j,

$$w_0 = v_0 \left(e^{-r_h(TSS_j - f_{ns}TSS_a)} - e^{-r_p(TSS_j - f_{ns}TSS_a)} \right), \quad (11)$$

TSS_j is the total suspended solids concentration in each of the ten considered layers of the settler, TSS_a is the TSS in the feed layer ($TSS_a = TSS_7$) and v_0, v_0', r_h, r_p and f_{ns} are the settling parameters [13].

The solids flux due to the bulk movement of liquid may be up or down, v_{up} and v_{dn} respectively, depending on its position relative to the feed layer, and consequently $v_{up} = Q_{ef}/A_s$ and $v_{dn} = (Q_r + Q_w)/A_s$. The subscript "r" is concerned with the recycled sludge and "w" refers to the wasted sludge.

The sedimentation flux, J_s, for the layers under the feed layer ($j = 7, \ldots, 10$) is given by $J_{s,j} = v_{s,j}TSS_j$ and above the feed layer ($j = 1, \ldots, 6$) the clarification flux, J_{clar}, is given by

$$J_{clar,j} = \begin{cases} v_{s,j}TSS_j & \text{if } TSS_{j+1} \leq TSS_t \\ \min(v_{s,j}TSS_j, v_{s,j+1}TSS_{j+1}) & \text{otherwise} \end{cases} \quad (12)$$

where TSS_t is the threshold concentration of the sludge. The resulting solids balances around each layer, considering steady state, are the following:

- for the layers above the feed layer ($j = 1, \ldots, 6$)

$$\frac{v_{up}(TSS_{j+1} - TSS_j) + J_{clar,j-1} - J_{clar,j}}{h/10} = 0, \quad (13)$$

- for the feed layer ($j = 7$)

$$\frac{\frac{Q\,TSS_a}{A_s} + J_{clar,j-1} - (v_{up} + v_{dn})TSS_j - \min(J_{s,j}, J_{s,j+1})}{h/10} = 0, \quad (14)$$

- for the intermediate layers under the feed layer ($j = 8, \ldots, 10$)

$$\frac{v_{dn}(TSS_{j-1} - TSS_j) + \min(J_{s,j}, J_{s,j-1}) - \min(J_{s,j}, J_{s,j+1})}{h/10} = 0. \quad (15)$$

By convention, $J_{clar,0} = J_{s,11} = 0$.

E. Flow and mass balances around the system

The system behavior, in terms of concentration and flows, may be predicted by balances. In order to achieve a consistent system, these balances must be done around the entire system and not only around each unit operation. This is crucial to reinforce the robustness of the model. Furthermore, these balances may not be a sum of the mass balances of the individual components since the PWWF events are contemplated in the ATV design included in the settler modelling. The balances were done to the suspended matter, dissolved matter and flows.

In the case of the suspended matter, the mass balances concern the organic (X) and inorganic (X_{II}) solids are $(1+r)\,Q_{inf}X_{in} = Q_{inf}X_{inf} + (1+r)\,Q_{inf}X - (V_aX)/(SRTX_r)\left(X_r - X_{ef}\right) - Q_{inf}X_{ef}$, and $Q_{inf}0.2\,TSS_{inf} = (V_aX_{II})(SRTX_r)\left(X_{II_r} - X_{II_{ef}}\right) + Q_{inf}X_{II_{ef}}$, where r is the recycle rate. The subscripts "inf" and "in" denote the influent and the entry of the aeration tank, respectively.

The balances of the dissolved matter are done for each one of the dissolved components S_S, S_O, S_{NO}, S_{NH}, S_{ND}, S_{alk}, as shown in the S_S case: $(1+r)\,Q_{inf}S_{S_{in}} = Q_{inf}S_{S_{inf}} + rQ_{inf}S_{S_r}$. Besides the mass balances, flow balances are also necessary: $Q = Q_{inf} + Q_r$ and $Q = Q_{ef} + Q_r + Q_w$.

F. System variables definition

To complete the model, some definitions are added:

- Sludge retention time: $SRT = (V_aX)/(Q_wX_r)$;
- Hydraulic retention time: $HRT = V_a/Q$;
- Recycle rate: $r = Q_r/Q_{inf}$; $r = TSS/(TSS_{r_{max}} - TSS)$;
- Recycle rate in a PWWF event: $r_p = 0.7TSS/(TSS_{max_p} - 0.7TSS)$;
- Recycle flow rate during a PWWF event: $Q_{r_p} = r_pQ_p$;
- Maximum overflow rate: $Q_p/A_s \leq 2$.

A fixed value for the relation between volatile and total suspended solids was considered $VSS/TSS = 0.7$,where $TSS_{r_{max}}$ is the maximum total suspended solids concentration allowed in the recycle flow and TSS_{max_p} is the maximum total suspended solids concentration allowed in the recycle flow during a PWWF event.

G. Simple bounds

All variables must be non-negative, although more restricted bounds are imposed to some of them due to operational consistencies, $0 \leq K_La \leq 300$, $0.05 \leq HRT \leq 2$, $800 \leq TSS \leq 6000$, $0.5 \leq r \leq 2$, $2500 \leq TSS_r \leq 10000$, $6 \leq S_{alk} \leq 8$, $6 \leq S_{alk_{in}} \leq 8$, and $S_O \geq 2$.

IV. Multi-objective approaches

In this paper we aim to optimize the WWTP design, in terms of a secondary treatment, in a way that the strict laws on effluent quality are accomplished. The WWTP design optimization plays, in essence, two roles:

i) minimizing simultaneously the variables that mostly influence the operation and investment costs of an activated sludge system - the influential variables - using them separately;
ii) minimizing the total cost, hereupon denoted by TC, which is the sum of investment and operation costs, and maximizing the effluent quality measured by a quality index function, represented by the variable QI.

The herein presented paradigm uses the output of the first approach to make the bi-objective problem easy to solve. For practical reasons, the observed trade-offs between the influential variables of the Pareto fronts are used to define tighter lower and upper bounds for those variables and help to generate an initial approximation closer to feasibility.

A. Minimizing the influential variables

To avoid the use of cost functions that are time and local dependent, four objective functions, each describing a variable that influences the investment and operation costs of a WWTP, in each unit operation, are used.

As far as the aeration tank is concerned, the variables that mostly influence the costs are the volume (V_a) and the air flow (G_S). In terms of investment, the first variable influences directly the cost of construction of the tank, and the second will influence the required power of the air pumps. In terms of operation, both variables will determine the power needed to aerate the sludge properly, as well as the maintenance, in terms of electromechanical and civil construction material, due to deterioration.

As to the secondary settler, and assuming that the settling process is only due to the gravity, the variables that most influence the costs are the sedimentation area (A_s) and the tank depth (h), for obvious reasons.

B. Minimizing the total cost function

The objective cost function represents the total cost and includes both investment and operation costs. In this paper, for the sake of simplicity, no pumps are considered, which means that all the flows in the system move by the effect of gravity. The total cost is given by the sum of the investment (*IC*) and operation (*OC*) costs. To obtain a cost function based on portuguese real data, a study was carried out with a WWTP building company. The basic structure of the model is $C = aZ^b$ [32], where a and b are parameters that depend on the region where the WWTP is being built, and have to be estimated. Variable Z is the characteristic of the unit operation that is influencing the cost, for example, V_a and G_S for the case of aeration tank. Parameters a and b were estimated by a least squares technique, giving the following investment cost function for the aeration tank $IC_a = 148.6V_a^{1.07} + 7737G_S^{0.62}$.

The operation cost is usually estimated on an annual basis, so it has to be updated to a present value using the updating term Γ:

$$\Gamma = \sum_{j=1}^{\mathscr{N}} \frac{1}{(1+i)^j} = \frac{1-(1+i)^{-\mathscr{N}}}{i}, \tag{16}$$

where i represents the discount rate (rate of return), i.e., the rate that is used to valuing a project using the concept of the time value of money, over a certain amount of time, for example, $\mathscr{N}$ years. This is also taken as the average life-expectancy of a WWTP. In this study, $i = 0.05$ and $\mathscr{N} = 20$ years are used. Since the collected data come from a set of WWTP in design, operation data are not available. However, from the company experience, the expected life span for the civil engineering construction works is 20 years and the maintenance expenses are around 1% of the investment costs during the first 10 years and around 2% during the remaining ones. Although the replacement costs of the electromechanical components are negligible, they are usually replaced after 10 years. The predominant cost comes from the energy used for pumping the air flow into the aeration tank. The power cost (P_c) in Portugal is 0.08 €$/kWh$. With this information and with the updating term Γ in equation (16), the operation cost of the aeration tank is $OC_a = \left[0.01\Gamma + 0.02\Gamma(1+i)^{-10}\right] 148.6V_a^{1.07} + (1+i)^{-10}7737G_S^{0.62} + 115.1\Gamma P_c G_S$.

The term $(1+i)^{-10}$ is used to bring to present a future value, in this case, 10 years from now.

Similarly, the least squares technique is used to fit the basic model to the available data, and the correspondent investment cost function, $IC_s = 955.5A_s^{0.97}$, and the operation cost function (concerned only with the maintenance for the civil construction), $OC_s = \left[0.01\Gamma + 0.02\Gamma(1+i)^{-10}\right] 148.6(A_s h)^{1.07}$, are obtained for the settling tank. The objective cost function (TC) is then given by the sum of all the previous functions:

$$\begin{aligned} TC &= 174.2V_a^{1.07} + 12487G_S^{0.62} + 114.8G_S + 955.5A_s^{0.97} \\ &\quad +41.3\,(A_s h)^{1.07}. \end{aligned} \tag{17}$$

C. Minimizing the quality index function

To be able to attain effluent quality at a required level, a quality index function may be used to measure the amount of pollution in the effluent.

The Quality Index (QI) defined by the BSM1 model [22] gives a measure of the amount of daily pollution, in average terms during seven days. It depends on the quality of the effluent in terms of TSS, COD, biochemical oxygen demand (BOD), total Kjeldahl nitrogen (TKN), S_{NO} and the effluent flow (Q_{ef}). The obtained function is

$$QI = (2TSS + COD + 2BOD + 20TKN + 2S_{NO})\frac{Q_{ef}}{1000}. \tag{18}$$

D. Multi-objective elitist genetic algorithm

Mathematically, a multi-objective optimization problem with q objectives and n real decision variables can be formulated as, without loss of generality:

$$\begin{array}{ll} \min\limits_{x\in\Omega} & f_k(x),\, k = 1,\ldots,q \\ \text{s.t.} & h_i(x) = 0,\, i = 1,\ldots,m,\; g_j(x) \le 0,\; j = 1,\ldots,p \end{array} \tag{19}$$

where x is an n dimensional vector and $\Omega \subset \mathbb{R}^n$ ($\Omega = \{x \in \mathbb{R}^n : l \le x \le u\}$), $f_k(x)$ are the objective functions, $h(x) = 0$ are the equality constraints and $g(x) \le 0$ are the inequality constraints. The vectors $l, u \in \mathbb{R}^n$ define the lower and upper bounds on x, respectively.

For a multi-objective minimization problem, a solution $x \in \mathbb{R}^n$ dominates $y \in \mathbb{R}^n$, i.e., $x \prec y$ if and only if, $\forall_{k\in\{1,\ldots,q\}} : f_k(x) \le f_k(y) \quad \wedge \quad \exists_{k\in\{1,\ldots,q\}} : f_k(x) < f_k(y)$. A solution $x \in \mathbb{R}^n$ is Pareto-optimal if and only if, there is no solution $y \in \mathbb{R}^n$ which dominates x, i.e., $\nexists_{y\in\mathbb{R}^n} : y \prec x$.

The main goal of a multi-objective algorithm is to find a good and balanced approximation to the Pareto-optimal set. In order to produce a good approximation to the Pareto-optimal front, evolutionary algorithms generate a population of points [29], [28]. Other recent strategies for multi-objective optimization are in [33], [34], [35]. We apply the Multi-objective Elitist Genetic Algorithm (MEGA), described in [36] as a solution method to the previously proposed multi-objective mathematical formulations. It can find, in a single run, multiple approximations to the solutions of the Pareto-optimal set without the need of fixing any weights and a well distributed representation of the Pareto-optimal frontier induced by the use of diversity-preservation mechanisms. We now shortly describe some technical features and the parameters of the MEGA paradigm (see Algorithm 1).

Algorithm 1 Multi-objective Elitist Genetic Algorithm

```
Require: e ≥ 1, s > 1, 0 < p_c < 1, η_c > 0, 0 < p_m < 1, η_m > 0,
    s_SP > s, σ_share > 0
k ← 0
Randomly generate the main population P ∈ Ω
while stopping criterion is not met do
    Fitness assignment FA(P, σ_share) for all points in P
    Update SP with the non-dominated points in P
    Introduce in P the elite with e points selected from SP
    Select by tournaments s points from P
    Apply SBX crossover to the s points, with probability p_c
    Apply mutation to the s points with probability p_m
    k ← k + 1
end while
Update SP with the non-dominated points in P
return Non-dominated points from SP
```

MEGA starts from a population of points P of size s. In our implementation, a real representation is used since we are leading with a continuous problem. Additionally, a secondary population SP that archives potential Pareto-optimal solutions found so far during the search process is maintained. The elitist technique implemented is based on the secondary population with a fixed parameter e ($e \geq 1$) that controls the elitism level, i.e., e is the maximum number of non-dominated solutions of the secondary population that will be introduced in the main population. Excessive levels of elitism can conduct to the stagnation of the search by loosing diversity in the main population. Conversely, if the elitism level is excessively low the convergence may be compromised [30].

In order to handle constraints, we implemented the constrained tournament method in which a new dominance relation is defined [37]. A solution $x \in \mathbb{R}^n$ constraint-dominates $y \in \mathbb{R}^n$, i.e., $x \prec_c y$ if and only if: x is feasible and y is not; x and y are unfeasible, but x has a smaller constraint violation; x and y are feasible, x dominates y, i.e., $x \prec y$. The constraint violation measure herein used is $viol = \sum_{i=1}^{m} |h_i(x)| + \sum_{j=1}^{p} \max(0, g_j(x))$, and if $viol \leq 10^{-2}$ then the solution is considered feasible. During the search, any generated point x that does not satisfy the box constraints is projected onto the set Ω, component by component (for $i = 1, \ldots, n$): $x_i = \max\{l_i, \min\{x_i, u_i\}\}$.

The solutions are evaluated according to a fitness assignment function $FA(P, \sigma_{\text{share}})$ (Algorithm 2) that is based on the constraint-dominance relation between points. All solutions are ranked in terms of dominance defining several fronts. In order to maintain diversity, a sharing scheme depending on an initial parameter σ_{share} is applied to the solutions belonging to the same front. For this purpose, an adaptive sharing scheme on objective space was adopted for diversity preservation as described in [29]. The fitness of a solution x belonging to front P_{rank} depends on two components: the rank of the front containing the solution ($rank$), and the niche count of the solution ($nc(x, \sigma_{\text{share}})$), i.e., the number of solutions in the same front that are within the radius σ_{share}. In each iteration, non-dominated points in main population

Algorithm 2 Fitness Assignment

Require: P, $\sigma_{\text{share}} > 0$
1: $fitness \leftarrow 0$; $rank \leftarrow 1$; $P_{current} \leftarrow P$
2: **while** $P_{current} \neq \emptyset$ **do**
3: Find the non-dominated solutions set P' according to $\prec_c$
4: $P_{rank} \leftarrow P_{current} \backslash P'$
5: Compute σ_{share} for P_{rank}
6: $F(x) \leftarrow fitness + rank \times nc(x, \sigma_{\text{share}})$ for each x in P_{rank}
7: $fitness \leftarrow \max(F(x))$ for all solutions x in P_{rank}
8: $P_{current} \leftarrow P_{current} \backslash P_{rank}$
9: $rank \leftarrow rank + 1$
10: **end while**
11: **return** Fitness values of solutions in P

are archived in SP. The SP update implies the determination of Pareto optimality of all solutions stored so far, in order to eliminate those that became dominated. As the size of SP grows, the time to complete this operation may become significant. So, in order to prevent the growing computation times, in general, a maximum $s_{SP} > s$ size is imposed.

A tournament selection that guarantees that better points are more likely to be selected was used to select points from the main population. New points in the search space are generated by the application of genetic operators (crossover and mutation) to the selected points from main population. A Simulated Binary Crossover (SBX) [38] that combines two points in order to generate new ones was implemented. Two points, $z^{(1)}$ and $z^{(2)}$, are randomly selected from the pool and, with probability p_c, two new points, $w^{(1)}$ and $w^{(2)}$ are generated according to $w_i^{(1)} = 0.5\left((1+\beta_i)z_i^{(1)} + (1-\beta_i)z_i^{(2)}\right)$ and $w_i^{(2)} = 0.5\left((1-\beta_i)z_i^{(1)} + (1+\beta_i)z_i^{(2)}\right)$ for $i = 1, \ldots, n$. The values of β_i are obtained from the following distribution:

$$\beta_i = \begin{cases} (2r_i)^{\frac{1}{\eta_c+1}} & \text{if} \quad r_i \leq 0.5 \\ \left(\frac{1}{2(1-r_i)}\right)^{\frac{1}{\eta_c+1}} & \text{if} \quad r_i > 0.5 \end{cases} \tag{20}$$

where $r_i \sim U(0,1)$ and $\eta_c > 0$ is an external parameter of the distribution. This procedure is repeated until the number of generated points equals the number of points in the pool.

A Polynomial Mutation is applied, with a probability p_m, to the points produced by the crossover operator. Mutation introduces diversity in the population guarantees that the probability of creating a new point $t^{(l)}$ closer to the previous one $w^{(l)}$ ($l = 1, \ldots, s$) is more than the probability of creating one away from it. It can be expressed by $t_i^{(l)} = w_i^{(l)} + (u_i - l_i)\iota_i$ for $i = 1, \ldots, n$. The values of ι_i are given by:

$$\iota_i = \begin{cases} (2r_i)^{\frac{1}{\eta_m+1}} - 1 & \text{if} \quad r_i < 0.5 \\ 1 - (2(1-r_i))^{\frac{1}{\eta_m+1}} & \text{if} \quad r_i \geq 0.5 \end{cases} \tag{21}$$

where $r_i \sim U(0,1)$ and $\eta_m > 0$ is an external parameter of the distribution. The search ends when a given stopping criterion is satisfied. The best approximations to the Pareto-optimal set are archived in SP.

V. Numerical results

In order to illustrate the performance of the above described solution method MEGA when solving both multi-objective mathematical formulations, the models were coded in the programming language MATLAB™ (MATLAB is a registered trademark of the MathWorks, Inc.). Both models have 113 variables, 103 equality constraints and one inequality constraint. All the variables are bounded below and above. During the experiments, the values set to all stoichiometric, kinetic and operational parameters are those available from the benchmark simulation model n. 1 [22] since they are usually found in real activated sludge based plants.

The MEGA algorithm was coded in MATLAB programming language and the numerical results were obtained with a Intel Core2 Duo CPU 1.8GHz with 2GB of memory. The MEGA parameters are: $s = 40$, $e = 4$, $p_c = 0.9$, $\eta_c = 20$, $p_m = 1/113$, $\eta_m = 20$, $s_{SP} = \infty$ and $\sigma_{\text{share}} = 0.1$. The maximum number of objective function evaluations allowed is 50 000. MEGA uses elitism based on a secondary population which proves to be very useful in finding good approximations to the Pareto-optimal set. The constrained tournament method is coupled with MEGA for an efficient handling of the linear and non-linear constraints of the problem. The importance of controlling the level of elitism with a secondary population has been emphasized on comparison studies [36], [30] with other evolutionary algorithms, like NSGA-II [18] and SPEA, developed by Zitzler and Thiele [39], when a set of small benchmark multi-objective problems is used. In the present study, where a real world application is considered and a highly constrained problem with linear and non-linear constraints is addressed, the MEGA algorithm has been shown to behave rather well. The previously stated 10% of elitism have reinforced the convergence properties of MEGA even with large and difficult problems. We remark that some commercial implementations of NSGA-II, namely the function `gamultiobj` in the Global Optimization Toolbox of MATLAB, do not provide any constraint-handling mechanism for non-linear constraints. Therefore, they are not suited for numerical comparisons with MEGA when solving the herein presented multi-objective optimization problems.

In both approaches, an initial approximation, x^0, obtained from the GPS-X simulator [13], when real influent data is provided to the simulator, was introduced in the initial population. Further, tighter bounds for the influential variables were provided to the bi-objective modelling, after the trade-offs between the influential variables of the Pareto fronts from the first approach have been analyzed. The remaining points of the main population were randomly generated. Several experiments were conducted without introducing this initial approximation in the population and the algorithm failed to achieve a feasible point within the previously defined maximum number of objective function evaluations. No feasible solution was found even when the algorithm was allowed to run for 1 000 000 function evaluations, corresponding to 387.5 seconds. As a consequence, these experiments were not further explored.

The characteristics of the influent to test the model are: $Q = 1050$ m^3/day, $Q_p = 108$ m^3/h, $BOD = 810$ g/m^3 $COD = 2000$ g/m^3, and $TSS = 750$ g/m^3. All the state variables are defined as function of the composite variables, according to the GPS-X Technical Reference [13]. The proportion of particulate inert inorganic material (X_{II}) in the TSS is set to 0.3.

A. *Influential variables minimized*

When the problem which consists of minimizing A_s, h, V_a and G_S is addressed, the following solutions are obtained. Figures 1a to 1c show the most important bi-dimensional projections of the Pareto front. All the plotted solutions are non-dominated and represent different compromises between the objectives.

Figure 1a shows that the sedimentation area and the aeration tank volume are conflicting and we have found four possible trade-offs between the two variables. We can see that the larger V_a, the smaller A_s is. In physical terms, this makes sense since when V_a is larger, the sludge is less concentrated allowing a smaller A_s. A similar behavior is observed in Figure 1b, since the sedimentation depth has the same kind of relation with the aeration tank volume. In Figure 1c we can see the relation between the settling area and depth. Because of the ATV model, there is a compromise between these two variables. The larger is the settling area, the smaller the depth needed, and vice-versa. This is a very important observation because, for example, if the land area available for building the WWTP is small, one can choose to build a deeper settling tank, achieving a similar design in terms of costs. We observed that the air flow and the aeration tank volume are not conflicting and so this projection showed only one point. This is also expectable since if the aeration tank is larger, a bigger amount of air is needed to maintain the required dissolved oxygen level. We remark that all the solutions are feasible. The total computational time is about 380 seconds.

Finally, an important remark concerned with the reduced number of points in the observed Pareto fronts. The points correspond to the non-dominated as well as feasible solutions. Since the decision variables of the problem are constrained mainly by equality constraints, the feasible region is rather small. Thus, very few non-dominated points are feasible. However, we highlight the fact that these points are well distributed in the space of the objectives.

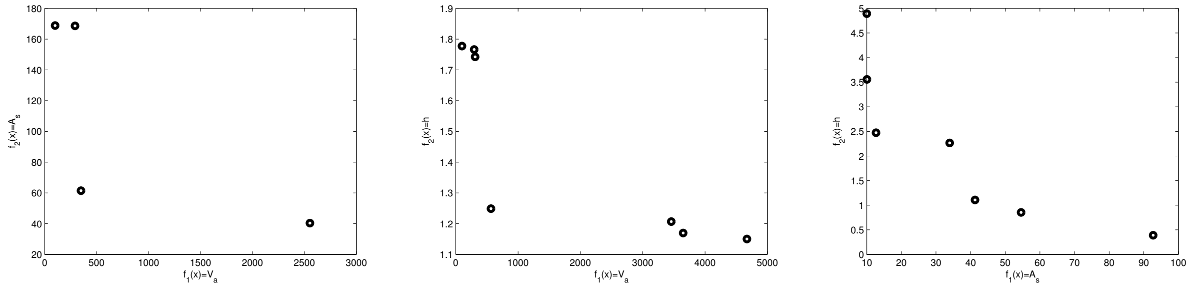
From Figure 1a we may conclude that only one solution is in the region of the good compromise solutions. Thus, the choice is obvious, unless other important issues are to be considered during decision making. A similar argument applies to the Figure 1b. Three good compromise solutions are depicted in Figure 1c. Our view is that the most right solution of the three is to be preferred, if a large building area is available, since depth excavation turns out to be harder and more expensive than width excavation. However, if a reduced area for building the WWTP is provided, then the left most solution is to be preferred.

B. *Cost and quality index minimized*

To analyze the effect of the population size on the performance of MEGA when solving the bi-objective problem, three values of s were tested: 40, 80 and 120. The larger the population size is the higher is the likelihood of having more non-dominated solutions. Figure 2a shows the Pareto-optimal front defined by the approximations to the Pareto-optimal solutions, for each of the three tested values of s. In this figure, the compromise solutions between QI and TC are plotted. The values of the TC are in millions of Euros (M €). We may observe that the quality of the solutions is not greatly affected by s, although the computational time increases hugely with s. Figure 2b aims to show that the pair QI and TC that corresponds to the slightly infeasible initial approximation x^0 (represented by '•', in full red) is closer to the Pareto front than the other pairs that come from randomly generated approximations (represented by '+').

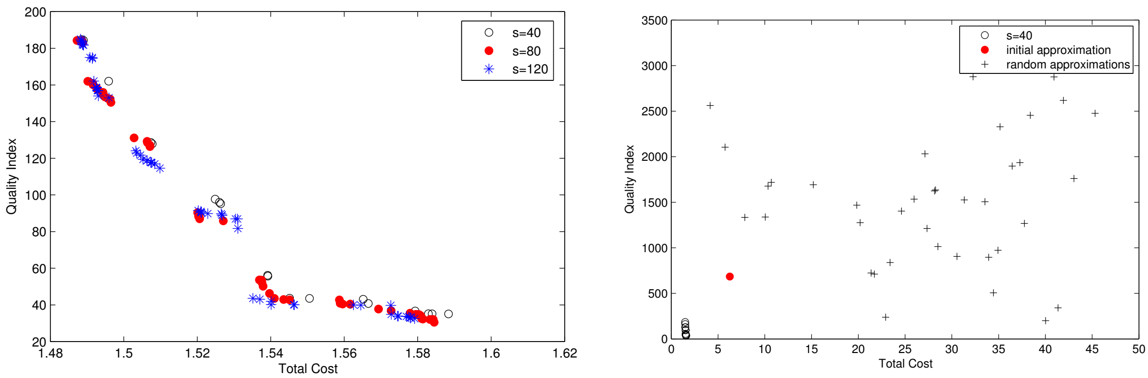
Due to the stochastic nature of the algorithm, the problem was solved more than once to confirm the consistency of the obtained results. Apart from the provided x^0, the other initial approximations required by MEGA were randomly generated, thus slightly different results although with similar trade-offs between the objectives were obtained.

In Table I, the most important decision variables of the obtained approximations to the Pareto-optimal solutions are presented, namely, the area and depth of the secondary settler, as well as COD and TSS, for the case $s = 40$. In all the obtained Pareto solutions, $V_a = 1567$, $G_S = 100$ and $N = 15$ were obtained. It can also be observed that the sedimentation area maintains almost the same value in all the non-dominated solutions, being the settler depth the variable with the most significant differences. The total computational time is about 190 seconds. We can observe that the non-dominated solutions indicate that the followed methodology produces improved results with physical meaning. The obtained WWTP designs represent compromises that are



(a) Projection of A_s *vs* V_a (b) Projection of h *vs* V_a (c) Projection of h *vs* A_s

Fig. 1: Bi-dimensional representation of the Pareto fronts



(a) TC and QI for $s = 40$, $s = 80$, and $s = 120$ (b) Initial and random approximations for $s = 40$

Fig. 2: Pareto curves for the Total Cost and Quality Index

TABLE I: Optimal values of the non-dominated solutions for the most important variables

A_s	h	COD	TSS	TC (M €)	QI
816	3.3	36.2	12.2	1.488	184.8
816	3.3	36.1	12.2	1.489	184.4
816	3.4	37.1	12.0	1.496	162.0
816	3.4	37.1	12.0	1.496	162.0
815	3.6	37.1	11.1	1.508	128.6
815	3.6	37.1	11.1	1.508	127.9
816	3.9	35.0	11.7	1.524	97.7
816	3.9	34.4	11.7	1.526	96.0
816	3.9	34.4	11.7	1.526	95.2
812	4.2	28.1	9.2	1.539	56.2
812	4.2	28.0	9.3	1.539	55.7
813	4.2	28.7	8.4	1.545	43.7
813	4.3	28.7	8.3	1.551	43.5
813	4.6	27.5	8.2	1.565	43.1
813	4.6	26.1	8.1	1.567	40.8
813	4.8	22.9	7.3	1.583	36.7
813	4.8	22.7	6.6	1.583	35.2
813	4.9	22.6	6.6	1.584	35.1
813	4.9	22.7	6.6	1.588	35.1

economically attractive with convenient quality indexes and satisfy the law limits. Moreover, these limits in terms of COD and TSS are below the law limits, showing the robustness of the solution. It is easily observed from Figure 2a that from a certain point on (corresponding to a total cost of about 1.54 M €) the solutions are no longer attractive, since a small improvement in the quality index implies a much higher raise in the total cost.

To validate the goodness of the proposed methodology based on two multi-objective approaches, a comparison with results obtained by a hybrid genetic algorithm for uni-objective programming is included [6]. The therein obtained minimum Total Cost is 1.04 M €. This solution was obtained after 1 035 200 function evaluations and corresponds to a Quality Index of 285, which is much higher than the herein reported value. Thus, the multi-objective approaches to the WWTP optimal design have indeed supplied a valuable procedure to identify good trade-offs between conflicting objectives.

VI. Conclusions

The paper presents new and expeditious methodologies to model and solve a WWTP design optimization problem that can be extended to any WWTP unit operation modelling, regardless adjusting to each particularity of the problem under study. Two multi-objective approaches are proposed. The influential variables minimization strategy is more suitable for a draft project, when the exact location and time where the WWTP is going to be built is still unknown. The decision-maker has, observing the Pareto-optimal solutions, a set of alternatives from which he/she can choose from. This information might help to elaborate a first version of the project, allowing to study all the alternatives, even with different unit operations. When the specific location and moment in time are defined, the analysis based on the minimization of the two objective functions – the total cost and the quality index – is to be preferred. With the information gather from the first approach, the decision-maker may provide tighter lower and upper bounds for the influential variables and other input data aiming to make the bi-objective optimization problem easy to solve and to improve the solution.

The results obtained in this study clearly show that the multi-objective modelling is an effective tool to WWTP design optimization. The achieved optimal solutions are meaningful in physical terms. Investment and operation costs are highly influenced by the optimized variables, meaning that the obtained solutions are economically attractive. Both multi-objective approaches to WWTP design optimization provide a set of non-dominated solutions from which the decision-maker can choose according to his/her preferences.

In the future we intend to introduce the game theory [40] to assist the solution selection process, defining supra-criteria to obtain a compromise solution acceptable to all the players.

REFERENCES

[1] R. Otterpohl, T. Rolfs, and J. Londong. "Optimizing operation of wastewater treatment plants by offline and online computer simulation," *Water Science and Technology*, vol. 30, no. 2, pp.165–174, 1994.

[2] A. Seco, J. Serralta, and J. Ferrer. "Biological nutrient removal model no.1 (BNRM1)," *Water Science and Technology*, vol. 50, no. 6, pp. 69–78, 2004.

[3] X. X. Zhang, D. Y. Zhao, Z. X. Wang, B. Wu, W. X. Li, and S. P. Cheng. "Environmental biological model based optimization of activated sludge process," *International Journal of Environmental Science and Technology*, vol. 6, no. 1, pp. 69–76, 2009.

[4] D. Tyteca, Y. Smeers, and E. J. Nyns. "Mathematical modeling and economic optimization of wastewater treatment plants," *CRC Critical Reviews in Environmental Control*, vol. 8, no. 1, pp. 1–89, 1977.

[5] L. Costa, I. A. C. P. Espírito Santo, E. M. G. P. Fernandes, R. Denysiuk. "Using a genetic algorithm to solve a bi-objective WWTP process optimization," In: B. Hu, K. Morasch, S. Pickl and M. Siegle (eds.), *Operations Research Proceedings 2010*, pp. 359–364, Springer-Verlag, 2011.

[6] I. A. C. P. Espírito Santo, L. Costa, R. Denysiuk, E. M. G. P. Fernandes. "Hybrid genetic pattern search augmented Lagrangian algorithm: application to WWTP optimization," In: M. Collan (ed.), *Proceedings of 2nd International Conference on Applied Operational Research*, Lecture Notes in Management Science, vol. 2, pp. 45–56, 2010.

[7] I. A. C. P. Espírito Santo, E. M. G. P. Fernandes, M. M. Araújo, and E. C. Ferreira. "An augmented Lagrangian pattern search method for optimal WWTP designs," *Proceedings of the 6th WSEAS International Conference on System Science and Simulation in Engineering (ICOSSSE'07*, pp. 313–318, 2007.

[8] M. Henze, C. P. L. Grady Jr, W. Gujer, G. V. R. Marais, and T. Matsuo. "Activated sludge model no. 1," *Technical Report 1, IAWPRC Task Group on Mathematical Modelling for design and operation of biological wastewater treatment*, 1986.

[9] G. A. Ekama, J. L. Barnard, F. W. Günthert, P. Krebs, J. A. McCrquodale, D. S. Parker, and E. J. Wahlberg. "Secondary settling tanks: Theory, modelling, design and operation," *Technical Report 6, IAWQ - International Association on Water Quality*, 1997.

[10] I. Takács, G. G. Patry, and D. Nolasco. "A dynamic model of the clarification-thickening process," *Water Research*, vol. 25. no. 10, pp. 1263–1271, 1991.

[11] I. A. C. P. Espírito Santo, E. M. G. P. Fernandes, M. M. Araújo, and E. C. Ferreira. "On the secondary settler models robustness by simulation," *WSEAS Transactions on Information Science and Applications, ISSN: 1790-0832*, vol. 3, pp. 2323–2330, 2006.

[12] J. Hakanen, K. Miettinen, and K. Sahlstedt. "Wastewater treatment: New insight provided by interactive multiobjective optimization," *Decision Support Systems*, vol. 51, no. 2, pp. 328–337, 2011.

[13] Hydromantis, Inc. *GPS-X User's Guide*, 2001.

[14] K. Miettinen. "IND-NIMBUS for demanding interactive multiobjective optimization," In: Tadeusz Trzaskalik (ed.), *Multiple Criteria Decision Making '05*. Karol Adamiecki University of Economics Press, Katowice, pp. 137–150, 2006.

[15] P. Biswas, P. Bose, and V. Tarein. "Optimal choice of wastewater treatment train by multi-objective optimization," *Engineering Optimization*, vol. 39, no. 2, pp. 125–145, 2007.

[16] N. Srinivas and K. Deb. "Multi-objective optimisation using non-dominated sorting in genetic algorithms," *Evolutionary Computation*, vol. 2, no. 3, pp. 221-248, 1994.

[17] J. Iqbal and C. Guria. "Optimization of an operating domestic wastewater treatment plant using elitist non-dominated sorting genetic algorithm," *Chemical Engineering Research and Design*, vol. 87, no. 11, pp. 1481–1496, 2009.

[18] K. Deb, A. Pratap, S. Agrawal, and T. Meyarivan. "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–198, 2002.

[19] X. Flores, A. Bonmati, M. Poch, I. R. Roda, L. Jimenez, and R. Banares-Alcantara. "Multicriteria evaluation tools to support the conceptual design of activated sludge systems," *Water Science and Technology*, vol. 56, no. 6, pp. 85–94, 2007.

[20] G. Fu, S.-T. Khu, and D. Butler. "Use of surrogate modelling for multiobjective optimisation of urban wastewater systems," *Water Science and Technology*, vol. 60, no. 6, pp. 1641–1647, 2009.

[21] B. Béraud, C. Lemoine, and J.-F. Steyer. "Multiobjective genetic algorithms for the optimisation of wastewater treatment processes," *Computational Intelligence Techniques for Bioprocess Modelling, Supervision and Control*, Studies in Computational Intelligence, vol. 218, pp. 163–195, 2009.

[22] J. Alex, L. Benedetti, J. Copp, K. V. Gernaey, U. Jeppsson, I. Nopens, M. N. Pons, C. Rosen, J. P. Steyer, and P. Vanrolleghem. "Benchmark simulation model no. 1 (BSM1)," *Technical report, IWA Taskgroup on Benchmarking of Control Strategies for WWTPs*, 2008.

[23] L. J. Alvarez-Vázquez, N. García-Chan, A. Martínez, and M. E. Vázquez-Méndezin. "Multi-objective Pareto-optimal control: an application to wastewater management," *Computational Optimization and Applications*, vol. 46, no. 1, pp. 135–157, 2008.

[24] J. O. H. Sendín, I. Otero, A. A. Alonso, and J. R. Banga. "Improved optimization methods for the multiobjective design of bioprocesses," *Industrial & Engineering Chemistry Research*, vol. 45, no. 25, pp. 8594–8603, 2006.

[25] F. Logist, B. Houska, M. Diehl, and J. F. Van Impe. "Fast pareto set generation for nonlinear optimal control problems with multiple objectives," *Structural and Multidisciplinary Optimization*, vol. 42, no. 4, pp. 591–603, 2010.

[26] F. Logist, B. Houska, M. Diehl, and J. F. Van Impe. "Robust multi-objective optimal control of uncertain (bio)chemical processes," *Chemical Engineering Science*, vol. 66, no. 20, pp. 4670–4682, 2011.

[27] D.E. Goldberg. *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley, Reading, Massachusetts, 1989.

[28] K. Deb. *Multi-Objective Optimization using Evolutionary Algorithms*. John Wiley and Sons, Ltd, Chichester, 2001.

[29] L. Costa and P. Oliveira. "An adaptive sharing elitist evolution strategy for multi-objective optimization," *Evolutionary Computation*, vol. 11, no. 4, pp. 417–438, 2003.

[30] L. Costa and P. Oliveira. "An elitist genetic algorithm for multiobjective optimization," In: M. G. C. Resende and J. Pinho de Sousa (eds.), *Metaheuristics*. Kluwer Academic Publishers, Norwell, MA, USA, pp. 217–236, 2004.

[31] G.-M. Zeng, S.-F. Zhang, X.-S. Qin, G.-H. Huang, and J.-B. Li. "Application of numerical simulation on optimum design of two-dimensional sedimentation tanks in the wastewater treatment plant," *Journal of Environmental Sciences*, vol. 15, no. 3, pp. 346–350, 2003.

[32] D. Tyteca. "Cost functions for wastewater conveyance systems," *Water Pollution Control Federation*, vol. 48, no. 9, pp. 2120–2130, 1976.

[33] C. C. Da Ronco, and E. Benini. "GeDEA-II: A simplex crossover based evolutionary algorithm including the genetic diversity as objective," *Engineering Letters*, vol. 21, no. 1, pp. 23-35, 2013.

[34] H. Zhang. "The performance analysis of an improved PSOIW for multi-objective optimization," *IAENG International Journal of Computer Science*, vol. 39, no. 1, pp. 34-41, 2012.

[35] H. Zhang. "An analysis of multiple particle swarm optimizers with inertia weight for multi-objective optimization," *IAENG International Journal of Computer Science*, vol. 39, no. 2, pp. 190-199, 2012.

[36] L. Costa and P. Oliveira. "An evolution strategy for multiobjective optimization," In: *Proceedings of the Congress on Evolutionary Computation (CD edition)*, Honolulu, USA, 2002.

[37] K. Deb. "An efficient constraint handling method for genetic algorithms," *Computer Methods in Applied Mechanics and Engineering*, vol. 186, no. 2–4, pp. 311–338, 2000.

[38] K. Deb and R. B. Agrawal. "Simulated binary crossover for continuous search space," *Complex Systems*, vol. 9, no. 2, pp. 115–148, 1995.

[39] E. Zitzler and L. Thiele. "Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach," *IEEE Transactions on Evolutionary Computation*, vol. 3, pp. 257–271, 1999.

[40] K. Annamdas and S. Rao. "Multi-objective optimization of engineering systems using game theory and particle swarm optimization," *Engineering Optimization*, vol. 41, no. 8, pp. 737–752, 2009.