A Study on Incipient Sediment Motion and the Stability Conditions of Rubble

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***Abstract*—The exploration for the stability conditions of rubble under the action of current or waves, is not only one of the issues demanding immediate settlement in the Kinematical Mechanics of Sediment, but also an important problem in rationalized design of riprap engineering. Based on the related research data at home and abroad, the paper analyses the force condition of rubble under current and waves. It concludes that Shields parameter, which is the ratio of the shear stress to the product of the effective specific weight and equivalent diameter of rubble in incipient motion, is a function of Reynolds number, Keulegan & Carpenter parameter and submarine rest angle. For the purpose of eliminating the interference of irregular stone shapes and supporting ways between stones, the author applies the fixed steel rings to hold up rubble in the wave flume experiment. The result confirms that the Shields parameter will leap when Reynolds number is greater than 105 when Keulegan & Carpenter parameter is over 10.**

***Index Terms*—Sediment; Rubble; Stability Condition**

I. Introduction

SEDIMENT movement mechanics is a subject that studies the law of sediment motion in the process of particles erosion, transportation and sedimentation under the water (or wind) action. The biggest particles studied involve gravel and pebbles, but no more than 200 mm. However, in the stone buildings in river or on coast, such as breakwaters, debris barriers, diversion dikes, spur dikes and revetments, etc., the weight of the protective stones is generally in tons, with a minimum of dozens of kilograms. The equivalent diameter ($d = 1.24(w_s / \gamma_s)^{1/3}$) is often more than 200 mm. At present, sediment movement mechanics and engineering design have their own theories and empirical methods with their respective systems.

Sediment particles and rubble are different in geometric scales, but both the sediment incipient motion rule and the solid from the perspective of fluid mechanics. These two independent systems, sediment movement mechanics and engineering design, are caused by the different subject development histories. It's unnecessary to confine the particles studied in sediment movement mechanics at small scales. Because, broadly speaking, rubble also belongs to the sediment. Studying the stability conditions of rubble in flow and waves is not only one of the issues demanding immediate settlement in sediment movement mechanics, but also an important factor to solve the problem of engineering design rationalization.

Shields (1936) and Iribarren(1938) respectively put forward the famous idea of Shields curve and the theory of armor stone in sloping breakwater. In the last decades many researchers have studied the stability of sloping breakwaters; see Bruun (1985) for reference. Experimentation in wave flume was the most common way to address this problem. According to the data of Qiantang river, Wang (2002) also referred to the characteristics of flow resistance coefficient in large Reynolds number ($\mathrm{Re} > 10^5$) to extend the Shields curve. They argued that when the grain Reynolds number ($\mathrm{Re}_* = u_* d / \upsilon$) is close to 10^5, Shields parameters would jump from 0.062 to 0.25. They also pointed out that the weight of protective stone obtained by the current engineering design formula is rather conservative.

This paper aims to prove whether Shields number will jump when grain Reynolds number is greater than 10^5 through systemic experiment.

II. Stability Conditions of the Rubbles

A. Incipient Sediment Motion in Wave Action

PCompared with unidirectional flow, wave motion has its own characteristics. In fluid water, the velocity and the pressure at each point oscillate inconstantly in time, and the forces on bed grains are also changing with time. When the velocity reaches the maximum, the bed shear force reaches its maximum, too. Sediment incipient motion depends on the maximum shear force.

①Particle Velocity And the Bed Shear Force

Generally, water particles velocity can be calculated according to the micro-amplitude wave theory, and horizontal velocity can be expressed as

$$u = \frac{\pi H}{T} \cdot \frac{chk(z+h)}{shkh} \cos(kx - \sigma t) \qquad (1)$$

Where

H is wave height,

T is wave period,

$k = 2\pi / L$ is wave number,

L is wave length,

$\sigma = 2\pi / T$ is circular frequency,

h is water depth,

z is vertical coordinates, with static water for zero.

The maximum horizontal velocity of water points on the bed is

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$$u_{bm} = \frac{TH}{Tshkh} \quad (2)$$

The horizontal amplitude of bottom water particles is

$$a_b = \frac{H}{2shkh} \quad (3)$$

The maximum shear force under wave action can be written as

$$\tau_{bm} = \frac{1}{2} f_w \rho u_{bm}^2 \quad (4)$$

Where f_w is wave resistance coefficient, or drag coefficient of sediment under wave action.

In the case of laminar flow, the wave resistance coefficient f_w can be expressed as

$$f_w = 2\sqrt{\frac{\nu}{a^2\omega}} \quad (5)$$

In the turbulent flow, can use Swart or Josson and Bagnold 's formula

$$\begin{cases} \left. \begin{aligned} & f_w = \exp\left[5.213\left(\frac{k_s}{a}\right)^{0.194} - 5.977\right] \\ & \text{or} \frac{1}{4\sqrt{f_w}} + \log\frac{1}{4\sqrt{f_w}} = -0.08 + \log\frac{a}{k_s} \end{aligned} \right\} & \text{when} \frac{a}{k_s} \geq 1.7 \\ f_w = 0.28 & \text{when} \frac{a}{k_s} < 1.7 \end{cases} \quad (6)$$

Where k_s is roughness, when bed is flat, equal to size of the grains that form the channel bed, mean $k_s = D$. When the bed is covered by sand ripple, the bed roughness is the magnitude order of sand wave height η. Ma (2012) gave the experience formula

$$\frac{k_s}{\eta} = 25\frac{\eta}{\lambda} \quad (7)$$

in which λ is sand wave length, accordingly

$$\frac{k_s}{\alpha} = 25\frac{\eta^2}{\lambda\alpha} \quad (8)$$

Josson studied the relationship among wave resistance coefficient f_w, wave Reynolds number $\mathrm{Re}(=u_{bm}a_b/\upsilon)$, and a_b/k_s. See Fig.1.

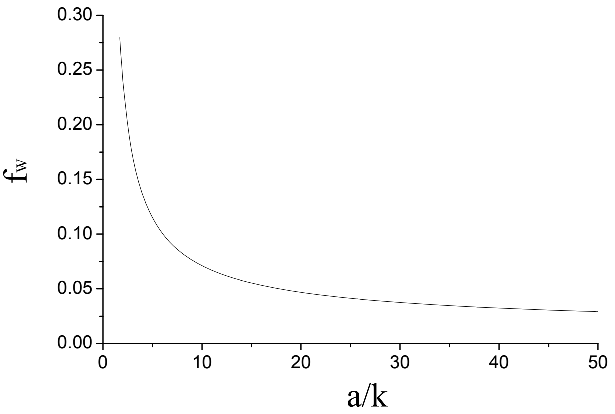

Fig.1 Relations between f_w and $\frac{a}{k}$

②Drag Coefficient And the Inertial Force Coefficient

The experimental results of constant flow have shown that C_D is a function of Reynolds number. For wave action, theoretically, we should identify the instantaneous Re, and then gain the corresponding C_D, but it is difficult to do it in this way.

According to such researches as Li(2014), Pang(2012), Cheng(2012), recommends the design values of the C_D, arguing that the values in waves and steady flow are roughly identical. The manual also recommends the inertial force coefficient:

$$C_M = \begin{cases} 2.0 & \mathrm{Re} < 2.5\times10^5 \\ 2.5 & 2.5\times10^5 < \mathrm{Re} < 5\times10^5 \\ 1.5 & \mathrm{Re} > 5\times10^5 \end{cases} \quad (9)$$

It should be noticed that some researchers think that C_D and C_M depend not only on Re, but also on k_c. $k_c = \frac{u_m T}{d}$ is Keulegan & Carpenter number, also known as Strouhal number. Using the micro-amplitude wave theory to make transformation, then

$$\begin{cases} \frac{u_m T}{d} > 63, & C_D \approx steady\ flow\ C_D \\ 6 < \frac{u_m T}{d} < 63, & C_D > steady\ flow\ C_D \end{cases} \quad (10)$$

Li and other researchers study the relations among C_D, C_M and $(k_c)_p$ under regular wave and irregular wave. They think when $(k_c)_p < 16$, dominated by inertial force, $C_D = 0$; when $(k_c)_p > 30$, dominated by drag force, $C_M = 0$.

B. *The Stability Conditions of the Rubble*

Similar with the sediment incipient, the instability of rubble mainly begins with rolling and sliding modes. The standard of rubble instability is generally based on whether it leaves the original location or not. The rubble stability conditions are the hydrodynamic conditions where rubble is in the state of critical instability.

In wave action, besides support action, rubble is acted by uplift force F_L, drag force F_D, effective specific weight W and inertia force F_I. Seeing Fig.2, each force can be expressed as follow

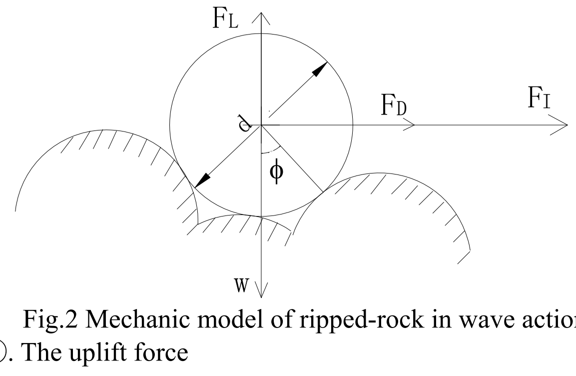

Fig.2 Mechanic model of ripped-rock in wave action

①. The uplift force

$$F_L = \frac{1}{2}\xi_1 \varepsilon C_L \rho d^2 u_d^2 u_d \qquad (11)$$

where C_L is uplift coefficient.

②. The drag force

$$F_D = \frac{1}{2}\xi_1 \varepsilon C_D \rho d^2 u_d^2 \qquad (12)$$

where u_d is the flow velocity that acts on the rock, ξ_1 is area coefficient, ε is exposure coefficient.

③. The effective gravitation

$$W' = \xi_2(\gamma_s - \gamma)d^3 \qquad (13)$$

where d is equivalent diameter, ξ_2 is volume coefficient.

④. The inertia force

$$F_I = \xi_2 \varepsilon C_M \rho d^3 \frac{\partial u_d}{\partial t} \qquad (14)$$

in which $\frac{\partial u_d}{\partial t}$ is acceleration of the rock.

Taking the instantaneous balance of torque, one can obtain

$$(F_D + F_I)\frac{d}{2}\cos\phi + (F_L - W')\frac{d}{2}\sin\phi = 0 \qquad (15)$$

where ϕ is repose angle, support angle. From eq.(15), one obtains

$$F_D + F_I = (W' - F_L)tg\phi \qquad (16)$$

It is generally believed that $f = tg\phi$. So eq.(16) is same as the formula with sliding balance.

According to linear wave theory, eq.(15) can be written as

$$\frac{\frac{1}{2}C_D \rho u_{dm}^2}{(\gamma_s - \gamma)d} = \frac{1}{\varepsilon}[(ctg\phi + \frac{C_L}{C_D})\xi \cos\sigma t\left|\cos\sigma t\right| + 2\pi \frac{C_M}{C_D} ctg\phi(\frac{u_{dm}T}{d})^{-1}\sin(\sigma t)]^{-1} \qquad (17)$$

Where u_{dm} is the maximum horizontal water particles velocity, t is the time when the right eq.(17) reaches its minimum, which is a function of Re and k_c. $\xi = \frac{\xi_1}{\xi_2}$ is comprehensive shape coefficients. Because C_D, C_M and C_L are all the functions of Re and k_c, when rubble is in the state of critical instability, u_{dmc} is used to express u_{dm}. When eq.(17) reaches its minimum, eq.(18) can be obtained approximately:

$$\frac{\frac{1}{2}C_D \rho u_{dm}^2}{(\gamma_s - \gamma)d} = f(\mathrm{Re}, k_c, \phi, \xi, \varepsilon) \qquad (18)$$

It is the stability condition of rubble in wave action. It means the ratio of instability shear force to $(\gamma_s - \gamma)d$ is the function of Reynolds number, Strouhal number and repose angle, which also contains rock shape and exposure coefficients. It is generally believed that the larger the area/volume ratio is, the more adequate the rubble exposes, and the easier the rubble loses stability. Since $\tau_c = \frac{1}{2}C_D \rho u_{dmc}^2$, $\mathrm{Re}_* = \sqrt{\frac{C_D}{2}}\mathrm{Re}$, eq.(18) can be written as

$$\Theta = \frac{\tau_c}{(\gamma_s - \gamma)d} = f_1(\mathrm{Re}_*, k_c, \phi, \xi, \varepsilon) \qquad (19)$$

In constant uniflow, no acceleration exists, no inertia force acts, so the rubble instability condition is

$$\Theta = f_1(\mathrm{Re}_*, \phi, \xi, \varepsilon) \qquad (20)$$

Under wave action, if the rock equivalent diameter is as small as the sediment size, k_c will be large. Referring to research results mentioned in part 2.1, at that time the inertia force term, the second term in the right hand of eq.(18) denominator, can be dropped. C_D and C_L are no longer related to k_c. Furthermore, the submarine rest angle shows little change, which can be considered as a constant. If it doesn't take the influence of shape and exposure into account, eq.(20) reduces to

$$\Theta = f_1(\mathrm{Re}_*) \qquad (21)$$

The results obtained by this and Shields are completely identical. It is also indicated that Shields parameter can reflect the law of sediment incipient under wave condition.

III. Experiment Research on Stability Condition of Sphere in Wave

According to the former theoretical analysis, the stability conditions of rubble under wave action have been obtained. The following will identify $f(\mathrm{Re}, k_c, \phi, \xi, \varepsilon)$ of the eq.(19) through systemic experimental study.

A. *Experiment Project*

The purpose of this experiment is to study the rubble stability condition after Reynolds number is increased to 10^5, thus to verify the extension of Shields curve made by Wang and Shen.

Due to irregular rock shape, the mechanical performances are different even in the same flow condition. Moreover, the various placements of the rocks cause different exposure of each rock. The above two factors are the main reasons to induce the uncertainty of experimental results.

To eliminate this complex disturbing factors, isolated cement sand balls are taken as samples. The experiment applies the fixed rims to simulate the natural supporting condition of rocks. Then $f(\mathrm{Re}, k_c, \phi, \xi, \varepsilon)$ reduce to $f(\mathrm{Re}, k_c, \phi)$.

However, it is difficult for common wave maker to make large scale rock lose stability. Therefore, the author sets the samples on the submerged breakwater that has certain front slope. The inadequacy of wave maker can be resolved to a certain extent.

According to the above plan, as ξ, ε are known, eq. (17) becomes

$$\frac{\frac{1}{2}C_D\rho u_{dm}^2}{(\gamma_s-\gamma)d}=\frac{2}{3}[(ctg\phi+\frac{C_L}{C_D})\cos\sigma t\,|\cos\sigma t\,|+\frac{4\pi}{3}\sin(-\sigma t)]^{-1} \quad (22)$$

Where u_{dm} is water particle velocity acting on sphere in submerged breakwater u_{dm} can be tested, or can be approximately calculated as follows

$$u_{dm}=\alpha u_{dm0}=\alpha\frac{\pi H}{T}\frac{chk(h-d/2)}{shkh} \quad (23)$$

Where α is velocity enhancement factor.

B. Experimental Setup

The experiment is conducted in a wave flume, which are 60m long, 3m wide and 2m high. The maximum wave height of wave maker is 40cm, period 0.80～4.0s.

Experimental Conditions

Cement sand balls are used in the experiment, $\gamma_s=2.1g/cm^3$, including five different sizes, D =5.35, 15.45, 16.65, 19.95, 35.0cm. Support angle ϕ=30°, 37°, 45°.

The height of submerged breakwater front slop, $m=2$. There are two kinds of dikes: some are the sand balls with a diameter of 5.35cm on a small dike (H=13.8cm), others on a large dike (H=50cm). Each ball is placed in such a position that the center of sphere is two times the diameter away from the front shoulder of dike, and the static water is tangent to the top of the ball.

Wave height and cycle tested is respectively recorded by capacitive sensor and light oscillograph. The general configuration of experimental setup is shown in Fig.3.

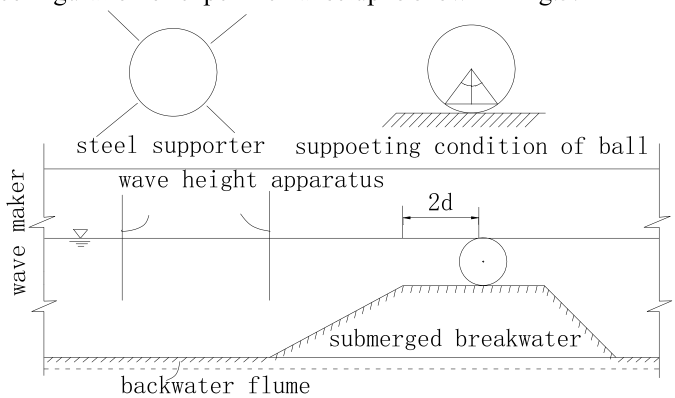

Fig.3 Experimental setup

IV. Results and Discussion

① Velocity

The velocity is not obtained from experimental investigations due to some restrictions of experimental conditions. The velocity around water points can be calculated by eq.(23). α is a coefficient over 1, and relevant to many factors, such as submerged breakwater height, water depth, the ratio of front slope and so on. For lack of data, $\alpha\approx1.5$ is obtained by eq.(22).

② Drag coefficient

Seeing Fig.4, when $10^3<\text{Re}<10^5, C_D=0.4\sim0.6$, the flow boundary is a well-separated laminar layer ; when $\text{Re}>10^5$, the boundary layer is of turbulent nature. However, C_D can't be completely read values from the wind tunnel test, and the effect of the bed should be also taken into account. According to the results (Josson) on f_w in rough turbulent flow region, when $\frac{a_b}{k_s}<1.57$ (large grain), $f_w=0.3$. In the experiment Re are all larger than 10^3, so let $C_D=0.3$.

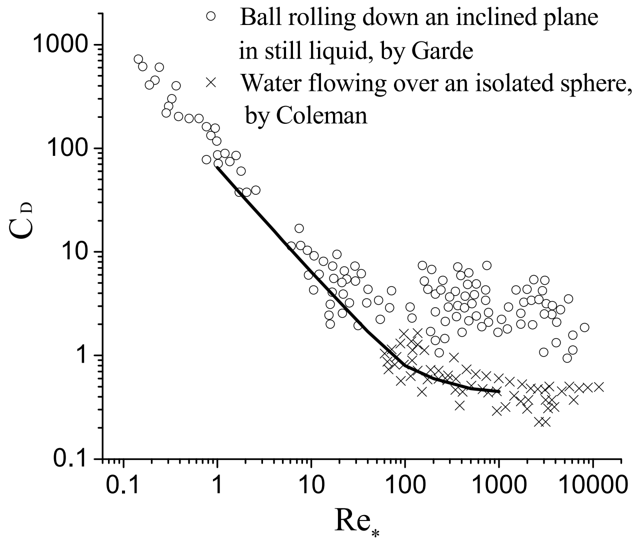

Fig.4 The variation of the drag coefficient

③ Extension of Shields Curves

In terms of experimental results, Shields curves have the characteristics as follows:

I. The Relation Between Θ and k_c

Experimental study on small diameter piles has found that when $k_c>20$, C_D and C_M are closed to a constant value. The relationship between Θ and k_c is shown in Fig.5 ($\text{Re}<10^5$).

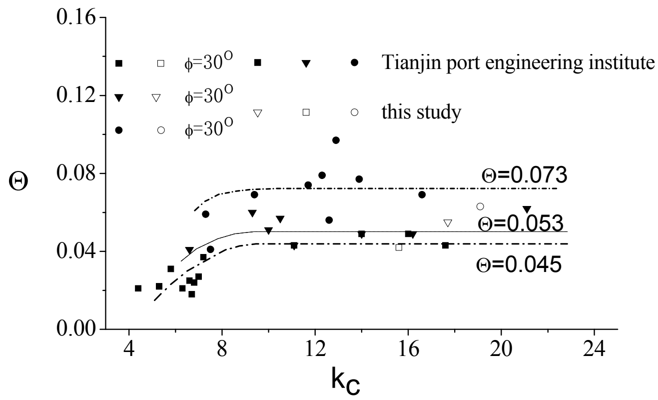

Fig.5 Relations between Θ and k_c

From Fig.5, the following are obtained:

$$\begin{cases}\text{when } k_c>10, & \Theta \text{ is constant}\\ \text{when } k_c<10, & \Theta \text{ decrease as } k_c\end{cases}$$

The relationship between Θ and k_c deserves further investigation.

II. The Relation Between Θ and ϕ

ϕ is the support angle. It represents the capability to keep itself stable. The larger the ϕ is, the more stable of the rubble is. Fig.5 shows that, when $k_c > 10$, for different ϕ, Θ are obviously different.

$$\begin{cases} when\ \phi = 30^0,\ \Theta = 0.045 \\ when\ \phi = 37^0,\ \Theta = 0.053 \\ when\ \phi = 45^0,\ \Theta = 0.073 \end{cases}$$

III.The Relation Between Θ and Re_*

Two conditions should be explained for the discussion on the relation of Θ and Re_*. First, it must satisfy the condition of $k_c > 10$, because Θ is independent with k_c at that moment; Then, it's necessary to discuss different ϕ respectively. The experimental curves (in Fig.6) satisfied the two conditions above

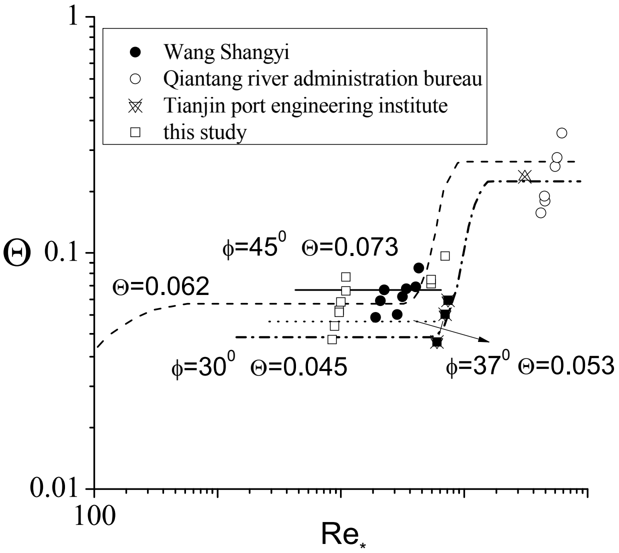

Fig.6 Verify the extension of Shields curve

When $\mathrm{Re}_* < 10^5$, Θ are closed to 0.062(different ϕ). In Fig.6, Θ are relatively small (except for $\phi = 45^0$). It is shown that fully-exposed droplets lose stability more easily than natural rock.

When $\mathrm{Re}_* > 10^5$, Θ increase evidently. It is consistent with the extension of Shields (Wang and Shen), which confirms that Shields parameter increases when Reynolds number is greater than 10^5 and Keulegan & Carpenter parameter is greater than 10. According to the data, the Re_* (when Θ is jumping) is larger than the estimates by Wang and Shen.

V. Conclusion

In current or wave, the rubble is acted by supporting action, effective specific weights, drag force, lift force and inertia force (without this term for steady flow). The Shields parameter, which is the ratio of shear stress to the multiplication of effective specific weights and equivalent diameter of the rubble in incipient motion, is a function of Reynolds number, Keulegan & Carpenter parameter and repose angle. It is also related to rubble shape and the degree of exposure.

When angle repose is certain, and Keulegan & Carpenter parameter is greater than 10(without this term for steady flow), the stability condition of rubble is only a function of Reynolds number (or grain Reynolds number). It is also confirmed that when grain Reynolds number is larger than 10^5, there will be a jump of Shields parameter.

NOTATION

The following symbols are used in this paper:

C_D, C_L, C_M = drag, uplift and inertial force coefficient, respectively;
d = equivalent diameter;
f_w = wave resistance coefficient;
F_D, F_L, F_M = drag force, lift force and inertial force, respectively;
g = Gravitational acceleration;
H = wave height;
h = water depth;
k = wave number;
k_s = roughness;
k_c = Keulegan & Carpenter number;
L = wave length;
Re, Re_* = Reynolds number, grain Reynolds number, respectively;
t = the time that the right of eq.(17) reach its minimum;
T = wave period;
u_d = the flow velocity that acts on the rock;
u_{dm} = the maximum horizontal water particles velocity;
z = vertical coordinates, with static water for zero;
α = velocity enhancement factor;
a_b = horizontal amplitude of bottom water particles;
σ = circular frequency;
ξ_1 = area coefficient;
ξ_2 = volume coefficient;
ε = exposure coefficient;
ξ = comprehensive shape coefficients;
λ = sand wave length;
ϕ = repose angle, support angle;
τ_{bm} = horizontal amplitude of bottom water particles;
Θ = Shields parameter.

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