

A New Combination Rule in Evidence Theory

Qianhui Dong, Qian Sun, Fei Yu

Abstract—Multi-target tracking system is used to distinguish the class from different targets and obtain the trajectory of all targets from multiple information systems. As a significant and novel information fusion method, evidence theory has been used in multi-target tracking system. The evidence combination rule which is the core of evidence theory is turning into a new research emphasis. Yager combination rule which defined in DST is an effective solution. PCR5 and PCR6 are proposed in DSMT which is an extension of DST. But because of the huge computation problem, the use of PCR5 and PCR6 in information fusion is restricted especially in real-time environment. In order to overcome the huge computation drawback of PCR5/6, a novel evidence combination rule, mixture combination rule (MCR) is proposed in this paper. MCR combines the advantages of PCR5/6 and YGR and switches its combination rule between PCR5/6 and YGR after making a judgment of decision stability. The simulation test on MCR is also done and the test results show that the computation load of MCR is largely reduced compared with PCR5/6.

Index Terms—multi-mobile robot system, observability analysis, graph, localization.

I. INTRODUCTION

The purpose of multi-target tracking (MTT) system is to classify the different targets into different classes and to obtain the trajectory of all targets. Because achievement of MTT is based on the information from different systems or sources, it belongs into the category of information fusion. Information fusion is defined as a process dealing with the association, correlation, and combination of data and information from single and multiple sources to achieve refined position and identity estimates, and complete and timely assessments of situations and threats, and their significance. The process is characterized by continuous refinements of its estimates and assessments, and the evaluation of the need for additional sources, or modification of the process itself, to achieve improved results [1], [2]. It has been an emerging technology which applied in robot navigation [3], [4], target identification [5], mobile robot vision [6], fault diagnosis [7] and other domains.

The main basic approach information fusion methods includes Bayesian probability theory, fuzzy logic and Dempster-Shafer (D-S) evidence theory [8], [9], [10]. For the Bayesian theory, it requires complete knowledge of combined conditional probabilities and specification of the priori

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knowledge of probability distribution. On the other hand, it cannot measure a body of evidence with an imprecision on probability measurement. The main advantage of the fuzzy fusion approach is that the evidence from multiple features can be combined using fuzzy logic operations, and uncertainty can be represented. The fuzzy set framework provides a lot of combination operators, which allows the user to adopt a fusion scheme and specify the data at hand. However, to our knowledge, the membership functions for the fuzzy set are not easy to obtain in real-world application systems.

For the evidence theory, there are three main advantages should be taken into account [11], [12]. First of all, since the D-S evidence theory supports the representation of both imprecision and uncertainty, it is considered to be a more flexible and general approach than the traditional probability theory. Secondly, evidence theory offers the possibility of coming up with the probabilities of a collection of hypotheses, whereas a classical probability theory only deals with one single hypothesis. Finally, the major strength of the evidence theory is its ability to deal with ignorance and missing information.

The combination rule is one of key technologies of evidence theory and the most common one is the Dempster's combination rule (DCR) [13], [14]. But since the counter-intuitive behavior of DCR [15], [16], many researchers are focus on it and proposed some effective combination methods to solve this problem. Partial conflict redistribution rule 5 (PCR5) and partial conflict redistribution rule 6 (PCR6) are two novel combination methods and result in good performance [17], [18]. But the large amount of computation needed in PCR5 and PCR6 [19] makes it difficult to implement in real time.

In order to solve this problem, a novel combination method which is called mixture combination rule (MCR) is proposed in this paper. MCR combines the advantages of DCR and PCR5/6 and has a better performance than DCR and lower computation load than PCR5/6 by shifting between DCR and PCR5/6 based on the condition of stable decision. MCR makes its judgement based on the basic probability assignment (BPA) and switches the combination rules. If the BPAs from each experts at the same time meet with the requirement of decision stability, the combination rule will be switched to DCR, otherwise the combination rule will be switched to PCR5/6.

This paper is organized as follows. In section 2, the background of evidence theory is briefly recalled. The evidence combination rules used in this paper are introduced in section 3. The decision stability of combination rules analyzed and a novel combination algorithm is proposed in section 4. Section 5 will make a conclusion and propose some challenging problems which need to be done in the future.

II. BACKGROUND ON EVIDENCE THEORY

A. Dempster-Shafer Theory

The Dempster-Shafer evidence theory was originally developed by Dempster, who concerned about the lower and upper probabilities, and later Shafer made his contribution by offering belief functions to model uncertain knowledge on the basis of mathematical foundations.

A key point of the D-S evidence theory is the basic probability assignment (BPA, or mass function) m which is defined on 2^Θ , the the power set of Θ , as

$$m : 2^\Theta \rightarrow [0, 1] \quad (1)$$

where m is defined for every element A of 2^Θ and $m(A)$ belongs to the $[0, 1]$ interval with the following property:

$$m(\phi) = 0, \quad \sum_{A \in 2^\Theta} m(A) = 1 \quad (2)$$

Θ is called a frame of discernment and is constituted with a set of mutually and exhaustive singleton hypotheses θ_i as

$$\Theta = \{\theta_1, \theta_2, \dots, \theta_N\} \quad (3)$$

B. Dezert-Smarandache Theory

In the context of the belief function theory, the DST is based on the use of functions defined on the power set 2^Θ (that is the set of all the disjunctions of the elements of Θ). Hence the experts can express their option not only on Θ but also on 2^Θ as in the probabilities theory. The extension of this power set into the hyper-power set D^Θ (that is the set of all the disjunction and conjunction of the elements of Θ) proposed by Dezert and Smarandache[17], gives more freedom to the expert. This extension of DST is call Dezert-Smarandache Theory (DSmT).

Hence BPA of DSmT is defined as

$$m : D^\Theta \rightarrow [0, 1] \quad (4)$$

where m is defined for every element A of D^Θ and $m(A)$ belongs to the $[0, 1]$ interval with the following property:

$$m(\phi) = 0, \quad \sum_{A \in D^\Theta} m(A) = 1 \quad (5)$$

III. EVIDENCE COMBINATION RULES

A. Dempster's combination rule

The Dempster's Combination Rule (DCR) is the most commonly used combination rule in D-S theory and is defined as

$$m_{1,2,\dots,s}^{DCR}(X) = \frac{1}{1-K} m_{1,2,\dots,s}(X) \quad (6)$$

where $K = \sum_{A_i \cap B_j = \emptyset} m_1(A_i)m_2(B_j) \neq 1$ is referred to as the conflict mass.

Suppose there are 4 different classes of targets and FoD is expressed as $\Theta_{\mathbb{B}} = \{B_1, B_2, B_3, B_4\}$. The basic probability assignments (BPA) reported by two different sensor systems are shown as Table I.

According to the Dempster's combination rule, the conflict mass can be calculated as $K = 0.47$, the combination results can be figured out as $m_{\Theta_{\mathbb{B}}}(B_1) = 0.453$; $m_{\Theta_{\mathbb{B}}}(B_2) = 0.132$; $m_{\Theta_{\mathbb{B}}}(B_3) = 0.057$; $m_{\Theta_{\mathbb{B}}}(B_4) = 0.283$; $m_{\Theta_{\mathbb{B}}}(\Theta) = 0.057$.

TABLE I
BASIC BELIEF ASSIGNMENTS

	B_1	B_2	B_3	B_4	Θ
Expert 1	0.3	0.1	0.15	0.3	0.15
Expert 2	0.4	0.2	0.0	0.2	0.2

TABLE II
BASIC BELIEF ASSIGNMENTS

	\emptyset	A	B	C	Θ
Expert 1	0.00	0.96	0.04	0.00	0.00
Expert 2	0.00	0.00	0.02	0.98	0.00

According to the Dempster's combining rule, the identification result supports that the target is B_1 . It seems that the decision result makes sense.

Another example shows as Table II and according to the Dempster's combination rule, the $K = 0.9992$, the combination results can be figured out as $m_{\Theta}(\emptyset) = 0.0$, $m_{\Theta}(A) = 0.0$, $m_{\Theta}(B) = 1.0$, $m_{\Theta}(C) = 0.0$, $m_{\Theta}(\Theta) = 0.0$. According to the Dempster's combining rule, the two system offer little belief to C . But the combination and identification result supports that the target is C . It does not make sense at all. So it is called counter-intuitive problem for this situation which is caused by the conflict evidence. To solve the conflict problem, several alternatives to the combination process have been proposed [20] which including a simple but effective combination rule, Yager combination rule (YGR). So some typical methods will be introduced in the following subsections.

B. Yager combination rule

Yager' idea is to assign the conflict mass in the null set to the base set Θ . That is, the non-null $m(\emptyset)$ is distributed among all the elements of the FoD rather than just the elements which happen to be intersections of the combining masses. Thus, the fused BPA is generated by the Yager's Rule (YGR) as:

$$m_{1,2,\dots,s}^{YGR}(X) = \begin{cases} m_{1,2,\dots,s}(X), & \text{for } X \neq \emptyset; \\ m_{1,2,\dots,s}(X) + K, & \text{for } X = \Theta \end{cases} \quad (7)$$

C. Partial conflict redistribution rule 5

PCR5 is a combination rule which defined in DSmT and the general formula when $s \geq 2$ sources is given as

$$m_{1,2,\dots,s}^{PCR5}(X) = m_{1,2,\dots,s}(X) + \frac{\sum_{\substack{2 \leq t \leq s \\ 1 \leq r_1, \dots, r_t \leq s \\ 1 \leq r_1 \leq r_2 \leq \dots \leq r_t = s}} \sum_{\substack{X_{j_2}, \dots, X_{j_t} \in G^\Theta \setminus \{X\} \\ \{j_2, \dots, j_t\} \in \mathcal{P}^{t-1}(\{1, \dots, n\}) \\ X \cap X_{j_2} \cap \dots \cap X_{j_s} = \emptyset \\ \{i_1, \dots, i_s\} \in \mathcal{P}^s(\{1, \dots, s\})}} (\prod_{k_1=1}^{r_1} m_{i_{k_1}}(X))^2 \prod_{l=2}^t (\prod_{k_l=r_{l-1}+1}^{r_l} m_{i_{k_l}}(X_{j_l}))}{(\prod_{k_1=1}^{r_1} m_{i_{k_1}}(X)) + \prod_{l=2}^t (\prod_{k_l=r_{l-1}+1}^{r_l} m_{i_{k_l}}(X_{j_l}))} \quad (8)$$

where i, j, k, r, s and t in (8) are integers. $m_{1,2,\dots,s}(X)$ corresponds to the conjunctive consensus on X between s sources and where all denominators are different from zero. If a denominator is zero, that fraction is discarded; $\mathcal{P}^k(\{1, 2, \dots, n\})$ is the set of all subsets of k elements from $\{1, 2, \dots, n\}$ (permutations of n elements taken by k), the order of elements does not count.

D. Partial conflict redistribution rule 6

The general formula of PCR6 proposed by Martin and Osswald is given as

$$m_{1,2,\dots,s}^{PCR6}(X) = m_{1,2,\dots,s}(X) + \sum_{i=1}^s m_i(X)^2 \sum_{\substack{\cap_{k=1}^{s-1} Y_{\sigma_i(k)} \cap X \equiv \emptyset \\ (Y_{\sigma_i(1)}, \dots, Y_{\sigma_i(s-1)}) \in (G^\Theta)^{s-1}}} \frac{\prod_{j=1}^{s-1} m_{\sigma_i(j)}(Y_{\sigma_i(j)})}{m_i(X) + \sum_{j=1}^{s-1} m_{\sigma_i(j)}(Y_{\sigma_i(j)})} \quad (9)$$

where σ_i counts from 1 to s avoiding i :

$$\begin{cases} \sigma_i(j) = j & \text{if } j < i \\ \sigma_i(j) = j + 1 & \text{if } j \geq i \end{cases} \quad (10)$$

After simplified previous two fusion rules when ($s = 2$), one finds that the simplified PCR5 and PCR6 are coincided as $m_{PCR5/6}(\cdot) = m_{1,2}^{PCR6}(\cdot) = m_{1,2}^{PCR5}(\cdot)$. in this case, the combination result is expressed as

$$m_{PCR5/6}(X) = \sum_{\substack{X_1, X_2 \in G^\Theta \\ X_1 \cap X_2 = \emptyset}} m_1(X_1)m_2(X_2) + \sum_{\substack{Y \in G^\Theta \setminus \{X\} \\ X \cap Y = \emptyset}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)} \right] \quad (11)$$

IV. A NOVEL COMBINATION METHOD: MIXTURE COMBINATION METHOD

A. The condition of stable decision between different combination rules

As we all know, the decision of evidence theory is based on the basic belief assignments obtained by combination rules, so the decisions vary with different combination rules. If the decision result do not change with combination rules, it is called stable decision. Conversely, the decision is unstable. As the stable decision is essential criteria of combination rules switch, this section will focus on the decision stability analysis of different combination rules.

The decision stability can be defined as followed:

If and only if

$$m_{\otimes}(B_i) > m_{\oplus}(B_j), \quad \forall B_j \in \Theta_B \text{ and } i \neq j$$

and

$$m_{\otimes}(B_i) > m_{\oplus}(B_j), \quad \forall B_j \in \Theta_B \text{ and } i \neq j$$

are true simultaneously, it is called stable decision, where \otimes and \oplus denotes different combination rules, respectively.

If the system is "two classes" situation, the condition of stable decision the between different combination rules can be simplified as

$$[m_{\otimes}(A) - m_{\otimes}(B)][m_{\oplus}(A) - m_{\oplus}(B)] > 0 \quad (12)$$

For a defined system, the number of elements is known and the condition of stable decision is also determined uniquely. Let's take a "two classes and two expert" as an example and show the determination method of stable decision.

The basic probability assignments are shown as Table III.

TABLE III
BASIC BELIEF ASSIGNMENTS

	\emptyset	A	B	Θ
Expert report 1	0	a_1	b_1	$1 - a_1 - b_1$
Expert report 2	0	a_2	b_2	$1 - a_2 - b_2$

The combination results of Yager's combination rule are

$$\begin{cases} m_{YGR}(A) = a_1 + a_2 - a_1 a_2 - K \\ m_{YGR}(B) = b_1 + b_2 - b_1 b_2 - K \\ m_{YGR}(\Theta) = (1 - a_1 - b_1)(1 - a_2 - b_2) + K \end{cases} \quad (13)$$

where $K = a_1 b_2 + a_2 b_1$. Eq.(13) can be transferred as

$$m_{YGR}(A) - m_{YGR}(B) = (a_1 + a_2 - a_1 a_2) - (b_1 + b_2 - b_1 b_2) \quad (14)$$

The combination results of PCR5/6 are

$$\begin{cases} m_{PCR5/6}(A) = a_1 + a_2 - a_1 a_2 + \frac{a_1^2 b_2}{a_1 + b_2} + \frac{a_2^2 b_1}{a_2 + b_1} - K \\ m_{PCR5/6}(B) = b_1 + b_2 - b_1 b_2 + \frac{b_1^2 a_2}{b_1 + a_2} + \frac{b_2^2 a_1}{b_2 + a_1} - K \\ m_{PCR5/6}(\Theta) = (1 - a_1 - b_1)(1 - a_2 - b_2) + K \end{cases} \quad (15)$$

And the simplified functions of Eq.(15) are expressed as

$$[m_{PCR5/6}(A) + K](a_1 + b_2)(a_2 + b_1) = (a_1 + b_2)(a_2 + b_1)(a_1 + a_2 - a_1 a_2) + a_1^2 b_2(a_2 + b_1) + a_2^2 b_1(a_1 + b_2) \quad (16)$$

$$[m_{PCR5/6}(B) + K](a_1 + b_2)(a_2 + b_1) = (a_1 + b_2)(a_2 + b_1)(b_1 + b_2 - b_1 b_2) + b_1^2 a_2(a_1 + b_2) + b_2^2 a_1(a_2 + b_1) \quad (17)$$

And Eq.(16) and Eq.(17) can be rewritten as

$$[m_{PCR5/6}(A) - m_{PCR5/6}(B)](a_1 + b_2)(a_2 + b_1) = (a_1 + b_2)(a_2 + b_1)[(1 - b_1)(1 - b_2) - (1 - a_1)(1 - a_2)] + a_1 b_2(a_2 + b_1)(a_1 - b_2) + a_2 b_1(a_1 + b_2)(a_2 - b_1) \quad (18)$$

The condition of stable decision between YGR and PCR5/6 can be expressed as

$$[m_{YGR}(A) - m_{YGR}(B)][m_{PCR5/6}(A) - m_{PCR5/6}(B)] > 0 \quad (19)$$

In order to determine the condition of stable decision, random tests are designed. a_1, b_1, a_2 and b_2 are designed as random numbers which ranged between 0 and 1, respectively and the test is repeated 500,000 times. The test results contain 4 unrelated variables so the results are projected to $a_1 - b_2, a_2 - b_1, a_1 - a_2, b_1 - b_2$ planes, respectively. The unstable points projected to each planes are shown as Fig. 1 to Fig. 4.

The dot-forming region denotes unstable decision area. It can be seen from Fig. 5 to Fig. 8 that the boundary of the area can be regard as an exponential curve approximately. MATLAB curve fitting tool is used to obtain the curve which can enclose the whole unstable region. The curve function expressions are expressed as

$$a_2 = 0.4e^{-57.3a_1} \quad (20)$$

$$b_2 = e^{-10.2b_1} \quad (21)$$

$$b_1 = 0.91e^{-29.3a_2} \quad (22)$$

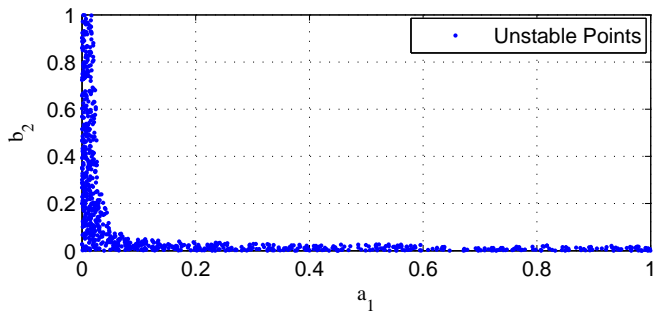


Fig. 1. Unstable points projected to $a_1 - b_2$ planes

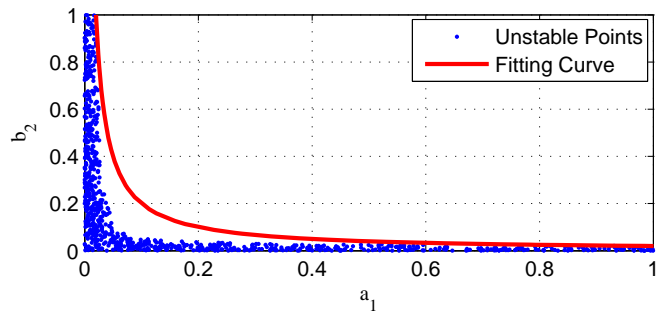


Fig. 5. Unstable region projected to $a_1 - b_2$ planes

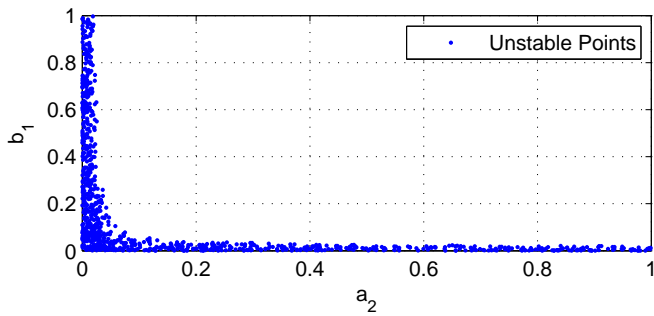


Fig. 2. Unstable points projected to $a_2 - b_1$ planes

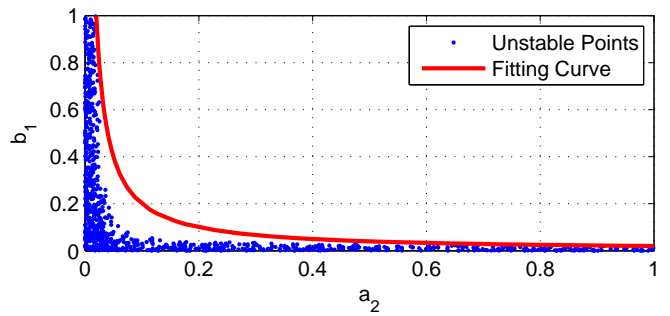


Fig. 6. Unstable region projected to $a_2 - b_1$ planes

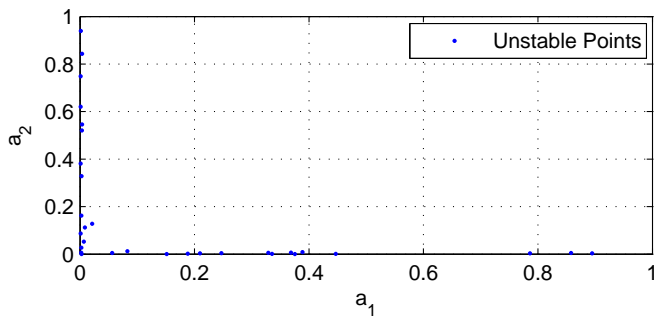


Fig. 3. Unstable points projected to $a_1 - a_2$ planes

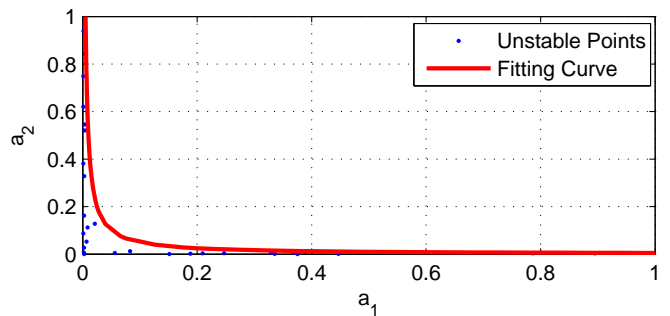


Fig. 7. Unstable region projected to $a_1 - a_2$ planes

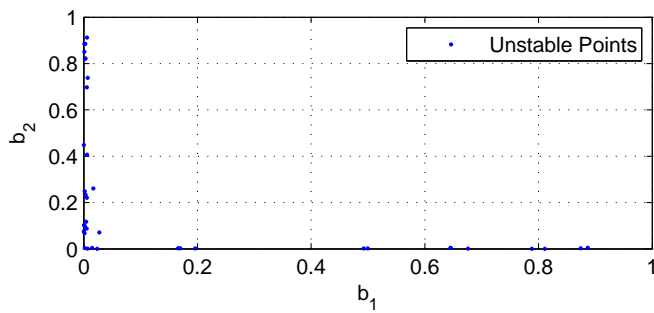


Fig. 4. Unstable points projected to $b_1 - b_2$ planes

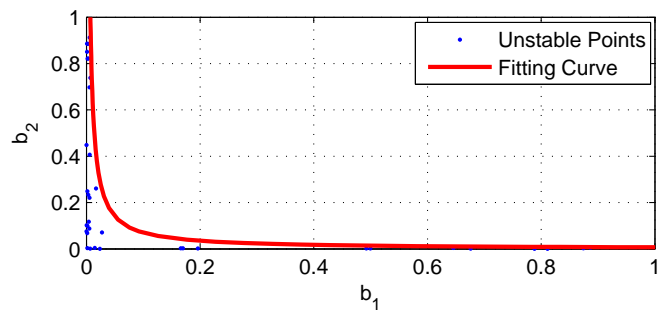


Fig. 8. Unstable region projected to $b_1 - b_2$ planes

TABLE IV
ALGORITHM OF MCR

Algorithm of MCR	
Inputs:	$m_1(A), m_1(B), m_2(A), m_2(B)$
	for $l = 1 : L$
	if (Decision is stable)
	$m_{MCR}(A) \leftarrow m_{YGR}(A);$
	$m_{MCR}(B) \leftarrow m_{YGR}(B);$
	else
	$m_{MCR}(A) \leftarrow m_{PCR5/6}(A);$
	$m_{MCR}(B) \leftarrow m_{PCR5/6}(B);$
	end
	end
Return	$m_{MCR}(A);$
Return	$m_{MCR}(B);$

$$b_2 = 0.89e^{-30.3a_1} \quad (23)$$

At last the stable condition between YGR and PCR5/6 in “two class, two experts” case is obtained and expressed as

$$\begin{cases} a_2 \geq 0.4e^{-57.3a_1} \\ b_2 \geq e^{-10.2b_1} \\ b_1 \geq 0.91e^{-29.3a_2} \\ b_2 \geq 0.89e^{-30.3a_1} \\ 0 \leq a_1, a_2, b_1, b_2 \leq 1 \end{cases} \quad (24)$$

For “arbitrary classes and arbitrary experts” case, the determination method of stable decision condition is the same as “two class, two experts” case. As a result, it is not needed to repeat here.

B. Mixture combination rule

As we all know that PCR5/6 suffers from the large computation load and it is difficult to be used in real multi-target tracking system especially for large amount and fast-moving targets. In order to overcome the drawback of PCR5/6, a mixture combination rule (MCR) is proposed in this paper. The kernel of MCR can be described as that the combination method switches between YGR and PCR5/6. As a result, MCR combines the advantages of YGR and PCR5/6, but the computation load is not enhanced. It makes its judgement on the decision stability. If the decision satisfies the condition of stability, the combination rule will be switched to YGR. Otherwise the combination rule will be PCR5/6. The algorithm of MCR is described as Table IV.

C. Case study

A “two experts and two class” example is used to illustrate the MCR algorithm. The basic probability assignments are shown as Table III. There is no need to calculate the results of both combination methods in MCR, it only need to make a judgement of the BPA. If the BPA a_1, b_1, a_2 and b_2 satisfy Eq.(24), the evidence combination rule will be YGR, otherwise the evidence combination rule will be switched to PCR5/6.

In order to verify the effectiveness which reducing the computation load of MCR, the Monte Carlo method is used. We sample 100,000 random numbers which are between 0 and 1 as the values of PBA. These values are combined with YGR, PCR5/6 and MCR, respectively. These tests repeat 20 times and the results are shown in Table V. The average run time of PCR5/6, YGR and MCR are 0.2667s, 0.1720s and

TABLE V
RUNNING TIME COMPARISON OF THESE THREE ALGORITHMS

Test	Run Time			Test	Run Time		
	PCR5/6	YGR	MCR		PCR5/6	YGR	MCR
1	0.0233	0.0173	0.0170	11	0.2798	0.1818	0.2065
2	0.0479	0.0352	0.0378	12	0.3060	0.1974	0.2243
3	0.0734	0.0530	0.0580	13	0.3325	0.2126	0.2421
4	0.0978	0.0714	0.0768	14	0.3573	0.2272	0.2605
5	0.1218	0.0872	0.0947	15	0.3837	0.2420	0.2797
6	0.1483	0.1022	0.1126	16	0.4090	0.2579	0.2978
7	0.1739	0.1176	0.1314	17	0.4341	0.2746	0.3161
8	0.1981	0.1336	0.1511	18	0.4596	0.2892	0.3355
9	0.2290	0.1493	0.1706	19	0.4881	0.3046	0.3526
10	0.2554	0.1664	0.1877	20	0.5139	0.3199	0.3714

0.1962s, respectively. The performance of MCR is same as PCR5/6 because of the design idea of MCR. Compared with PCR5/6, the running time of MCR is reduced by 26%.

V. CONCLUSIONS

The evidence theory and evidence combination rules are introduced in this paper. After the comparison of YGR and PCR5/6, it is known that PCR5/6 has a better performance in combination than YGR, but the computation load of PCR5/6 is larger than YGR. The stability of decision between YGR and PCR5/6 is also studied in this paper. At last, a novel combination rule, mixture combination rule (MCR) which combines the virtues of YGR and PCR5/6 is proposed in this paper. The kernel of MCR can be described as that the combination method switches between YGR and PCR5/6. The Monte Carlo analysis result show that the computation load of MCR is largely reduced than PCR5/6.

REFERENCES

- [1] A. White, “Data fusion lexicon. joint directors of laboratories, technical panel for C³,” *Naval Ocean Systems Center, San Diego, Tech. Rep*, 1987.
- [2] A. A. Fathima, S. Vasuhi, N. N. Babu, V. Vaidehi, and T. M. Treesa, “Fusion framework for multimodal biometric person authentication system,” *IAENG International Journal of Computer Science*, vol. 41, no. 1, pp. 18–31, 2014.
- [3] R. R. Murphy, “Dempster-shafer theory for sensor fusion in autonomous mobile robots,” *Robotics and Automation, IEEE Transactions on*, vol. 14, no. 2, pp. 197–206, 1998.
- [4] Z. Qi and Q. Wang, “Error analysis and the development of an error mitigation approach for use in the rotation fiber optic gyro inertial navigation system,” *Engineering Letters*, vol. 21, no. 4, pp. 203–211, 2013.
- [5] B. Ristic and P. Smets, “Target identification using belief functions and implication rules,” *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 41, no. 3, pp. 1097–1103, 2005.
- [6] D. Riahi and G.-A. Bilodeau, “Multiple feature fusion in the dempster-shafer framework for multi-object tracking,” in *Computer and Robot Vision (CRV), 2014 Canadian Conference on*, May 2014, pp. 313–320.
- [7] O. Basir and X. Yuan, “Engine fault diagnosis based on multi-sensor information fusion using dempster-shafer evidence theory,” *Information Fusion*, vol. 8, no. 4, pp. 379 – 386, 2007.
- [8] G. Shafer, *A Mathematical Theory of Evidence*. Princeton university press Princeton, 1976, vol. 1.
- [9] J. R. Boston, “A signal detection system based on dempster-shafer theory and comparison to fuzzy detection,” *Systems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on*, vol. 30, no. 1, pp. 45–51, 2000.
- [10] S. Le Hegarat-Masclé, I. Bloch, and D. Vidal-madjar, “Application of dempster-shafer evidence theory to unsupervised classification in multisource remote sensing,” *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 35, no. 4, pp. 1018–1031, 1997.
- [11] P. Soundappan, E. Nikolaidis, R. T. Haftka, R. Grandhi, and R. Canfield, “Comparison of evidence theory and bayesian theory for uncertainty modeling,” *Reliability engineering & System safety*, vol. 85, no. 1, pp. 295–311, 2004.

- [12] T. L. Wickramaratne, "An analytical framework for soft and hard data fusion: A dempster-shafer belief theoretic approach," DTIC Document, Tech. Rep., 2012.
- [13] F. Sebbak, F. Benhammadi, A. Chibani, Y. Amirat, and A. Mokhtari, "New evidence combination rules for activity recognition in smart home," in *Information Fusion (FUSION), 2013 16th International Conference on*, July 2013, pp. 46–52.
- [14] D. Han, J. Dezert, C. Han, and Y. Yang, "New dissimilarity measures in evidence theory," in *Information Fusion (FUSION), 2011 Proceedings of the 14th International Conference on*, July 2011, pp. 1–7.
- [15] L. A. Zadeh, "A simple view of the dempster-shafer theory of evidence and its implication for the rule of combination," *AI magazine*, vol. 7, no. 2, pp. 85–90, 1986.
- [16] A. Burkov, S. Paquet, G. Michaud, and P. Valin, "An empirical study of uncertainty measures in the fuzzy evidence theory," in *Information Fusion (FUSION), 2011 Proceedings of the 14th International Conference on*, July 2011, pp. 1–8.
- [17] F. Smarandache and J. Dezert, *Advances and Applications of DS_mT for Information Fusion (Collected works), second volume: Collected Works*. Infinite Study, 2006, vol. 2.
- [18] —, "On the consistency of pcr6 with the averaging rule and its application to probability estimation," in *Information Fusion (FUSION), 2013 16th International Conference on*, July 2013, pp. 1119–1126.
- [19] R. Haenni and N. Lehmann, "Resource bounded and anytime approximation of belief function computations," *International Journal of Approximate Reasoning*, vol. 31, no. 1 - 2, pp. 103 – 154, 2002.
- [20] I. Bloch, "Some aspects of dempster-shafer evidence theory for classification of multi-modality medical images taking partial volume effect into account," *Pattern Recognition Letters*, vol. 17, no. 8, pp. 905–919, 1996.