The Analysis on Transmission Characteristics of the 25Hz Phase-sensitive Track Circuit in the Regulated State

Feng Chang, Ruicheng Zhao, Wenzhen Kuang

***Abstract*—A 25Hz phase-sensitive track circuit is used in electrified railway stations allowing determination of whether a train occupies a track section and avoiding the interference of traction current to signal circuit. The transmission characteristics of a 25Hz phase-sensitive track circuit in the regulated state were studied, allowing the development of a model of choke transformer and track circuit. Considering the impedance bond, a four-terminal network model that adjusts the state of track circuit was built, and a distributed-parameter method was applied to solve the four-terminal network coefficient of the track circuit in the regulated state. The sinusoidal steady-state responses can be obtained directly from the complex frequency-domain solution, and the DC steady-state response can be calculated using the final value theorem of Laplace transform theory. This steady-state analysis of track circuit transmission lines allows the characteristic analysis of track circuit during steady state operation. This work also can serve as guidance to help ensure signal transmission without distortion. Finally, the transmission characteristics of the track circuit were simulated by Matlab. The simulation results show that the transmission characteristics of the track circuit in regulated state is closely related to the length of the track circuit and ballast resistance.**

***Index Terms*—phase-sensitive track circuit, transmission characteristic, distributed-parameter, regulated state**

I. INTRODUCTION

Two-element and two-position relay with frequency and phase selectivity as a receiver was used in a 25Hz phase-sensitive track circuit [1]. To allow it to work reliably, the voltage and current of the track circuit must be transmitted in compliance with the requirements of frequency, phase, and other factors. Therefore, it is of major significance to study the transmission characteristics of the track circuit. Because a rail line is a circuit with uniform distribution parameters, the distributed parameter method can be used to establish a model of rail transmission line and calculate the coefficient of the four terminal network [2-3], to easily evaluate the impact of a parameter on the voltage and current of the track circuit. Given this information, it is straightforward to propose improvement programs and ensure that the track circuit is safe, reliable, and efficient.

The numerical method includes use of the time and frequency domain method to solve two major categories of rail transport line. The transient analysis of the transmission line equation of the track circuit is based on the time domain method [4-6]. In the time domain, the partial differential equations group is used based on the theory of partial differential equation numerical solution to establish differential formation of track circuits. According to the boundary conditions, the time domain voltage response of the adjustment state can be determined. This method allows the solving of the complex equations, but also allows consideration of the convergence and stability. In order to obtain the general solution of the transmission line in frequency domain, the Laplace transform is applied to the transmission line equation of the track circuit. In the case of shorting of the track circuit, the time domain solution can be obtained by the Laplace inverse transform. The steady state response of the track transmission line is not considered, only the transient response of the track is analyzed.

The stability of the signal transmission of the track circuit directly affects the safety of the train operation. To ensure the quality of signal transmission, the variation law of the voltage and current on the track transmission line was analyzed using the theory of distributed parameter circuit. When there is a track circuit failure, the precise location of the fault point is particularly important because the track circuit transmission line is very long. This is illustrated by the theory of distributed parameters [7-8]. The steady state analysis of the track circuit transmission line allows the analysis of the characteristics of the stable operation of the track transmission line. In particular, it provides the ability to ensure that the signal is not distorted. However, the solution obtained using the time domain method has many uncertain solutions and is unable to determine the only uncertainty under steady state conditions. Solving the steady state parameters directly in the frequency domain must be done by Laplace transform. The solving process is simple, and there is either convergence or no convergence, and it is either stable or not stable, so it may not be necessary to perform the complex and complicated process of Laplace inverse transform. This can be applied directly to a

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computer simulation program, allowing improved accuracy.

II. Solution of Four-terminal Network Coefficients of Track Circuit

A. Distributed Parameter Model of Track Circuit [9-10]

A track circuit is a circuit with uniform distribution parameters, and is characterized by two-way asymmetric leakage current. One way current from the rail can leak into the earth directly and the current in the other direction passes through ballast and sleepers and there can be leakage from a single rail to the other.

We can regard the two rails and the earth as a circuit composed of three wires by using the distributed parameter method for the track circuit modeling. The earth can be considered as the conductor of the infinite area. These three wires are connected by a uniform distribution of leakage of g_1, g_2 and g_{12}. The equivalent circuit of a short circuit dx of the track circuit, as shown in Figure 1.

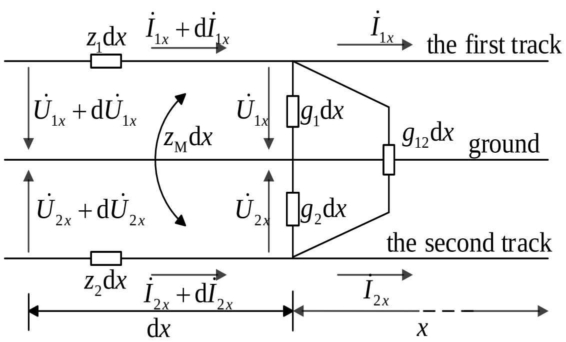

Fig. 1. Equivalent Circuit of Short dx for Track Line

In Figure 1, z_1, z_2 for the single rail impedance ratio (Ω), g_1 and g_2 are the specific conductance of the rail to the ground (S). g_{12} is the conductance of the ballast surface and sleepers (S). z_{M} is the mutual impedance between two rails (Ω). x is the distance between the connecting load and the rail line terminal (m). $\dot{I}_{1x}$ and $\dot{I}_{2x}$ respectively represent the current in the two rails, which is in the positive direction by the supply terminal to load (A). $\dot{U}_{1x}$ and $\dot{U}_{2x}$ respectively represent the voltage of the two rails to the ground, in the positive direction from the rail to the ground (V).

According to Kirchhoff's law, a short dx of the rail line can be written as the following equations:

$$\begin{cases} \dot{U}_{1x} + \mathrm{d}\dot{U}_{1x} - \dot{U}_{1x} = z_1 \mathrm{d}x \dot{I}_{1x} + z_{\mathrm{M}} \mathrm{d}x \dot{I}_{2x} \\ \dot{U}_{2x} + \mathrm{d}\dot{U}_{2x} - \dot{U}_{2x} = z_2 \mathrm{d}x \dot{I}_{2x} + z_{\mathrm{M}} \mathrm{d}x \dot{I}_{1x} \\ \dot{I}_{1x} + \mathrm{d}\dot{I}_{1x} - \dot{I}_{1x} = g_1 \mathrm{d}x \dot{U}_{1x} + g_{12} \mathrm{d}x \dot{U}_{1x} - g_{12} \mathrm{d}x \dot{U}_{2x} \\ \dot{I}_{2x} + \mathrm{d}\dot{I}_{2x} - \dot{I}_{2x} = g_2 \mathrm{d}x \dot{U}_{2x} + g_{12} \mathrm{d}x \dot{U}_{2x} - g_{12} \mathrm{d}x \dot{U}_{1x} \end{cases} \tag{1}$$

The formula (1) is obtained as follows:

$$\dot{U}_{1x} = A_1 \cosh \gamma_1 x + A_2 \sinh \gamma_1 x + A_3 \cosh \gamma_2 x + A_4 \sinh \gamma_2 x \tag{2}$$

$$\dot{U}_{2x} = M\left(A_1 \cosh \gamma_1 x + A_2 \sinh \gamma_1 x\right) + N\left(A_3 \cosh \gamma_2 x + A_4 \sinh \gamma_2 x\right) \tag{3}$$

$$\dot{I}_{1x} = y_{11}\left(A_1 \sinh \gamma_1 x + A_2 \cosh \gamma_1 x\right) + y_{12}\left(A_3 \sinh \gamma_2 x + A_4 \cosh \gamma_2 x\right) \tag{4}$$

$$\dot{I}_{2x} = y_{21}\left(A_1 \sinh \gamma_1 x + A_2 \cosh \gamma_1 x\right) + y_{22}\left(A_3 \sinh \gamma_2 x + A_4 \cosh \gamma_2 x\right) \tag{5}$$

In the formula (2) to (5):

$$\gamma_1 = \sqrt{\frac{1}{2} a_1 - \sqrt{\frac{1}{4} a_1^2 - a_2}}, \ \gamma_2 = \sqrt{\frac{1}{2} a_1 + \sqrt{\frac{1}{4} a_1^2 - a_2}},$$

A_1, A_2, A_3, A_4 is a constant of integration;

$$M = \frac{\gamma_1^2 - z_1\left(g_1 + g_{12}\right) + z_{\mathrm{M}} g_{12}}{z_{\mathrm{M}}\left(g_2 + g_{12}\right) - z_1 g_{12}},$$

$$N = \frac{\gamma_2^2 - z_1\left(g_1 + g_{12}\right) + z_{\mathrm{M}} g_{12}}{z_{\mathrm{M}}\left(g_2 + g_{12}\right) - z_1 g_{12}},$$

$$a_1 = g_{12}\left(z_1 + z_2 - 2z_{\mathrm{M}}\right) + g_1 z_1 + g_2 z_2$$

$$a_2 = \left(z_1 z_2 - z_{\mathrm{M}}^2\right)\left(g_1 g_2 + g_1 g_{12} + g_2 g_{12}\right),$$

$$y_{11} = \gamma_1 \frac{z_2 - \mathrm{M} z_{\mathrm{M}}}{z_1 z_2 - z_{\mathrm{M}}^2}, \ y_{12} = \gamma_2 \frac{z_2 - N z_{\mathrm{M}}}{z_{12} - z_{\mathrm{M}}^2},$$

$$y_{21} = \gamma_1 \frac{M z_1 - z_{\mathrm{M}}}{z_1 z_2 - z_{\mathrm{M}}^2}, \ y_{21} = \gamma_2 \frac{N z_1 - z_{\mathrm{M}}}{z_1 z_2 - z_{\mathrm{M}}^2}.$$

B. Solving the State Adjustment Coefficients of Four-terminal Network

The equivalent circuit of the four-terminal network of the track circuit is shown in Figure 2. Considering the influence of the adjacent track section in the beginning and end of the rail line, the Z_{HBX} and Z_{KBX} are separately connected with the neutral point of the ideal self-coupling transformer. Z_{HBX} and Z_{KBX} represent the input impedance of an adjacent track section between the transformer and choke point of the earth. Therefore, the equivalent input impedance Z_{BXH} and Z_{HBX} of the elements and the ideal autotransformer are connected to the beginning of the equivalent circuit. The equivalent output impedance of the Z_{BXK} and Z_{KBX} of the various components that are connected with the ideal self-coupling transformer are also present in the terminal.

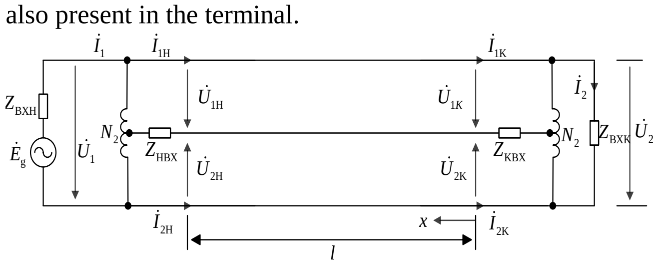

Fig. 2. Equivalent Circuit of the Four-terminal Network of the Track Circuit

Figure 3 and 4 show the equivalent circuit of the track circuit power supply and the receiving end, respectively.

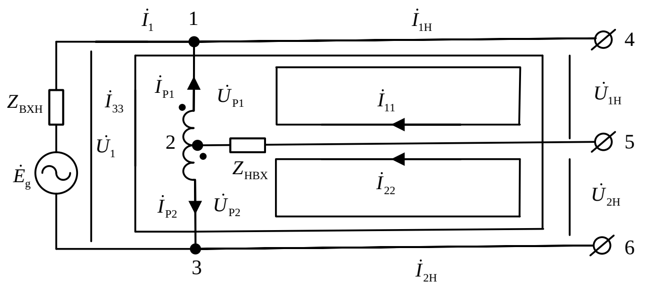

Fig. 3 Equivalent Circuit of the Power Supply Terminal of Track Circuit

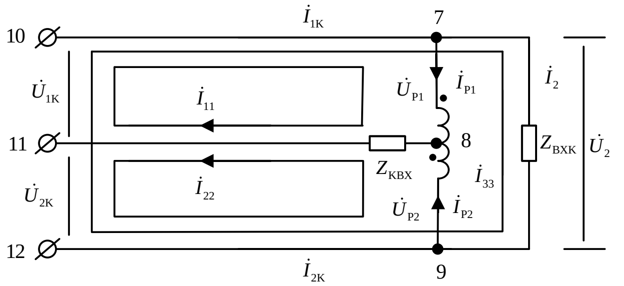

Fig. 4 Equivalent Circuit of the Receiving Terminal of Track Circuit

The loop current method allows the following:

$$\dot{I}_{11}=\dot{I}_{P1},\dot{I}_{22}=\dot{I}_{P2},\dot{I}_2=\dot{I}_{33},\dot{I}_{KBX}=\dot{I}_{11}+\dot{I}_{22},$$
$$\dot{I}_{1K}=\dot{I}_{11}+\dot{I}_{33},\dot{I}_{2K}=\dot{I}_{22}-\dot{I}_{33}$$

It follows that:

$$\dot{I}_{1K}+\dot{I}_{2K}=\dot{I}_{KBX} \tag{6}$$

In Figure 3, the node 7 has:

$$\frac{\dot{I}_{1K}+\dot{I}_{2K}}{2}+\dot{I}_2=\dot{I}_{1K} \tag{7}$$

The voltage and current of the coil of the self-coupling transformer are presented by $\dot{U}_{P1}$, $\dot{I}_{P1}$ and $\dot{U}_{P2}$, $\dot{I}_{P2}$ respectively:

$$\dot{U}_{P1}=\dot{I}_{P1}Z_{P1}-\dot{I}_{P2}Z_{P12} \tag{8}$$

$$\dot{U}_{P2}=-\dot{I}_{P2}Z_{P2}+\dot{I}_{P1}Z_{P12} \tag{9}$$

In formula (8) and (9), the impedances of Z_{P1} and Z_{P2} are coupled with the upper and lower half coil of the self-coupling transformer, respectively. Z_{P12} is the mutual inductance between the upper and lower windings of the self-coupling transformer. If the entire impedance of the coil represented by Z_P, there is:

$$Z_{P1}=Z_{P2}=\frac{Z_P}{4}、\ Z_{P12}=\frac{Z_P}{4} \tag{10}$$

The formula (10) can be substituted into the formula (8) and (9), to obtain the following:

$$\dot{U}_{P1}=\dot{I}_{P1}\frac{Z_P}{4}-\dot{I}_{P2}\frac{Z_P}{4}=\frac{\dot{U}_P}{2} \tag{11}$$

$$\dot{U}_{P2}=-\dot{I}_{P2}\frac{Z_P}{4}+\dot{I}_{P1}\frac{Z_P}{4}=\frac{\dot{U}_P}{2} \tag{12}$$

In Figure 3, we can write the following equations for the loop 1-4-5-2 and 2-5-6-3:

$$\begin{cases}\dfrac{\dot{U}_P}{2}=Z_{HBX}\left(\dot{I}_{1H}+\dot{I}_{2H}\right)+\dot{U}_{1H}\\ \dfrac{\dot{U}_P}{2}=-Z_{HBX}\left(\dot{I}_{1H}+\dot{I}_{2H}\right)-\dot{U}_{2H}\end{cases} \tag{13}$$

The formula (8) is equal to the right side of these two equations:

$$\dot{U}_{1H}+\dot{U}_{2H}=-2Z_{HBX}\left(\dot{I}_{1H}+\dot{I}_{2H}\right) \tag{14}$$

In Figure 3, the following equations describe the loop 7-10-11-8 and 8-11-12-9:

$$\begin{cases}\dfrac{\dot{U}_P}{2}=\dot{U}_{1K}-Z_{KBX}\left(\dot{I}_{1K}+\dot{I}_{2K}\right)\\ \dfrac{\dot{U}_P}{2}=Z_{KBX}\left(\dot{I}_{1K}+\dot{I}_{2K}\right)-\dot{U}_{2K}\end{cases} \tag{15}$$

The right side of the above formula can be written as:

$$\dot{U}_{1K}+\dot{U}_{2K}=2Z_{KBX}\left(\dot{I}_{1K}+\dot{I}_{2K}\right) \tag{16}$$

We can use formula (7):

$$\dot{I}_{1K}-\dot{I}_{2K}=2\dot{I}_2 \tag{17}$$

The formula (18) can be obtained from the left and right sides of formula (15):

$$\dot{U}_{1K}-\dot{U}_{2K}=\dot{U}_P=\dot{U}_2 \tag{18}$$

$\dot{U}_2$ and $\dot{I}_2$ are the current and voltage of the receiving terminal, respectively.

Next, $x=0$ is substituted into formulas (2) to (5) to obtain $\dot{U}_{1K},\dot{U}_{2K},\dot{I}_{1K}$ and $\dot{I}_{2K}$, and then substitute $x=l$ into formulas (2) to (5) to obtain $\dot{U}_{1H},\dot{U}_{2H},\dot{I}_{1H}$ and $\dot{I}_{2H}$. Then substitute these results into formulas (14), (15), (17) and (18) to obtain the following formulas:

$$(1-M)A_1+(1-N)A_3=\dot{U}_2 \tag{19}$$

$$(y_{11}-y_{21})A_2+(y_{12}-y_{22})A_4=2\dot{I}_2 \tag{20}$$

$$\begin{aligned}&(1+M)A_1-2Z_{KBX}(y_{11}+y_{21})A_2+(1+N)A_3-\\&2Z_{KBX}(y_{12}+y_{22})A_4=0\end{aligned} \tag{21}$$

$$k_1A_1+k_2A_2+k_3A_3+k_4A_4=0 \tag{22}$$

In formula (22):

$$\begin{cases}k_1=(1+M)\cosh\gamma_1 l+2Z_{HBX}(y_{11}+y_{21})\sinh\gamma_1 l\\ k_2=(1+M)\sinh\gamma_1 l+2Z_{HBX}(y_{11}+y_{21})\cosh\gamma_1 l\\ k_3=(1+N)\cosh\gamma_2 l+2Z_{HBX}(y_{12}+y_{22})\sinh\gamma_2 l\\ k_4=(1+N)\sinh\gamma_2 l+2Z_{HBX}(y_{12}+y_{22})\cosh\gamma_2 l\end{cases} \tag{23}$$

In the case of rail line symmetry:

$$M=1,N=-1,$$
$$y_{11}=y_{21}=\frac{1}{Z_{B1}},\ y_{12}=-y_{22}=\frac{1}{Z_{B2}} \tag{24}$$

$$Z_{\mathrm{B1}} = \frac{1}{2} E Z_{\mathrm{B}} \sqrt{1+P}\ , Z_{\mathrm{B2}} = \frac{1}{2} Z_{\mathrm{B}} ,$$

$$Z_{\mathrm{KBX}} = Z_{\mathrm{HBX}} = \frac{1}{2} Z_{\mathrm{B1}} \tag{25}$$

In formulas (24) and (25), Z_{B1} is the characteristic impedance between the rails and the earth of the symmetrical rail line, Z_{B2} is the phase circuit impedance of the symmetrical rail transmission line, Z_{B} is the characteristic impedance of the symmetrical rail line, and P is the surface conductance coefficient. The formula (24) is put into formulas (19) to (22) to obtain:

$$\begin{cases} 2A_3 = \dot{U}_2 \\ \dfrac{1}{Z_{\mathrm{B2}}} A_4 = \dot{I}_2 \\ A_1 - \dfrac{2Z_{\mathrm{KBX}}}{Z_{\mathrm{B1}}} A_2 = 0 \\ k_1 A_1 + k_2 A_2 + k_3 A_3 + k_4 A_4 = 0 \end{cases} \tag{26}$$

Finishing formula (26) can be drawn as follows:

$$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \dfrac{1}{Z_{\mathrm{B2}}} \\ 1 & -\dfrac{2Z_{\mathrm{KBX}}}{Z_{\mathrm{B1}}} & 0 & 0 \\ k_1 & k_2 & k_3 & k_4 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} \dot{U}_2 \\ \dot{I}_2 \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

The upper formula is solved to obtain:

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \frac{Z_{\mathrm{B2}}}{2k_2} \begin{bmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{21} & A_{21} \\ \dfrac{k_2 Z_{\mathrm{B1}} + 2k_1 Z_{\mathrm{KBX}}}{Z_{\mathrm{B2}} Z_{\mathrm{B1}}} & 0 & A_{33} & A_{43} \\ 0 & \dfrac{2k_2 Z_{\mathrm{B1}} + 4k_1 Z_{\mathrm{KBX}}}{Z_{\mathrm{B1}}} & A_{34} & A_{44} \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ \dot{I}_2 \\ 0 \\ 0 \end{bmatrix} \tag{28}$$

The formula (28) is further simplified:

$$A_3 = \frac{k_2 Z_{\mathrm{B1}} + 2k_1 Z_{\mathrm{KBX}}}{2k_2 Z_{\mathrm{B1}}} \dot{U}_2 \tag{29}$$

$$A_4 = \frac{Z_{\mathrm{B2}} \left(2k_2 Z_{\mathrm{B1}} + 4k_1 Z_{\mathrm{KBX}} \right)}{2k_2 Z_{\mathrm{B1}}} \dot{I}_2 \tag{30}$$

From formulas (24) and (25), we can find:

$$k_1 = k_2 \tag{31}$$

The formulas (25) and (31) are substituted into formulas (29) and (30), to give:

$$A_3 = \dot{U}_2 \tag{32}$$

$$A_4 = Z_{\mathrm{B}} \dot{I}_2 \tag{33}$$

For $\dot{U}_2$ and $\dot{I}_2$, a similar approach allows determination of I:

$$\dot{U}_1 = \dot{U}_{1\mathrm{H}} - \dot{U}_{2\mathrm{H}} \tag{34}$$

$$\dot{I}_1 = \frac{1}{2} \left(\dot{I}_{1\mathrm{H}} - \dot{I}_{2\mathrm{H}} \right) \tag{35}$$

Substitute the $x = l$ into formula (2) and (5) to determined $\dot{U}_{1\mathrm{H}}$, $\dot{U}_{2\mathrm{H}}$, $\dot{I}_{1\mathrm{H}}$, $\dot{I}_{2\mathrm{H}}$, to allow:

$$\begin{aligned} \dot{U}_1 = {} & (1-M)\left(A_1 \cosh \gamma_1 l + A_2 \sinh \gamma_1 l \right) + \\ & (1-N)\left(A_3 \cosh \gamma_2 l + A_4 \sinh \gamma_2 l \right) \end{aligned} \tag{36}$$

$$\begin{aligned} \dot{I}_1 = {} & \frac{1}{2} [\left(y_{11} - y_{21} \right)\left(A_1 \sinh \gamma_1 l + A_2 \cosh \gamma_1 l \right) + \\ & \left(y_{12} - y_{22} \right)\left(A_3 \sinh \gamma_2 l + A_4 \cosh \gamma_2 l \right)] \end{aligned} \tag{37}$$

The formula (24) and $\gamma_2 = \gamma$ can be combined in formulas (36) and (37) to obtain:

$$\dot{U}_1 = 2\left(A_3 \cosh \gamma l + A_4 \sinh \gamma l \right) \tag{38}$$

$$\dot{I}_1 = 2\frac{1}{Z_{\mathrm{B}}} \left(A_3 \sinh \gamma l + A_4 \cosh \gamma l \right) \tag{39}$$

Then formulas (32) and (33) are substituted into formulas (38) and (39) to obtain:

$$\dot{U}_1 = 2\cosh(\gamma l)\dot{U}_2 + 2Z_{\mathrm{B}} \sinh(\gamma l)\dot{I}_2 \tag{40}$$

$$\dot{I}_1 = 2\frac{1}{Z_{\mathrm{B}}} \sinh(\gamma l)\dot{U}_2 + 2\cosh(\gamma l)\dot{I}_2 \tag{41}$$

According to formulas (40) and (41), the coefficient of the four-terminal network of the symmetrical rail line can be written as:

$$A = D = 2\cosh(\gamma l)\ ,\ B = 2Z_{\mathrm{B}} \sinh(\gamma l)\ ,$$

$$C = 2\frac{1}{Z_{\mathrm{B}}} \sinh(\gamma l) \tag{42}$$

III. Adjustment Mode Choke Transformer Combined with the Track Circuit to Solve a Four-terminal Network Coefficients

The 25Hz phase sensitive track circuit is used to transmit the signal current to the adjacent section of the rail using the polar crossing method. The two choke point is shown in Figure 5, and the flow of traction transformer coil 3 and 4 have the same potential, so there is no potential difference [11]. Therefore, the signal current of this section will not flow over the insulation section to the adjacent track section.

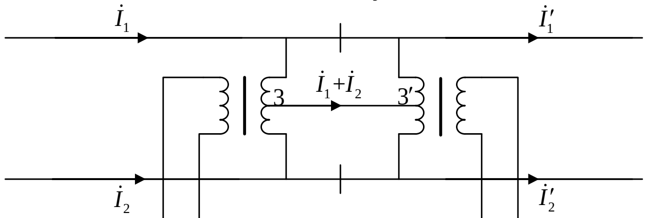

Fig. 5. Diagram of the Working Principle of the Choke Transformer

The secondary side of the transformer no-load was measured as Z_{m}=2.369Ω, and the secondary side short circuit was measured as $Z_1 = Z_2' = 0.017\Omega$ [12]. We know from Figure 5 that the four-terminal network of the choke transformer is a type of T network, as shown in Figure 6.

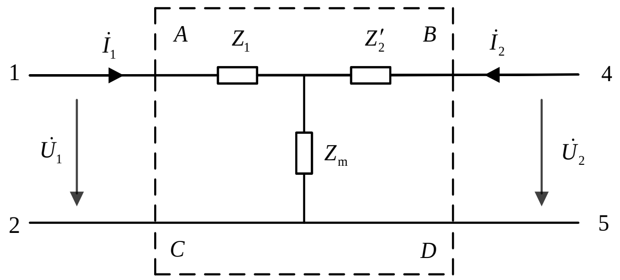

Fig. 6. T-type Equivalent Circuit of Choke Transformer

For a T-type network, because we know the value of the excitation and leakage impedance, we can express the four-terminal network choke transformer coefficient as:

$$\begin{cases} A = \left.\dfrac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_2=0} = \dfrac{Z_1 + Z_m}{Z_m} = 1.007 \\ B = \left.\dfrac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{U}_2=0} = Z_1 + Z_2' + \dfrac{Z_1 \cdot Z_2'}{Z_m} = 0.034 \\ C = \left.\dfrac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{I}_2=0} = \dfrac{1}{Z_m} = 0.422 \\ D = \left.\dfrac{\dot{I}_1}{\dot{I}_2}\right|_{\dot{U}_2=0} = \dfrac{Z_m + Z_2'}{Z_m} = 1.007 \end{cases} \tag{43}$$

The choke transformer equivalent circuit is shown in Figure 7. The equivalent circuit consists of three parts: the ideal autotransformer, T-type equivalent circuit, and an ideal transformer turn ratio of $(2N_T : N_2)$. Where, Z_1 is the magnetic flux leakage impedance of the traction winding (the number of turns is $2N_T$), Z_2' is the magnetic flux leakage impedance of the signal winding as converted to traction windings, and Z_m is converted to the traction windings of the excitation impedance [13].

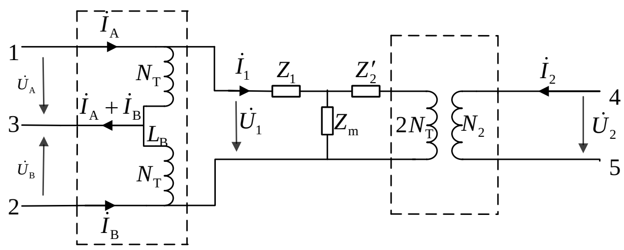

Fig. 7. Equivalent Circuit of the Choke Transformer

After state adjustment, the equivalent circuit of the combination of track circuit and choke transformer is shown in Figure 8. The left and right ends of the rail lines were connected with the sender and receiver choke transformer, respectively.

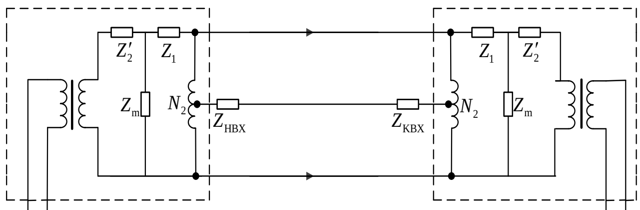

Fig. 8. Equivalent Circuit of the Adjustment Mode Choke Transformer Combined with Track Circuit

The transmission terminal choke transformer is represented by the four-terminal network N_{FE}, the receiving terminal of the choke transformer is represented by the four-terminal network N_{JE}, and the middle part of the rail network circuit is represented by the four-terminal network N_G. Then Figure 8 can be considered equivalent to Figure 9.

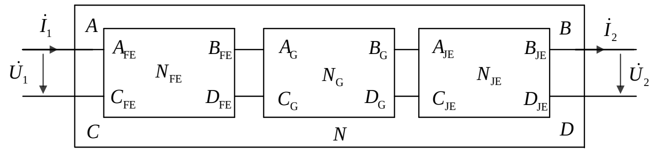

Fig. 9. Cascade Adjustment Mode Choke Transformer and the Track Circuit's Four-terminal Network

It can be determined from Figure 9 that the overall coefficient of a four-terminal network is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_{FE} & B_{FE} \\ C_{FE} & D_{FE} \end{bmatrix} \cdot \begin{bmatrix} A_G & B_G \\ C_G & D_G \end{bmatrix} \cdot \begin{bmatrix} A_{JE} & B_{JE} \\ C_{JE} & D_{JE} \end{bmatrix} \tag{44}$$

In Figure 8, for the sending terminal of the choke transformer, the signal coil is considered the start terminal. For the receiving terminal of the choke transformer, the signal coil is calculated as the end terminal. Therefore, the choke transformer of the four-terminal network's energy transfer direction is changed. When the energy transfer direction is changed for a four terminal network, the position of the four terminal network coefficient matrix A and D will also be swapped. However, before the choke transformer coefficient matrix $A = D$, so then the sending and receiving terminal of the choke transformer's four-terminal network matrix remains the same. The formula (42) and (43) are substituted into (44) to obtain the adjustment of the overall four terminal network coefficient as:

$$\begin{cases} A = (2.028 + 0.850Z_B)\sinh\gamma l + 0.029\cosh\gamma l + \dfrac{0.068}{Z_B} \\ B = (0.068 + 2.028Z_B)\sinh\gamma l + 0.068\cosh\gamma l + \dfrac{0.002}{Z_B} \\ C = (0.850 + 0.356Z_B)\sinh\gamma l + 0.850\cosh\gamma l + \dfrac{2.028}{Z_B} \\ D = (0.029 + 0.850Z_B)\sinh\gamma l + 2.028\cosh\gamma l + \dfrac{0.068}{Z_B} \end{cases} \tag{45}$$

IV. STEADY STATE RESPONSE ANALYSIS OF TRACK CIRCUIT TRANSMISSION

A. DC Steady State Response of Track Circuit's Adjustment State

We start with the track transmission line as shown in Figures 1 and 2 [14-16]. Assume that the track transmission line start terminal is connected with the resistance $Z_{BX0}(s)$ of the voltage source E_s, and the end terminal accesses an arbitrary load $Z_{BXL}(s)$. According to previous results from the literature [5], combined with the equations in this paper, for the excitation of the transmission line within the given boundary conditions, the voltage and current in the frequency domain solutions $E(x,s)$ and $I(x,s)$ can be written as:

$$\begin{cases} E(x,s)=\dfrac{e^{\gamma(s)(2l-x)}+n_2(s)e^{\gamma(s)x}}{e^{2\gamma(s)l}-n_1(s)n_2(s)}\bullet k(s)\bullet\dfrac{E_s}{s} \\ I(x,s)=\dfrac{e^{\gamma(s)(2l-x)}-n_2(s)e^{\gamma(s)x}}{[e^{2\gamma(s)l}-n_1(s)n_2(s)]\bullet Z_C(s)}\bullet k(s)\bullet\dfrac{E_s}{s} \end{cases} \tag{46}$$

Where,

$$k(s)=\frac{Z_C(s)}{Z_{BX0}(s)+2Z_C(s)},\ n_1(s)=\frac{Z_{BX0}(s)-2Z_C(s)}{Z_{BX0}(s)+2Z_C(s)},$$
$$n_2(s)=\frac{Z_{BXL}(s)-2Z_C(s)}{Z_{BXL}(s)+2Z_C(s)}$$

The final value theorem of Laplace transform is then applied to the $E(x,s)$ in formula (46), and the steady state response of the rail line is obtained as follows:

$$\begin{aligned} E(x) &= \lim_{t\to+\infty} e(x,t) \\ &= \lim_{s\to 0} s\bullet E(x,s) \\ &= \lim_{s\to 0}\frac{e^{\gamma(s)(2l-x)}+n_2(s)\bullet e^{\gamma(s)x}}{e^{2\gamma(s)l}-n_1(s)n_2(s)}\bullet k(s)\bullet E_s \end{aligned} \tag{47}$$

When $s\to 0$, the characteristic impedance of the rail transmission line $Z_C(0) = R_C = \dfrac{\sqrt{R_0(g_1+g_2)}}{2g}$, propagation constant $\gamma(0)=\alpha=\sqrt{R_0(g_1+g_{12})}$, and the line voltage of the DC steady state response is:

$$E(x)=\frac{e^{\alpha(2l-x)}+[Z_{BXL}(0)-R_C]/[Z_{BXL}(0)+R_C]\bullet e^{\alpha x}}{e^{2\alpha l}-\dfrac{Z_{BX0}(0)-R_C}{Z_{BX0}(0)+R_C}\bullet\dfrac{Z_{BXL}(0)-R_C}{Z_{BXL}(0)+R_C}}\times \frac{R_C}{Z_{BX0}(0)+2R_C}\bullet E_s \tag{48}$$

When the track circuit is in the adjustment state, then $Z_{BXL}(s)=0$, and then:

$$E(x)=\frac{e^{\alpha(2l-x)}-e^{\alpha x}}{e^{2\alpha l}+\dfrac{Z_{BX0}(0)-R_C}{Z_{BX0}(0)+R_C}}\bullet\frac{R_C}{Z_{BX0}(0)+2R_C}\bullet E_s \tag{49}$$

With the same method the following can be obtained:

$$I(x)=\frac{e^{\alpha(2l-x)}+e^{\alpha x}}{(e^{2\alpha l}+\dfrac{Z_{BX0}(0)-R_C}{Z_{BX0}(0)+R_C})\bullet R_C}\bullet\frac{R_C}{Z_{BX0}(0)+2R_C}\bullet E_s \tag{50}$$

B. Sinusoidal Steady State Response of Track Circuit Adjustment

Sinusoidal excitation of the track circuit start terminal is $E_s(t)=\sqrt{2}E\sin(\omega t+\theta)$, represented by the phasor that is $\dot{E}_s=E\angle\theta$. Therefore, the voltage response of the track transmission line is:

$$\begin{aligned} \dot{E}(x) &= \frac{E(x,j\omega)}{E_s(j\omega)}\bullet\dot{E}_s \\ &= \frac{e^{\gamma(j\omega)(2l-x)}+n_2(j\omega)e^{\gamma(j\omega)x}}{e^{2\gamma(j\omega)l}-n_1(j\omega)n_2(j\omega)}\bullet k(j\omega)\bullet\dot{E}_s \end{aligned} \tag{51}$$

$E(x,j\omega)$ denotes a function at sinusoidal excitation voltage response of the transmission line and $E_S(j\omega)$ is a function of the excitation source.

When $x=0$:

$$\dot{E}(0)=\frac{e^{\gamma(j\omega)2l}+n_2(j\omega)}{e^{2\gamma(j\omega)l}-n_1(j\omega)n_2(j\omega)}\bullet k(j\omega)\bullet\dot{E}_s \tag{52}$$

According to $\cosh\gamma(j\omega)\bullet x=\dfrac{e^{\gamma(j\omega)x}+e^{-\gamma(j\omega)x}}{2}$ and $\sinh\gamma(j\omega)\bullet x=\dfrac{e^{\gamma(j\omega)x}-e^{-\gamma(j\omega)x}}{2}$, and substitution into the formulas (51) and (52) results in:

$$\dot{E}(x)=\dot{E}(0)\cosh(\gamma x)-Z_C\bullet\dot{I}(0)\bullet\sinh(\gamma x) \tag{53}$$

In the same way, it can be written as follows:

$$\dot{I}(x)=\dot{I}(0)\cosh(\gamma x)-\frac{1}{Z_C}\bullet\dot{E}(0)\bullet\sinh(\gamma x) \tag{54}$$

The 25Hz phase sensitive track circuit used in the station and the parameters of the track transmission line were calculated according to its steady state response. The steady state analysis of the track circuit transmission line has great theoretical value for the analysis of the stable operation of the track. Additionally it has important application value for the transmission of the signal without distortion. In this paper, the frequency domain solution of the track transmission line equation was derived from the literature [5]. This section describes the basis of this response for steady state DC using the final value theorem of Laplace transform to derive the orbit of the transmission line. The sinusoidal steady state response of the track transmission line was derived from the solution of the frequency domain. The relationship of the sinusoidal steady state response to the hyperbolic function was verified. For the theoretical derivation of the track circuit in the frequency domain, the process is simpler than in the time domain. The solution is unique and there are no uncertainty solutions. In the frequency domain, the results can be directly input into a computer program without the cumbersome and complex process of the Laplace inverse transform. Therefore, the practical value of engineering application is higher.

V. SIMULATION ANALYSIS ON THE ADJUSTMENT STATE OF THE TRACK CIRCUIT'S TRANSMISSION CHARACTERISTICS

MATLAB is the abbreviation of Matrix Laboratory, and it was developed around 1980. After continuous development, the MATLAB has become more than just a "matrix laboratory", but is widely used for engineering calculations and numerical analysis [17-18].

The process was simulated with a MATLAB program. We already know that rail impedance $z = 0.62\angle 42°\Omega / km$, the voltage signal receiving coil terminals of the choke transformer is $\dot{U}_2 = 1.30095\angle 73.23°$ V, and the current is $\dot{I}_2 = 0.2130\angle 65.07°$ A. First, the ballast resistance has a fixed value of $r_d = 0.6\Omega\bullet km$, and simulation of the signal coil of the sending terminal choke transformer's voltage and current can change with the transmission distance of the curve. Then, a fixed value is used for the transmission distance, $l = 1.5km$, and simulation of the signal coil of the sending terminal choke transformer's voltage and current can change with the ballast resistance of the curve. The simulation results are shown in Figures 10 to 13.

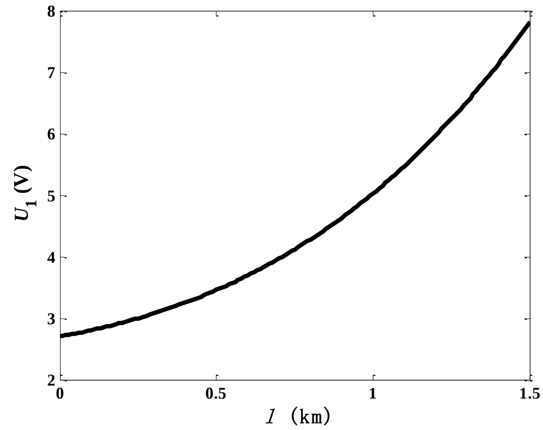

Fig. 10. Change in the Sending Terminal Voltage with Transmission Distance

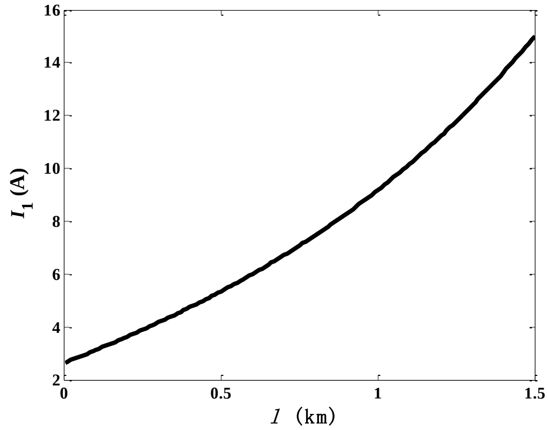

Fig. 11. Change in the Sending Terminal Current with Transmission Distance

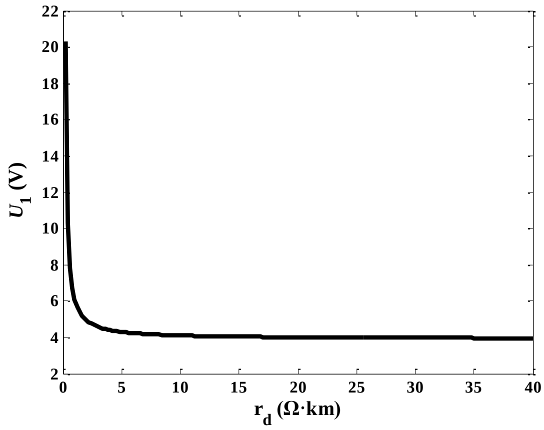

Fig. 12. Change in the Sending Terminal Voltage with Ballast Resistance

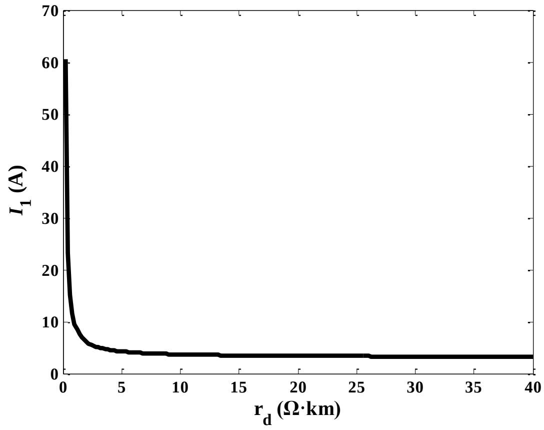

Fig. 13. Change in the Sending Terminal Current with Ballast Resistance

From Figures 10 and 11, the sending terminal's voltage and current increases as transmission distance increases. From the reverse perspective, the receiving terminal's voltage and current will decrease with the increase of transmission distance. This is consistent with the actual situation, as transmission distance increases, there is a larger impedance of the rail and the loss is greater. In order to maintain the surface voltage and current of the rail, it is necessary to increase the sending terminal's voltage and current. As shown in Figures 12 and 13, the receiving terminal's voltage and current decreases with the increase of ballast resistance. Then on the other hand, the receiving terminal's voltage and current will increase with the increasing of ballast resistance. When the ballast resistance increases, the leakage resistance between the rail and the ground is reduced. To keep the receiving terminal at constant voltage and current, the voltage and current of the sending terminal must decrease. This illustrates that to improve the receiving terminal's voltage and current, the transmission distance must be reduced or the ballast resistor should be improved. Of course, the track circuit must maintain a certain length, so it is necessary to increase the minimum ballast resistance. Overall, this allows energy savings and also increases stability.

VI. Conclusion

The system structure of the 25Hz phase sensitive track circuit was introduced, and its working principle was described. The choke transformer and the track circuit was modeled and the equivalent circuit for four terminal network's coefficient was solved. The overall choke transformer and track circuit composed of four-terminal network's coefficients was described and solved. The transmission line equation of the track circuit was solved in the frequency domain, the steady state response of the track transmission line was derived, and the theoretical basis of the steady state response of the track circuit was provided. Finally, MATLAB was used to program the simulation. The simulation results show that the transmission characteristics and the length l and the ballast resistance r_d of track circuit are closely related. This work can help guide improved methods for ensuring the stability of the rail and allowing energy conservation.

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