Rendezvous of Non-Cooperative Spacecraft and Tug Using a Tether System

Vladimir S. Aslanov, Ruslan S. Pikalov

Abstract— **This paper focuses on a docking of two bodies by means of a tether in a force-free field which is studied in the framework of space debris removal problem. The two bodies are connected by the viscoelastic tether. One of the bodies (tug) is under constant thrust. In the initial time the bodies are at some distance from each other. Analysis of the tethered bodies dynamics is carried out assuming that the tether's length can be changed according to a prescribed law. The goal is to find the control law that allows to realize impactless docking of two bodies. Analysis of the consistency of the proposed law with the requirements of safe rendezvous of the bodies is conducted. The obtained results can be applied to study the properties and possible configurations of the tethered of spacecraft docking system.**

Index Terms— **Spacecraft rendezvous, tethered system, active space debris removal, space tug, tethered control**

I. Introduction

In the recent years, the problems of space debris have came to the forefront of modern astronautics. According to the forecast made by Donald J. Kessler [1], space debris can put an end to further space exploration. There are more than 15,000 large objects on the orbits around the Earth. Only 7% of these are active spacecraft, 17% are nonfunctional spacecraft and 13% are orbital stages of the rockets. All these objects are tracked and an active spacecraft or a space station can avoid collision with such objects [2-4]. Collisions of the large space debris with other debris can significantly increase numbers of the small space junk on the Earth orbit. There are a large number of papers devoted to this problem [1-23]. There are various solutions to this problem [5-9]. Many modern spacecraft are equipped with de-orbiting facilities [24-27].

Various approaches have been offered to eliminate already running units. These approaches include powerful lasers to create pressure on the satellite to reduce its orbital velocity [5], electrodynamics tether system to create a drag force [6], starting devices (e.g. chaser, debritor) to capture of debris, and to lead away from the orbit [7-21, 28-31]. Such methods are called the active debris removal [7-21]. In the papers [7-9] it were proposed to use a rocket upper stage as tethered tug for space debris removal. It is assumed that this upper stage have been completed its main mission and its secondary goal is to use the remaining fuel reserves to rendezvous with space debris with similar orbital parameters. Capturing of the space debris is performed by the tether released from the tug. The tug and debris are pulled together using the tether. The next stage is an orbital momentum changing which lowers the periapsis of both objects (tug and debris).

This study focuses on the stage of pulling space debris by the tether. The goal is to find the control law that allow one to realize the impactless docking of the tug and debris.

The paper is divided into three main parts. In the Section 2 a length control strategy for the two-body system with an inextensible tether is considered. In the Section 3 an analysis of the proposed tether length control laws is given in terms of possible implementation. Section 4 focuses on the study of behavior of the two-body with a viscoelastic tether.

II. Length Control Strategy for the Two-Body System with an Inextensible Tether

The two-body system includes two points m_1 (the space tug) and m_2 (the space debris) connected by the tether which has the initial length L_0 as shown in Fig.1.

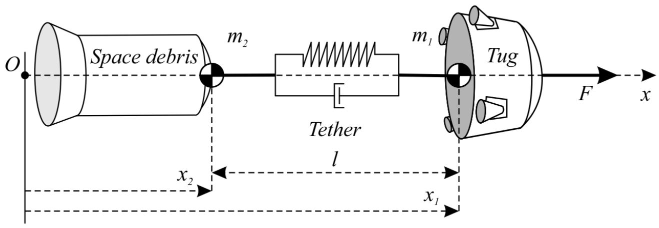

Fig.1. Scheme the two-body system

The position of the material points m_1 and m_2 can be defined by the coordinates x_1 and x_2 respectively. It is assumed that controlled motion is considered in a very short time interval compared to the orbital period. We suppose that all external forces significantly less than the thrust tug F and this force coincides with the line connecting the two material points (Fig.1). Internal interaction of the bodies is determined by the tether tension force T. The dynamics of the system, taking into account

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the gravitational forces of the Earth, will be discussed in the following papers.

In addition, we assume that the tether is weightless. Taking into account the accepted assumptions, the motion equations of the two-body system with the inextensible tether can be written as [32]:

$$\ddot{x}_1 = \frac{1}{m_1}(F - T), \quad \ddot{x}_2 = \frac{1}{m_2}T. \tag{1}$$

Using the current length of the tether as a new variable

$$l = x_1 - x_2 \tag{2}$$

and according to the equations (1) the equation of the relative motion of the two tethered bodies can be written as

$$\ddot{l} = \frac{1}{m_1}(F - T) - \frac{1}{m_2}T. \tag{3}$$

To ensure the safe rendezvous, the variable (2), must satisfy the following boundary conditions

$$\begin{aligned} t = 0: &\quad \dot{l} = 0, \quad l = l_0, \\ t = t_k: &\quad \dot{l} = 0, \quad l = 0. \end{aligned} \tag{4}$$

Note that the relative velocity $\dot{l}$ of the two bodies at the end of pulling maneuver (4) must be zero. The conditions (4) are satisfied for the following series

$$L = \frac{L_0}{2}\left(1 + \sum_{i=0}^{n} a_{2i+1}\cos\left[(2i+1)\omega t\right]\right), \tag{5}$$

where

$$\omega = \frac{\pi}{t_k}, \tag{6}$$

t_k is the duration of the maneuver, a_{2i+1} is the series coefficient that satisfy to following relationship

$$\sum_{i=1}^{n} a_{2i+1} = 1. \tag{7}$$

For the case when $n = 0$, the series (5) has a very simple form

$$L = \frac{L_0}{2}(1 + \cos\omega t). \tag{8}$$

Fig.2 shows the function (5) in cases:

$n = 0, \quad a_1 = 1$ - solid line,

$n = 2, \quad a_1 = 0.5, \quad a_3 = a_5 = 0.25$ - dotted line,

$n = 2, \quad a_1 = a_3 = 0.2, \quad a_5 = 0.6$ -dashed line.

Thus, the series (5) satisfies the boundary conditions (4).

Differentiating twice the equation (5) and equating the result to the equation (3), we get

$$\begin{aligned} &\frac{1}{m_1}(F - T) - \frac{1}{m_2}T = \\ &-\frac{L_0\omega^2}{2}\left(\sum_{i=0}^{n} a_{2i+1}(2i+1)^2\cos\left[(2i+1)\omega t\right]\right). \end{aligned} \tag{9}$$

The expression for the tension force, in which the boundary conditions are satisfied (4), can be written as

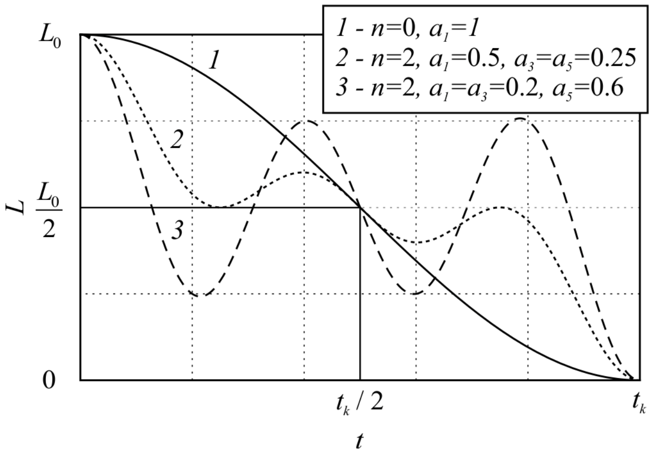

Fig.2 The dependence of the tether length control law from time

$$\begin{aligned} T = M\Bigg(&\frac{L_0\omega^2}{2}\left(\sum_{i=0}^{n} a_{2i+1}(2i+1)^2\cos\left[(2i+1)\omega t\right]\right) + \\ &+\frac{F}{m_1}\Bigg), \end{aligned} \tag{10}$$

where $M = m_1 m_2 / m$ is the reduced mass of the system, $m = m_1 + m_2$ is the total mass of the two-body system. Substituting the expression (10) in the equations (1) we obtain the equations of motion of the two-body system with the inextensible tether

$$\begin{aligned} \ddot{x}_1 &= \frac{F}{m_{tot}} - \frac{M_2 L_0 \omega^2}{2}\sum_{i=0}^{n} a_{2i+1}(2i+1)^2\cos\left[(2i+1)\omega t\right], \\ \ddot{x}_2 &= \frac{F}{m_{tot}} + \frac{M_1 L_0 \omega^2}{2}\sum_{i=0}^{n} a_{2i+1}(2i+1)^2\cos\left[(2i+1)\omega t\right], \end{aligned} \tag{11}$$

where $M_1 = m_1 / m$ and $M_2 = m_2 / m$ are the non-dimensional masses of the tug and the space debris respectively.

For the case when $n = 0$ the expression (10) has the following form:

$$T = M\left(\frac{L_0\omega^2}{2}\cos\omega t + \frac{F}{m_1}\right), \tag{12}$$

and the equations (11) can be written as:

$$\begin{aligned} \ddot{x}_1 &= \frac{F}{m_{tot}} - \frac{M_2 L_0 \omega^2}{2}\cos\omega t, \\ \ddot{x}_2 &= \frac{F}{m_{tot}} + \frac{M_1 L_0 \omega^2}{2}\sin\omega t. \end{aligned} \tag{13}$$

III. TETHER LENGTH CONTROL LAWS

Let us examine the possibility of implementing of the tether length control laws (5). All numerical calculations will be performed for the two-body system with the parameters listed in Table 1.

We also use the following initial conditions for numerical simulation:

$$x_1(0) = L_0, \quad x_2(0) = 0, \quad \dot{x}_1(0) = 0, \quad \dot{x}_2(0) = 0. \tag{14}$$

A. The requirements to the maneuver pulling

The rendezvous of the spacecrafts should occur without the collisions. This is unacceptable for the active debris removal missions, as this may lead to serious consequences, for example, to damage of the tug, the formation of new debris, and increase of the debris environment. To prevent it we formulate the following conditions:

1. Any collisions should be avoided, i.e.

$$\forall\, t < t_k : x_1 \neq x_2. \quad (15)$$

2. The magnitude of the tension force T should be as low as possible. The less the tension force T, the smaller would be the mass of the mechanism that implements this force. That is why we are introducing this limitation.

3. A work of the tension force T on the time interval $t \in [0, t_k]$ should be minimal:

$$W = \int_0^{L_0} T\delta L \to \min. \quad (16)$$

This requirement is due to the energy capabilities of the space tug.

4. The tether throughout the maneuver must be strained, i.e.

$$T > 0. \quad (17)$$

Otherwise, it can lead to negative consequences, such as tangling the tether.

TABLE I
PARAMETERS OF THE TWO-BODY SYSTEM

Symbol	Parameters	Value
L_0	initial length	20 m
m_1	tug mass	756 kg
m_2	space debris mass	1720 kg
t_k	duration of the maneuver	50 s
EA	stiffness of the tether	6000 N
C	damping of the tether	4000 N·s
F	the thrust tug	100 N

B. Choice of the thrust tug

According to the equation (10) and the condition (17), to the thrust tug F should satisfy the condition

$$F > -\frac{1}{2} m_1 L_0 \omega^2 \left(\sum_{i=0}^{n} a_{2i+1} (2i+1)^2 \cos\left[(2i+1)\omega t\right] \right). \quad (18)$$

As the case of constant thrust is considered in this paper, the tether remains strained during the maneuver if

$$F > -\frac{m_1 L_0 \omega^2}{2} \sum_{i=0}^{n} a_{2i+1} (2i+1)^2. \quad (19)$$

For the case when $n = 0$, the condition (19) takes the form

$$F > \frac{m_1 L_0 \omega^2}{2}. \quad (20)$$

From (20) it follows that the decrease of the required thrust tug F can be achieved by increasing the maneuver time t_k.

C. Influence of the coefficients a_{2i+1} on the tension force T

For example, we consider the effect of the coefficient a_{2i+1} on the value of the tension force T when $n = 1$.

Fig.3 shows that decreasing value of the coefficient a_1 results in negative value of the tension force T (for instance $a_1 = 0.6$, $a_3 = 1 - a_1 = 0.4$). Such cases do not correspond to the condition (17).

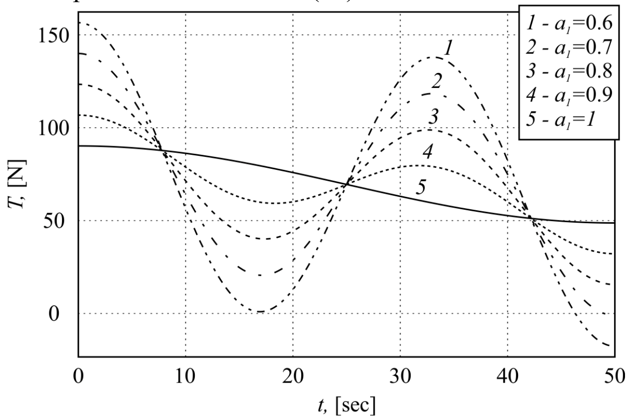

Fig. 3 The dependence of the tension force T from time

D. Work of the tension force T

To calculate the integral (16) we rewrite it in the form

$$W = \int_0^{L_0} T\delta l = \left[\delta l = \frac{dl}{dt} dt \right] = \int_{t_k}^{0} T \frac{dl}{dt} dt. \quad (21)$$

Substituting (10) and the derivative of the equation (5) in (21) we get

$$W = -\frac{M L_0 \omega}{2} \left\{ \frac{F}{m_1} \int_{t_k}^{0} \left(\sum_{i=0}^{n} a_{2i+1} (2i+1) \sin\left[(2i+1)\omega t\right] \right) dt + \right.$$
$$+ \frac{L_0 \omega^2}{2} \int_{t_k}^{0} \left[\left(\sum_{i=0}^{n} a_{2i+1} (2i+1)^2 \cos\left[(2i+1)\omega t\right] \right) \right. \quad (22)$$
$$\left. \left. \left(\sum_{i=0}^{n} a_{2i+1} (2i+1)^2 \sin\left[(2i+1)\omega t\right] \right) \right] \right\}.$$

The first integral in (22) can be easily computed using the conditions (7)

$$\int_{t_k}^{0} \left(\sum_{i=0}^{n} a_{2i+1} (2i+1) \sin\left[(2i+1)\omega t\right] \right) dt =$$
$$-\frac{2}{\omega} \sum_{i=0}^{n} a_{2i+1} = -\frac{2}{\omega}.$$

The subintegral function of the second integral in (22) is the odd periodic function of time t as the product of the sums of even and odd functions. Therefore, the second integral is zero. Finally, the work of the tension force T (22) takes the following form

$$W = M_2 L_0 F = \frac{m_2}{m_1 + m_2} L_0 F. \quad (23)$$

It follows that the work of the tension force T (23) depends only on the masses of the tug and of the debris, the total unstretched tether length and the thrust tug.

IV. EQUATIONS OF MOTION WITH A VISCOELASTIC TETHER

In this section we consider behavior of the two-body system under the length control law (8), taking into account the viscoelastic properties of the tether. The equations of motion can be written as:

$$\ddot{x}_1 = \frac{1}{m_1}\left[F - H_v\left(T_S + T_d\right)\right],$$
$$\ddot{x}_2 = \frac{1}{m_2} H_v\left(T_S + T_d\right), \qquad (24)$$

where

$$H_v = \begin{cases} 0, & \left|x_1 - x_2\right| \le L, \\ 1, & \left|x_1 - x_2\right| > L. \end{cases}$$

T_S is a tension force and T_d is a damping force:

$$T_S = EA\varepsilon, \qquad (25)$$

$$T_d = C\dot{\varepsilon}, \qquad (26)$$

$$\varepsilon = \frac{\left|x_1 - x_2\right|}{L} - 1, \qquad (27)$$

$$\dot{\varepsilon} = \frac{\left(x_1 - x_2\right)\left(\dot{x}_1 - \dot{x}_2\right)}{\left|x_1 - x_2\right| L} - \frac{\dot{L}}{L}\left(1 + \varepsilon\right), \qquad (28)$$

where EA is stiffness of the tether, C is a damping constant. L is the length control law (8). By substituting the expressions (25) - (28) into the equations (24), we get the equations of motion system with a viscoelastic tether.

In order to study the influence of the viscoelastic properties of the tether, several numerical techniques are used. The numerical results are based on the numerical integration of the equations (24) - (28) using an explicit fourth-order Runge-Kutta method. Table 1 presents parameters of the two-body system.

Fig.4 depicts the results of simulation for four the different initial conditions:

$$\begin{aligned} &(a) \quad x_1(0) = L_{0,} \quad x_1(0) = 0, \\ &(b) \quad x_1(0) = 0.85L_{0,} \quad x_1(0) = 0, \\ &(c) \quad x_1(0) = 1.15L_{0,} \quad x_1(0) = 0, \\ &(d) \quad x_1(0) = 1.3L_{0,} \quad x_1(0) = 0. \end{aligned} \qquad (29)$$

Fig.4b shows that the viscoelastic properties of the tether do not affect the behavior of the system in contrast to the cases (Figs. 4a, 4c, 4d).

In the Fig. 4d we have seen the collision between the tug and the debris at moments of time: $t_1 \approx 13s, t_2 \approx 33s$. In order to prevent the collision the thrust tug should be increase (Fig. 5).

V. CONCLUSION

The tether length control law for the docking of two bodies by means of the viscoelastic tether in a force-free field is proposed. The general analysis shows that, the requirements of safe rendezvous of the two body using the proposed law can be implemented. It was also showed that one can realize a soft docking of the two bodies. We discovered that the elastic properties of the tether make it difficult to the soft docking. It is shown that a change in the thrust tug can compensate the action of the elastic properties of the tether. The obtained results can be applied to study the properties and possible configurations of the tethered spacecraft docking system.

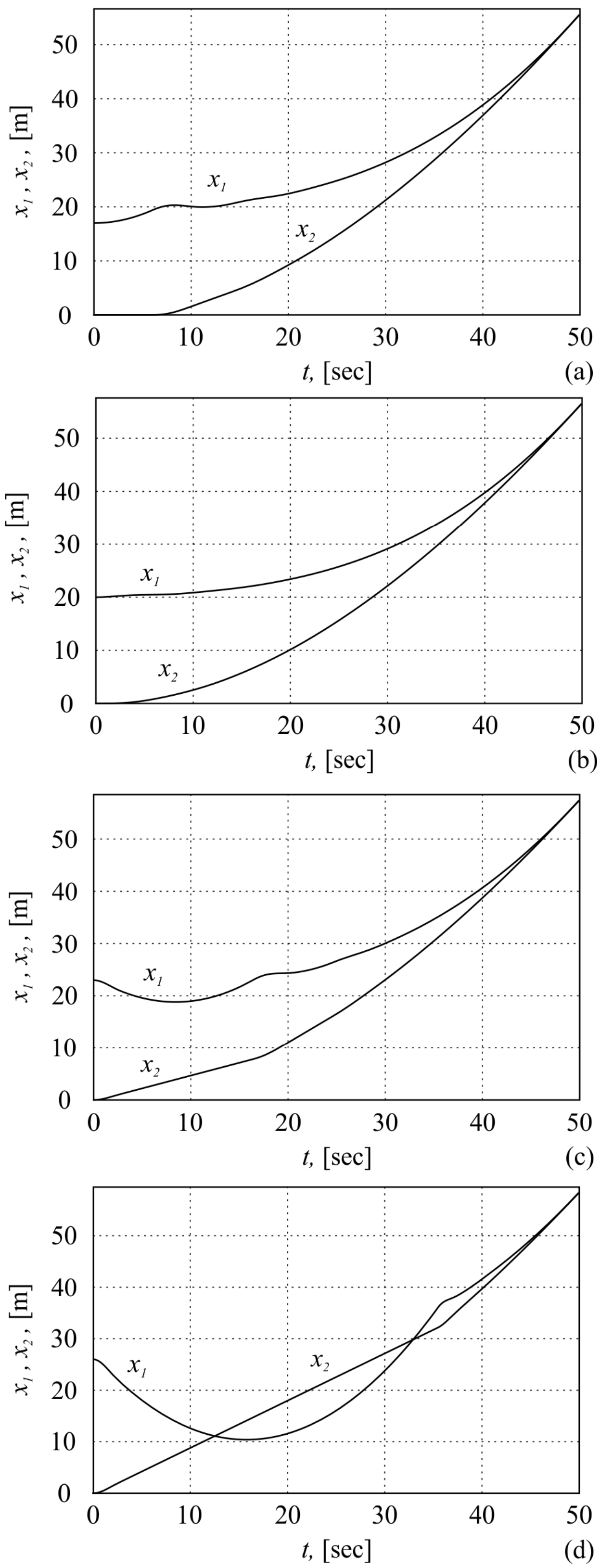

Fig.4. The dependence of the coordinates x_1 and x_2 from time ($F = 100N$)

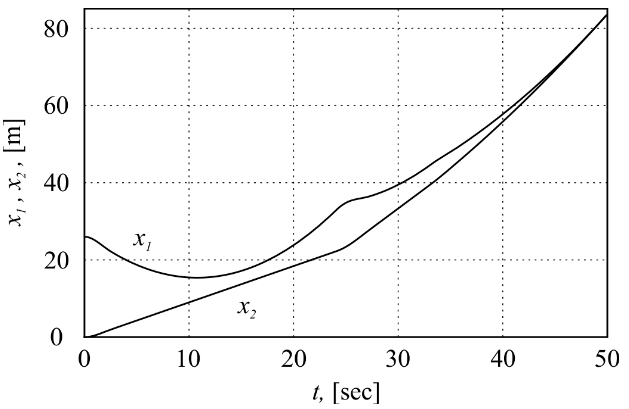

Fig.5 The dependence of the coordinates x_1 and x_2 from time ($F = 150N$)

REFERENCES

[1] Kessler D. J., Cour-Palais B. G., "Collision frequency of artificial satellites: the creation of a debris belt," *Journal of geophysical research*, vol. 83, pp 2637-2646, 1978

[2] Anselmo L., Pardini C., "Ranking upper stages in low Earth orbit for active removal," 6th European conference for aeronautics and space sciences (EUCASS), Krakow, Poland, 2015

[3] Anselmo L., Pardini C., "Analysis of the consequences in low earth orbit of the collision between cosmos 2251 and iridium," *Research Gate*, access https://www.researchgate.net/publication/228975104/.

[4] Bolonkin A., "New methods of removing space debris," access http://www.rxiv.org/pdf/1403.0670v1.pdf/.

[5] Phipps C. R., Baker K. L., Libby S. B., Liedahi D. A., Olivier S.S., "Removing orbital debris with lasers," *Advanced in Space Research*, vol. 49, no. 9, pp 1283-1300, 2012

[6] Nishida S., Kawamoto S., "Strategy for capturing of a tumbling space debris," *Acta Astronautica*, vol. 68, no. 1-2, pp 113-120, 2011

[7] Trushlyakov V., Makarov J., Raykunov G., Shatrov J., Baranovo D., "The development of autonomous onboard systems for the controlled deorbiting of stages separating parts of space launch vehicle," 2nd European Workshop on Active Debris Removal, Quentin, Paris, France, 2012

[8] Makarov Y., Ronse A., Trushlyakov V., "The use of adapted upper stages for the removal of satellite and rocket body debris from unstable orbital regions," 62nd International Astronautical Congress, Cape Town, South Africa, 2011

[9] Lee E.Z.J., Seubert C.R., Schaub H., Trushkyakov V., Yutkin E., "Tethered tug for large low earth orbit debris removal," AAS/AIAA Astrodynamics Specialists Conference, Charleston, SC, 2012

[10] Cougnet C., Alary D., Gerber B., Utzmann J., Wagner A., "The debritor: an "off the shelf" based multimission vehicle for large space debris removal," Proc. of the 63rd International Astronautical Congress, 2012

[11] Benvenuto R., Salvi S., Lavagna M., "Dynamics analysis and GNC design of flexible systems for space debris active removal," *Acta Astronautica*, vol.110, pp 247-265, 2015

[12] Lee E.Z.J., Schaub H., "Input shaped large thrust maneuver with a tethered debris object," *Acta Astronautica*, vol. 96, pp 128-137, 2014

[13] Lee E.Z.J., Schaub H., "Tethered towing using open-loop input-shaping and discrete thrust levels," *Acta Astronautica*, vol. 105, pp 373-384, 2014

[14] DeLuca L.T., Bernelli F., Maggi F., Tadini P., Pardini C., "Active debris removal by a hybrid propulsion module," *Acta Astronautica*, vol. 91, pp 20-33, 2013

[15] Flodin L., "Attitude and orbit control during deorbit of tethered space debris," access http://www.diva-portal.org/smash/record.jsf?pid=diva2%3A812509&dswid=7351/.

[16] Pang Z., Yu B., Jin D., "Chaotic motion analysis of a rigid spacecraft dragging a satellite by an elastic tether," *Acta Mechanica*, vol. 226, pp 2761-2771, 2015

[17] Wen H., Zhu Z.H., Jin D., Hu H., "Constrained tension control of a tethered space-tug system with only length measurement," *Acta Astronautica*, vol. 119, pp 110-117, 2016

[18] Saunders C., Forshaw J.L., Lappas V.J., Chiesa A., Parreira B., Biesbroek R., "Mission and systems design for the debris removal of massive satellites," 65th International Astronautical Congress, Toronto, Canada, 2014.

[19] Aslanov V.S., Yudintsev V.V., "Dynamics of large debris connected to space tug by a tether," *Journal of Guidance, Control, and Dynamics*, vol. 36, no. 6, pp 1654-1660, 2013

[20] Aslanov V.S., Yudintsev V.V., "Dynamics of large space debris removal using tethered space tug," *Acta Astronautica*, vol. 91, pp 149-156, 2013

[21] Aslanov V.S., Yudintsev V.V., "Dynamics, analytical solutions and choice of parameters for towed space debris with flexible appendages," *Advances in Space Research*, vol. 2, no. 2, pp 660-667, 2015

[22] Kobayashi T., Tsuda S., "Sliding mode control of space robot for unknown target capturing," *Engineering Letters,* vol.19, no. 2, pp 105-111, 2011

[23] Ishkov S.A., Filippov G.A., "A disposal of the space debris with special spacecraft debris collector using low trust," *Engineering Letters,* vol. 23, no.2, pp 98-109, 2015

[24] Aslanov V.S., Ledkov A.S., "Dynamics of the tethered satellite system," Elsevier Science: Burlington, 2012

[25] Russian Federal Space Agency, VOLGA Upper Stage description, access http://samspace.ru/products/upper_stages/bv_volga/.

[26] NPO Lavochkin, description the upper stage Fregat, access http://www.laspace.ru/rus/fregat.php/.

[27] SpaceX Falcon 9 description, access http://www.spacex.com/falcon9/.

[28] Muhammand A.K., Okamoto S., Lee J.H., "Comparisons of proportional and active-force controls on vibration of a flexible link manipulator using a piezoelectric actuator through calculations and experiments," *Engineering Letters,* vol. 22, no.3, pp 134-141, 2014

[29] Mahamood M.R., "Improving the performance of adaptive PDPID control of two-link flexible robotic manipulator with ILC," *Engineering Letters*, vol.20, no. 3, pp 259-270, 2012

[30] Wang D., Huang P., Cai J., Meng Z., "Coordinated control of tethered space robot using mobile tether attachment point in approaching phase," *Advances in Space Research,* vol. 54, no. 6, pp 1077-1091, 2014

[31] Huang P., Zhang F., Xu X., Meng Z., Liu Z., Hu Y., "Coordinated coupling control of tethered space robot using releasing characteristics of space tether," *Advanced Space Research,* vol.57, no. 7, pp 1528–1542, 2016

[32] Strauch D., "Classical Mechanics An Introduction," Springer-Verlag Berlin Heidelberg, 2009