Dynamic Output Feedback Guaranteed Cost Control for Linear Nominal Impulsive Systems

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Abstract—**This paper proposes an approach to dynamic output feedback guaranteed cost control problem for linear nominal impulsive systems. Sufficient conditions for the existence of the guaranteed cost control are presented in terms of linear matrix inequalities (LMI). Upon satisfaction of the conditions presented, a dynamic output feedback controller is easily calculated. Finally, an example is given to illustrate the effectiveness of the results.**

Index Terms—**linear nominal impulsive system; asymptotic stability; dynamic output feedback; linear matrix inequality (LMI).**

I. Introduction

It is known to all that, in any control design, a controller is sought not only to stabilize the system but also to ensure satisfactory performance. The guaranteed cost control aims at stabilizing the system while maintaining an adequate level of performance represented by the quadratic cost. Great efforts were made to investigate guaranteed cost control problems; for example, the optimal guaranteed cost control of uncertain linear systems was studied in [1], while the designed controller for uncertain discrete-time systems with both state and input delays was obtained in [2], and an LMI approach of robust H_∞-control for uncertain impulsive systems was presented with state feedback control; more researches, see [3,4,5] and relevant references therein. On the other hand, all the states of a system are not always observed in practical designs, so the dynamic feedback designs need to be considered, see [6,7,8].

The control of impulsive or nonlinear systems received more recently researchers' special attention due to their applications. However, in many literatures, the results obtained are based on the assumption that the state jumping at the impulsive time instant has a special form; see, for example, [9]. This assumption is not satisfied for most impulsive systems.

The main purpose of this paper is to propose a new design method for guaranteed cost control for linear nominal impulsive systems. The proposed guaranteed cost control method can be said general in a sense that it can also be applied to uncertain impulsive systems.

The organization of this paper is as follows. In section 2, problem formulation and some preliminaries are given. In section 3, guaranteed cost control for linear nominal impulsive systems is considered. In section 4, our main results on constructing a dynamic feedback controller design were presented in order to optimize the quadratic upper bound. A numerical example is provided in section 5.

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II. Problem formulation and preliminaries

Consider linear nominal impulsive systems represented by the following state equations

$$\begin{array}{rcl} \dot{x}(t) &=& A_1x(t)+B_1u_c(t), \quad t\neq t_k, \\ \Delta x(t) &=& (A_2-I)x(t)+B_2u_d(t), \quad t=t_k, \\ y_c(t) &=& C_1x(t), \quad t\neq t_k, \\ y_d(t) &=& C_2x(t), \quad t=t_k, \\ x(t_0) &=& x_0, \end{array} \tag{1}$$

where $x(t)\in R^n$ is the state vector, $y_c(t)\in R^{r_c}$ and $y_d\in R^{r_d}$ are the measurable outputs, $u_c(t)\in R^{l_c}$ and $u_d(t)\in R^{l_d}$ are the control inputs, $A_1, A_2, B_1, B_2, C_1, C_2$ are all real constant matrices.

Given positive-definite symmetric matrices Q_1, Q_2, R_1, R_2 and a scalar $d>0$, we shall consider a cost function represented by

$$\begin{array}{rcl} J &=& \displaystyle\int_0^\infty [x^T(t)Q_1x(t)+u_c^T(t)R_1u_c(t)]dt \\ && \displaystyle +\frac{1}{d}\sum_{j=1}^{\infty}[x^T(t_j)Q_2x(t_j)+u_d^T(t_j)R_2u_d(t_j)]. \end{array} \tag{2}$$

Associated with the cost (2), the guaranteed cost controller is defined as follows:

Definition 1. *Consider the uncertain system (1), if there exist control laws $u_c(t), u_d(t)$ and a positive scalar r, such that the closed-loop system is asymptotically stable and the closed-loop value of the cost function (2) satisfies $J\leq r$, then r is said to be a guaranteed cost and $u_c(t), u_d(t)$ are said to be guaranteed cost controllers for the system (1).*

In this paper, the problem we consider is that determining a dynamic output feedback controller of the form:

$$\begin{array}{rcl} \dot{\hat{x}}(t) &=& A_{c_1}\hat{x}(t)+B_{c_1}y_c(t), \quad t\neq t_k, \\ \hat{x}(t^+) &=& A_{c_2}\hat{x}(t)+B_{c_2}y_d(t), \quad t=t_k, \\ u_c(t) &=& C_c\hat{x}(t), \quad t\neq t_k \\ u_d(t) &=& C_d\hat{x}(t), \quad t=t_k, \end{array} \tag{3}$$

where $\hat{x}(t)\in R^n$ is the controller state, then we will obtain the closed-loop systems by applying the controller (3) to system (1)

$$\begin{array}{rcl} \dot{\bar{x}}(t) &=& \bar{A}_{c_1}\bar{x}(t), t\neq t_k \\ \bar{x}(t^+) &=& \bar{A}_{c_2}\bar{x}(t), t=t_k, \end{array} \tag{4}$$

where

$$\bar{A}_{c_1}=\begin{bmatrix} A_{c_1} & B_{c_1}C_1 \\ B_1C_c & A_1 \end{bmatrix}, \bar{A}_{c_2}=\begin{bmatrix} A_{c_2} & B_{c_2}C_2 \\ B_2C_d & A_2 \end{bmatrix},$$

$$\bar{x}(t)=\begin{bmatrix} \hat{x}(t) \\ x(t) \end{bmatrix},$$

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and the cost function (2) became the following form

$$\begin{aligned} J &= \int_o^\infty [x^T(t)Q_1x(t)+u_c^T(t)R_1u_c(t)]dt \\ &\quad +\frac{1}{d}\sum_{i=1}^{\infty}[x^T(t_j)Q_2x(t_j)+u_d^T(t_j)R_2u_d(t_j)] \\ &= \int_o^\infty [\bar{x}^T(t)\bar{C}_1^T\bar{C}_1\bar{x}(t)dt \\ &\quad +\frac{1}{d}\sum_{i=1}^{\infty}[\bar{x}^T(t_j)\bar{C}_2^T\bar{C}_2\bar{x}(t_j)], \end{aligned}$$

where

$$\bar{C}_1=\begin{bmatrix} Q_1^{\frac{1}{2}} & 0 \\ 0 & R_1^{\frac{1}{2}}C_c \end{bmatrix}, \bar{C}_2=\begin{bmatrix} Q_2^{\frac{1}{2}} & 0 \\ 0 & R_2^{\frac{1}{2}}C_c \end{bmatrix},$$

that asymptotically stabilizes the uncertain system (1) and satisfies $J \le r$.

III. Guaranteed cost control

We are interested in find the least upper bound for the cost function. The following theorem presents an asymptotical stability condition with a guaranteed cost.

Theorem 1. *If for prescribed scalars $\beta>0, \mu\in(0,1]$, there exists a positive definite symmetric matrix $P\in R^{2n\times 2n}$ such that the following matrix inequalities hold:*

$$\bar{A}_{c_1}^TP+P\bar{A}_{c_1}+\frac{\ln\mu}{\beta}P+\bar{C}_1^T\bar{C}_1<0, \tag{5}$$

$$\bar{A}_{c_2}^TP\bar{A}_{c_2}-\mu P+\bar{C}_2^T\bar{C}_2\le 0, \tag{6}$$

then the closed-loop system (4) is asymptotically stable for any impulsive time sequence $\{t_k\}$ satisfies $\sup_k\{t_k-t_{k-1}\}\le\beta$ when $d\ge\mu$ such that the cost function (2) satisfies the following bound $J\le\frac{1}{\mu}trace(P\bar{x}_0\bar{x}_0^T)$ and for any impulsive time sequence t_k satisfies $\sup_k\{t_k-t_{k-1}\}\le\beta$ when $d<\mu$ such that the cost function (2) satisfies the following bound $J\le\frac{1}{d}trace(P\bar{x}_0\bar{x}_0^T)$.

Proof: From (5), there exists a sufficient small $\delta>0$, such that

$$\bar{A}_{c_1}^TP+P\bar{A}_{c_1}+(\frac{\ln\mu}{\beta}+\delta)P+\bar{C}_1^T\bar{C}_1<0. \tag{7}$$

Define a Lyapunov function as follows

$$V(t)=\bar{x}^T(t)P\bar{x}(t), \tag{8}$$

for all $t\in(t_k,t_{k+1}]$. Calculating the derivative of $V(t)$ along the solution of system (4), we can conclude

$$\dot{V}(t)=\bar{x}^T(t)(\bar{A}_{c_1}^TP+P\bar{A}_{c_1})\bar{x}(t)<0. \tag{9}$$

Applying (5) and (7) to (8) yields

$$\dot{V}(t)<-(\frac{\ln\mu}{\beta}+\delta)V(t), t\in(t_k,t_{k+1}], \tag{10}$$

which implies that

$$V(t)<V(t_k^+)e^{-(\frac{\ln\mu}{\beta}+\delta)(t-t_k)}, \tag{11}$$

that is

$$V(t)>V(t_{k+1})e^{\frac{\ln\mu}{\beta}(t_{k+1}-t)}, \tag{12}$$

or

$$V(t)<V(t_k^+)e^{-\frac{\ln\mu}{\beta}(t-t_k)}. \tag{13}$$

Using (6), we have

$$V(t_k^+)=\bar{x}^T(t_k)\bar{A}_{c_2}^TP\bar{A}_{c_2}\bar{x}(t_k)\le\mu V(t_k). \tag{14}$$

On the basis of (11) and (14), we obtain

$$\begin{aligned} V(t) &< \mu^k e^{(-\frac{\ln\mu}{\beta}+\delta)(t-t_0)}V(t_0) \\ &= \frac{1}{\mu}e^{(k+1)\frac{\ln\mu}{\beta}[\beta-\frac{t-t_0}{k+1}]-\delta(t-t_0)}, \end{aligned} \tag{15}$$

where $t\in(t_k,t_{k+1}]$.

The above inequality shows that system (4) is asymptotically stable for $\mu\in(0,1]$ and $\sup_k\{t_k-t_{k-1}\}\le\beta$. In the case of $\mu=1$, (11) becomes $V(t)\le e^{-\delta(t-t_0)}V(t_0), t\ge t_0$, which implies system (4) is asymptotically stable for any impulsive time sequence t_k.

Now let us consider the cost function

$$\begin{aligned} J &= \int_0^T \left[x^T(t)Q_1x(t)+u_c^T(t)R_1u_c(t)\right]dt \\ &\quad +\frac{1}{d}\sum_{j=1}^{\infty}\left[x^T(t_j)Q_2x(t_j)+u_d^T(t_j)R_2u_d(t_j)\right], \end{aligned}$$

where $T\in[t_k,t_{k+1})$.

On the basis of (5), (6), (9) and (10), we obtain

$$\begin{aligned} J &\le -\int_0^T\left[\dot{V}(t)+\frac{\ln\mu}{\beta}V(t)\right]dt \\ &\quad +\frac{1}{d}\sum_{j=1}^{k}\left[\mu V(t_j)-V(t_j^+)\right] \\ &\le V(t_0)-V(T)-\frac{\ln\mu}{\beta}\int_0^T V(t)dt \\ &\quad +\sum_{j=1}^{k}\left[(1-\frac{1}{d})V(t_j^+)+(\frac{\mu}{d}-1)V(t_j)\right] \\ &\le V(t_0)-V(T)-\frac{\ln\mu}{\beta}[\int_0^{t_1}e^{-\frac{\ln\mu}{\beta}(t-t_0)}V(t_0^+)dt \\ &\quad +\int_{t_1}^{t_2}e^{-\frac{\ln\mu}{\beta}(t-t_1)}V(t_1^+)dt+\cdots \\ &\quad +\int_{t_{k-1}}^{t_k}e^{-\frac{\ln\mu}{\beta}(t-t_{k-1})}V(t_{k-1}^+)dt \\ &\quad +\int_{t_k}^{T}e^{-\frac{\ln\mu}{\beta}(t-t_k)}V(t_k^+)dt] \\ &\quad +\sum_{j=1}^{k}\left[(1-\frac{1}{d})V(t_j^+)+(\frac{\mu}{d}-1)V(t_j)\right] \\ &\le V(t_0)-V(T) \\ &\quad +\sum_{j=1}^{k}[(1-\frac{1}{d})V(t_j^+)+(\frac{\mu}{d}-1)V(t_j)] \\ &\quad +\sum_{j=1}^{k-1}(e^{-\frac{\ln\mu}{\beta}(t_{j+1}-t_j)}-1)V(t_j^+) \\ &\quad +(e^{-\frac{\ln\mu}{\beta}(t_1-t_0)}-1)V(t_0) \end{aligned}$$

$$
\begin{aligned}
&\le \frac{1}{\mu}V(t_0) - V(T) \\
&\quad + \sum_{j=1}^{k}[(\frac{1}{\mu}-\frac{1}{d})V(t_j^+) + (\frac{\mu}{d}-1)V(t_j)] \\
&= \frac{1}{\mu}V(t_0) - V(T) \\
&\quad +(d-\mu)\sum_{j=1}^{k}[\frac{1}{\mu d}V(t_j^+) - \frac{1}{d}V(t_j)]. \qquad (16)
\end{aligned}
$$

If $d \ge \mu$,

$$J \le \frac{1}{\mu}V(t_0) - V(T).$$

If $d < \mu$,

$$
\begin{aligned}
J &\le \frac{1}{\mu}V(t_0) - V(T) \\
&\quad +(d-\mu)\sum_{j=1}^{k}[\frac{1}{\mu_d}V(t_j^+) - \frac{1}{d}V(t_j)] \\
&\le \frac{1}{\mu}V(t_0) - V(T) \\
&\quad +(d-\mu)\sum_{j=1}^{k}[\frac{1}{\mu_d}V(t_j^+) - \frac{1}{\mu_d}V(t_{j-1}^+)] \\
&\le \frac{1}{\mu}V(t_0) - V(T) + \frac{d-\mu}{\mu_d}V(t_k^+) \\
&\le \frac{1}{\mu}V(t_0) - V(T) \\
&\quad +\frac{d-\mu}{\mu_d}\exp\frac{In\mu}{\beta}(T-t_k)V(T) \\
&\le \frac{1}{\mu}V(t_0) - \frac{\mu}{d}V(T). \qquad (17)
\end{aligned}
$$

Then we have $J \le \frac{1}{d}V(t_0)$ if $d < \mu$ and $J \le \frac{1}{\mu}V(t_0)$ if $d \ge \mu$. The proof is complete.

Next we consider the case of $\mu \in (1, \infty)$.

Theorem 2. *If for prescribed scalars $\beta > 0$, $\mu \in (1,\infty)$, there exists a positive definite symmetric matrix $P \in R^{2n\times 2n}$, such that the matrix inequalities (5) and(6) hold, then the closed-loop system (4) is asymptotically stable for any impulsive time sequence $\{t_k\}$ satisfying $\inf_k\{t_k - t_{k-1}\} \ge \beta$. Moreover, the cost function (2) satisfies the following bounds:*

$$J \le trace(P\bar{x}_0\bar{x}_0^T), d \ge 1,$$

and

$$J \le \frac{1}{d}trace(P\bar{x}_0\bar{x}_0^T), d < 1.$$

Proof: Similar to the proof of Theorem 1. The inequalities (11)-(15) show that the closed-loop system (4) is asymptotically stable for $\mu > 1$ and $\inf_k\{t_k - t_{k-1}\} \ge \beta$.

Now let us consider the cost function

$$
\begin{aligned}
J &= \int_o^{\infty}[\bar{x}^T(t)\bar{C}_1^T\bar{C}_1\bar{x}(t)dt \\
&\quad +\frac{1}{d}\sum_{j=1}^{\infty}[\bar{x}^T(t_j)\bar{C}_2^T\bar{C}_2\bar{x}(t_j)],
\end{aligned}
$$

where $T \in (0,\infty)$.

On the basis of (5), (6), (12), (13), (14), we imply

$$
\begin{aligned}
J &\le -\int_0^T[\dot{V}(t) + \frac{\ln\mu}{\beta}V(t)]dt \\
&\quad +\frac{1}{d}\sum_{i=1}^{k}[\mu_V(t_j) - V(t_j^+)] \\
&\le V(t_0) - V(T) - \frac{\ln\mu}{\beta}\int_0^T V(t)dt \\
&\quad +\sum_{i=1}^{k}[(1-\frac{1}{d})V(t_j^+) + (\frac{\mu}{d}-1)V(t_j)] \\
&\le V(t_0) - V(T) - \frac{\ln\mu}{d}\{\int_0^{t_1} e^{\frac{\ln\mu}{\beta}(t_1-t)}V(t_1)dt \\
&\quad +\int_{t_1}^{t_2}\exp\{\frac{\ln\mu}{\beta}(t_2-t)\}V(t_2)dt \\
&\quad +\cdots+\int_{t_{k-1}}^{t_k}\exp\{\frac{\ln\mu}{\beta}(t_k-t)\}V(t_k)dt \\
&\quad +\int_{t_k}^{T}\exp\{\frac{\ln\mu}{\beta}(t_{k+1}-t)\}V(t_{k+1})dt \\
&\quad +\sum_{i=1}^{k}[(1-\frac{1}{d})V(t_j^+) + (\frac{\mu}{d}-1)V(t_j)] \\
&\le V(t_0) - V(T) \\
&\quad +\sum_{i=1}^{k}[(1-\frac{1}{d})V(t_j^+) + (\frac{\mu}{d}-1)V(t_j)] \\
&\quad +\sum_{i=1}^{k}[(1-\exp\{\frac{\ln\mu}{\beta}(t_j-t_{j-1})\})V(t_j)] \\
&\quad +[e^{\frac{\ln\mu}{\beta}(t_{k+1}-T)} - e^{\frac{\ln\mu}{\beta}(t_{k+1}-t_k)}]V(t_{k+1})\} \\
&\le V(t_0) - V(T) \\
&\quad +\sum_{i=1}^{k}[(1-\frac{1}{d})V(t_j^+) + (\frac{\mu}{d}-\mu)V(t_j)] \\
&\quad +(e^{\frac{\ln\mu}{\beta}(t_{k+1}-T)} - \mu)V(t_{k+1}) \\
&\le V(t_0) - V(T) \\
&\quad +\frac{d-1}{d}\sum_{i=1}^{k}[V(t_j^+) - \frac{\mu}{d}V(t_j)] \\
&\quad +(e^{\frac{\ln\mu}{\beta}(t_{k+1}-T)} - e^{\frac{\ln\mu}{\beta}(t_{k+1}-t_k)})V(t_{k+1}).
\end{aligned}
$$

which follows that $J \le V(t_0) - V(T)$ for $d \ge 1$ and for $d < 1$,

$$
\begin{aligned}
J &\le V(t_0) - V(T) + \frac{d-1}{d}\sum_{j=1}^{k}[V(t_j^+) - \mu V(t_j)] \\
&\quad +[e^{\frac{\ln\mu}{\beta}(t_{k+1}-T)} - e^{\frac{\ln\mu}{\beta}(t_{k+1}-t_k)}]V(t_{k+1}) \\
&\le V(t_0) - V(T) + \frac{d-1}{d}\sum_{i=1}^{k}[V(t_j^+) - V(t_{j-1}^+)] \\
&\quad +[e^{\frac{\ln\mu}{\beta}(t_{k+1}-T)} - e^{\frac{\ln\mu}{\beta}(t_{k+1}-t_k)}]V(t_{k+1}) \\
&\le \frac{1}{d}V(t_0) - V(T) + \frac{d-1}{d}V(t_k^+) \\
&\quad +[e^{\frac{\ln\mu}{\beta}(t_{k+1}-T)} - e^{\frac{\ln\mu}{\beta}(t_{k+1}-t_k)}]V(t_{k+1}) \\
&\le \frac{1}{d}V(t_0) + (\mu - 1 - \frac{\mu}{d})V(T). \qquad (18)
\end{aligned}
$$

Then we have $J \leq V(t_0)$ for $d \geq 1$ and $J \leq \frac{1}{d}V(t_0)$ for $d \in (0,1)$. The proof is complete.

Remark 1. *Theorem 1 and Theorem 2 can be used to dynamic out put feedback guaranteed cost control for linear uncertain impulsive systems, and the similar results can be obtained.*

Remark 2. *It is easy to see the condition in (5) and (6) is not an LMI with respect to the parameters $P > 0$, and an LMI can be obtained in the following steps (see Section IV).*

IV. Existence condition and parameterization

In this section, we present sufficient conditions for the existence of guaranteed cost controller for linear nominal impulsive systems using the positive definite solutions of LMI's.

Using the Schur complement [10] to (5), we have

$$\begin{bmatrix} \bar{A}_{c_1}^T P + P\bar{A}_{c_1} + \frac{\ln\mu}{\beta}P & \bar{C}_1^T \\ \bar{C}_1 & -I \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} -\mu P & \bar{A}_{c_2}^T P & \bar{C}_2^T \\ P\bar{A}_{c_2} & -P & 0 \\ \bar{C}_2 & 0 & -I \end{bmatrix} \leq 0. \tag{20}$$

Let

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}, P^{-1} = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix},$$

where P_{11}, P_{22}, S_{11}, $S_{22} \in R^{n\times n}$ and definite the matrix

$$M = \begin{bmatrix} S_{11} & I \\ S_{12}^T & 0 \end{bmatrix}, N = \begin{bmatrix} I & P_{11} \\ 0 & P_{12}^T \end{bmatrix},$$

then $P_{12}S_{12}^T = I - P_{11}S_{11}$, $PM = N$, and $M^T PM = \begin{bmatrix} S_{11} & I \\ I & P_{11} \end{bmatrix}$.

Pre- and Post-multiplying (19) by diag$\{M^T, I\}$, combining with Schur complement yields

$$\begin{bmatrix} \Gamma_{11} & A_1 + \hat{A} + \frac{\ln\mu}{\beta}I & S_{11}Q_1^{\frac{1}{2}} & \hat{C}^T R^{\frac{1}{2}} \\ * & \Gamma_{22} & Q_1^{\frac{1}{2}} & 0 \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{21}$$

where $\hat{A} = S_{11}A_1^T P_{11} + S_{11}C_1^T B_{c_1}^T P_{12}^T + S_{12}C_c^T B_1^T P_{11} + S_{12}A_{c_1}^T P_{12}^T$, $\hat{B} = P_{12}B_{c_1}$, $\hat{C} = C_c S_{12}^T$ and $\Gamma_{11} = A_1 S_{11} + B_1\hat{C} + (A_1 S_{11} + B_1\hat{C})^T$, $\Gamma_{22} = P_{11}A_1 + \hat{B}C_1 + (P_{11}A_1 + \hat{B}C_1)^T + \frac{\ln\mu}{\beta}P_{11}$.

Similarly, Pre- and Post-multiplying (20) by diag$\{M^T, M^T, I\}$, combining with Schur complement yields

$$\begin{bmatrix} -\mu S_{11} & -\mu I & S_{11}A_2^T + \hat{C}_d B_2^T & \hat{A}_d^T & S_{11}Q_2^{\frac{1}{2}} & \hat{C}_d R^{\frac{1}{2}} \\ * & -\mu P_{11} & A_2^T & A_2^T P_{11} + C_2^T \hat{B}^T & Q_2^T & 0 \\ * & * & -S_{11} & -I & 0 & 0 \\ * & * & * & -P_{11} & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0. \tag{22}$$

where $\hat{A}_d^T = S_{11}A_2^T P_{11} + S_{11}C_2^T B_{c_2}^T P_{12}^T + S_{12}C_d^T B_2^T P_{11} + S_{12}A_{c_2}^T P_{12}^T$, $\hat{B}_d = P_{12}B_{c_2}$, $\hat{C} = S_{12}C_d^T$.

Then we obtained $S_{11}, \hat{C}, \hat{B}, \hat{A}, P_{11}$ from the LMIs (21)-(22), so we can get P_{12}, S_{12} from $I - P_{11}S_{11} = P_{12}S_{12}^T$ and the solutions of the controller

$$\begin{aligned} A_{c_1}^T &= S_{12}^{-1}(\hat{A}^T - S_{11}A_1^T P_{11} \\ &\quad - S_{11}C_1^T B_{c_1}^T P_{12}^T - S_{12}C_c^T B_1^T P_{11})(P_{12}^T)^{-1}, \\ B_{c_1} &= P_{12}^{-1}\hat{B}, \\ C_c &= \hat{C}(S_{12}^T)^{-1}, \\ A_{c_2}^T &= S_{12}^{-1}(\hat{A}_d{}^T - S_{11}A_{21}^T P_{11} \\ &\quad - S_{11}C_2^T B_{c_2}^T P_{12}^T - S_{12}C_d^T B_2^T P_{11})(P_{12}^T)^{-1}, \\ B_{c_2} &= P_{12}^{-1}\hat{B}_d, \\ C_d^T &= (S_{12}^T)^{-1}\hat{C}. \end{aligned}$$

Now we suggest the following nonlinear minimization problem involving LMI conditions instead of the original convex minimization problem

when $\beta > 0, \mu \in (0,1], d \geq \mu$, we have

$$\min_{P_{11},S_{11},d,\mu} \frac{1}{\mu} trace(P\bar{x}_0\bar{x}_0^T)$$

subject to

(i) (21)-(22); (ii) $\begin{bmatrix} S_{11} & I \\ I & P_{11} \end{bmatrix} < 0.$

when $\beta > 0, \mu \in (0,1], d < \mu$, we have

$$\min_{P_{11},S_{11},d,\mu} \frac{1}{d} trace(P\bar{x}_0\bar{x}_0^T)$$

subject to

(i) (21)-(22); (ii) $\begin{bmatrix} S_{11} & I \\ I & P_{11} \end{bmatrix} > 0.$

when $\beta > 0, \mu > 1, d \geq 1$, we have

$$\min_{P_{11},S_{11},d,\mu} trace(P\bar{x}_0\bar{x}_0^T)$$

subject to

(i) (21)-(22); (ii) $\begin{bmatrix} S_{11} & I \\ I & P_{11} \end{bmatrix} < 0.$

when $\beta > 0, \mu > 1, d < 1$, we have

$$\min_{P_{11},S_{11},d,\mu} \frac{1}{d} trace(P\bar{x}_0\bar{x}_0^T)$$

subject to

(i) (21)-(22); (ii) $\begin{bmatrix} S_{11} & I \\ I & P_{11} \end{bmatrix} > 0.$

V. Numerical Example

Consider the linear nominal impulsive systems (1) with parameters as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.3 & 0.1 \\ 0 & 1.3 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.5 & 1 \\ 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0.5 \\ 0 & 0.5 \end{bmatrix}.$$

Assume that $\beta = 0.2$, $\mu = 0.8$, the initial state $\bar{x}(0) = (-2.5, 2.1)^T$. By using Matlab2014a, we get the solutions of the guaranteed cost controller

$$
\begin{aligned}
P &= \begin{bmatrix} 0.6105 & -0.0028 \\ -0.0028 & 0.7518 \end{bmatrix}, \\
A_{c_1} &= \begin{bmatrix} -12.1178 & -0.1817 \\ -217.5018 & 188.7909 \end{bmatrix}, \\
A_{c_2} &= \begin{bmatrix} -18.7039 & -0.2553 \\ -135.3866 & 8.8084 \end{bmatrix}, \\
B_{c_1} &= \begin{bmatrix} -1.0517 & -1.8333 \\ -168.0540 & -60.2795 \end{bmatrix}, \\
B_{c_2} &= \begin{bmatrix} -8.0523 & -11.7441 \\ -129.4567 & -53.5444 \end{bmatrix}, \\
C_c &= \begin{bmatrix} 6.0479 & 0 \\ 0 & 11.0301 \end{bmatrix}, \\
C_d &= \begin{bmatrix} 1.9476 & 0 \\ 0 & 1.6740 \end{bmatrix}.
\end{aligned}
$$

Furthermore,

$$
\begin{aligned}
&\bar{A}_{c_1}^T P + P\bar{A}_{c_1} + \frac{\ln\mu}{\beta}P + \bar{C}_1^T\bar{C}_1 = -0.0167 < 0, \\
&\bar{A}_{c_2}^T P\bar{A}_{c_2} - \mu P + \bar{C}_2^T\bar{C}_2 = -0.0813 \leq 0.
\end{aligned}
$$

From Theorem 1, the closed-loop system (4) is asymptotically stable for any impulsive time sequence $\{t_k\}$ satisfies $\sup_k\{t_k - t_{k-1}\} \leq 0.2$. Let $d = 1$, $d \geq \mu$, the cost function (2) satisfies $J \leq 17.7$. Let $d = 0.5$, $d < \mu$, the cost function (2) satisfies $J \leq 20.3$.

VI. Conclusion

This paper studied an approach to dynamic output feedback guaranteed cost control problem for linear nominal impulsive systems. The existence results of the guaranteed cost control are obtained. Our method is helpful to improve the existing technologies used in the analysis and control for linear nominal impulsive systems. Moreover, it is important to notice that the methods and technologies used in this paper can be extended to many other types of dynamic systems with impulses; see, for example, [11-15]. Future work will include impulsive dynamic systems modeling and analysis.

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