Mode Adaptive Demixing Based on EMD in FMCW System

Xinyu Dao, Min Gao, and Cheng Cheng

Abstract—**The problem of mode mixing in the empirical mode decomposition (EMD) vastly restricts extraction of the useful signal and the performance of denoising. An adaptive and convenient demixing algorithm to promote the separation ability of noise and expected signal is proposed. After decomposing the original signal, a multiple-input and multiple-output mechanism is designed to realize optimal reconstruction and decomposition of the first order intrinsic mode functions (imfs). The correlation of each imf is utilized to ascertain appropriate reconstruction order without additive introduction of parameters and components. Compared with EMD, the proposed algorithm maintains the better demixing performance indicated in the measurement results of synthetic signals and real-life frequency modulated continuous wave radar signal.**

Index Terms—**Mode mixing, Empirical mode decomposition (EMD), Adaptive demixing, Frequency modulated continuous wave**

I. Introduction

THE empirical mode decomposition (EMD) has been extensively employed to analyse non-linear and non-stationary signals. Recent works have demonstrated that the empirical mode decomposition acts essentially as a dynamic filter bank. It can split composite signals adaptively into narrow subbands. However, as an off-line approach, the main problems of EMD are low frequency resolution, mode mixing and inseparability of informative components and noise. To improve performance of denoising, many EMD-based denoising approaches have been proposed while the limitation of denoising effect is still the mode mixing [1-5]. Thus, the salient task in this paper is to pursuit an adaptive and effective method to solve the mode mixing.

Mode mixing, a well-recognized limitation in EMD, means that one or more components appear in a single intrinsic mode function (imf) [6], [7]. The phenomenon is caused by intermittency of a signal component or closely spaced spectral tones. When mode mixing occurs, the imfs obtained by decomposition can be devoid of any physical meaning and it is not benefited for signal denoising or frequency analysis. To solve the problem of mode mixing, Huang et al [8] proposed a noise-assisted method, namely ensemble empirical mode decomposition (EEMD). R. Deering and J. F. Kaiser proposed a masking signal method to improve EMD [6]. EMD combined with independent component analysis was put forward in [9]. However, except EEMD, other methods could overcome mode mixing merely under certain conditions [10]. The EEMD deals with noise interference that has been shown to change the zero and extremes point of signal distribution [11]. EEMD, considering the Gauss white noise, alleviates affection of mode mixing whereas the ensemble number and parameters of noise are difficult to choose. Meanwhile, the ensemble average procedure of white noise inevitably reduces the computational efficiency. Actually, the goal of decomposition is to separate noise and desired signal effectively so as to acquire useful information. Hence, we design a multiple-input and multiple-output (MIMO) system to realize second order decomposition. Without redundant addition of other components, the method could also be applied commendably to extract target information in the frequency modulated continuous wave (FMCW) radar system.

In this study, the relevant theory of EMD is firstly introduced in section II. Then, the proposed demixing method is presented in section III. In section IV, to verify the effect of the proposed method, three typical measured signals contaminated by noise were simulated to compare performance with different techniques. Finally, the practical FMCW signal was conducted to verify the feasibility of the proposed method.

II. Basis of EMD

EMD is a time-frequency analysis approach that adaptively decomposes a signal into a series of intrinsic mode functions permuted in the descending order of frequency. Normally, any one-dimensional discrete signal with multiple modes of oscillation can be decomposed into different imfs and a residual component after EMD. The imf must satisfy two conditions: (1) Zero-crossing condition, namely the number of extrema(including maximum and minimum) and zero-crossings is required to be equal or differs by one at most. (2) Mean condition, it means that the mean value of the envelope constructed by the local maximum and local minimum is zero at any point. The EMD is a total data processing method based on the local characteristics of the time scale, which the basis function or the parameters of filters are not determined in advance. Therefore, the instantaneous frequency obtained by EMD possesses a strong

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physical meaning. The sifting process is described as Table. I. Detail introduction can be found in [12].

TABLE I
Sifting process of EMD

Algorithm: Sifting process of EMD
Step1: Find out all local maximum and local minimum value of the original signal $s(t)$
Step2: Form the upper envelope $e_u(t)$ and lower envelope $e_d(t)$ using the cubic spline interpolation curve constructed by the local maximum and minimum of $s(t)$ respectively
Step3: Calculate the local mean $m_1(t)$ of the $s(t)$,namely $m_1(t)=(e_u(t)+e_d(t))/2$
Step4: Calculate the difference between $s(t)$ and $m_1(t)$: $h_1(t)=s(t)-m_1(t)$
Step5: Repeat Step1~Step4 until $h_1(t)$ satisfy the conditions of imf
Step6: After obtaining the first imf $h_1(t)$, calculate the residue $r_1(t)$: $r_1(t)=s(t)-h_1(t)$
Step7: If the $r_1(t)$ is either monotonous or a constant, terminate the decomposition. If not, repeat Step1~Step5

After decomposing, the original signal can be expressed as follows:

$$s(t)=\sum_{i=1}^{n} h_i(t)+r_n(t) \tag{1}$$

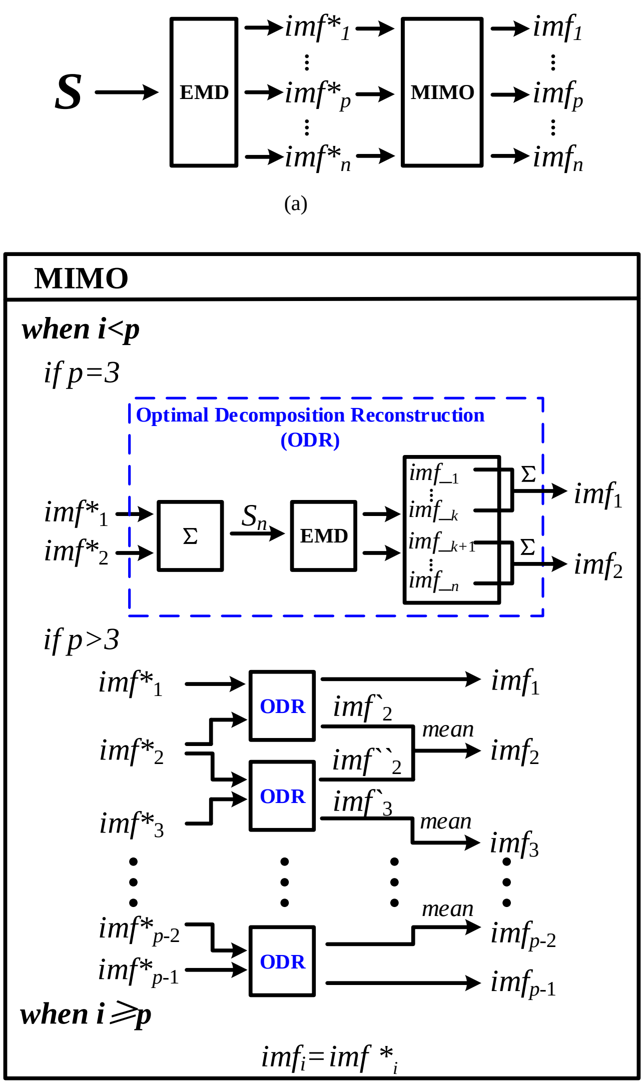
Where $h_i(t)$ represents the imf, $r_n(t)$ is the residual component.

III. Proposed Demixing Method

Conventional EMD processing is regarded as an example of Single-Input and Multiple–Output system. It means that an input signal produces multiple imfs after decomposing. However, the obtained imfs inevitably suffer from mode mixing problem. Inspired by the solution of interference problem in multi-channel digital transmission system, we design a Multiple-Input and Multiple-Output system to perform the demixing.

The proposed method consists of the consecutive application for each pair of adjacent imfs. The kernel of the MIMO is to determine the segment that needs reconstruction. Utilizing the correlation between imfs, the proper remixing is performed adaptively. The whole procedure of the MIMO can be described briefly as follows: remixing, applying classic EMD, splitting new imfs into non-overlapping subsets, and reconstructing to acquire the demixed imfs. The entire procedure is also called the optimal decomposition reconstruction (ODR) in this proposed method.

The first order imfs are reconstructed to different new signals (S_n) according to the correlation between the original signal and the estimated signal that has been removed noise component. Each pair of adjacent imfs that are decided to remix need to experience classical EMD again so that the overlapped information is separated as much as possible. The new imfs are divided into two parts with the smallest correlation. Then, sum each imf within respective part to obtain the ultimate imfs. The detailed description of the proposed method is shown in Fig. 1.



(a)

(b)

Fig. 1 Schematic diagram of proposed method
(a) Procedure of demixing
(b) Principle of MIMO

The correlation is depicted by the correlation coefficient (CC) that is defined as follows:

$$\rho(m)=\sum_{t=0}^{N} s(t)s_m^*(t) \Big/ \sqrt{\sum_{t=0}^{N} s^2(t)\sum_{t=0}^{N}(s_m^*)^2(t)} \tag{2}$$

Where $s_m^*(t)$ is the estimated signal given by [13]:

$$s_m^*(t)=s(t)-\sum_{i=1}^{m} h_i(t) \tag{3}$$

Where m=p-1, the value for p is the critical order that determines whether imf need remix. Generally, p is the last value in CC bigger than C. As mentioned in [13], C belongs to [0.75, 0.85], here, C is set to 0.85 in the proposed method due to its optimal performance [14].

After the second decomposition, the key of reconstruction is to find a critical position in the set of imfs. Hence, the value for k denotes the minimum correlation between the sum of higher order imfs and the sum of lower order imfs. It is defined as follows [15]:

$$k=\arg\min_{1<k<n} \left|corr\left(\sum_{1\le i\le k} imf_{\_i},\ \sum_{k<i\le n} imf_{\_i}\right)\right| \tag{4}$$

From Fig. 1, it can be found that the lower orders imfs after the p-th imf are not altered severely. Considering less noise contained in lower orders has a little effect on the desired signal, we sum the lower orders imfs after the *p*-th imf directly to maintain the calculation efficiency of the system.

IV. Verification

A. Classical Test Signals Verification

To explore the performance of the proposed method, three synthetic signals, namely 'Bumps', 'Doppler' and 'Blocks' signals with different signal-noise-ratio (SNR), are conducted to simulate by MATLAB. As an example, signals contaminated by white Gaussian noise are simulated in this paper, where the number of samples is 1000 and the input SNR is 5dB. The three synthetic signals are demonstrated in Fig. 2(a). For the decomposition results, here, we merely demonstrate 'Bumps' signal as shown in Fig. 2(b) and Fig. 2(c) due to limitation of the paper length.

The aim of the simulation experiment is to compare the conventional method of the EMD-based denoising with the proposed method in this study. The decomposition results of 'Bumps' signal applying classic EMD and proposed method are given in Fig. 2(b) and Fig. 2(c), respectively. From Fig. 2(b) (Only the first five imfs are illustrated), we can notice that the phenomenon of mode mixing is extremely serious especially in the lower order components with the classic EMD. Most useful information is still engulfed in the noise. However, it is obvious that the characters of signal are manifested clearly in the proposed method shown in the Fig. 2(c).

To verify the performance of separating noise in different decomposition methods (EMD, EEMD and Proposed method), three kinds of synthetic signals are simulated with the input SNR varied from -3dB to 7 dB in steps of 2dB. Discarding the same high order noise, the output SNR results after denoising are presented in Table II~Table IV. As seen in the Tables, each technology significantly denoises the signal, but it can be found that the proposed method provides the highest output SNR and the better performance compared with other methods. For example, the denoised Bumps signals are illustrated in Fig. 3. When the input SNR is set to 5 dB with the noisy Bumps signal, the output SNR achieved by EMD, EEMD and proposed method are 10.9590, 11.9418, 12.9543 dB, respectively.

The root mean square error (RMSE) with different input SNR, index of orthogonality (IO) and costing time are also used to evaluate the effect of different methods. The RMSE can be computed as:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(x(n)-x'(n))^2} \tag{5}$$

The RMSE is usually employed to compute the error of the decomposed component with the real component. Less RMSE value means a more accurate component.

The IO is defined as [11]:

$$\text{IO} = \sum_{t=0}^{T}\left(\sum_{j=1}^{n}\sum_{i=1}^{n} I_j(t)I_i(t)/x^2(t)\right) \tag{6}$$

Where $x(t)$ denotes the original signal, $I_i(t)$ denotes the imf component. When $i=n$ or $j=n$, namely the last $I_n(t)$ represents the residue $r_n(t)$. Less IO means that the decomposing result is more orthogonal with less frequency mixing [10].

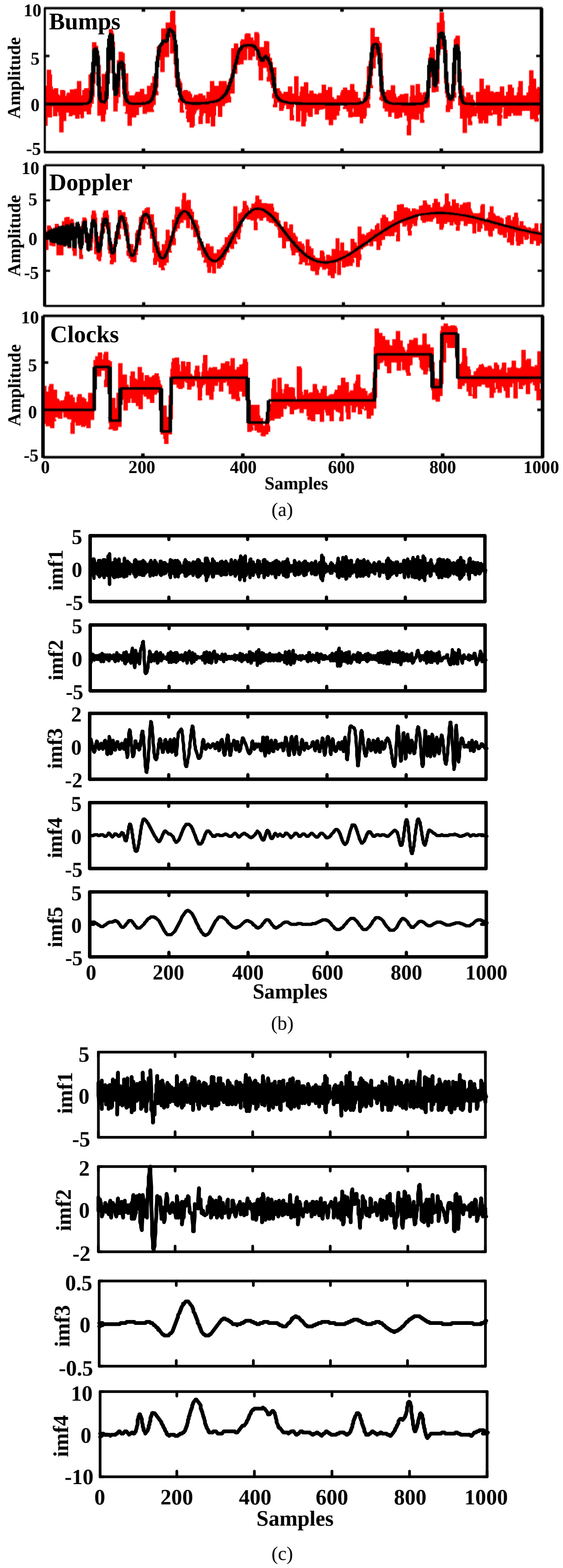


Fig. 2 Three signals and corresponding results of decomposition
(a) Original 'Bumps', 'Doppler' and 'Clocks' signals (The black line is pure signal and the red lines are contaminated by noise signal with the SNR=5)
(b) Decomposition results of Bumps signal with classic EMD
(c) Decomposition results of Bumps signal with proposed method

TABLE II
Results of three methods on the noisy Bumps signal

SNR(dB)	EMD(dB)	EEMD(dB)	**Proposed method(dB)**
-3	0.8897	0.9723	**1.1180**
-1	1.5472	1.9892	**3.0032**
1	5.3428	6.5342	**7.9934**
3	8.4106	9.4445	**11.4598**
5	10.9590	11.9418	**12.9543**
7	11.6988	12.2955	**14.1852**

TABLE III
Results of three methods on the noisy Doppler signal

SNR(dB)	EMD(dB)	EEMD(dB)	**Proposed method(dB)**
-3	0.5301	0.3944	**0.3898**
-1	1.7831	2.0034	**3.1087**
1	5.2865	6.3525	**8.0211**
3	8.4245	9.4087	**10.7728**
5	10.5630	11.1069	**11.8592**
7	11.4036	12.0507	**13.9096**

TABLE IV
Results of three methods on the noisy Clocks signal

SNR(dB)	EMD(dB)	EEMD(dB)	**Proposed method(dB)**
-3	3.2673	3.2623	**3.4073**
-1	1.9235	2.4463	**3.9987**
1	5.7638	6.4826	**7.9956**
3	8.1392	9.1021	**10.2110**
5	10.1636	10.6916	**10.6284**
7	11.3874	12.45388	**14.0042**

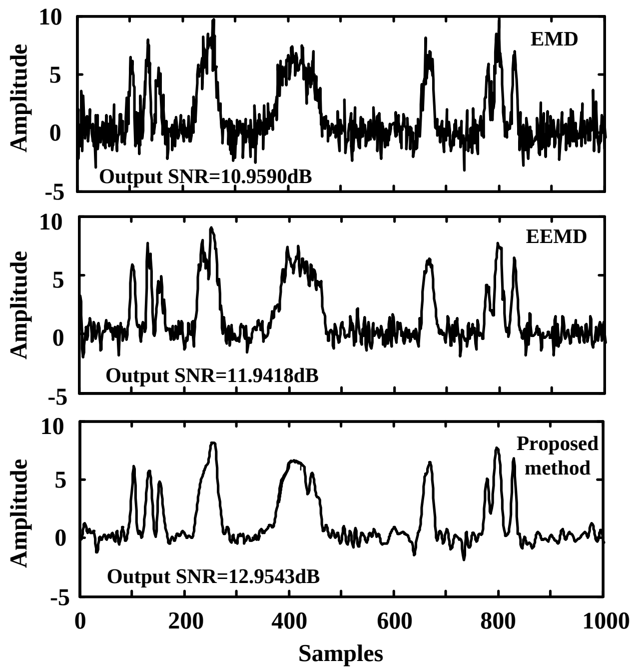


Fig.3 Denoised Bumps signal with the input SNR=5dB

We choose the input SNR varied from -3dB to 13 dB in steps of 2dB to conduct tests. The RMSE of three synthetic signals are shown in Fig. 4~Fig.6. Moreover, operate the three methods to decompose the three signals in the same computer with CPU: Intel(R) Core(TM), Memory: 4.0 GB, and compute the IO and the costing time.

The IO of the three signals with different input SNR is depicted in Fig.7~Fig.9. The costing time of operating is presented in Table V.

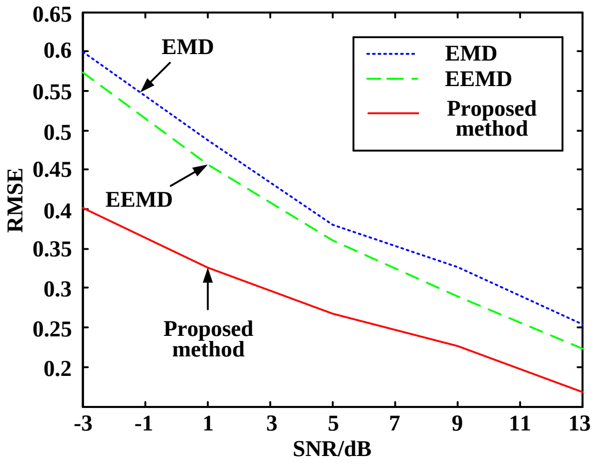


Fig. 4 The RMSE of different methods in 'Bumps' signal

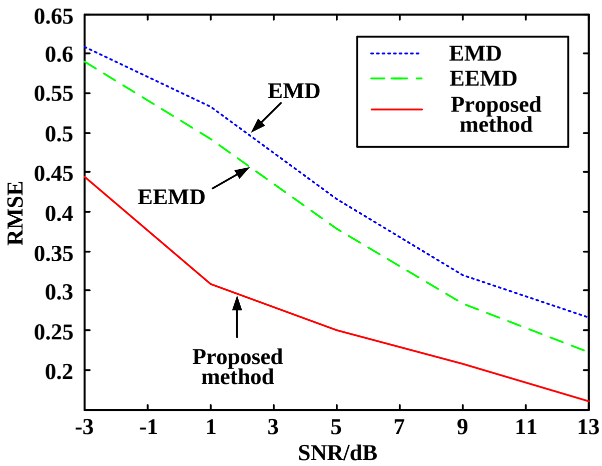


Fig. 5 The RMSE of different methods in 'Doppler' signal

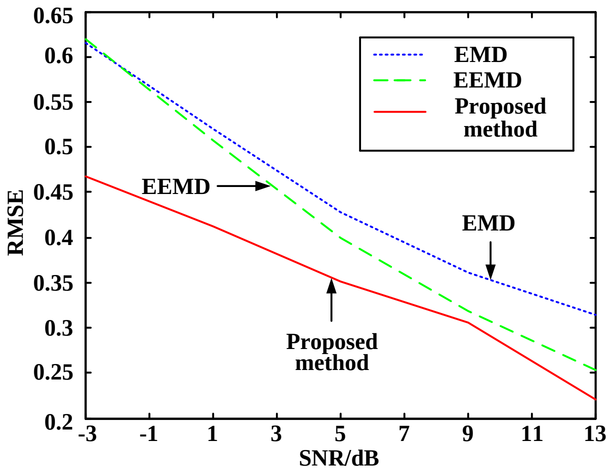


Fig. 6 The RMSE of different methods in 'Blocks' signal

TABLE V
The costing time of three methods in three signals

	EMD(ms)	EEMD(ms)	**Proposed method(ms)**
Bumps	1.05	38.42	**5.32**
Doppler	2.43	39.72	**6.73**
Blocks	2.78	39.89	**7.95**

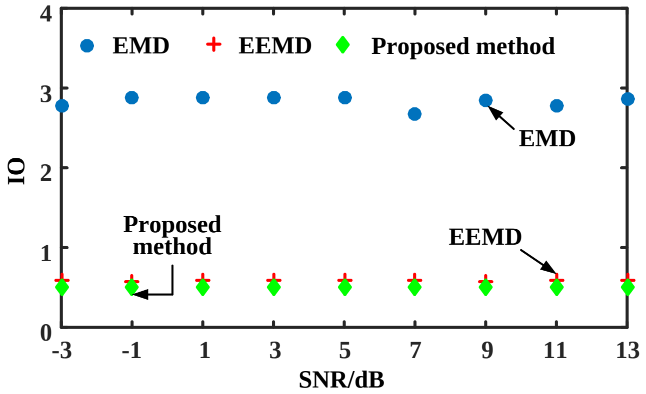


Fig. 7 The IO of 'Bumps' signal with different input SNR

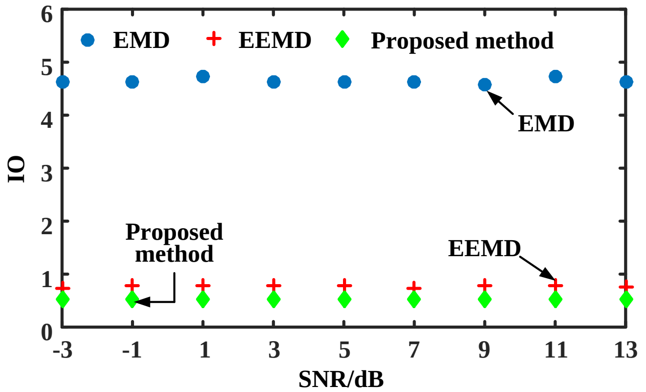


Fig. 8 The IO of 'Doppler' signal with different input SNR

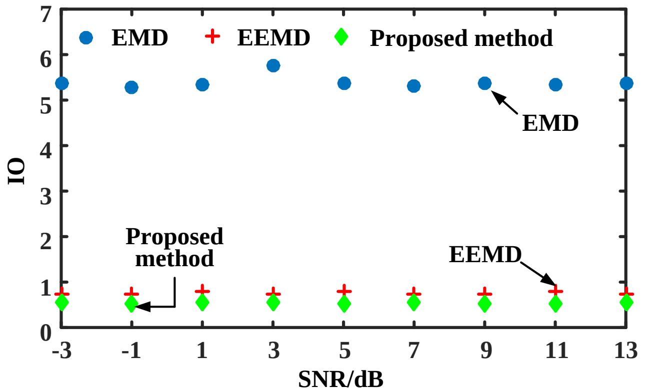


Fig. 9 The IO of 'Blocks' signal with different input SNR

From Fig.4~Fig.6, it can be found that the proposed method has the least RMSE. That means the proposed method maintains more accurate component. The results of IOs presented in Fig.7~Fig.9 indicate that the proposed method has the best orthogonality and the least IO value among the three methods for the three signals. Additionally, the IO value varies little with the input SNR. It implies that the proposed method could reserve orthogonality no matter what the input SNR is.

From the Table V, it is obvious that the EEMD consumes the most time while the proposed method costs the least time. Without abundant ensemble computing like EEMD, the proposed method only conducts twice EMD. Thus, it saves more computing time

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B. Synthetic Signals Verification

In order to illustrate the capability of our proposed method in mode demixing in synthetic signals, the test signal composed of multiple monotone signals is continuously simulated. Considering that most signals could be decomposed into several monotone signals, here, we set the test signal containing three different frequency as an illustration. The synthetic signal is described as,

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t) + \cos(2\pi f_3 t) + n(t) \quad (7)$$

Where, $f_1 = 5$ Hz , $f_2 = 10$ Hz , $f_3 = 20$ Hz and $n(t)$ is the additive white Gaussian noise. The samples length is set to 300 and the sampling frequency satisfy the Nyquist sampling frequency. The input SNR is set at 5dB. The synthetic signal is processed by the three methods and merely the frequency spectrum of decomposition is given due to our concern in frequency demixing. The input signal is shown in Fig.10 and the frequency spectrum of three decomposition methods is presented in Fig.11 ~Fig.13, respectively.

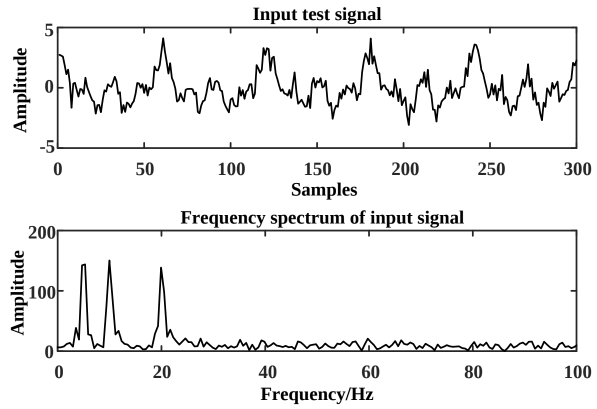


Fig. 10 The input signal and its frequency spectrum

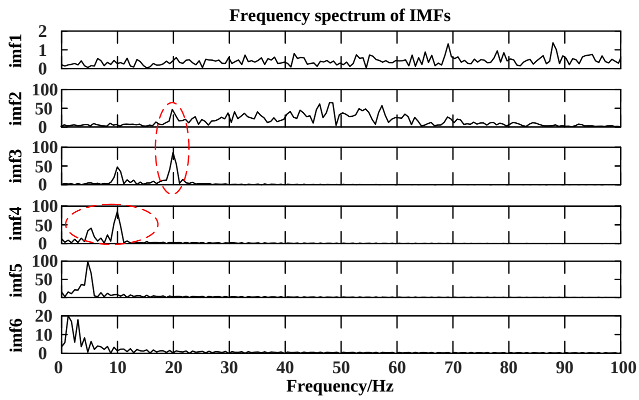


Fig. 11 The frequency spectrum of EMD

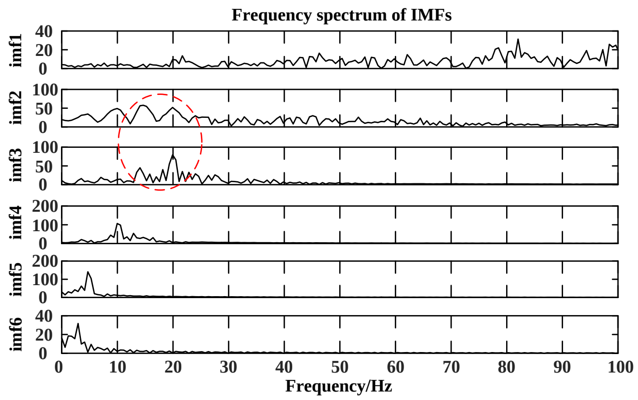


Fig. 12 The frequency spectrum of EEMD

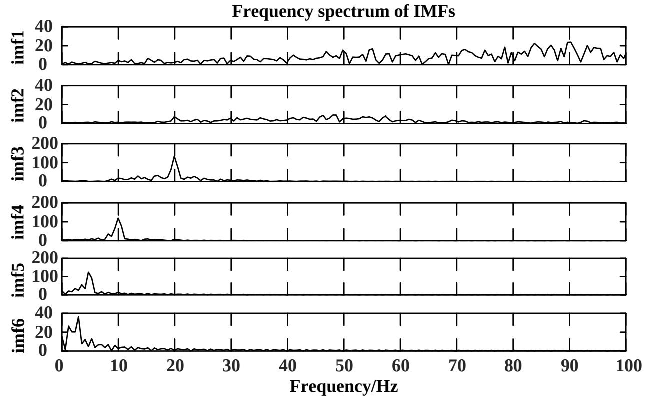


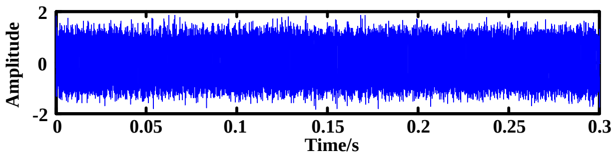
Fig. 13 The frequency spectrum of proposed method

From the Fig.11~Fig. 13, it can be observed that the mode mixing of the test signal after EMD is the most serious. Not only the same frequency component appears in different imfs, but also multiple frequency components appear in the same imf. That case has been alleviated in EEMD while the same frequency components still appear in imf2 and imf3. Obviously, it is not conducive to the extraction of the desired signal. However, in the Fig.13, we notice that the phenomenon of mode mixing is relived very well. The imf1 and imf2 is dominated by noise and the three different frequencies are clearly separated in imf3~imf5.

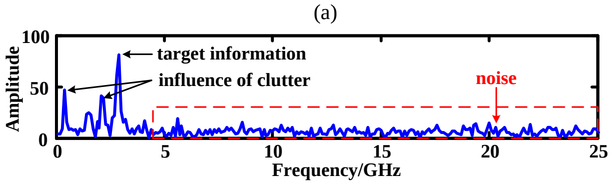
C. Real FMCW Radar Signal Verification

To verify the application of the proposed method in real-life condition, we take two tests to achieve the different purpose. Firstly, we collect the echo signal to acquire the range information in real FMCW radar system. Then, we investigate the ability of reducing the noise from the echo signal.

The original carrier frequency is set to be 24GHz, the modulated period is 10 μs , the maximum frequency deviation is 2GHz, and the original distance from radar to target is 20m. The echo signal in receiver is presented in Fig.14. For the echo signal, the three methods are implemented to decompose the signal. The decomposition results of EMD, EEMD and the proposed method are shown in Fig.15 (a), Fig.15 (b) and Fig.15 (c), respectively.

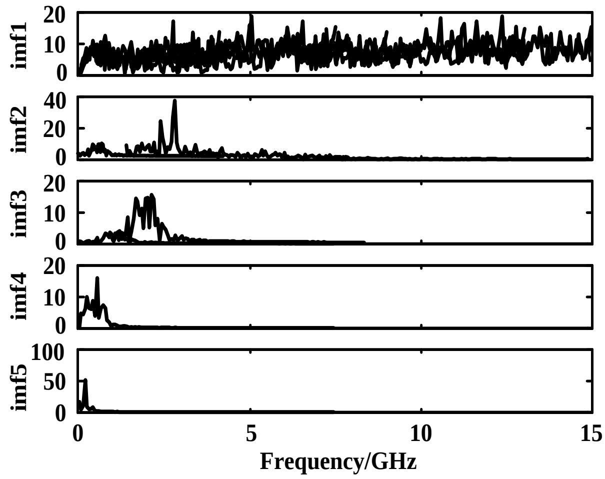


(a)

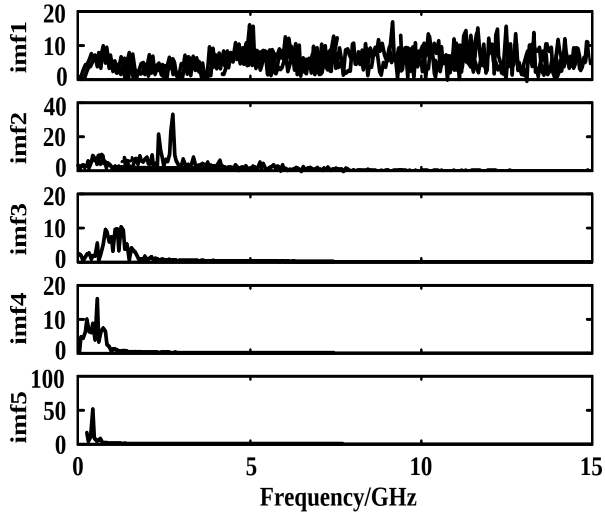


(b)

Fig. 14 Realistic echo signal in FMCW
(a) Echo signal in time domain
(b) Echo signal in frequency domain



(a)



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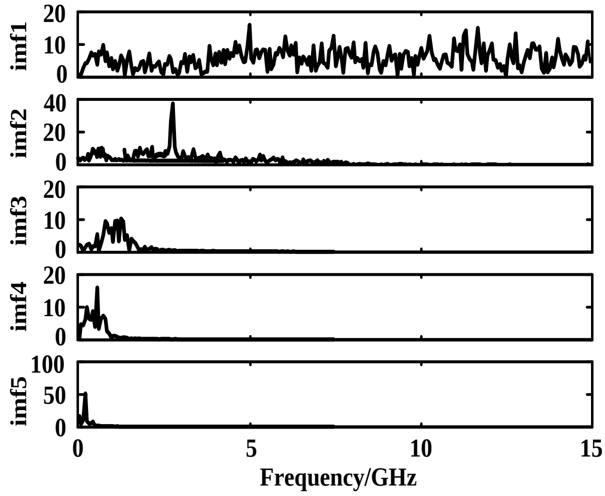


(c)

Fig. 15 Decomposition results of echo signal with the three methods
(a) Decomposition results of EMD
(b) Decomposition results of EEMD
(c) Decomposition results of the proposed method

The echo signal contains amount of noise and serious ground clutter interference beside desired target information shown in the Fig.14. Thus, in order to acquire the precise target information, it is essential to separate the useful signal from interference source. From the Fig.15, we can notice that the echo signal is divided into five imfs in the frequency domain. The Fig.15 (a) and Fig.15 (b) show that the mode mixing occurs in the imf2 or imf3. However, the target information is differentiated greatly from the noise and clutter in the Fig.15 (c). The first order is dominated by the system noise. The influence of clutter is reflected from imf3 to imf5. The range information of target is mainly contained in imf2. By designing a proper narrow band filter, it is easy to extract the range information.

We continue to investigate the capability of denoising in the outdoor condition. The experimental environment is described as Fig.16. The distance from FMCW radar to land is 10m and the angle of incidence is 45°. The collected signal is processed by the three methods mentioned in this paper. Decompose the echo signal by the three methods and the results are shown in Fig.17~Fig.19. It is clear that the main difference is the high order components. The noise leads that the mode mixing occurs more easily during the whole procedure of decomposition.

Discard the same high order noise and reconstruct all residues. The signals processed by three methods are presented in Fig.20 (a) ~ Fig.20 (c). In order to observe the characteristic of the processed signal more clearly, the signals which samples are from 5000 to 5200 are illustrated in Fig.20 (d). It can be found that the signal processed by the proposed method is similar to original frequency modulated signal. By computing the SNR of the processed signals, the proposed method has the highest SNR with 15.678dB while the SNR of EMD and EEMD is 10.438dB and 12.653dB, respectively.

Fig. 16 Experimental scenario

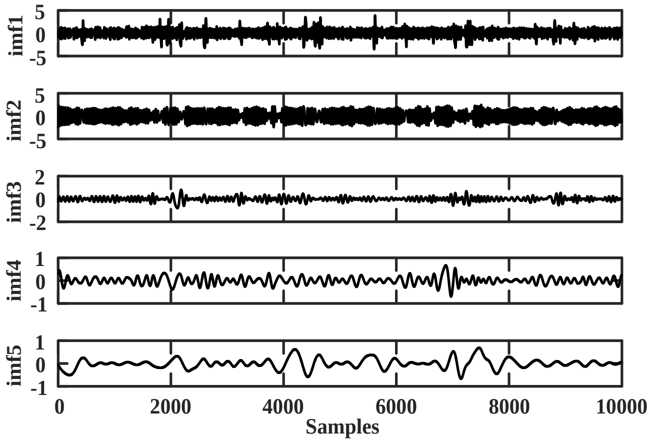


Fig. 17 Decomposition results of EMD

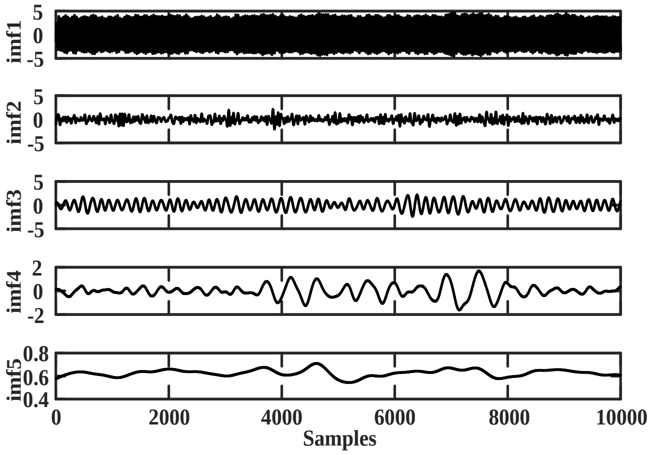


Fig. 18 Decomposition results of EEMD

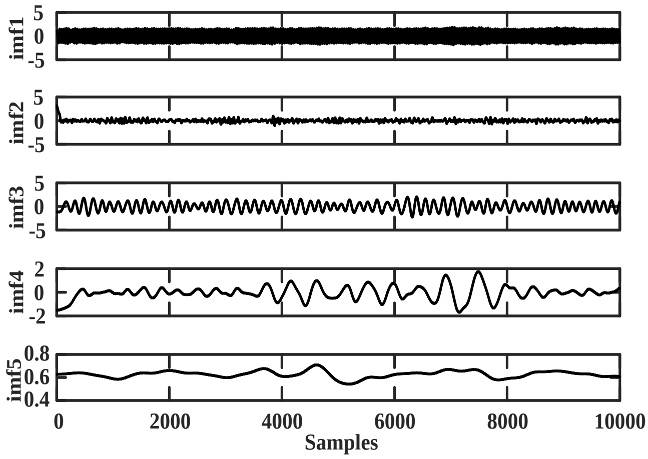
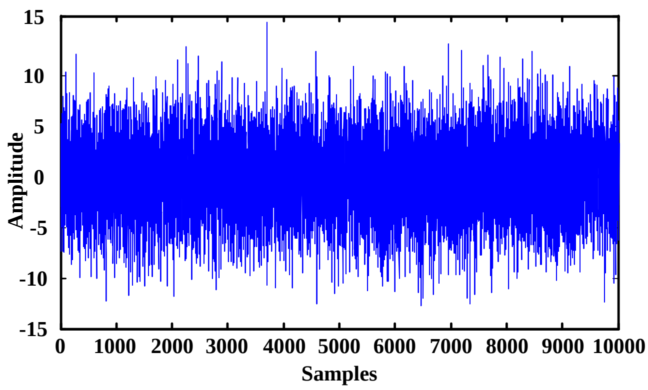
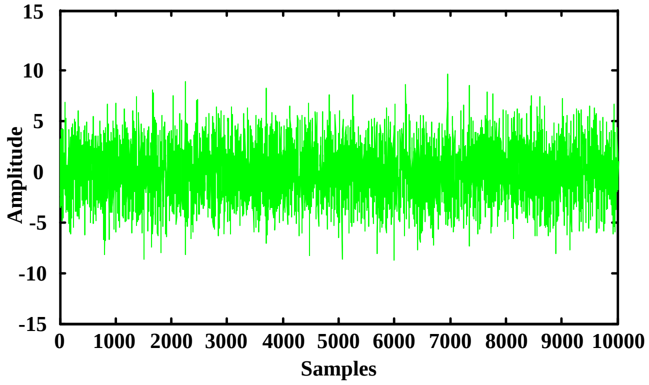


Fig. 19 Decomposition results of the proposed method



(a)



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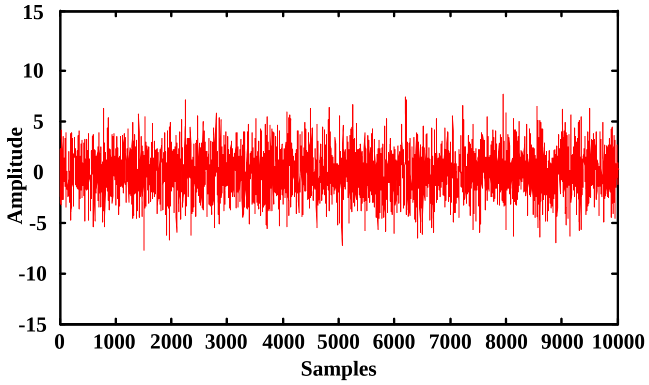
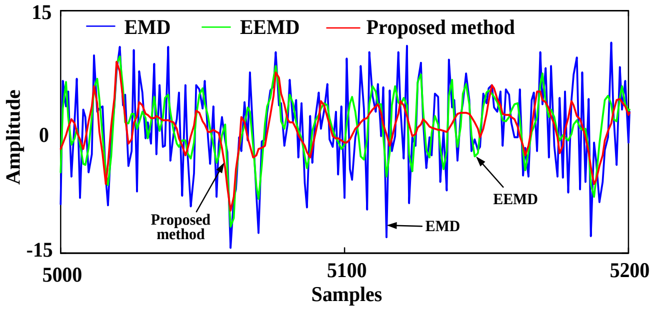


(c)



(d)

Fig. 20 Processing results of echo signal with the three methods
(a) Processing results of EMD
(b) Processing results of EEMD
(c) Processing results of the proposed method
(d) Partial signals (samples from 5000 to 5200)

V. CONCLUSION

In this paper, a MIMO system is proposed to improve the ability of demixing in EMD. The correlation between disparate imfs is utilized to reconstruct and decompose signal. Different from other remixing methods, the proposed algorithm only reconstructs two adjacent imfs once a time so that the overlapped information is greatly distinguished. Moreover, this method avoids the chosen of the noise parameters and ensemble number like EEMD. The simulation results show that the approach in this paper manifests better demixing performance. In real experiment of FMCW radar system, target information is also separated and extracted easily by this algorithm.

Theoretically speaking, the proposed method is more convenient to achieve and can reduce more calculation time. It indicates that the method could pave the way for the application of rapid signal processing with the requirement of high precision.

In the future, we will intend to apply the proposed method to more complex signals and improve the calculation efficiency ulteriorly so that the computational cost could be reduced to deal with the real-time signals.

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