Real Time Reliability Evaluation and Residual Life Prediction for Individual Product based on Gamma Degradation Process

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Abstract—In this paper, a real time reliability evaluation and residual life prediction methods for an individual product based on Gamma degradation process are studied. In order to obtain the real time reliability assessment, a conditional reliability model is established and a Bayesian updating theory is used to update the model parameters. In order to obtain the residual life of individual product, the lifetime distribution is approximated by using a Birnbaum-Saunders distribution. Considering the model is very complicated, the estimators of the unknown parameters are obtained via Markov Chain Monte Carlo method. At last, a numerical example about laser device is given to verify the validity of the present model and method.

***Index Terms*—Gamma process, Residual life, Bayesian method, Real time reliability Evaluation**

I. Introduction

THE traditional reliability assessment methods completely relied on the lifetime data[1]. However, most modern products have long lifetime and high reliability, therefore, the lifetime data of these products are often hard to obtain. From an economical and practical viewpoint, degradation data can be used as an alternate resource for lifetime analysis [2]. In the last decades, degradation data has played an important role in reliability assessment.

For most electrical and mechanical products, degradation is a common phenomenon, and it can be described by a continuous performance process in terms of time [3]. Because the stochastic process can very flexibly characterize the failure generating mechanisms and runtime environment properties, many articles have used the stochastic process approach to model the degradation path, such as Markov process, Wiener process, and Gamma process, et al [4-6].

Among those processes, Gamma process has been widely studied. Bagdonavicius and Nikulin [7] have used gamma process to describe the degradation path of products. Lawless and Crowder [8] have discussed covariates and random effects about gamma process. Park and Padgett [9] have provided several new degradation models that incorporate an accelerated test based on stochastic processes such as gamma process. Crowder and Lawless [10] have used gamma process to illustrate their single-inspection policy for the maintenance of automobile brake pads. Noortwijk [11] has surveyed the application of gamma process in maintenance.

In reliability study, beyond evaluating products' reliability, how to obtain the residual lifetime of a product is also of great interest. In this paper, Gamma process is proposed to describe the degradation path. The real time reliability assessment and residual life method for an individual product are obtained. Considering the Markov Chain Monte Carlo (MCMC) method is convenient and efficient to sample from complex distributions, it is used to estimate parameters of the model.

The rest of the paper is organized as follows. In Section 2, we introduce the Gamma degradation process and obtain the distribution function of the lifetime. Then, the real time reliability assessment and residual life estimation models are presented in Section 3. Parameter estimation is obtained via MCMC method in section 4. In Section 5, to verify the validity of the present method, a numerical example about laser device is given. Finally, some conclusions are made in Section 6.

II. The Gamma degradation process

The gamma process is a continuous time stochastic process with independent, non-negative increments. As mentioned earlier, gamma process is more suitable to describing a monotone increasing degradation path. A well-adopted form for Gamma process $\{X(t), t \geq 0\}$ can be expressed as

$$X(t) \sim Gamma(\alpha t, \beta) \tag{1}$$

where Gamma(αt, β) is a Gamma distribution with shape parameter αt and scale parameter β, and the Gamma process has the follow properties

(1) $X(0) = 0$ with probability one;

(2) For all $t_2 > t_1 > s_2 > s_1$, the increments $X(t_2) - X(t_1)$ and $X(s_2) - X(s_1)$ are independent;

(3) $\Delta X(t) = X(t+\Delta t) - X(t) \sim Gamma(\alpha \Delta t, \beta)$, for all $\Delta t > 0$.

where the probability density function (PDF) of $\Delta X(t)$ is

$$\Delta X(t) \sim f_{GA}(x \mid \alpha\Delta t, \beta) = \frac{\beta^{\alpha\Delta t}}{\Gamma(\alpha\Delta t)} x^{\alpha\Delta t-1} \exp[-\beta x], x > 0 \tag{2}$$

where $\Gamma(s) = \int_0^{+\infty} y^{s-1} \exp(-y) dy$ is a complete Gamma function.

Let $D > 0$ denote the critical level for the degradation product. Similarly to Ref [12], the product's lifetime T can be defined as the time when the degradation path $X(t)$ firstly crosses the critical level D, that is

$$T = \inf\{t \mid X(t) \geq D\} \tag{3}$$

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Suppose that the degradation path of product is governed by Equation (1). Since the Gamma process has a monotone increasing degradation path, the cumulative distribution function (CDF) of the lifetime T can be expressed as

$$F(t)=P(T<t)=P(X(t)>D)$$
$$=1-\frac{1}{\Gamma(\alpha t)}\int_0^{D\beta} z^{\alpha t-1}\exp[-z]dz \quad (4)$$

We can get the PDF of the lifetime T and the reliability at time t as

$$f(t)=F'(t)=\frac{\alpha}{\Gamma(\alpha t)}\int_0^{D\beta}\left[\frac{\Gamma'(\alpha t)}{\Gamma(\alpha t)}-\log z\right]z^{\alpha t-1}\exp(-z)dz \quad (5)$$

and

$$R(t)=\frac{1}{\Gamma(\alpha t)}\int_0^{D\beta} z^{\alpha t-1}\exp[-z]dz \quad (6)$$

III. Real time reliability assessment model and residual life estimation model

Although Equation (6) provides a basic method for product reliability evaluation, this evaluation method only is related to current degradation data.

From the Ref [13], the reliability function in Equation (6) cannot effectively quantify the reliability of product under varying environment conditions. In order to solve this problem, the condition reliability is used to bridge the connection.

In this section, a real time conditional reliability evaluation method is given.

A. The conditional reliability function

Suppose that product has operated until time t_k without failure, and $X(t_k)$ is the corresponding degradation quantity at time t_k, the conditional reliability can be formulated as

$$R\left(t\mid X(t_k)=x\right)=P\left(X(t)\le D\mid X(t_k)\right) \quad (7)$$

If the degradation path $X(t)$ is given by Equation (1), and $X(t_k)$ is the observation degradation value until time t_k. Given α and β, let $\Delta X(t-t_k)=X(t)-X(t_k)$, $l=t-t_k$, we can have

$$\Delta X(t-t_k)\sim Gamma\left(\alpha l,\beta\right) \quad (8)$$

and the corresponding PDF is given by

$$\Delta X(t-t_k)\sim f_{Ga}\left(z\mid \alpha l,\beta\right)=\frac{\beta^{\alpha l}}{\Gamma(\alpha l)}z^{\alpha l-1}\exp[-\beta z] \quad (9)$$

Based on Equations (8) and (9), the conditional reliability can be formulated as

$$R\left(t\mid X(t_k)\right)=\Pr\left(T>t\mid X(t_k)=x\right)$$
$$=\Pr\left(X(t)<D\mid X(t_k)=x\right)$$
$$=\Pr\left(X(t)-X(t_k)<D-x\mid X(t_k)=x\right)$$
$$=\Pr\left(\Delta X(t-t_k)<D-x\right)$$
$$=\int_0^{D-X(t_k)}\frac{\beta^{\alpha l}}{\Gamma(\alpha l)}z^{\alpha l-1}\exp[-\beta z]dz \quad (10)$$

B. Updating random effects

Considering that each item possibly experiences different source of variation during its operation, for a degradation model to be realistic, it is more appropriate to incorporate item to item variability in the degradation process. Here, the Bayesian method is used to reach this end.

To capture heterogeneities within a population, Refs [14, 15] suppose that α is a fixed parameter which is common to all products, β is a random effect representing between item variations. In addition, suppose that the random effect β follows Gamma (η, γ). The ideas of random effect and Gamma assumption are widely used in degradation model in Ref [16].

Assume that the history degradation data of the product is $X_{1:k}=[x(t_1), x(t_2), \dots, x(t_k)]$. The likelihood function of $X_{1:k}$ can be expressed as

$$L(X_{1:k}\mid\alpha,\beta)=\prod_{j=1}^{k}\frac{\beta^{\alpha\Delta t_j}}{\Gamma(\alpha\Delta t_j)}\left(\Delta x_j(t)\right)^{\alpha\Delta t_j-1}\exp\left[-\beta\Delta x_j(t)\right] \quad (11)$$

where

$t_0=0$, $\Delta x(t_j)=x(t_j)-x(t_{j-1})$, $\Delta t_j=t_j-t_{j-1}$.

According to the Bayesian theory, the posterior PDF of β can be formulated as

$$\pi(\beta\mid X_{1:k})=\frac{L(X_{1:k}\mid\alpha,\beta)\pi(\beta)}{\int_0^{+\infty}L(X_{1:k}\mid\alpha,\beta)\pi(\beta)d\beta}$$
$$\propto L(X_{1:k}\mid\alpha,\beta)\pi(\beta)$$
$$\propto \beta^{\eta+\alpha\sum_{j=1}^{k}\Delta t_j-1}\exp\left[-\beta\left(\gamma+\sum_{j=1}^{k}\Delta x_j(t)\right)\right] \quad (12)$$

Let

$$\eta^*=\eta+\alpha\sum_{j=1}^{k}\Delta t_j=\eta+\alpha t_k,\ \text{and}\ \gamma^*=\gamma+\sum_{j=1}^{k}\Delta x_j(t)=\gamma+X(t_k) \quad (13)$$

Based on Equations (12) and (13), we can get

$$\pi\left(\beta\mid X_{1:k}\right)=Ga\left(\eta^*,\gamma^*\right) \quad (14)$$

Given the value $X_{1:k}=[x(t_1), x(t_2), \dots, x(t_k)]$, under squared error loss function, the Bayesian estimation of parameter β can be obtained as

$$\beta^*=E\left[\beta\mid X_{1:k}\right]=\frac{\eta^*}{\gamma^*}=\frac{\eta+\alpha t_k}{\gamma+X(t_k)} \quad (15)$$

where $E(\cdot)$ stands for the expectation.

C. Evaluating the real time reliability

Along with the random-effect updating, the real-time reliability of the target product can be evaluated. Based on Equations (10) and (15), the real-time conditional reliability function can be expressed as

$$R'\left(t\mid X(t_k)\right)=\int_0^{D-X(t_k)}\frac{\left(\beta^*\right)^{\alpha l}}{\Gamma(\alpha l)}z^{\alpha l-1}\exp\left[-\beta^* z\right]dz$$
$$=\int_0^{D-X(t_k)}\left(\frac{\eta+\alpha t_k}{\gamma+X(t_k)}\right)^{\alpha l}\frac{z^{\alpha l-1}}{\Gamma(\alpha l)}\exp\left[-\frac{\eta+\alpha t_k}{\gamma+X(t_k)}z\right]dz \quad (16)$$

D. Residual life estimation model

Suppose the degradation measurement data for a particular unit is $X(t_k)$ at time t_k. From the definition of the lifetime T, the residual life (RL) T_1 of the particular unit at time t_k can be expressed as

$$T_1=\inf\{t: X(t+t_k)\ge D\} \quad (17)$$

From Ref [12], we know that the key for estimating RL is to derive the PDF of lifetime. According to the independent increments property of the Gamma process, we can get

$$T_1=\inf\{t: X(t+t_k)\ge D\}$$
$$=\inf\{t: X(t+t_k)-X(t_k)\ge D-X(t_k)\}$$
$$=\inf\{t: X(t)\ge D-X(t_k)\} \quad (18)$$

From Expression (5), we know that the PDF of the lifetime T is complicated and computationally quite intractable. Following the work of Park and Padgett [9], the failure time distribution in this situation can be approximated by a Birnbaum-Saunders distribution with CDF

$$F_T(t) \cong \Phi\left[\frac{1}{\theta}\left(\sqrt{\frac{t}{\delta}}-\sqrt{\frac{\delta}{t}}\right)\right] \tag{19}$$

where

$$\theta=\frac{1}{\sqrt{\beta D}}, \text{ and } \delta=\frac{\beta D}{\alpha}$$

and the corresponding PDF is

$$f_T(t) \cong \frac{1}{2\sqrt{2\pi}\theta\delta}\left[\left(\frac{t}{\delta}\right)^{-1/2}+\left(\frac{t}{\delta}\right)^{-3/2}\right]\exp\left[-\frac{1}{2\theta^2}\left(\frac{t}{\delta}-2+\frac{\delta}{t}\right)\right] \tag{20}$$

Based on Equations (18) and (20), along with the random-effects updating, the PDF of RL of the particular unit at time t_k can be written as

$$f_{T_1}(t)=\frac{1}{2\sqrt{2\pi}\theta^{(k)}\delta^{(k)}}\left[\left(\frac{t}{\delta^{(k)}}\right)^{-1/2}+\left(\frac{t}{\delta^{(k)}}\right)^{-3/2}\right] \times\exp\left[-\frac{1}{2(\theta^{(k)})^2}\left(\frac{t}{\delta^{(k)}}-2+\frac{\delta^{(k)}}{t}\right)\right] \tag{21}$$

where

$$\theta^{(k)}=\frac{1}{\sqrt{\beta^*\left(D-X(t_k)\right)}}, \quad \delta^{(k)}=\frac{\beta^*\left(D-X(t_k)\right)}{\alpha},$$

and

$$\beta^*=\frac{\eta+\alpha t_k}{\gamma+X(t_k)} \tag{22}$$

IV. PARAMETERS ESTIMATION VIA MCMC METHOD

According to Equations (16) and (21), we know that the Gamma degradation model is characterized by the three unknown parameters in the vector

$$\rho=(\alpha,\eta,\gamma)$$

To achieve parameters estimation, we assume that there are n units are tested, and $X_i(t_{ij})$ denotes the cumulative degradation values of product i at time t_{ij}, for $i = 1,2,\ldots, n$; $j = 0,1,2,\ldots, m$.

Let

$$\Delta X_i(t_{ij})=X_i(t_{ij})-X_i(t_{i(j-1)}), \Delta t_{ij}=t_{ij}-t_{i(j-1)}, \quad t_{i0}=0,\ X_i(t_{i0})=0 \tag{23}$$

From Equation (2), when the random effect β follows gamma distribution Gamma (η, γ), we can get

$$f_{GA}(\Delta x_{ij})=\int_0^{+\infty} f_{GA}(\Delta x_{ij}\mid\alpha\Delta t_{ij},\beta)\pi(\beta)d\beta =\frac{\Gamma\left(\alpha\Delta t_{ij}+\eta\right)}{\Gamma\left(\alpha\Delta t_{ij}\right)\Gamma(\eta)}\frac{\gamma^\eta(\Delta x_{ij})^{\alpha\Delta t_{ij}-1}}{(\gamma+\Delta x_{ij})^{\alpha\Delta t_{ij}+\eta}} \tag{24}$$

Due to the independence assumption of the degradation measurements of different product, the likelihood function can be obtained as

$$l(\rho)=\prod_{i=1}^{n}\prod_{j=1}^{m}\frac{\Gamma\left(\alpha\Delta t_{ij}+\eta\right)}{\Gamma\left(\alpha\Delta t_{ij}\right)\Gamma(\eta)}\frac{\gamma^\eta(\Delta x_{ij})^{\alpha\Delta t_{ij}-1}}{(\gamma+\Delta x_{ij})^{\alpha\Delta t_{ij}+\eta}} \tag{25}$$

By maximizing the likelihood function in Equation (25), we can obtain the estimation of the unknown parameters. Here, instead of directly maximizing the likelihood function, the Bayesian MCMC method is used to estimate the value of ρ.

The MCMC method is a simulation method in which the analytical posterior distribution is difficult to be computed. By using the MCMC method, it is possible to generate samples from the posterior distribution and to use these samples to estimate the desired features of the posterior distribution. Gibbs sampler is an algorithm to generate a sequence of samples from the full conditional probability distribution of two or more random variables. We can use the Gibbs sampler to generate a sample, and then the unknown parameters are estimated.

In this paper, the normal distribution is selected as the prior distribution of μ_β, and the gamma distribution was selected as the prior distribution of the parameters b, σ_B and σ_β, where the gamma distribution can be conveniently made as a non-informative distribution. Furthermore, those selected prior distributions can easily implement MCMC simulation because those prior distributions lead to a conjugate posterior distribution. By using the MCMC method, we can numerically evaluate the posterior distributions of the parameters in the nonlinearity diffusion process model.

The algorithm of parameters estimation via the Gibbs sampling can be summarized as follow:

Step 1: Initialize $\rho^{(0)}=(\rho_1^{(0)},\rho_2^{(0)},\cdots,\rho_k^{(0)})$;

Step 2: Set $t=1$;

Step 3: Generate $\rho_1^{(t)}$ from conditional distribution $\pi_1^*(\rho_1\mid\rho_2,\rho_3,\cdots,\rho_k,X)$;

Step 4: Generate $\rho_2^{(t)}$ from conditional distribution $\pi_2^*(\rho_2\mid\rho_1,\rho_3,\cdots,\rho_k,X)$;

Step 5: Generate $\rho_j^{(t)}$ from conditional distribution $\pi_j^*(\rho_j\mid\rho_1,\cdots,\rho_{j-1},\rho_{j+1},\cdots,\rho_k,X)$;

Step 6: Generate $\rho_k^{(t)}$ from conditional distribution $\pi_k^*(\rho_k\mid\rho_1,\rho_2,\cdots,\rho_{k-1},X)$;

Step 7: Set $t=t+1$, and repeat Steps 3-7, $t=1,2,\cdots,N_1$;

Step 8: Estimate desired features based on the simulate samples $\rho^{(1)},\rho^{(2)},\cdots,\rho^{(N_1)}$.

We can use the Bayesian software package WinBUGS (see in Ref. [17]) to carry out the Gibbs sampling, and then we can obtain the estimator of the model parameters.

V. NUMERICAL EXAMPLE

In this section, in order to illustrate the usefulness and validity of the proposed model and method, a laser device example is presented. Generally, the operational current is an important performance for a laser device. Usually, the laser device is supposed to have failed if the operational current reaches at a predefined threshold level. The initial laser devices data can be found in Meeker and Escobar [2], in which 15 products were tested, and their degradation processes are shown in Tab. I. The measured frequency is 250 hours, and the experiment is terminated at 4000 hours. Similar to Si et al. [12], the threshold is set as $\xi = 10$.

TABLE I
THE LASER DATA

	Operating current					
	0	250	500		3750	4000
1	0	0.47	0.93	…	9.87	10.94
2	0	0.71	1.22	…	8.91	9.28
3	0	0.71	1.17	…	6.45	6.88
4	0	0.36	0.62	…	5.95	6.14
5	0	0.27	0.61	…	7.10	7.59
6	0	0.36	1.39	…	10.4	11.0
7	0	0.36	0.92	…	6.1	7.17
8	0	0.46	1.07	…	5.81	6.24
9	0	0.51	0.93	…	7.20	7.88
10	0	0.41	1.49	…	11.28	12.21
11	0	0.44	1.00	…	6.96	7.42
12	0	0.39	1.80	…	7.37	7.88
13	0	0.30	0.74	…	7.85	8.09
14	0	0.44	0.70	…	6.51	6.88
15	0	0.51	0.83	…	6.16	6.62

A. *Model checking*

To check the validity of the proposed degradation model, a graphic method is presented to assess the goodness of fit. Let $\Delta X(t_{ij}) = X(t_{ij}) - \Delta X(t_{i(j-1)})$, $\Delta t_{ij}= t_{ij} - t_{i(j-1)}$, and let $(\hat{\alpha},\hat{\beta})$ be the estimator of (α,β) . According to the independent increment properties of Gamma process, we have

$$\Delta X(t_{ij}) \sim Gamma\left(\hat{\alpha}\Delta t_{ij},\hat{\beta}\right)$$

To verify the reasonability of the Gamma degradation model, the increments of laser degradation path follows a gamma distribution. From the Fig 1, we find that the Gamma probability plot approximates a straight line, it indicates this model is a good fit.

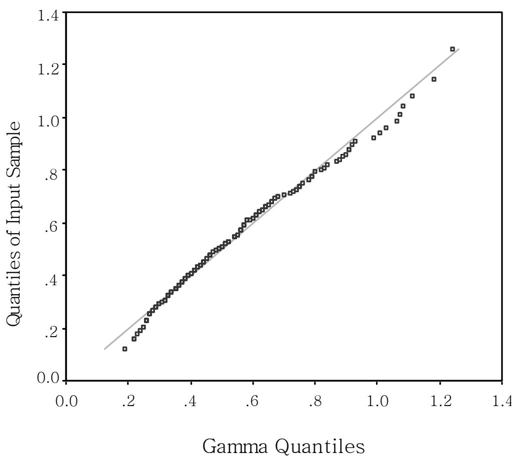

Fig 1 The Gamma probability plots for the degradation data

B. *Individual real time reliability assessment*

Based on the above data, by using MCMC method, the estimation of the unknown parameters can be obtained. Via WinBUGS software package, we generate 50,000 realizations of ρ from the posterior distribution and use the last 40,000 in the estimation of mean, standard deviation, MCMC error and quantile of parameters. Table II presents the computational results.

In order to obtain the real time reliability evaluation of individual degradation product, the degradation history information of particular laser device (named unit 1) is given in Table III.

Then, we can get the updated parameters about the specific degradation unit 1, and Table IV shows the updated parameters at the four different degradation times. Once the parameters in the model are updated, the real time conditional reliability assessment and the PDF of the estimated RL for the specific individual degradation product can be calculated at each time point.

Firstly, set the threshold ξ=10, by using the evaluation procedure described in Section 3, we can obtain the evaluation results of different individual products. As shown in Fig 2, the real time reliability curves are plotted to compare the unit 1 and unit 5 at updating time points 2500h. Therefore, we can find that the real time reliability of different individual product has larger difference.

TABLE II
MEAN, STANDARD ERRORS, MC ERROR AND 95% HPD INTERVAL

PARAMETER	MEAN	STANDARD ERRORS	MC ERROR	95% HPD INTERVAL
α	0.037	0.0038	0.0001	(0.0395, 0.0474)
η	25.00	10.990	0.3935	(9.1990, 51.250)
γ	1.258	0.5785	0.0206	(0.4430, 2.6580)

TABLE III
DEGRADATION HISTORY OF A PARTICULAR LASER DEVICE

t	250	500	750	1000	1250	1500	1750	2000
x(t)	0.47	0.93	2.21	2.72	3.51	4.34	4.91	5.48
t	2250	2500	2750	3000	3250	3500	3750	4000
x(t)	5.99	6.72	7.13	8.00	8.92	9.49	9.87	10.94

TABLE VI
UPDATING OF DEGRADATION PARAMETERS FOR UNIT 1

t	500	2000	2500	3000
η	84.355	104.14	123.925	143.71
γ	5.598	6.783	7.978	9.258

From Fig 2, we can find that unit 1 has smaller reliability value than unit 5. This phenomenon is consistent with their degradation trend and their current performance values. In fact, the reliability of unit 1 will be falling more quickly than unit 5 because the current degradation levels at 2500h of unit 1 and unit 5 are 6.72 and 4.63, respectively.

Hence, different individual products have their own real time reliability characteristic.

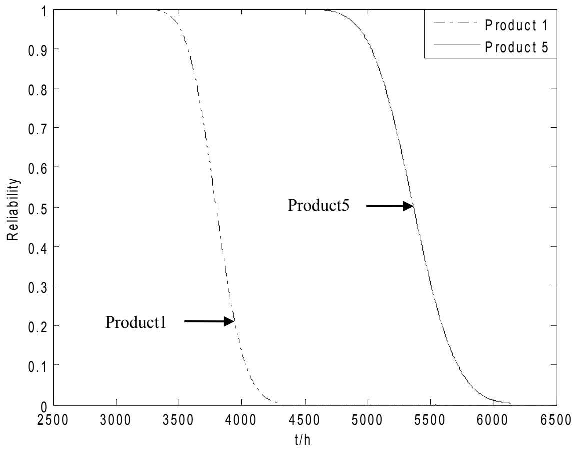

Fig 2 Real time reliability for product 1 and 5, survived to 2500 h

C. *Prediction of residual life*

In Table III, we find that the degradation history information of unit 1 is 9.87 at the tested time 3750 h. It can be found that this degradation value is very close to the failure threshold ξ=10. Note that the initial laser device data of unit 1 is zero according to Ref. [2], therefore, we can consider that the degradation data captures full life cycle history (useful life) of the laser devices of unit 1. In other words, the actual lifetime of the laser device of unit 1 is approximated to be 3800 h and the actual RL at each tested point is known from the full life cycle data.

By using the parameters updated result in Table IV, the PDF of the estimated RL can be calculated at four different time points in Fig. 3. As shown in Fig. 3, the actual RL (denoted by square) falls within the range of the estimated PDF of the RL at each tested point from our model. In addition, from right to left, we can find that the estimated PDF of the RL becomes more and more sharply as the degradation data is accumulated, so that its uncertainty becomes smaller when the degradation parameters are updated. This implies that the uncertainty of the estimated RL is reduced since more data are utilized during estimating the model parameters.

When we use our predictive model for RL estimation at a given test point, we use the tested data up to that tested point. In other words, if we estimate the RL at t_k, the data $X_{1:k}$ = $[x(t_1),\ x(t_2),\ \ldots\ ,\ x(t_k)]$ are used to update the model parameters and obtain the estimation of RL. In order to obtain accurate prediction results, the RL should be re-estimated once new degradation information is available. The more degradation information is used, the more accurate the RL estimation is obtained.

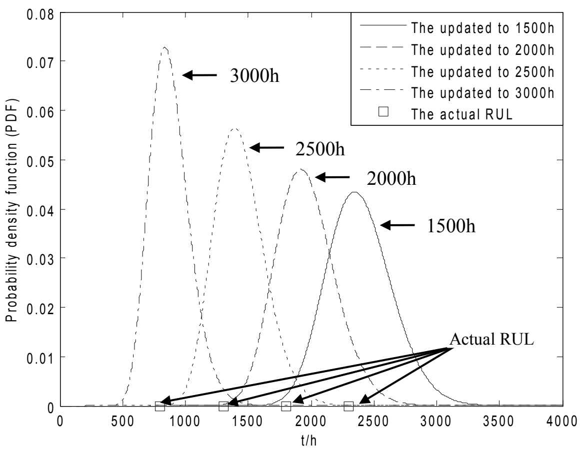

Fig. 3 The PDF of RL for unit 1

VI. CONCLUSION

In this paper, a real time reliability evaluation method for a single product is given under gamma degradation process, and we discuss how to update the model parameters and obtain the residual life prediction. Considering the model is very complicated, MCMC method is used to estimate the unknown parameters. A case study of the lasers data is given to validate the effectiveness of the proposed model.

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