Contrast Measurement with Histograms of Second-order Derivatives of Pixels for Magnetic Resonance Images

W. T. Chan, K. S. Sim, and F. S. Abas

***Abstract*— Second-order derivatives are used to obtain more information from the pixels in a magnetic resonance image. This information is the difference between the values of pixels, which is a factor in the measurement of the level of contrast in the image. This information is presented in the form of a histogram of second-order values, which expresses differences in pixel values caused by contrast between details in the image. The proposed method of contrast measurement utilizes a Laplace curve together with the histogram, both of which are generated with image data. The curve and histogram are used to determine a measurement quantity that expresses the level of contrast and the presence of details in the image. The measurement quantity, which is a score, describes the change of contrast in the image as contrast enhancement is applied. This score is compared with other established contrast scores that utilize data from an individual image, without comparison to another image. These other scores are average local contrast, root mean square contrast and Michelson contrast. The goal of the comparison is to determine the performance of the proposed score at detecting the occurrence of excessive gain as contrast enhancement is applied on an MRI image. Sample MRI images, including images from established databases, are used for the testing and comparison. The proposed score performs better than average local contrast and root mean square contrast and provides results that do not diverge as gain is increased.**

***Index Terms*—second-order derivatives, histograms, Laplace distribution curves, contrast measurement**

I. Introduction

MRI images provide information about the interiors of objects, such as the human body in the field of medicine. For the information to be presentable and useful, the MRI images must have sufficient levels of contrast among the image details. This contrast is improved with the use of contrast media and imaging techniques [1]. It can also be improved in post-processing [2]. In the case of post-processing, the improvement in contrast is measured through the use of quantities such as the contrast-to-noise ratio [3]. This paper proposes a new quantity of contrast measurement through the use of the second-order derivatives of pixels in the MRI images. This contrast measurement is similar to measurements such as average local contrast, root mean square contrast and Michelson contrast, which use statistical calculations based on data in individual images.

Manuscript received June 1, 2018; revised January 31, 2019.

W. T. Chan is with the Faculty of Engineering and Technology (FET), Multimedia University (MMU), Jalan Ayer Keroh Lama, Bukit Beruang, 75450 Melaka, Malaysia. (phone: +606-2523185; fax: +606 - 231 6552; e-mail: wtchan@mmu.edu.my).

K. S. Sim is also with the FET of MMU. (e-mail: kssim@mmu.edu.my).

F. S. Abas is also with the FET of MMU. (e-mail: fazly.salleh.abas@mmu.edu.my).

MRI images are generally presented as grayscale images. The differences between the gray values of pixels in the image provide the contrast that is needed to determine details in the image. The differences between the gray values of the pixels can be expressed by the second-order derivatives of each pixel and its neighbours. Second-order variables have been used for the analysis of MRI images, such as in texture analysis [4] and for noise reduction [5]. This paper proposes the use of second-order derivatives that are generated with the Laplacian operator for the calculation of a contrast score.

The second-order derivatives are presented as histograms; the reason for this is described in the Methodology section. In order to differentiate these histograms from histograms of gray values, these histograms of second-order derivatives are henceforth referred to as “second-order histograms”.

Research involving MRI images have utilized the profiles of histograms before. An example is the utilization of the curvature of intensity histograms of MRI images that have their contrast enhanced through brightness changes, in order to detect tumours [6]. In the case of the method that is proposed in this article, it utilizes the profile of the Laplace distribution curve together with the aforementioned second-order histograms for the generation of a contrast score.

II. Methodology Of Contrast Measurement

A. Second-order Derivatives

A 2-dimensional Laplacian operator with a 3×3 mask is used on every pixel to produce a second-order derivative [7]. In the case of pixels that are on the edges of the image, a 3×2 or 2×3 mask is used, whichever appropriate. A 2×2 mask is used for pixels at the corners of the image. In these cases, the equation for the operator is changed to accommodate the masks.

B. MRI Image Samples

The development of the method uses a set of 150 MRI images drawn from multiple sources, including databases such as Radiopaedia and The Cancer Imaging Archives. The images are drawn from multiple sources for the purpose of diversity in the images that are used to test the proposed method. The tests utilize two means of changing the contrast of the images: applying gain on the images in progressive steps to produce additional images with different levels of contrast, and applying gray histogram equalization. The

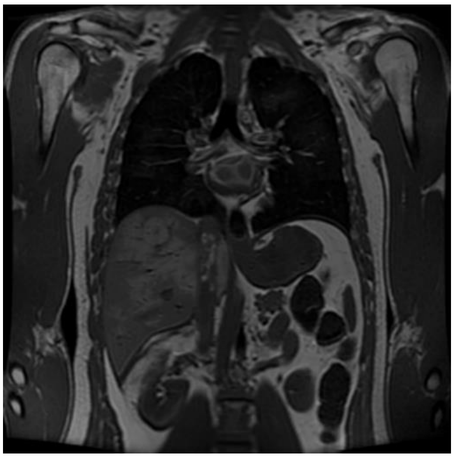

Fig. 1(a): MRI image of an abdomen

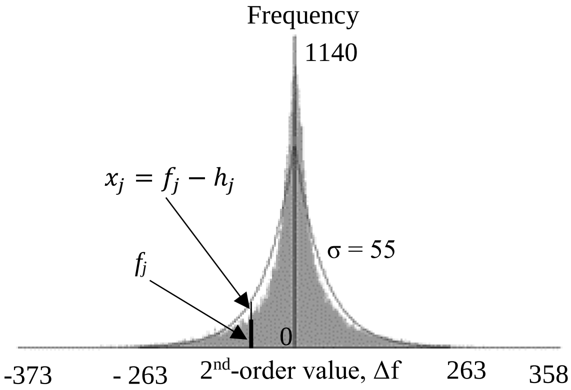

Fig. 1(b): Second-order histogram of MRI image in Fig. 1(a)

samples also include images with labels and arrows to test the method's response to the presence of detail markers.

C. *Laplace Distribution Profile*

Figure 1(a) shows an example of an MRI image. Figure 1(b) shows the second-order histogram of the image in Figure 1(a). The profile of the histogram closely follows a Laplace distribution. Figure 1(b) also includes a Laplace curve. The curve is generated through Equation (1) using the standard deviation of the distribution of second-order derivative values of the pixels in the image. Equation (1) is as follows:

$$f(x|\mu, b) = N\left(\frac{1}{2b} exp\left(-\frac{|x-\mu|}{b}\right)\right) \qquad (1)$$
$$b = \frac{\sigma}{2^{0.5}}$$

where x represents a second-order derivative value,
f is the frequency of x in the image, b is the diversity variable,
σ is the standard deviation of the second-order derivative,
μ is the mean of the second-order derivatives, and
N is the number of pixels in the image.

The method uses the curve to calculate the variables that determine the contrast score. This is similar to the methodology in the technique that reduces noise by using histograms of second-order derivatives [8].

D. *Hypothesis on the Utilization of the Second-order Histogram and Curve*

Research and development of image processing techniques includes the utilization of the comparisons of distribution curves and histogram profiles. For example, Lai et al. used Gaussian curves and histograms of gray values to enhance contrast of images [9]. There has also been the utilization of histograms that have the profile of a Laplace distribution function, such as the use of a Laplacian operator to produce a histogram that provides data on the edges of objects in an image [10].

The proposed method uses a Laplace curve on histograms of second-order derivatives. The hypothesis behind this is that the differences in the heights of the curve and the frequencies in the histogram profile represent a way to quantify the contrast of details in the image. The hypothesis further states that when an image has good contrast, the differences are small, such that the second-order histogram resembles the Laplace curve.

Thus, the proposed method focuses on the regions in a second-order histogram where the curve has a height greater than the corresponding interval in the histogram profile. The method calculates the differences in the heights of the curve and the frequencies in the histogram profile with Equation (2). Figure 1(b) shows an example of x_j and j.

$$x_j = f_j - h_j \qquad (2)$$

where j is the second-order derivative in the histogram,
f_j is the frequency of j,
h_j is the height of the curve at the location of j, and
x_j is the difference between the frequency of j and the corresponding height of the curve h_j.

Equation (3) uses Equation (2) to calculate the contrast score that this article proposes.

$$S_p = \frac{1}{N}\sum_{n}^{m}\left(x_j | (x_j < 0)\right) \qquad (3)$$

where x_j and j have been described in Equation (2),
N is the number of pixels in the image,
m and n are the boundaries of the aforementioned range of j,
and S_p is the proposed contrast score.

The condition $x_j < 0$ ensures that regions where the curve height is greater than the height of the corresponding histogram interval are selected for the score calculation.

E. *Problem Statement*

Contrast enhancement may lead to excessive increases or decreases in the values of a pixel, thus resulting in the loss of details in an image when the pixel values overflow. The contrast score for the image should indicate this loss of detail. There are established contrast scores such as root mean square contrast (RMSC), average local contrast score (ALC) and Michelson contrast (MC) that involve statistical calculations of the data from the image [11]. However, these contrast scores may not indicate the aforementioned losses.

F. *Contrast Scores Used for Comparison*

The following equations are used for comparisons against the proposed contrast score. Equation (4) shows the equation for RMSC.

$$RMSC = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(k_i - k_{avg}\right)^2} \quad (4)$$

where i is the raster coordinate of a pixel,
k_i is its gray value, and
k_{avg} is the average gray value of all pixels in the image, and
N is the number of pixels in the image.

Equations (5) to (7) show the equations for ALC.

$$ALC = \frac{1}{N}\sum_{i=1}^{N} lc_i \quad (5)$$

$$lc_i = \frac{|L_i - L_{i-1}| + |L_i - L_{i+1}| + |L_i - L_{i-w}| + |L_i - L_{i+w}|}{4} \quad (6)$$

$$L_i = 100\sqrt{\left(\frac{k_i}{255}\right)^{\gamma}} \quad (7)$$

where i is the raster coordinate of a pixel,
N is in Equation (1),
w is the width of the image,
lc_i is the local contrast score for a pixel,
L_i is its perceptual luminance,
γ is the gamma correction factor, which is set to 2.2 as per the suggestion of Matković et al [11], and
i-w, i+w, i-1 and i+1 are the coordinates for other pixels that are adjacent to the current pixel in the cardinal directions.

For parity with the other contrast scores, which uses data from every pixel in the image, the Michelson contrast score uses a 3×3 mask for the region of interest, applied on every pixel, and then averaged. Equation (8) shows the equation for averaged Michelson contrast (AMC):

$$AMC = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}}\right) \quad (8)$$

where i is the raster coordinate of a pixel,
I_{max} is the highest value among the neighbouring pixels in the 3×3 region of interest centered on pixel i,
I_{min} is the lowest value among the neighbouring pixels, and
N is the number of pixels in the image.

These established contrast scores have been selected for comparisons against the proposed contrast score because of their capability for calculations using data from a single image alone, without reference to another image.

G. *Calculation of Excessive Gain*

After applying contrast enhancement, there may be pixels that have their values set to or increased beyond the maximum of the scale, e.g. 255 on grayscale, or set to or decreased below the minimum, e.g. 0 on grayscale. However, these pixels can only have a maximum gray value of 255, or a minimum of 0. Therefore, in the case of these pixels, they are marked as having lost detail due to excessive gain. The justification is that they are no longer distinguishable from other pixels that have values at the maximum or minimum. The methodology collects these pixels into a set E, as expressed by Equation (9). Equation (10) determines the extent of detail loss from excessive gain.

$$e_i = f(o_i) \quad (9)$$

$$E = \{e_i \mid ((e_i \geq 255) \cup (e_i \leq 0)) \wedge (0 < o_i < 255)\}$$

$$p = |E|$$
$$P = \frac{p}{N} \times 100\% \quad (10)$$

where o_i is the original value of the pixel,
e_i is the value of the pixel after contrast enhancement,
E is the set that has e_i pixels that fulfill the conditions of excessive gain as described above,
p is the number of pixels that have excessive gain,
N is the number of pixels in the image, and
P is the percentage of pixels that have excessive gain.

The methodology considers the occurrence of excessive gain as P in Equation (10) rising above 0%.

H. *Hypothesis Testing with Incremental Application of Gain*

The idea behind this test is that the first peaking of a score indicates the occurrence of excessive gain. After excessive gain has occurred, the contrast score should decrease to account for the loss of detail due to excessive gain. For a contrast score to be accurate, its first peaking should occur close to the occurrence of excessive gain.

The methodology considers that the first peaking is the first maximum that the score achieves before decreasing. The testing uses the number of contrast increments between the occurrence of excessive gain and the first peaking of a score as the measurement for how accurately the score detects the occurrence excessive gain.

The contrast increments are implemented through the application of gain. The gain that is applied on the images is expressed as a multiplication of the values of the pixels. For example, a gain of ×1.1 multiplies the original value of a pixel by a factor of 1.1, with the result being the new value of the pixel. To test the response of the contrast score to varying levels of contrast enhancement, the applied gain is implemented in a series of steps. In each step, the applied gain is incremented by + 0.1, up to a maximum of ×3.0. The contrast scores for the image are calculated for each step.

I. *Hypothesis Testing with Gray Histogram Equalization*

Histogram equalization of gray levels is an established category of methods to enhance contrast in an image. These methods make use of data that is already in an image, such as the maximum and minimum pixel values [12], so the contrast scores that are described in part F are appropriate because the scores, and the proposed score, make calculations using data that is already in the image.

However, this contrast enhancement method can cause excessive gain too. Ideally, the contrast scores in part F and the proposed score should be able to detect this and hence decrease in order to indicate the loss of detail due to excessive gain. This test intends to measure their performance against each other.

After subjecting the sample images that were described in part B to gray histogram equalization (henceforth called

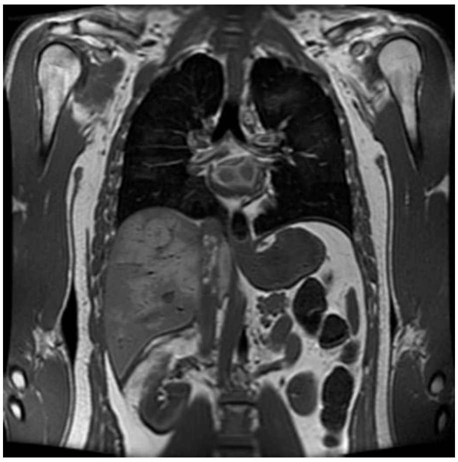

Fig. 2(a): Version of MRI Image in Fig. 1(a) with gain of ×1.8

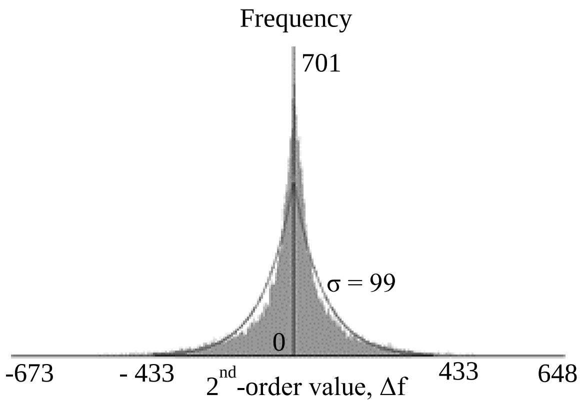

Fig. 2(b): Second-order histogram of the image in Fig. 2(a)

"GHE" for ease of writing), the excessive gain in the resulting images and their contrast scores are calculated as well. Table I shows the types of results that may be had for each type of contrast score from the tests with GHE-applied images, and the classification of the results according to the goals of this test.

III. Results & Discussion

A. Example of Proposed Score Measurement

Figure 2(a) shows a version of the image in Figure 1(a) with a gain of ×1.8. Figure 2(b) shows the second-order histogram of the image in Figure 2(a). The histogram in Figure 2(b) has a wider range of second-order derivatives than Figure 1(b), which is expected because the application of gain increases the differences between the values of the pixels. In the case of Figure 1(a), its proposed score S_p is 0.8696. As for Figure 2(a), its S_p is 0.8684. Excessive gain has affected 0.007% of Figure 2(a). Consequently, the change in S_p is small, but the change is still a decrease.

Figure 3(a) is a version of the image in Figure 3(a) with a gain of 3.0. Figure 3(b) is its second-order histogram. Excessive gain has affected 12.411% of 3(a). It has removed details such as the porosity in the bones and the borders between the layers of subcutaneous fat. The intervals in the histogram of Figure 3(b) also become less contiguous than Figure 1(b) and Figure 3(b). Since Equation (3) calculates the score using regions where the curve is higher than the histogram profile, a lower score is expected. Consequently, the proposed score of Figure 3(b) is 0.3416, which is lower than the score for Figure 1(a). The change in the score has accounted for the excessive gain in this case.

B. Example of Hypothesis Test with Incremental Application of Gain

Table II shows an example of the incremental gain application on the MRI image in Figure 1(a). There are 20 increments; each row presents the results of an increment. However, several rows are omitted from in Table II for display convenience. Excessive gain begins in the image at a gain of ×1.8, as shown in Table II.

The RMSC score continues to increase as the applied gain increases, i.e. it does not have a clear peaking. This happens regardless of how much excessive gain has affected the image. The ALC score achieves its first peak much later, at an applied gain of ×2.9. The AMC score achieve its first peak much earlier, at an applied gain of ×1.1. The first peak of the proposed score also achieves its first peak much earlier, specifically before gain is applied.

C. Results from Test with Incremental Application of Gain

Table III shows that the RMSC score performs poorly at detecting the occurrence of excessive gain because it continues to increase with applied gain for more than half of the sample images. The ALC score performs better than the RMSC score, but does not achieve peaking for some sample

Fig. 3(a): Version of MRI image in Fig. 1(a) with gain of 3.0

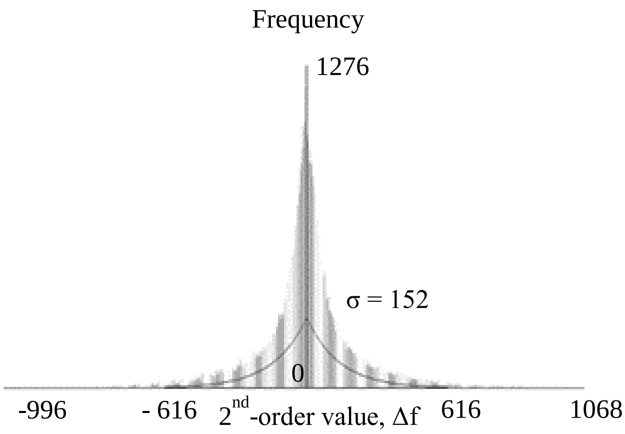

Fig. 3(b): Second-order histogram of the image in Figure 3(a) images. However, the AMC and the proposed scores do achieve peaking for all sample images.

The ALC score performs better than the AMC and the proposed scores in a small number of images in detecting the occurrence of excessive gain. Across all the images, the proposed score peaks closer to the occurrence of excessive gain than the other scores, but at a slightly higher standard deviation than AMC. These results suggest that the proposed score has slightly better performance than the Michelson score at detecting the occurrence of excessive gain.

D. Results of Testing with Histogram Equalization

Table IV shows the results of this test, grouped according to the occurrence of excessive gain or its non-occurrence. The proposed score rarely express a result of type (iii), which is a false positive. However, it is the least effective at expressing results of type (i), which is a true positive. This means that the proposed score is a conservative score, e.g. an image tends to score lower on the scale of this contrast measurement score than it would on the scales of the other scores.

In both tests, the proposed score does not show any noticeably different results for either MRI images with text and symbols or images without them.

IV. Conclusion

The method that is proposed in this article measures contrast in an image without using other images or local regions in the image as base comparisons, like ALC, RMC and AMC do. These measurement methods do not include the factor of human perception, which involves the observer's subjectivity [13]. This paper compares the proposed score against the other scores, because it does not include this factor too.

As shown in the results of the tests that are described in this article, the contrast score of this method does not diverge as contrast enhancement is gradually increased, unlike ALC and RMC. The proposed score is conservative, more so than even AMC. Consequently, it detects the occurrence of excessive gain earlier than the other contrast scores, but at the cost of misidentifying contrast improvements that occurred without excessive gain. On the other hand, a more complex score could be developed through having traits of the other scores incorporated into the proposed score, so as to improve its performance.

TABLE I
Types of Cases for a Contrast Score in Tests with Gray Histogram-Equalized (GHE) Images

Case Type	(i)	(ii)	(iii)	(iv)
Excessive Gain*	No	Yes	Yes	No
Inferences on GHE	GHE did not cause loss of detail.	GHE caused loss of detail.	GHE caused loss of detail.	GHE did not cause loss of detail.
Contrast Score	Increase	Decrease	Increase	Decrease
Inferences on Contrast Score	Contrast score correctly expressed positive outcome.	Contrast score correctly expressed negative outcome.	Contrast score failed to express negative outcome.	Contrast score failed to express positive outcome.
Result Class	True Positive	True Negative	False Positive	False Negative
Desirable	Yes	Yes	No	No

*Described in part *II.G*.

TABLE II
Averaged Results across Versions of the MRI Image in Figure 1(a) with Different Levels of Applied Gain

Gain applied	ALC	RMSC	AMC	Proposed score	Excessive gain in image (%)
Original	4.849	0.113	0.2955	**0.8696**	-
× 1.1	5.115	0.124	**0.2996**	0.8693	0
× 1.2	5.335	0.135	0.2990	0.8703	0
…	…	…	…	…	….
<u>× 1.7*</u>	<u>6.355</u>	<u>0.192</u>	<u>0.2987</u>	<u>0.8698</u>	<u>0</u>
× 1.8	6.529	0.203	0.2979	0.8684	0.007
× 1.9	6.718	0.214	0.2988	0.8514	0.036
…	…	…	…	…	….
× 2.7	7.629	0.288	0.2907	0.8622	6.684
× 2.8	7.647	0.293	0.2880	0.8573	8.381
× 2.9	**7.669**	0.297	0.2863	0.8027	10.630
× 3.0	7.637	0.300	0.2815	0.3416	12.411

*indicates increment before occurrence of excessive gain

TABLE III
Averaged Results on the Performance of the Contrast Scores across 150 Sample MRI Images

	ALC	RMSC	AMC	Proposed score
Percentage of images in which the score continues to increase without any peaking (%)	12.00	56.67	0	0
Average number of gain increments between occurrence of excessive gain and first peaking	7.92	13.02	1.25	1.05
Standard deviation of gain increments between occurrence of excessive gain and first peaking	4.71	4.49	1.36	2.31
Percentage of images in which the score is closer to the occurrence of excessive gain than the other scores (%)	2.50	0	46.67	50.83

TABLE IV
Percentage of Images where the Scores Expressed Results of the Corresponding Types, across 150 Sample Gray Histogram Equalized MRI Images

Excessive gain occurred	Case Types*	Percentage of images (%)			
		ALC	RMSC	AMC	Proposed score
No	True Positive	82.26	87.10	27.42	4.84
	False Negative	17.74	12.90	72.58	95.16
Yes	True Negative	13.64	9.09	53.41	94.32
	False Positive	86.36	90.91	46.59	5.68

*Refer to Table I for the explanation of the case types.

REFERENCES

[1] S. Maniam & J. Szklaruk, "Magnetic resonance imaging: Review of imaging techniques and overview of liver imaging," World Journal of Radiology, vol. 2, no. 8, pp. 309–322, 2010.

[2] J. Schulz-Menger, D.A. Bluemke, J. Bremerich, S.D. Flamm, M.A. Fogel, M.G. Frederich, R.J. Kim, F, von Knobelsdorff-Brenkenhoff, C.M. Kramer, D.J. Pennell, S. Plein, & E. Nagel, "Standardized image interpretation and post processing in cardiovascular magnetic resonance: Society for Cardiovascular Magnetic Resonance (SCMR) Board of Trustees Task Force on Standardized Post Processing," Journal of Cardiovascular Magnetic Resonance, vol. 15, no. 1, id. 35, 2013.

[3] M. Welvaert & Y. Rosseel, "On the Definition of Signal-To-Noise Ratio and Contrast-To-Noise Ratio for fMRI Data," PLOS One, vol. 8, no. 11, id. e77089, 2013.

[4] D. Avola, L. Cinque & G. Placidi, "Customized First and Second Order Statistics Based Operators to Support Advanced Texture Analysis of MRI Images," Computational and Mathematical Methods in Medicine, vol. 2013, id. 213901, 2013.

[5] W.T Chan, K.S. Sim & F.S. Abas, "Contrast measurement for MRI images using histogram of second-order derivatives," International Conference on Robotics, Automation and Sciences (ICORAS), pp. 1-5, 2017.

[6] C. Ciulla, D. Veljanovski, U.R. Shikoska & F.A. Risteski, "Intensity-Curvature Measurement Approaches for the Diagnosis of Magnetic Resonance Imaging Brain Tumors," Journal of Advanced Research, vol. 6, no. 6, pp. 1045-1069, 2015.

[7] R.C. Gonzalez & R.E. Woods, "Digital Image Processing, 4th Edition", London, Pearson, 2017.

[8] W.T. Chan, K.S. Sim & F.S. Abas, "Pixel Filtering and Reallocation with Histograms of Second-order Derivatives of Pixel Values for Electron Microscope Images," International Journal of Innovative Computing Information and Control, vol. 14, no. 3, pp. 915-928, 2018.

[9] Y.R. Lai, K.L. Chung, G.Y. Lin & C.H. Chen, "Gaussian mixture modeling of histograms for contrast enhancement," Expert Systems with Applications, vol. 39, no. 8, pp. 6720–6728, 2012.

[10] I. F. Jafar & K. A. Darabkh, "Image Contrast Enhancement Based on Equalization of Edge Histograms," IAENG International Journal of Computer Science, vol. 38, no. 3, pp. 192-204, 2011.

[11] K. Matković, L. Neumann, A. Neumann, T. Psik & W. Purgathofer, "Global Contrast Factor – a New Approach to Image Contrast," Computational Aesthetics in Graphics, Visualization and Imaging, 1st Eurographics Conference 2005, pp. 159-167, 2005.

[12] K. Iqbal, R. A. Salam, A. Osman and A. Z. Talib, "Underwater Image Enhancement Using an Integrated Colour Model," IAENG International Journal of Computer Science, vol. 34, no. 2, pp. 239-244, 2007.

[13] D.G. Pelli & P. Bex, "Measuring contrast sensitivity," Vision Research, vol. 90, pp. 10–14, 2013.