Inverse Gaussian Distribution for Modeling the Tensile Strength and Fatigue Life Data of Sn-3.0Ag-0.5-0.5Cu Solder Joint

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***Abstract*—In this paper, we propose to use inverse Gaussian distribution for modeling the tensile strength and fatigue life data of Sn-3.0Ag-0.5-0.5Cu solder joint. The Kolmogorov-smirnov statistic with its p-value and quantile-quantile plots show the inverse Gaussian fits these data well. Inverse Gaussian model may statistically analyze on the thermal aging effect of tensile and fatigue SAC305 solder joints.**

***Index Terms*—Inverse Gaussian, tensile strength, fatigue life.**

I. Introduction

IN the last several years, Pb-free solder alloys were extensively studied, in order to avoid the environmental pollution by lead-rich solders. The Sn-Ag-Cu (SAC) solder alloys have been recognized as the most promising candidates in electronics industry [1]. However, the exact compositions of the near-ternary SAC solder alloys are still in dispute depending on their applications. Kim et al [2] investigated the effects of the fourth elements, i.e., Fe, Ni, Co, Mn and Ti, on microstructural features, under cooling characteristics, and monotonic tensile properties of Sn-3 wt.% Ag-0.5 wt.% Cu lead-free solder. They found that the Sn-3Ag-0.5Cu-0.1Ni alloy was more reliable solder alloy with improved properties for all tests. Anderson and Harringa [3] reported elevated temperature aging of solder joints based on Sn-Ag-Cu: effects on joint microstructure and shear strength. Pang et al [4] investigated thermal cycling aging effects on a lead-free 95.5Sn-3.8Ag-0.7Cu solder joint. Cho et al [5] investigated the effects of Zn additions to Sn-0.7Cu and Sn-3.8Ag-0.7Cu (all in wt.% unless specified otherwise) Pb-free solders on the interfacial reactions with Cu substrates. More researches on SAC alloys were discussed [6-12].

The inverse Gaussian distribution has received considerable attention by researchers due to its broad applications in areas of quality control, fatigue and crack in fracture mechanics, mechanical engineering, rock engineering, management science, biology, reliability, economics, duration and failure time etc modeling. The behavior of a system or product often needs to be characterized by an appropriate lifetime distribution model. For example, Kalaba et al [13] used simple and complex Weibull distribution to describe and evaluate a thermal power system. Rabie and Li [14] used Burr-X to fit electron mobility. In the paper, we use inverse Gaussian distribution to analysis on the thermal aging effect Sn-3.0Ag-0.5Cu (wt% SAC305) solder joints tensile and fatigue at different temperature.

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II. Inverse Gaussian distribution and estimation method

The probability density function (PDF) and the cumulative distribution function (CDF) of inverse Gaussian (IG) distribution are given by

$$f(x,\mu,\lambda)=(\frac{\lambda}{2\pi x^3})^{1/2}\mathrm{e}^{\frac{-\lambda(x-\mu)^2}{2\mu^2 x}}, x>0,\mu,\lambda>0, \quad (1)$$

$$F(x,\mu,\lambda)=\Phi(\sqrt{\frac{\lambda}{x}}(\frac{x}{\mu}-1))+\mathrm{e}^{\frac{2\lambda}{\mu}}\Phi(-\sqrt{\frac{\lambda}{x}}(\frac{x}{\mu}+1)). \quad (2)$$

where μ is the location parameter, λ is the shape parameter and $\Phi(\cdot)$ denotes CDF of the standard normal.

We use maximum likelihood method to estimate the parameters of the IG distribution. Let $x=\{x_1,x_2,\cdots,x_n\}$ be a random sample of size n from IG distribution. The log-likelihood function is given by

$$l(\lambda,\mu|x)=\frac{n}{2}\log(\frac{\lambda}{2\pi})-\frac{3}{2}\sum_{i=1}^{n}\log(x_i)-\frac{\lambda}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}. \quad (3)$$

The maximum likelihood estimation (MLE) of λ and μ, say $\hat{\lambda}$ and $\hat{\mu}$, respectively, can be obtained by solving the following log-likelihood equations

$$\frac{\partial l}{\partial\lambda}=\frac{n}{2\lambda}-\frac{1}{2\mu^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}=0, \quad (4)$$

$$\frac{\partial l}{\partial\mu}=\frac{\lambda}{\mu^3}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{x_i}+\frac{\lambda}{\mu^2}\sum_{i=1}^{n}\frac{x_i-\mu}{x_i}=0. \quad (5)$$

By solving the log-likelihood equations (4) and (5), we obtain $\hat{\mu}=\bar{x}$ and $\hat{\lambda}=\frac{n\bar{x}^2}{\sum_{i=1}^{n}\frac{(x_i-\bar{x})^2}{x_i}}$.

III. Tensile and fatigue of SAC305 solder joints data

In this study, tensile and fatigue of SAC305 solder joints data sets at different temperature are taken from Wang et al's experiment data [12]. The data are listed table I–III. In Table I–III, where T represents the aging temperature (in Kelvin) and T_0 equals to 217°C, which is the melting point of SAC305. For more details see [12].

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TABLE I
TENSILE STRENGTH (IN MPA) OF SOLDER JOINTS AT DIFFERENT AGING TEMPERATURES AFTER AGING FOR 24 H (SERIES 1)

T/T_0	0.157	0.455	0.495	0.537	0.598	0.659	0.700	0.741	0.782	0.822	0.863	0.945
S1	41.10	45.20	47.40	45.10	49.53	43.32	41.32	38.55	40.20	39.50	38.83	33.58
S2	42.82	45.39	49.37	46.00	49.91	46.31	43.07	40.20	40.74	40.93	39.31	33.77
S3	43.26	49.75	50.52	50.42	52.43	47.40	44.66	42.59	41.54	42.49	39.53	34.38
S4	44.64	53.99	51.28	51.22	52.49	48.38	49.18	43.29	44.44	43.67	39.95	35.14
S5	47.01	54.18	53.67	53.38	53.92	49.18	50.29	47.59	44.72	45.04	40.08	35.56
S6	51.60	54.69	54.27	55.51	55.32	51.60	50.90	49.53	45.87	49.18	40.68	38.01
S7	52.36	56.21	55.93	55.90	55.48	55.99	51.47	51.73	48.29		43.04	41.70
S8	55.93			57.55					50.42		43.19	43.42
S9											45.71	46.06

Where the first horizontal row represents the normalization aging temperature and the first vertical column represents different sample numbers at the same aging temperature.

TABLE II
TENSILE STRENGTH (IN MPA) OF SOLDER JOINTS AT DIFFERENT AGING TEMPERATURES AFTER AGING FOR 72 H (SERIES 2)

T/T_0	S1	S2	S3	S4	S5	S6	S7	S8
0.157	44.53	45.79	47.40	49.56	50.68	50.75	53.01	56.01
0.455	41.20	41.54	42.55	43.85	43.98	48.88	48.99	49.89
0.598	38.12	39.34	40.15	41.02	41.34	42.66	42.96	43.48
0.945	31.11	31.94	33.44	34.02	34.60	35.81	38.92	

TABLE III
FATIGUE LIFE (IN CYCLE) OF SOLDER JOINTS AT DIFFERENT AGING TEMPERATURES AFTER AGING FOR 72 H (SERIES 3)

T/T_0	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15
0.157	874	947	1013	1038	1176	1279	1516	1603	1781	2137	2246	2456	2528	2754	3243
0.455	462	502	607	758	909	994	1024	1149	1298	1497	1576	1792	1881	2112	2236
0.598	416	469	520	554	609	770	806	853	934	1159	1261	1391	1522	1534	1799
0.945	191	293	337	359	402	537	689	764	787	923	986	1189	1319	1334	1379

IV. RESULTS AND DISCUSSION

In this section, we use IG distribution to model Tables I–III data. We check the validity of the IG model based on Kolmogorov-Smirnov (K-S) test. We compute the K-S distance between the empirical distribution and fitted distribution functions, and corresponding p-values based on the MLEs. The estimated parameters, K-S statistic values with their p-values are presented in Tables IV–VI. In order to assess if the model is appropriate, another useful graphical approach is to use the quantile-quantile (Q-Q) plot for each data set. A Q-Q plot depicts the points $\{F^{-1}(\frac{i-0.5}{n};\hat{\lambda},\hat{\mu}),x_i,i=1,2,\cdots,n\}$, where $\hat{\lambda},\hat{\mu}$ are the MLEs of λ,μ. Since the IG CDF does not have an explicit inverse $F^{-1}(x,\lambda,\mu)$, the roots of $F(x,\hat{\lambda},\hat{\mu})=\frac{i-0.5}{n},i=1,2,\cdots,n$, are solved numerically. We use the uniroot function in the R software to find the roots of the above equation. The corresponding Q-Q plots are shown in Figures 1–5.

From Tables IV–VI, it is clear that IG model fits quite well to in Tables I–III data sets. The corresponding Q-Q plots also show the IG distribution fit these data quite well and can be adequate for modeling these data. From these plots and K-S statistic values with the corresponding p-values can conclude that the IG distribution fits the experimental data well. Hence, the IG distribution can be used to statistically analyzed on thermal aging effect of SAC305 solder joints.

TABLE IV
MLES, K-S AND P-VALUES OF FITTING IG MODEL WITH SERIES 1 DATA

T/T_0	$\hat{\lambda}$	$\hat{\mu}$	p-value	K-S
0.157	4344.0	47.34	0.87	0.195
0.455	7176.7	51.34	0.43	0.309
0.459	17918.7	51.78	0.94	0.185
0.537	7301.8	51.89	0.93	0.176
0.598	29818.4	52.73	0.92	0.190
0.659	8551.8	48.88	0.97	0.167
0.700	6872.1	47.27	0.60	0.270
0.741	4322.0	44.78	0.94	0.183
0.782	7718.8	44.53	0.90	0.185
0.822	8736.0	43.47	1	0.130
0.863	15310.0	41.15	0.25	0.530
0.945	3016.8	37.96	0.52	0.256

TABLE V
MLES, K-S AND P-VALUES OF FITTING IG MODEL WITH SERIES 2 DATA

T/T_0	$\hat{\lambda}$	$\hat{\mu}$	p-value	K-S
0.157	9859.1	49.72	1	0.122
0.455	8334.8	45.11	0.63	0.246
0.598	22397.5	41.13	0.93	0.177
0.945	7222.1	34.26	0.99	0.144

V. THERMAL AGING LIFE DATA FOR ENAMELLED WIRES WITH COMPOSITE INSULATION LAYERS

In this section, to illustrate the flexibility of the IG distribution, we use IG distribution to fit failure data of thermal aging life for enameled wires with composite insulation layers. The data set is given by Pan et al [15]. The data set is listed in Table VII. The estimated parameters, K-S statistic values with their p-values are listed in Table VIII. From Table VIII, IG distribution may well model failure data of thermal aging life for enameled wires with composite insulation layers. Therefore, IG distribution may be used to

TABLE VI
MLEs, K-S and p-values of fitting IG model with Series 3 data

T/T_0	$\hat{\lambda}$	$\hat{\mu}$	p-value	K-S
0.157	9654.3	1772.7	0.88	0.143
0.455	4702.8	1253.1	0.95	0.126
0.598	4268.6	973.1	0.94	0.127
0.945	1792.2	765.9	0.82	0.153

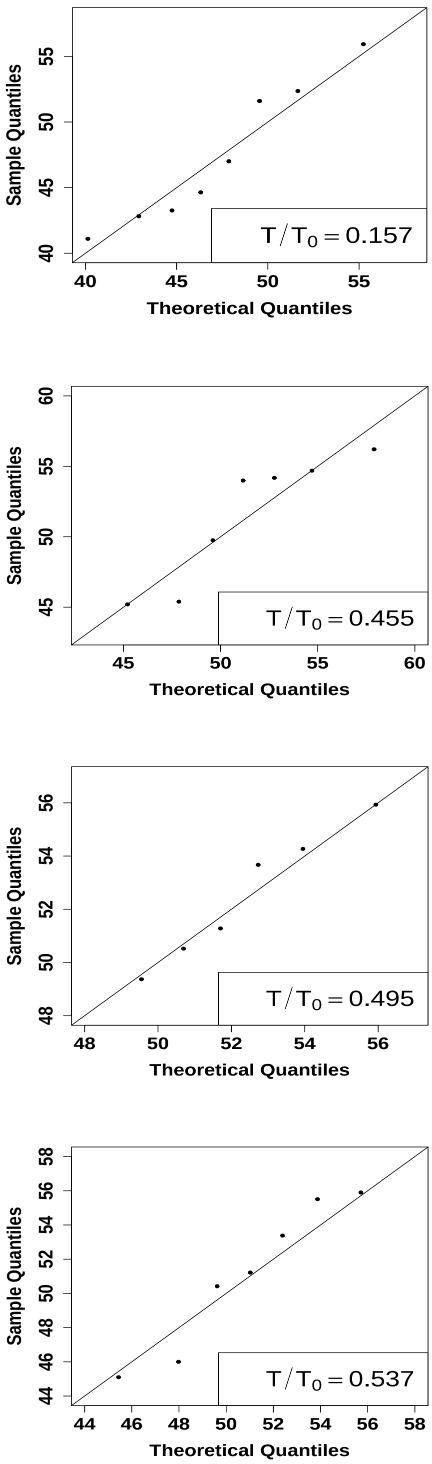

Fig. 1. Q-Q plot of the fitted IG distribution for tensile strength of solder joints at different aging temperatures after aging for 24 h under $T/T_0 = (0.157, 0.455, 0.495, 0.537)$.

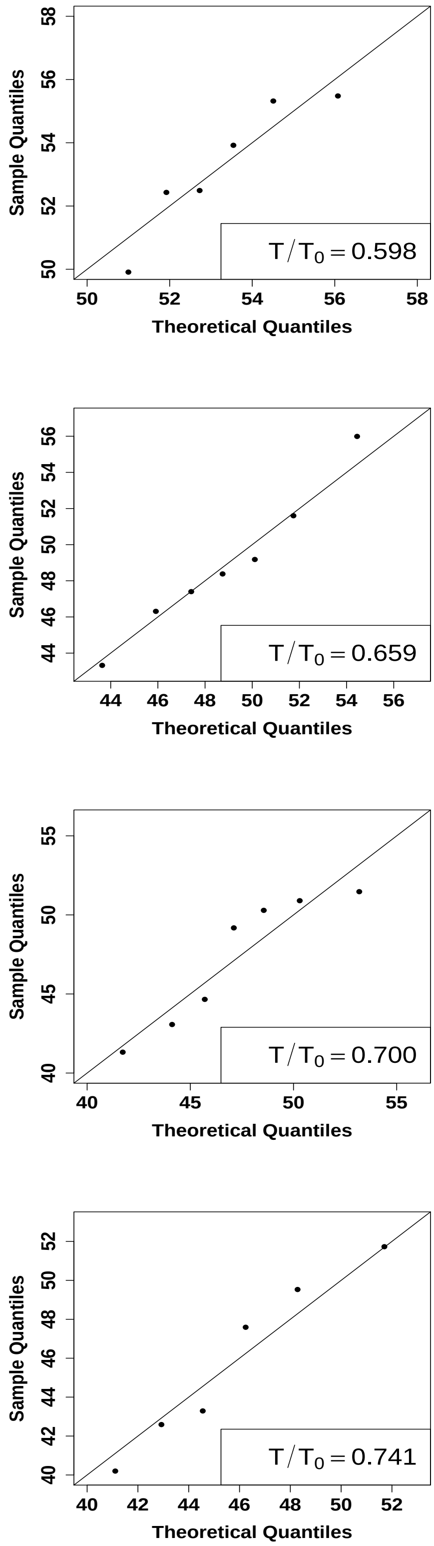

Fig. 2. Q-Q plot of the fitted IG distribution for tensile strength of solder joints at different aging temperatures after aging for 24 h under $T/T_0 = (0.598, 0.659, 0.700, 0.741)$.

model many complex life data sets.

VI. Conclusion

In this paper, we use IG model to fit tensile and fatigue of SAC305 solder joint data at different temperature. The results

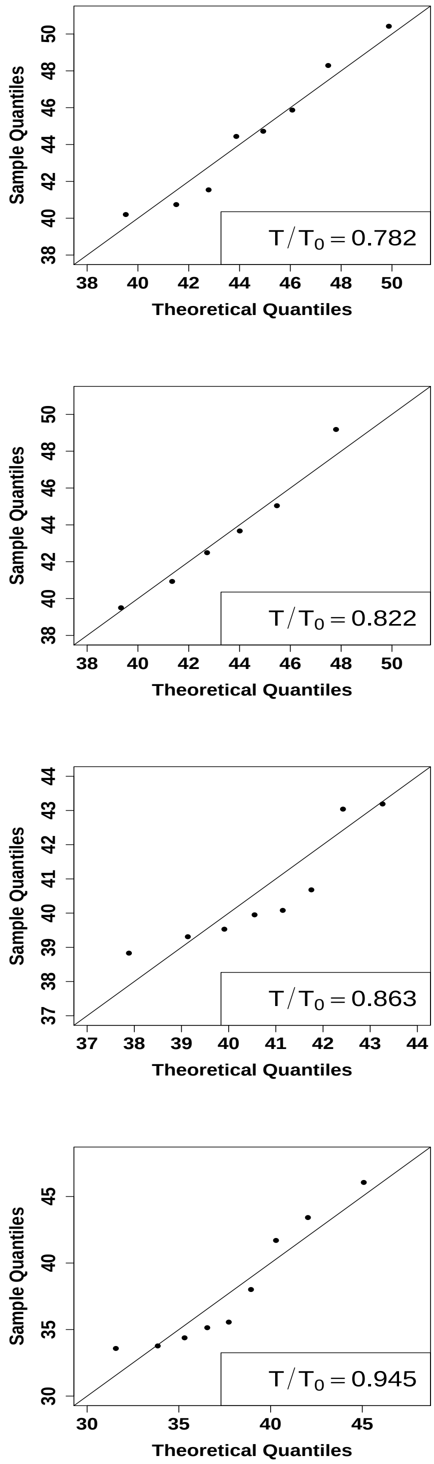

Fig. 3. Q-Q plot of the fitted IG distribution for tensile strength of solder joints at different aging temperatures after aging for 24 h under $T/T_0 = (0.782, 0.822, 0.863, 0.945)$.

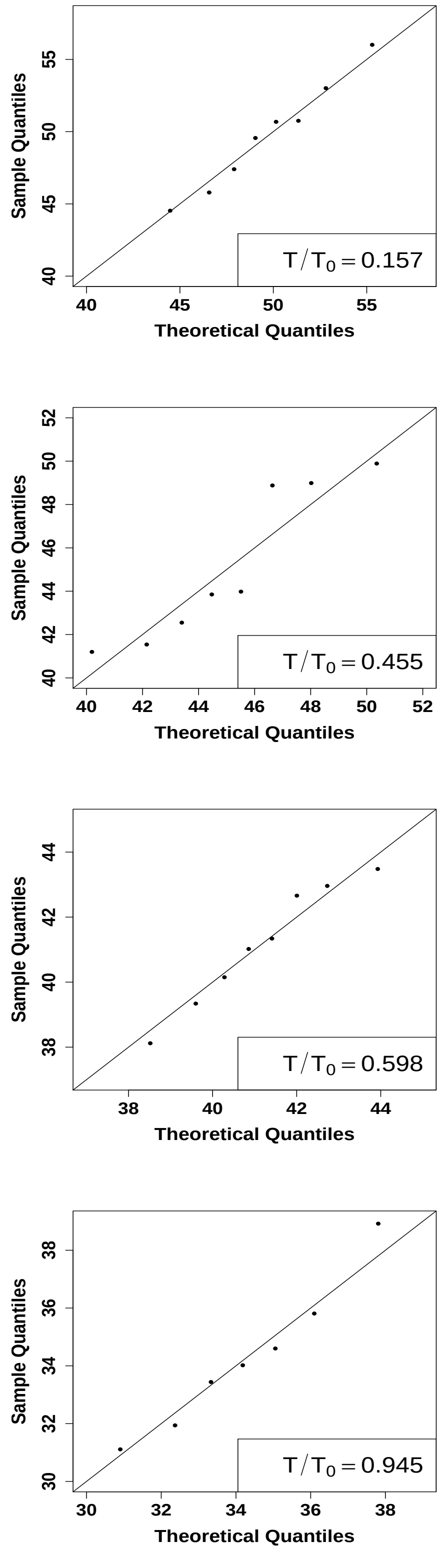

Fig. 4. Q-Q plot of the fitted IG distribution for tensile strength of solder joints at different aging temperatures after aging for 72 h under $T/T_0 = (0.157, 0.455, 0.598, 0.945)$.

show that the IG distribution agrees well with the experiment data. We can conclude that IG model is a very good choice for modeling tensile and fatigue of SAC305 solder joint data at the different temperature. The IG distribution is used to fit thermal aging life data to illustrate the flexibility of the IG distribution. The fitting results show that IG distribution can be used to model many complex life data.

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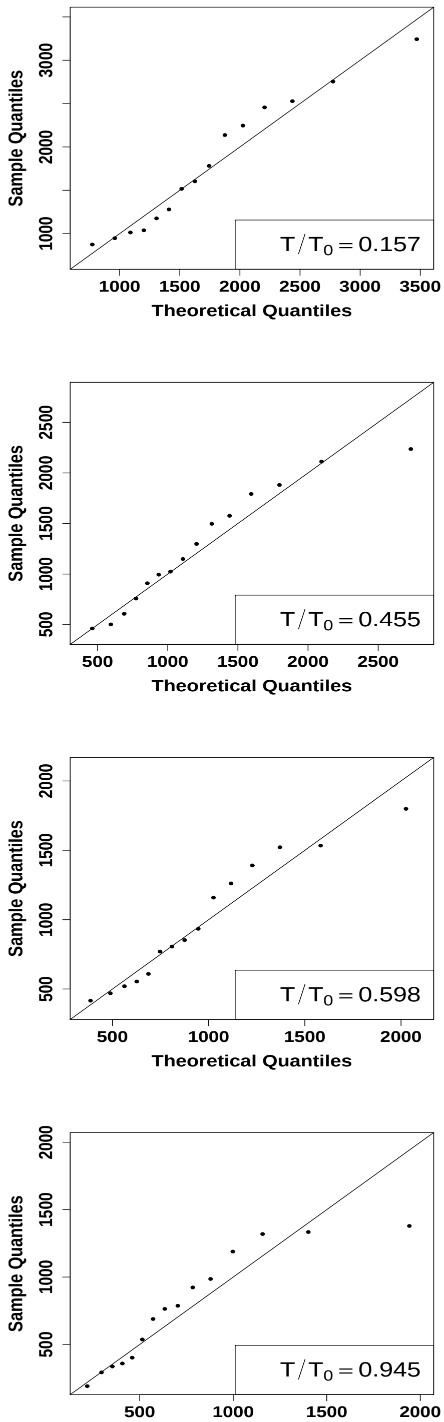

Fig. 5. Q-Q plot of the fitted IG distribution for fatigue life of solder joints at different aging temperatures after aging for 72 h under $T/T_0 = (0.157, 0.455, 0.598, 0.0.945)$.

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TABLE VII
FAILURE DATA OF THERMAL AGING LIFE FOR ENAMELED WIRES WITH COMPOSITE INSULATION LAYERS

T	Failure times(min)						
210°C	3853	3853	4523	5193	5193	5193	5863
	5863	5863	5863	5863	5863	5863	5863
	5863	6871	6871	7375	7015	8023	8023
230°C	1086	1420	1587	1754	1921	2255	2255
	2422	2422	2422	2589	2589	2589	2589
	3093	3237	3237	3237	4101	4101	4101
250°C	406.5	268.5	268.5	383.5	291.5	268.5	452.5
	245.5	475.5	268.5	475.5	406.5	475.5	314.5
	291.5	567.5	383.5	475.5	521.5	406.5	429.5

TABLE VIII
MLES, K-S AND P-VALUES OF FITTING IG MODEL WITH FAILURE DATA OF THERMAL AGING LIFE DATA

T	$\hat{\lambda}$	$\hat{\mu}$	p-value	K-S
210°C	153912.80	5940.62	0.232	0.2263
230°C	21409.62	2619.38	0.673	0.1577
250°C	5778.68	384.60	0.615	0.1653