Multi-objective Robust Optimization Model for Spare Parts Supply in Wartime

Yadong Wang, Quan Shi

***ABSTRACT* — High uncertainty is one of the challenges faced by wartime spare parts supply because of the diversity of combat scenarios, the fluctuation of spare parts demands and the supply risk. Considering the uncertainty of scenarios and such parameters, a robust optimization model of wartime spare parts supply is constructed. In the model, the shortest lead time and the minimum shortage costs are considered simultaneously, which is formulated as a multi-objective optimization problem. The adaptive penalty function method is used for the unconstrained processing. A multi-objective differential evolution algorithm with improved evolution strategy is used to solve the model. The results of the example show that, on the one hand, the improved evolution strategy improves the performance of the algorithm to some extent. On the other hand, the optimal solution of the robust optimization model can guarantee the feasible of wartime spare parts supply in the "worst case", that is, the model has a good robustness.**

***Index Terms*— wartime, spare parts supply, uncertainty, robust optimization, multi-objective optimization,**

I. INTRODUCTION

IN the modern war, how to overcome the uncertainty in battlefield condition and send spare parts to the front line quickly and accurately is an important issue. There are many differences between spare parts supply in wartime and peacetime. In the peacetime, the factors, such as demands of spare parts and the risk of disruption, could be known by analysis and prediction. In the wartime, it is difficult to accurately predict the demands of spare parts. That is because that, first of all, the diversity of combat patterns lead to the regulation of equipment usage and damage are different in different combat scenarios. Secondly, the loss of spare parts in wartime is a competitive failure under both the action of degradation and battle damage [1].

There are lots of works focus on the spare parts supply in peacetime, but just few of researches have been done to deal with the spare parts supply in wartime. Ren et.al. assumed that the demands rate of spare parts obeyed a known probability distribution, and studied the spare parts supply optimization problem under the uncertain demand by using stochastic optimization [2]. Liu established a stochastic programming model of irreparable spare parts supply in wartime by chance constrained programming [3]. It can be seen that, in terms of optimizing the supply of spare parts in wartime, uncertainty is one of the most challenges. There are few research works study the optimization of spare parts support in wartime from the aspects of uncertainty of spare parts demands, battle scenario and supply risk simultaneously. It is usually assumed that the demand of spare parts obeys the known probability distribution, which is not consistent with the real consumption of spare parts in wartime.

When dealing with the uncertainty of logistics, the methods of stochastic optimization, fuzzy optimization and robust optimization are often used. Reference [4] dealt with the uncertainty of transportation cost and product quantity in reverse supply by establishing a multi-product and multi-level stochastic optimization model. However, in stochastic optimization, it is usually necessary to know or assume the distribution of uncertain parameters in advance. In reference [5], the influence of uncertainty of cost and demand rate on supply network was studied by using fuzzy optimization method. However, fuzzy optimization is usually difficult to solve, and it is difficult to guarantee the feasibility of the solution in the worst case. Thus, the robust optimization was proposed because it doesn’t rely on the distribution of uncertain parameters, and can guarantees the feasibility of the solution in the worst case. Reference [6] used robust optimization to solve discrete scenario uncertainties in logistics network design, and three robust optimization models: maximum cost, maximum repentance and relative repentance were compared. Reference [7] considered the static scenario uncertainty optimization method in supply networks, in which the random parameters belong to box-shaped or ellipsoidal uncertain sets. In reference [8], a hybrid robust optimization model for optimal design of biofuel supply chain was proposed. The uncertainty of the technology was expressed as an imprecise conversion rate, and expressed as a probability scenario. The literature [9] proposed a stochastic robust optimization model for a closed-loop supply chain network design that considered the lateral transfer as a response strategy to the operational and interrupt risk so that the network is resilient when the supply is interrupted.

At present, robust optimization includes continuous uncertainty and discrete uncertainty, and most of research works just focus on single kind of uncertainty and single objective optimization problems. There is still a lot of improvement for combinational multi-objective robust optimization. In this paper, the optimization problem with continuous uncertainty such as spare parts demands and risk, as well as, discrete uncertainty of combat scenarios is studied. A multi-objective robust optimization model of spare parts supply in wartime is established, considering the minimum spare parts supply lead time and minimum shortage cost as the optimization objectives at the same time.

Manuscript received May, 20,2019; revised June, 19, 2019
Yadong Wang is currently pursuing the Ph.D. degree in armament science and technology from Army Engineering University of PLA. His research interests include equipment maintenance engineering, logistics management, optimization algorithm, artificial intelligence, and so on.
E-mail: xwzj0003@gmail.com

Quan Shi is a professor in Army Engineering University of PLA. His research interests include equipment maintenance engineering, battle damage assessment, PHM, artificial intelligence, and so on.
E-mail: junshiyc@163.com

II. SUPPLY OPTIMIZATION MODEL WITHOUT UNCERTAINTY

A. Problem Description

In the classical three-echelon supply network, the spare parts are supplied from rear warehouses to the front line to meet the demands of battlefield repair. The field warehouses serve as distribution centers to receive spare parts from the rear warehouses and distribute them to combat units.

This paper sets up a finite discrete set of task scenario $S=\{1,2,\cdots,s\}$. In different scenario, the values of parameters such as cost and risk are quite different. Even in the same scenario, parameters such as the demands of spare parts is neither determined nor subject to a specific distribution.

The model of spare parts supply in wartime must be based on the following assumptions: (1) this paper takes the supply of single kinds of spare parts as an example. In order to ensure the efficiency of emergency repair, replacement maintenance is adopted as the main maintenance method, and the demands of spare parts are equal to the quantity of replacement spare parts; (2) lateral transport between nodes is not taken into account; (3) in the supply network, the locations of each node are known; the transport time between rear warehouses to the field warehouses and between field warehouses to the combat units are known and fixed; (4) the risk of enemy attack only occur in the transportation between field warehouses and combat units, and the value of risk is uncertain. The overall maximum risk threshold of the whole supply network is known; (5) the rear warehouses have unlimited capacity and sufficient spare parts. The maximum capacity of each field warehouse is known and fixed; (6) the priority of combat units is the same, and the demand of spare parts of each combat unit is uncertain. The threshold of minimum spare parts fill rate of each combat unit is known. (7) since the more spare parts are supplied, the greater the risk is, this paper assumes that the risk is proportional to the number of spare parts; (8) in order to meet the requirement of spare parts for urgent repair task in the shortest time, only time and risk factors are considered, and other factors such as economy are not taken into account in military.

B. Parameter Description

L : index of rear warehouses， $l=1,2,\cdots,L$; I : index of field warehouses, $i=1,2,\cdots,I$; J : index of combat units， $j=1,2,\cdots,J$; S :index of scenarios, $s=1,2,\cdots,S$; U_i : maximum capacity of the field warehouse i ; $\tilde{d}_j^{\,s}$: The actual demand of spare parts of combat unit j under the scenario s ; T_{li} : transport time of per unit spare parts supplied from rear warehouse l to field warehouse i ; T_{ij} : transport time of per unit spare parts supplied from field warehouse i to combat unit j ; $\tilde{r}_{ij}^{s}$: actual risk of supply from field warehouse i to combat unit j in scenario s ; r^s : risk threshold for the entire supply network in scenario s ; $c_j^{\,s}$: unit shortage cost of the combat unit j in scenario s ; C^s : total shortage cost of the entire supply network in scenario s ; μ^s : the lowest threshold of demand fill rate, $\mu^s\in(0,1]$; x_{li} : the amount of spare parts supplied from rear warehouse l to field warehouse i ; y_{ij} : the amount of spare parts supplied from field warehouse i to combat unit j ;

C. Modeling

The following mixed integer programming model is established by considering the model under certain conditions:

$$\min T=\sum_{l}\sum_{i}(\mathrm{sgn}(x_{li})\times T_{li})+\sum_{i}\sum_{j}(\mathrm{sgn}(y_{ij})\times(T \tag{1}$$

$$\min \ C^s=\sum_{i}\sum_{j}c_j^s(\tilde{d}_j^s-y_{ij}) \tag{2}$$

s.t.

$$\mu^s-\frac{\sum_i y_{ij}}{\tilde{d}_j^{\,s}}<0 \tag{3}$$

$$\sum_i\sum_j(y_{ij}\times\tilde{r}_{ij}^s)-r^s<0 \tag{4}$$

$$\sum_l x_{li}-U_i<0 \tag{5}$$

$$\sum_j y_{ij}-\sum_l x_{li}\le 0 \tag{6}$$

$$y_{ij}\ge 0;x_{li}\ge 0,y_{ij},x_{li}\in N^{+} \tag{7}$$

$$s\in 1,2,\cdots,S \tag{8}$$

The objective function (1) indicates that the total lead time of the supply network should be minimum in order to complete the supply task in the shortest time, $\mathrm{sgn}(x)=\frac{2}{\pi}\int_0^{+\infty}\frac{\sin xt}{t}dt$ is a step function, when $x>0$, the value of the function is equal to 1, when $x=0$, the value of function is equal to 0, and the value of function is -1 while $x<0$; The objective function (2) indicates that the total cost of spare parts in the supply network should be minimum in order to ensure the implementation of the emergency repair task; Formula (3) indicates that the total fill rate of spare parts should not be lower than the minimum threshold; Formula (4) indicates that the overall risk of spare parts supplying from field warehouses to combat units should not exceed the maximum risk threshold; Formula (5) indicates the capacity limit of each field warehouse when the spare parts arrive from the rear warehouses to the field warehouses; Formula (6) indicates that the output of spare parts in each field warehouse should not exceed the amount of input. Constraint (7) defines the type of decision variable. In wartime spare parts supply, constraints (3)~(7) are hard constraints, that is, in any situation, the worst case within any range of uncertainty must be satisfied.

III. THE ROBUST OPTIMIZATION MODEL UNDER UNCERTAINTY

In the above model, the demand of spare parts, the supply risk, the shortage costs of the combat units and the minimum fill rate of the spare parts are different in the different scenarios, which meet the discrete uncertainty. Meanwhile, even in the same scenario, due to the complexity of the battlefield environment, the actual spare parts demands and the actual risks of the supply are uncertain, which can be formulate as: $\tilde{d}_j^{\,s} = d_j^{\,s} + \rho \cdot \bar{d}_j^{\,s}$, $\tilde{r}_{ij}^s = r^s{}_{ij} + \rho \cdot \bar{r}_{ij}^s$, where, $d_j^{\,s}$ and $r^s{}_{ij}$ are nominal values, ρ is the continuous uncertainty parameter. Robust optimization needs to deal with the two kinds of continuous and discrete uncertainty in the model simultaneously.

A. Robust Optimization Model with Uncertain Scenarios

In the above model, x_{li} and y_{ij} are decision variables, $\tilde{d}_j^{\,s}$, $\tilde{r}_{ij}^s$, $c_j^{\,s}$ and μ^s are uncertain control variables. In each discrete scenario s , $\left\{\tilde{d}_j^{\,s}, \tilde{r}_{ij}^s, c_j^{\,s}, \mu^s\right\}$ is a set of constraints, and P_s is the probability of occurrence for each scenario, $\sum_{s=1}^{S} P_s = 1$. If the decision variables keep the objective function of the model optimal in any scenario, the *optimality robustness* is defined. If the decision variables make the model basically feasible in any scenario, the *feasibility robustness* is defined. In robust optimization, a trade-off between optimality robustness and feasibility robustness of the model is required.

In the robust optimization model proposed by Mulvey, the balance between robustness of the solution and model are controlled by penalty function. Its objective function is represented as follows:

$$\min\ \sigma(\mathrm{x}, \mathrm{y}_1, \mathrm{y}_2, \cdots, \mathrm{y}_s) + \omega\rho(\mathrm{z}_1, \mathrm{z}_2, \cdots, \mathrm{z}_s) \quad (9)$$

where the first term $\sigma(\mathrm{x}, \mathrm{y}_1, \mathrm{y}_2, \cdots, \mathrm{y}_s)$ is used to measure the robustness of solution. The second term $\rho(\mathrm{z}_1, \mathrm{z}_2, \cdots, \mathrm{z}_s)$ is feasible penalty function which is used to describe the robustness of the model. $\rho(\bullet)$ is penalty function, $\{\mathrm{z}_s\}$ is the set of error vectors, ω is penalty coefficient.

In this paper, the following mean variance model is used in robust optimization:

$$\sigma(\bullet) = \sum_{s\in S} P_s \cdot c^s + \lambda \sum_{s\in S} P_s \cdot (c^s - \sum_{s\in S} P_s \cdot c^s)^2 \quad (10)$$

$$\rho(\bullet) = \sum_{s\in S} P_s \cdot (d^s - y^s)^2 \quad (11)$$

where the first term in formula (10) is the expected shortage cost in discrete scenario, and the second term is the variance of shortage cost which is used to measure the range of variation of objective function. The formula (11) is the variance between the supply and the actual demand, which is used to punish the deviation of the scenario parameters.

So the robust counterpart of objective function (2) is:

$$\min\ C^s = \sum_s \sum_i \sum_j P_s c_j^s (\tilde{d}_j^s - y_{ij}) + \lambda \cdot \left[\sum_s \sum_i \sum_j P_s (c_j^s (\tilde{d}_j^s - y_{ij}) - \sum_s \sum_i \sum_j P_s c_j^s (\tilde{d}_j^s - y_{ij}))^2 \right] + \omega \cdot \sum_s \sum_i \sum_j P_s (\tilde{d}_j^s - y_{ij})^2 \quad (12)$$

B. Robust Optimization Model with Uncertain Parameters

Even in a certain scenario, the demand of combat units $\tilde{d}_j^{\,s}$ and the transport risk $\tilde{r}_{ij}^s$ are still uncertain parameters, which can neither be expressed as constants nor be described by specific distributions. Thus, robust optimization is adopted to ensure that the requirements in the "worst case" can be met.

Without losing generality, the basic form of linear robust optimization is as follows:

$$\begin{array}{llcll} & \min\ CX & & & \min\ t \\ s.t. & AX \le b & \Rightarrow & s.t. & CX \le t \\ & l \le x \le u & & & AX \le b \\ & & & & l \le x \le u, t \in R \end{array} \quad (13)$$

where, the coefficient matrix C , A and the vector b contain uncertain parameters. Taking uncertain coefficient matrix $A = (a_{ij}) = (a_1, a_2, \cdots, a_m)$ as an example, let J_i is a set of subscripts for all columns of uncertain data in the ith row of the coefficient matrix a_{ij}, and the uncertain parameters in the matrix can be expressed as $a_{ij}, j \in J_i$ .Therefore, the uncertain parameters in formula (13) are represented as follows: $\tilde{a}_{ij} = a_{ij} + \rho_{ij}\bar{a}_{ij}$, $\tilde{c}_j = c_j + \rho_{j0}\bar{c}_j$ and $\tilde{b}_i = b_i + \rho_i \bar{b}_i\ \forall j \in J_i$.

Where $\tilde{a}_{ij}$, $\tilde{c}_j$ and $\tilde{b}_i$ represent the uncertain parameters; a_{ij} , c_j , b_i are defined as nominal values; $\bar{a}_{ij}$, $\bar{c}_j$ and $\bar{b}_i$ are used as uncertainty scale; ρ_{ij} , ρ_{j0} and ρ_i represent the uncertainty level, $\rho \in U$, where U is uncertainty set [10].

Then the robust counterpart of model (13) is as follow [11]:

$$\begin{array}{ll} \min\ t & \\ s.t. & \sum_j c_j x_j + [\max_{\rho\in \mathrm{U}} \left\{ \sum_{j\in J_i} \rho_{j0} \bar{c}_j x_j \right\}] \le t \\ & \sum_j a_{ij} x_j + [\max_{\rho\in \mathrm{U}} \left\{ \sum_{j\in J_i} \rho_{ij} \bar{a}_{ij} x_j \right\}] \le b_i - \max_{\rho\in \mathrm{U}} (\rho_i \bar{b}_i) \end{array} \quad (14)$$

There are several types of uncertain sets in linear robust optimization, such as box, polyhedral, ellipsoidal, et al [12]. Their uncertainty sets are as follows, respectively:

$$U_{box} = \left\{\rho \middle| \|\rho\|_\infty \le \varphi\right\} = \left\{\rho \middle| |\rho| \le \varphi, \forall j \in J_i\right\}$$

$$U_{polyhedral} = \left\{\rho \middle| \|\rho\|_1 \le \varphi\right\} = \left\{\rho \middle| \sum_{J_i} |\rho| \le \Gamma, \forall j \in J_i\right\}$$

$$U_{ellipsoidal} = \left\{\rho \middle| \|\rho\|_2 \le \varphi\right\} = \left\{\rho \middle| (\sum_{J_i} |\rho|^2)^{1/2} \le \Gamma\right\}$$

In the box-type uncertain set, when φ is equal to 1, the uncertain set is interval uncertainty. In this paper, the demands and risks of spare parts in wartime are all expressed by interval uncertainty. Then, $\max_{\rho \in U} \left\{ \sum_{j \in J_i} \rho_{j0} \bar{c}_j x_j : \|\rho\|_\infty \le 1 \right\} = \sum_{j \in J_i} \bar{c}_j |x_j|$. So the corresponding linear interval robust optimization equivalent model of model (13) is as follows:

$$\begin{aligned} &\min\ t \\ &s.t. \quad \sum_j c_j x_j + \sum_{j \in J_i} \bar{c}_j |x_j| \le t \qquad (15) \\ &\qquad \sum_j a_{ij} x_j + \sum_{j \in J_i} \bar{a}_{ij} |x_j| \le b_i - \bar{b}_i \end{aligned}$$

Based on the above derivation, the robust counterpart of objective function (12) is as follows:

$$\begin{aligned} &\min\ t \\ &\sum_s \sum_i \sum_j P_s c_j^s [(d_j^s + \bar{d}_j^s) - y_{ij}] - \\ &\lambda \cdot \left[\sum_s \sum_i \sum_j P_s \left\{ c_j^s [(d_j^s - \bar{d}_j^s) - y_{ij}] - \sum_s \sum_i \sum_j P_s c_j^s [(d_j^s + \bar{d}_j^s) - y_{ij}] \right\} \right]^2 \\ &+ \omega \cdot \sum_s \sum_i \sum_j P_s ((d_j^s + \bar{d}_j^s) - y_{ij})^2 - t \le 0 \end{aligned} \tag{16}$$

The robust counterpart of constraint (3) is:

$$\sum_j (d_j^{\ s} + \bar{d}_j^{\ s}) \mu^s - \sum_i \sum_j y_{ij} < 0 \tag{17}$$

The robust counterpart of constraint (4) is:

$$\sum_i \sum_j (y_{ij} \times r_{ij}^s + \bar{r}_{ij}^s |y_{ij}|) - r^s < 0 \tag{18}$$

In this case, a constrained multi-objective robust optimization model for wartime spare parts supply under discrete scenarios is obtained as equation (19).

$$\begin{aligned} &\min\ T = \sum_l \sum_i (\mathrm{sgn}(x_{li}) \times T_{li}) + \sum_i \sum_j (\mathrm{sgn}(y_{ij}) \times T_{ij}) \\ &\min\ t \\ &s.t. \quad \sum_s \sum_i \sum_j P_s c_j^s [(d_j^s + \bar{d}_j^s) - y_{ij}] + \\ &\lambda \cdot \sum_s \sum_i \sum_j P_s \left\{ c_j^s [(d_j^s - \bar{d}_j^s) - y_{ij}] - \sum_s \sum_i \sum_j P_s c_j^s [(d_j^s + \bar{d}_j^s) - y_{ij}] \right\}^2 \\ &+ \omega \cdot \sum_s \sum_i \sum_j P_s ((d_j^s + \bar{d}_j^s) - y_{ij})^2 - t \le 0 \\ &\qquad (d_j^{\ s} + \bar{d}_j^{\ s}) \mu^s - \sum_i y_{ij} < 0 \\ &\qquad \sum_i \sum_j (y_{ij} \times r_{ij}^s + \bar{r}_{ij}^s |y_{ij}|) - r^s < 0 \\ &\qquad \sum_l x_{li} - U_i < 0 \\ &\qquad \sum_j y_{ij} - \sum_l x_{li} \le 0 \\ &\qquad y_{ij} \ge 0; x_{li} \ge 0, x_{li}, y_{ij} \in N^+, t \in R \\ &\quad i = 1, 2, \cdots, I;\ j = 1, 2, \cdots, J;\ s = 1, 2, \cdots, S \end{aligned} \tag{19}$$

C. Constraints Handling

Before solving the fitness function constructed by the model (19) by using meta-heuristic algorithm, constrained handling process should be carried out firstly. Constrained handling is a key problem in multi-objective optimization. At present, the most common used methods can be classed in penalty function method, feasible rule based method, multi-objective method, and so on [13]. It is often difficult to select a reasonable rule for the feasible rules based method to balance the dominance of solutions and the degree of violation of constraints. The multi-objective method transforms the constraints into objectives, which greatly increases the difficulty of solving the problem. As the most common used algorithm, penalty function method has the characteristics of high feasibility and easy to be understood and calculated. Penalty function method includes stationary penalty function method, dynamic penalty function method and adaptive penalty function method. In literature [14], an adaptive penalty function method for evolutionary computation is presented. The constrained multi-objective optimization problem is transformed into an unconstrained multi-objective optimization, and then the multi-objective evolutionary algorithm is used to solve the constrained multi-objective optimization problem.

Defining a new fitness function consists of distance and penalty terms:

$$f_i(\bar{z}) = D_i(\bar{z}) + M \cdot P_i(\bar{z}) \tag{20}$$

where $f_i(\bar{z})$ represents the i th fitness function, $D_i(\bar{z})$ is the distance item, M is penalty coefficient, $P_i(\bar{z})$ is penalty term and $\bar{z}$ is individual vector.

The constraint violation is defined as follows:

$$v_i(\bar{z}) = \sum_{j=1}^{m} c_j(\bar{z}) \tag{21}$$

where, for inequality constraints, $c_j(\bar{z}) = \max(0, g_j(\bar{z}))$, and $g_j(\bar{z}) \le 0$ represents the j th unequal constraint. For equality constraints, $c_j(\bar{z}) = \max(0, |h_j(\bar{z}) - \varepsilon|)$, and $h_j(\bar{z}) \le 0$ represents the j th equal constraint, ε is a very small positive number.

The parameter r_f is used to represent the proportion of feasible individuals in the population.

$$r_f = \frac{\text{the number of feasible individual in population}}{\text{the size of population}}$$

Then the distance item is defined as follows:

$$D_i(\bar{z}) = \begin{cases} v_i(\bar{z}) & \text{if } r_f = 0 \\ \sqrt{obj_i(\bar{z})^2 + v_i(\bar{z})^2} & otherwise \end{cases} \tag{22}$$

where obj_i represents the i th objective function.

Define the punishment term as follows:

$$P_i(\bar{z}) = (1-r_f)X_i(\bar{z}) + r_f Y_i(\bar{z}) \tag{23}$$

where $X_i(\bar{z}) = \begin{cases} 0 & \text{if } r_f = 0 \\ v_i(\bar{z}) & otherwise \end{cases}$,

$Y_i(\bar{z}) = \begin{cases} 0 & \text{if } \bar{z} \text{isfeasible} \\ obj_i(\bar{z}) & \text{otherwise} \end{cases}$.

IV.SOLUTION ALGORITHM

Multi-objective evolutionary algorithm (MOEA), as a meta-heuristic algorithm, can solve multi-objective optimization problems effectively. Differential evolution (DE) algorithm is one of the widely used MOEA. In this paper, the improved multi-objective differential evolution algorithm is used to obtain the optimal solutions of the multi-objective optimization model by comparing the Pareto dominance relation between individuals.

There are four basic steps when using differential evolution: initialization, mutation, crossover, and selection.

Step 1 Initialization. The initial population is randomly generated, and the size of the population is N, the size of individual dimension is D. The elements, from x_{i1} to x_{iD} on the i th individual, represent the decision variable of the model. The individual elements of the initial population are as follows:

$$x_{ij}^0 = x_j^l + rand(0,1)\cdot(x_j^u - x_j^l) \tag{24}$$

where x_j^u and x_j^l are the upper and lower bounds of decision variables, respectively. $rand(0,1)$ represents a uniform distributed random number between $[0,1]$.

Step 2 Mutation. $x_i^t = x_{i1}^t, x_{i2}^t, \cdots, x_{iD}^t$ is the i th individual vector in the t th iteration. The mutation individual $v_i^t = v_{i1}^t, v_{i2}^t, \cdots, v_{iD}^t$ is generated according to the mutation strategy.

At present, the most common used differential evolutionary mutation strategies are as follows [16]:

(1) DE/rand/1:

$$v_i = x_{r1} + F\times(x_{r2} - x_{r3})$$

(2) DE/rand/2 :

$$v_i = x_{r1} + F\times(x_{r2} - x_{r3}) + F\times(x_{r4} - x_{r5})$$

(3) DE/best/1 :

$$v_i = x_{best} + F\times(x_{r1} - x_{r2})$$

(4) DE/best/2 :

$$v_i = x_{best} + F\times(x_{r1} - x_{r2}) + F\times(x_{r3} - x_{r4})$$

(5) DE/current-to-best/1 :

$$v_i = x_i + F\times(x_{best} - x_i) + F\times(x_{r1} - x_{r2})$$

(6) DE/current-to-rand/1 :

$$v_i = x_i + rand\times(x_{r1} - x_i) + F\times(x_{r2} - x_{r3})$$

where $r1 \neq r2 \neq r3$, and $F \in [0,1]$ is the mutation coefficient.

In the proposed mutation strategy, an idea of Archive Strategy was draw from the NSGA-II algorithm [17]. In this strategy an external archive is constructed to save and update the non-dominated individuals so far, and the niche technology is used to sort the non-dominated individuals in the archive. The process of mutate is as follows:

$$\begin{aligned} v_i = \bar{x}_c + F_1\times(x_{elite} - x_{rand}) + \\ F_2\times(x_{elite} - v_i) + F_3\times(x_{rand} - v_i) \end{aligned} \tag{25}$$

where x_i is the i th individual, and v_i is the corresponding mutation individual; x_{elite} is an elite individual selected from the external archive by niche technology, x_{rand} is a randomly selected individual from the external archive, and $x_{elite} \neq x_{rand}$. $\bar{x}_c = \omega_1\times x_{elite} + \omega_2 x_{rand} + \omega_3 x_i$, $\omega_i \geq 0$ and $\sum_{i=1}^{3}\omega_i = 1 \quad i=1,2,3$. $\omega_i = p_i / \sum_{i=1}^{3} p_i \quad i=1,2,3$, and $p_1=1$, $p_3=rand(0.7,1)$, $p_2=rand(0.5,p_3)$ 。

Eq. (25) can make full use of the information of the current population, as well as the information of the external archive, to ensure that the population moves to the elite individuals.

Step 3 Crossover. The crossover individual vector u_i^t is generated from the original individual x_i^t and the mutation individual v_i^t by the crossover operation:

$$u_{ij}^t = \begin{cases} v_{ij}^t, & if\ (rand_{ij} \leq \mathrm{CR}) \\ x_{ij}^t, & otherwise \end{cases} \tag{26}$$

where, $rand_{ij}$ is an uniform distributed number between $[0,1]$, and $\mathrm{CR} \in [0,1]$ is the crossover rate. It can be see that the greater the value of CR, the greater the probability of cross operation is.

Step 4 Selection. The values of fitness functions of the parental individual x_{ij}^t and the crossover individual u_{ij}^t were calculated respectively. According to the Pareto dominance relationship between them, the non-dominant individual is selected as the offspring individuals x_{ij}^{t+1}.

Step 5 If the termination condition is satisfied, the individuals in the external archive are selected as the final non-dominated solution set. Conversely, return to step 2.

V. EXAMPLE ANALYSIS

A. Mission Scenario

The three-echelon supply network is composed of one rear warehouse, three field warehouses and five combat units. There is ether offensive scenario or defensive scenario that may be happened after the beginning of the battle. In the defensive scenario, enemy attack is fierce, equipment damage is serious, and the urgent repair task is heavy. So the demand of spare parts is larger, and risk of disruption of transportation is higher. In the offensive scenario, our side is in a dominant situation, and less vulnerable to enemy attack which lead to relative relaxed in demand for spare parts, as well as, relative low in the risk of supply disruptions. However, the shortage in offensive scenario will have great effect on the result of battle. So the shortage costs are larger in offensive scenario. The information about the supply network and its nodes are shown from Table I to Table VI, respectively.

TABLE.I
LEAD TIME OF UNIT SPARE PARTS BETWEEN NODES

	field warehouses 1	field warehouses 2	field warehouses 3
rear warehouse	20	15	18
combat units 1	1.5	1	1
combat units 2	0.8	1.2	1
combat units 3	0.6	1	1.2
combat units 4	1.5	1.5	1.6
combat units 5	0.5	0.3	0.6

TABLE II
INVENTORY CAPACITY OF THE DISTRIBUTION CENTER

	field warehouses 1	field warehouses 2	field warehouses 3
Maximum capacity	55	50	60

TABLE III
SPARE PARTS DEMANDS OF COMBAT UNITS

	scenario 1	scenario 2
combat units 1	[16,26]	[4,10]
combat units 2	[11,21]	[2,6]
combat units 3	[14,22]	[6,10]
combat units 4	[22,30]	[8,12]
combat units 5	[26,36]	[9,15]

TABLE IV
RISK OF UNIT SPARE PARTS BETWEEN NODES

scenario 1 / scenario 2	field warehouses 1	field warehouses 2	field warehouses 3
combat units 1	(0.4,0.6)	(0.3,0.7)	(0.5,0.6)
	(0.2,0.5)	(0.1,0.3)	(0.2,0.3)
combat units 2	(0.1,0.3)	(0.3,0.6)	(0.4,0.7)
	(0.1,0.2)	(0.1,0.3)	(0.3,0.5)
combat units 3	(0.5,0.8)	(0.6,0.9)	(0.3,0.7)
	(0.1,0.2)	(0.2,0.5)	(0.1,0.3)
combat units 4	(0.5,0.9)	(0.2,0.6)	(0.2,0.4)
	(0.3,0.7)	(0.1,0.4)	(0.2,0.3)
combat units 5	(0.2,0.4)	(0.4,0.7)	(0.6,0.8)
	(0.1,0.2)	(0.2,0.5)	(0.3,0.6)

TABLE V
SHORTAGE COSTS OF UNIT SPARE PARTS IN COMBAT UNITS

	scenario 1	scenario 2
combat units 1	800	1000
combat units 2	1600	2500
combat units 3	1000	1800
combat units 4	900	1600
combat units 5	1500	2000

TABLE VI
THRESHOLD OF RELATED PARAMETERS OF THE SUPPLY NETWORK

	scenario 1	scenario 2
risk threshold	100	80
total shortage loss	18000	22000
Fill rate	0.90	0.95

The simulation studies were carried out in a MATLAB 2014b platform on an ASUS laptop with 5-6300HQ 2.3GBz CPU, 4GB RAM in Windows 7.0(64-bit) environment. The relevant experimental parameters are set as follows: scenario probability $P_{s1}=0.3$, $P_{s2}=0.7$, scenario penalty coefficient $\omega=1000$, $\lambda=10000$ [18], $M=10000$, population size $N=100$, the maximum iterations $iter_{\max}=300$, crossover probability $\mathrm{CR}=0.95$ [19], mutation factor $(F_1,F_2,F_3)=rand(0,1)$.

B. Result Analysis

The multi-objective differential evolution algorithm is used to solve the model and the non-dominated solution set of the model is obtained. The evolutionary strategy proposed in this paper is compared with the other three traditional evolutionary strategies. Experiments are carried out 15 times under each algorithm, and their non-dominated solution sets are finally obtained. The distribution of the non-dominated solution in the objective space is shown in Figure 1. The transverse axis and the longitudinal axis represent two fitness functions, respectively.

As can be seen directly from Figure 1, the results obtained by the evolutionary strategy proposed in this paper are widely and uniformly distributed in the objective space and also dominate the results of other strategies. That to say they have better convergence and distribution [20], and the improved mutation strategy improves the performance of the differential evolution algorithm and can well solve the proposed multi-objective robust optimization model.

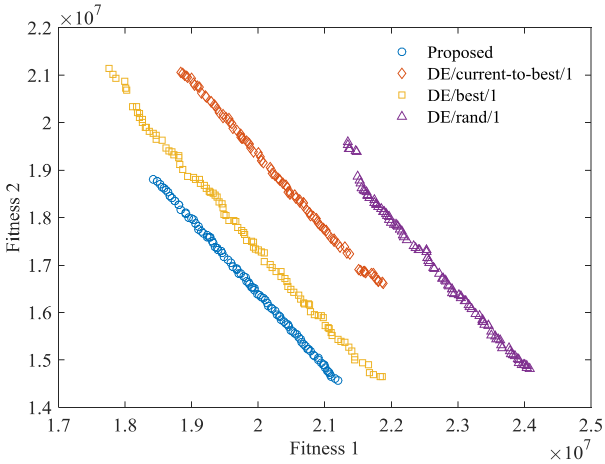

Fig.1 Comparison of non-dominated solutions

Each non-dominated solution corresponds to a supply scheme, which can be substituted into the model to calculate the lead time of spare parts supply, the shortage costs, and the risk in each scenario. Due to the large number of non-dominated solutions, the solutions can't be enumerated one by one, so the mean and variance of the results are shown in Table 7 and Table 8.

As can be seen from the Table VII, the shortage costs and risks of each algorithm are below the prescribed threshold, and all of non-dominant solutions are feasible solutions. A negative shortage indicates that the number of supplied spare parts is greater than the demand. The improved algorithm can get the shortest average lead time of spare parts, and the risk and shortage of each scenario is the smallest, which shows that the improved algorithm is superior to other algorithms. On the other hand, it can be seen from Table VIII that the variance of the corresponding indexes with our strategy is smaller than that of other strategies, which shows that the proposed algorithm has good adaptability and stability to the proposed robust optimization model.

TABLE VII
MEAN VALUE OF THE RESULTS BY EACH ALGORITHM

	Lead time	shortage cost in scenario 1	shortage cost in scenario 2	Risk in scenario 1	Risk in scenario 2	NO. of non-dominant
DE/rand/1	57.2	-10600	-20400	87.84	48.64	78
DE/best/1	57.9	-10300	-21900	86.32	47.21	100
DE/current-to-best/1	60.8	-8700	-16200	97.23	53.45	32
Proposed strategy	52.6	-18600	-27500	72.31	39.03	100

TABLE VIII
STANDARD DEVIATION OF THE RESULTS BY EACH ALGORITHM

	Lead time	shortage cost in scenario 1	shortage cost in scenario 2	Risk in scenario 1	Risk in scenario 2
DE/rand/1	9.22	2.0953	3.1612	11.62	6.37
DE/best/1	8.96	2.0879	3.2225	11.36	6.54
DE/current-to-best/1	5.62	1.5307	2.0805	10.54	5.96
Proposed strategy	6.01	1.5221	1.9870	9.88	5.92

In order to verify the robustness of the robust optimization model in the case of scenario uncertainty, the optimal solutions of the robust optimization model are substituted into the deterministic model just considering scenario 1 or scenario 2, respectively. The results are shown in figure 2. It can be seen that the shortage and the risk does not exceed the prescribed threshold, that is, the robust optimal solution is feasible in the deterministic models. The proposed model has good robustness.

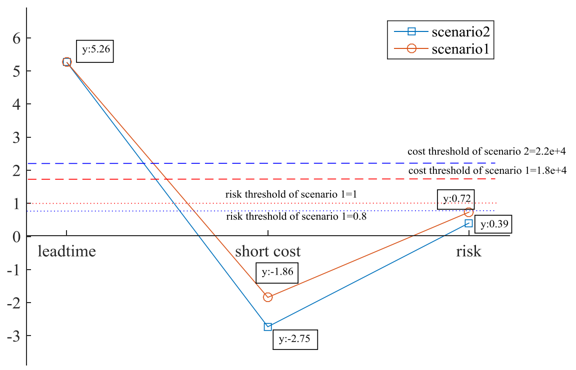

Fig.2 Feasibility analysis of the optimal solution of robust optimization in each scenario

In order to verify the robustness with parameter uncertainty, the optimal solutions of the robust optimization model are substituted into the deterministic model of parameter determination. In this paper, the robust optimization model is compared with three deterministic models. The parameters of the deterministic models are named the nominal value, the minimum value and the maximum value respectively. The results of lead time, shortage, and risk are shown in Table IX.

It can be seen from Table IX that the risk and shortage in each deterministic model are lower than the prescribed threshold, and the optimal solutions of the robust optimization model are the feasible in each deterministic model. At the same time, the risk and shortage of the robust optimization model is higher than that of other deterministic models, which indicates the conservatism of the robust optimization model, that is, the feasible scheme to satisfy the worst case.

TABLE IX
CALCULATION RESULTS CORRESPONDING TO DIFFERENT UNCERTAINTY LEVEL OF DEMAND

		Lead time	Shortage cost	risk
Nominal model	scenario 1	52.6	-18600	72.31
	scenario 2	52.6	-27500	39.03
Minimum model	scenario 1	52.6	-19500	56.92
	scenario 2	52.6	-28200	32.74
Maximum model	scenario 1	52.6	-17600	75.82
	scenario 2	52.6	-24700	41.89
Robust model	scenario 1	52.6	-17500	76.16
	scenario 2	52.6	-24000	42.50

VI. CONCLUSION

In this paper, the high uncertainty of spare parts supply in wartime is considered. The robustness analysis is carried out from two aspects, scenario uncertainty and parameter uncertainty, at the same time. A multi-objective robust optimization model for spare parts supply in wartime is constructed, taking the minimum lead time and shortage cost as the optimization objectives, and take the fill rate and the risk conditions as constraints. The meta-heuristic intelligent algorithm with improved evolution strategy is used to solve the model. The effectiveness and advantage of the improved algorithm are verified by comparison experiments. By analyzing the results, it is found that the optimal solutions of the robust optimization model are feasible in the worst case of the conditions of scenario and parameter uncertainty. That to say the robust optimization model has good robustness when dealing with uncertain conditions of spare parts supply in wartime. The robust optimization model established in this paper provides a certain decision-making frame for the optimization of spare parts supply under uncertain conditions in wartime.

References

[1] X. C. Liu, L. S. Zhu, W. Zhang. "Demand Forecasting of Fuzzy Inference-based Wartime Spares," *Acta Armamentarii*, vol.34, no. 9, pp. 1197-1200, Sep. 2013.

[2] J. Ren , X. L. Zheng, Y. J. Tan. "Supply Planning for Military Spare Parts in Battlefield with Broken Parts Remanufacturing Consideration," *Journal of Ordnance Engineering College*, no. 2, pp. 16-20, Feb. 2014.

[3] X. C. Liu. "Study on planning models and dynamic coordination mechanisms for multi-stage spare parts support under demand uncertainties," Ph.D. dissertation, National University of Defense Technology, Hunan 2009.

[4] B. Ayvaz, B. Bolat, N. Aydın. "Stochastic reverse logistics network design for waste of electrical and electronic equipment," *Resources, Conservation and Recycling*, vol. 104, pp. 391–404, 2015.

[5] M. Talaei, M. B. Farhang, M. S.Pishvaee, et.al. "A robust fuzzy optimization model for carbon-efficient closed-loop supply chain network design problem: a numerical illustration in electronics industry," *Journal of Cleaner Production*, vol. 113, pp. 662–673, 2016.

[6] P. Peng, L. V. Snyder, L. Andrew, et.al. "Reliable logistics networks design with facility disruptions," *Transportation Research Part B: Methodological*, vol. 45, no. 8, pp. 1190–1211, Aug. 2011.

[7] F. Maggioni, F. A. Potra, M. Bertocchi. "A scenario-based framework for supply planning under uncertainty: stochastic programming versus robust optimization approaches," *Computational Management Science*, vol. 14, no. 1, pp. 5–44, Jan. 2017.

[8] S. Bairamzadeh, M. M. Saidi, M. S. Pishvaee. "Modelling different types of uncertainty in biofuel supply network

design and planning: A robust optimization approach," *Renewable Energy*, vol. 116, pp. 500–517, 2018.

[9] A. Jabbarzadeh, M. Haughton, A. Khosrojerdi. "Closed-loop supply chain network design under disruption risks: A robust approach with real world application," *Computers & Industrial Engineering*, vol. 116, pp. 178–191, 2018.

[10] M. S. Pishvaee, M. Rabbani, S. A. Torabi. "A robust optimization approach to closed-loop supply chain network design under uncertainty," *Applied Mathematical Modelling*, vol. 35, no. 2, pp. 637–649, Feb. 2011.

[11] K. M. Shafiei, M. Solimanpur, A. Doniavi. "An integrated supply chain configuration model and procurement management under uncertainty: A set-based robust optimization methodology," *Applied Mathematical Modelling*, vol. 40, no.17, pp.7928–7947, 2016.

[12] B. L. Gorissen, İ. Yanıkoğlu, D. Hertog. "A practical guide to robust optimization," *Omega*, vol. 53, pp. 124–137, 2015.

[13] A. Panda, S. Pani. "A Symbiotic Organisms Search algorithm with adaptive penalty function to solve multi-objective constrained optimization problems," *Applied Soft Computing*, vol. 6, pp. 344–360, 2014.

[14] B. Tessema, G. G. Yen. "An Adaptive Penalty Formulation for Constrained Evolutionary Optimization," *IEEE Transactions on Systems, Man, and Cybernetics - Part A*: Systems and Humans, vol. 39, no. 3, pp. 565–578, Mar. 2009.

[15] T. Elmihoub, A. A. Hopgood, L. Nolle, et.al. "Hybrid Genetic Algorithms: A Review," *Engineering Letters*, vol. 13, no. 3, pp. 124 – 137, 2006.

[16] Y. Wang, Z. Z. Liu, J. Li, et.al. "On the selection of solutions for mutation in differential evolution," *Frontiers of Computer Science*, vol. 12, no. 2, pp. 297–315, Feb. 2018.

[17] K. Deb, S. Agrawal, A. Pratap, et.al. "A Fast Elitist Non-dominated Sorting Genetic Algorithm for Multi-objective Optimization: NSGA-II," *Lecture Notes in Computer Science*, pp. 849–858, 2000.

[18] M. L. Li, X. F. Wang, Q. X. Sun, et.al. "Research on Allocation Strategies of Multimodal Transportation for Emergency Resources Based on Robust Optimization," *Journal of the China railway Society*, vol. 39, no. 7, pp. 1-9, Jul. 2017.

[19] A. W. Mohamed. "A novel differential evolution algorithm for solving constrained engineering optimization problems," *Journal of Intelligent Manufacturing*, vol. 29, no. 3, pp. 659–692, Mar. 2017.

[20] R. Fabregat, Y. Donoso, B. Baran, et.al. "Multi-objective optimization scheme for multicast flows:a survey, a model and a MOEA solution," *International Ifip/acm Latin American Conference on Networking*, pp. 73-86, 2005.

Yadong Wang was born in Henan province ,China in 1992. He received the B.S. and M.S. degrees in control science and engineering from the Air Force Logistics Academy, Xuzhou, in 2017 . He is currently pursuing the Ph.D. degree in armament science and technology from Army Engineering University, Shijiazhuang.

His research interests include equipment maintenance engineering, logistics management, optimization algorithm, artificial intelligence, and so on.

Quan Shi was born in Hebei province ,China in 1966. He received the B.S. and M.S. degrees in mechanical engineering from the Academy of Armored Corps Engineering, Beijing, in 1989 and the M.S. degree in armament science and technology from Army Engineering University, Shijiazhuang, in 2005.

His research interests include equipment maintenance engineering, battle damage assessment, artificial intelligence, and so on.