A Simple Numerical Model for Water Quality Assessment with Constant Absorption around Nok Phrao Island of Trang River

Witsarut Kraychang and Nopparat Pochai

***Abstract*—Nok Phrao Island is given to the island by the indigenous people. It is located near the Trang estuary with several rivers flowing through. Since this island is the assembly point of different river currents, it consequently causes problems with the quality of water. This island area accumulates different pollution such as municipal and industrial pollution. This problem affects today life of the local people such as their occupation, healthy, and ecosystem. However, there are some problems which are still unsolved because of many influential factors which cannot yet be identified. In order to overcome these obstacles, mathematical models could be useful before conducting operations on the Nok Phrao Island. In this research, mathematical models were applied to assess the water quality at Nok Phrao Island by assuming observation points in the models to get a pollutant concentration before and after the implementation of biological treatment from the wastewater treatment station. The mathematical models proposed in this research could be recommended for the assessment of the standard water quality at Nok Phrao Island to account for the increase in population or growth of industries around Nok Phrao Island in the future.**

***Index Terms*— Water quality, hydrodynamic model, dispersion model, wastewater treatment, biological purifier**

I. Introduction

NOK PHRAO is the name of an island in the Trang estuary, Trang Province, situated in the middle with the water flowing through island and the water flows out of the island as shown in Fig. 1-2. This island is an important area of the communities for fishing and consumption. There are several research studies on water flow and water quality at important water source. The major causes of water quality are such disposal of community waste into the water resources, wastewater discharge from factories and biological reaction of plants in water resources which lead to decrease water quality. Several studies on wastewater treatment methods, have been shown its efficiency to overcome these abovementioned problems. One of the most widely used methods for solving environmental problems is mathematical model as early stage, to measure and assess the water and air quality. Moreover, [21] proposed a mathematical model to measure air pollutant concentration in area under a Bangkok sky train platform. In addition, three-dimensional mathematical model to measure air pollutant concentration in industrial areas was also proposed [22]. In addition, numerical methods are important in solving governing equation in mathematical model, which was developed along with mathematical models by researchers [23]-[25]. In this research, mathematical models were used to analyze the water quality at Nok Phrao Island, where wastewater was from the north of the island and biological reaction of plants comes from around Nok Phrao Island, with the assumption that wastewater treatment from biological purifier released from the wastewater treatment station to improve the water quality. The models using in this current study were divided into two models: i) water quality model used to describe pollutant concentration and ii) wastewater treatment model used to measure after the release of biological purifier for wastewater treatment.

There are many methods for monitoring the level of pollutants in the water, mostly conducted by a field measurement and a mathematical simulation. The shallow water mass transport's problems were presented in [1], as the method of characteristics that has been reported applied. In [9], [10], the finite element method for solving steady and unsteady water pollution measurements were introduced.

Fig. 1. Google Earth: Nok Phrao Island, Trang River in the southern in Thailand.

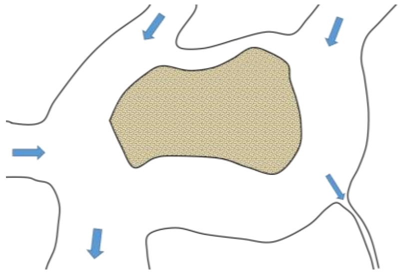

Fig. 2. Sketch the Nok Phrao Island with five sub rivers around island and middle of the island.

Manuscript received September 16, 2019; revised February 18, 2020.

Witsarut Kraychang is with Department of Mathematics and Computer Science, Navaminda Kasatriyadhiraj Royal Air Force Academy, Paholyothin Road, Saimai, Khlongthanon, Bangkok 10220, Thailand.
(e-mail: Witsarut_popmath@hotmail.com)

Nopparat Pochai is with Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand and Centre of Excellence in Mathematics, Commission on Higher Education, Si Ayutthaya Road, Bangkok 10400, Thailand.
(e-mail: nop_math@yahoo.com)

The various numerical techniques of solving the uniform flow of stream water quality model were presented in [6], [8]. The numerical methods of approximating the solution of the two-dimensional advection-diffusion-reaction equation are proposed in [3], [4].

Most non-uniform flow models need the input data concerned with the velocity of the current at any point and any time in the domain. The hydrodynamic model provide the velocity field and the elevation of the water. In [2], [8], the hydrodynamic model and advection-diffusion equation were used to approximate the velocity of the water current in a bay and a channel. In [4], [7], the results from hydrodynamic model were used as data for the non-uniform flow of the advection-diffusion-reaction equation, which provide the pollutant concentration field. The term of the friction forces occurred thanks to the drag of sides of the uniform reservoir. The theoretical solution of the model was found at the ending point of the domain and the analytical solution to check the accuracy of our approximate solution used. In [4], the Lax-Wendroff method with stability analysis to solve the two-dimensional hydrodynamic model with a rectangular domain was proposed. In [14], developed mathematical models and numerical methods for approximating water flow directions and pollutant concentration level in the Rama 9 reservoirs in opened with two parallel canals and assuming bottom topography of reservoir was flat. The Lax-Wendroff method was subsequently used in non-dimensional form of a shallow water equation to approximate the velocity of water and elevation of water; we used the forward difference in time and backward difference in space of advection diffusion equation. In [16], [17], the Lax-Wendroff method for solving the dimensional form of shallow water equation in rectangular model and spherical model with MATLAB program are proposed, respectively. In [18], combining two existing mathematical models, a hydrodynamic model which was used to describe the water current in an opened-closed positioning of reservoir and a dispersion model which was used to describe the diffusion of the pollutant concentration of water in an opened-closed reservoir. This was to make the proposed model suitable for the reservoir. The shallow water equation of the hydrodynamic model was assumed by averaging the equation over the depth with anisotropic bottom topography, and discarding the term regarding the Coriolis force, surface wind effect and external forces, resulting in the calculated velocity used in the dispersion model to approximate the concentration levels of the pollutants.

Determination of steady-state pollutant levels in a water reservoir causing by wastewater discharge from industrial plants and other external sources could be accurately accomplished by field sampling of the water randomly. However, it was difficult to get samples from every spot in the reservoir and highly cost to analyze all of the collected samples. Mathematical simulation was potentially a valuable tool that could be used to simulate the pollutant levels of the water at every spot of the whole reservoir from a relatively small set of collected samples, greatly reducing the total analytical cost. In [12], the authors proposed a mathematical simulation to deal with a lake water quality problem in China. They reported a great match between their calculated and measured pollutant levels. In [13], the authors presented a mathematical model for analyzing the hydrodynamics of and pollutant dispersion in river-type systems. They were able to report changes in the pollutant concentration in the river with time. In a mathematical modeling study of water-quality in the Rama 9 reservoir, Pathum Thani District, Thailand [14], two mathematical models were used to simulate its pollutant level. The first model was a hydrodynamic model that used the Lax-Wendroff method to provide the velocity vector of water flow and its elevation. The second model was a dispersion model that used a forward-in-time and central-space finite difference scheme to calculate the pollutant concentration. The resultant water velocity vector field, elevation, and pollutant concentration were reported in contour graphs. In [31], two-dimensional hydraulic and pollution models were used to simulate the transport of the pollutant. After the pollutant level at every location has been mathematically determined, it could be input into an optimization model to find the minimum cost that an industrial plant has to expend to initially treat its wastewater to an acceptably low pollutant level before discharging it into a reservoir. Simplex optimization method is a good mathematical model for determining minimum cost. In [5], a mathematical model was proposed for optimally controlling pollutant level in wastewater discharge that would reduce initial water treatment cost to a minimum. In [20], the authors proposed mathematical models and optimal control techniques for solving some problems in environmental engineering. In [15], two mathematical models were proposed: a hydrodynamic model and a steady-state pollutant dispersion model. They were used to calculate the pollutant level in a connected-pond reservoir system that has an entrance and an exit gate to open water of a canal and to determine the optimal pollutant levels in the wastewaters discharged from nearby industrial plants that would cost the plants minimally to pre-treat.

II. WATER QUALITY MODELS

In this section, to mention mathematical models used to describe the spread of water pollution in natural water sources that were opened. The first model was a hydrodynamic model, a model which describes the velocities field of the water and elevation of water. The second model was a dispersion model, to explain the spread of the pollutant concentration level at any displacements with any times.

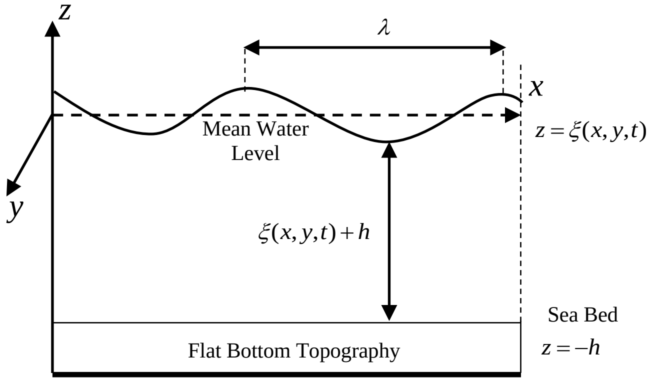

Fig. 3. Cross section in vertical of water resource.

A. *Hydrodynamic model*

In this section, a mathematical model was presented. The governing equation of this model was non-dimensional form of two-dimensional shallow water equations (SWEs), which described the velocities and elevation of water flow.

a) *Two-dimensional shallow water equations*

It was well known widely, two-dimensional shallow water equations were a system of equation that consist of the equation of conservation of mass and equation of conservation of momentum, could be obtained by depth averaging the Navier-Stokes equations. The two shallow water equations could be used to describe the water flows in the horizontal scale of the flow was much smaller than the depth of the water.

In this study, we present two-dimensional shallow water equations, neglecting the diffusion of momentum due to turbulence and viscosity, Coriolis factor, wind effects and shearing stresses at water surface, assuming water elevation to be much smaller than the depth of the domain and bottom topography was flat, the governing equations were as follows, the continuity equation:

$$\frac{\partial \xi}{\partial t}+h\frac{\partial u}{\partial x}+h\frac{\partial v}{\partial v}=0, \tag{1}$$

and the momentum equations:

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=0, \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=0, \tag{3}$$

where h is the depth measured from the mean water level to the sea bed (m), ξ is water elevation (m) as shown in Fig. 3, g is acceleration due to gravity (m/s^2) and u,v are the velocities in x and y directions (m/s), respectively, with initial condition $u=0, v=0, \xi=0$, while the boundary condition at horizontal sides $u=0, \frac{\partial v}{\partial y}=0, \xi=0$ and boundary condition at vertical sides $\frac{\partial u}{\partial x}=0, v=0, \xi=0$.

b) *Non-dimensional form*

We will transform (1)-(3) into non-dimensional form of two-dimensional shallow water equation, which reduce complexity of the calculations, following by linearly transformations: $U=u/\sqrt{gh}$,$V=v/\sqrt{gh}$, $X=x/l$, $Y=y/l$, $Z=\xi/h$ and $T=t\sqrt{gh}/l$ with derivative of variable by chain rule into (1)-(3), we have

$$\frac{\partial Z}{\partial T}+\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0\,, \tag{4}$$

$$\frac{\partial U}{\partial T}+\frac{\partial Z}{\partial X}=0\,, \tag{5}$$

$$\frac{\partial V}{\partial T}+\frac{\partial Z}{\partial Y}=0\,. \tag{6}$$

c) *Numerical Method*

Equations (4)-(6) can be written to the first-order hyperbolic equation in two-dimensional space by

$$\frac{\partial \mathbf{H}}{\partial T}=\mathrm{A}\frac{\partial \mathbf{H}}{\partial X}+\mathrm{B}\frac{\partial \mathbf{H}}{\partial Y}\,, \tag{7}$$

where 3×1 column vector

$$\mathbf{H}=\begin{pmatrix} Z \\ U \\ V \end{pmatrix} \text{ and } \mathrm{A}=\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathrm{B}=\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

are 3×3 dimensional matrices, using the Lax-Wendroff method for solving (7) by explicit finite difference method:

$$\begin{aligned}
&\mathbf{H}_{i,j}^{k+1}=\mathbf{H}_{i,j}^{k}+\mathrm{A}\frac{p}{2}\left(\mathbf{H}_{i+1,j}^{k}-\mathbf{H}_{i-1,j}^{k}\right)+\mathrm{B}\frac{p}{2}\left(\mathbf{H}_{i,j+1}^{k}-\mathbf{H}_{i,j-1}^{k}\right)+\\
&\mathrm{A}^2\frac{(p)^2}{2}\left(\mathbf{H}_{i+1,j}^{k}-2\mathbf{H}_{i,j}^{k}+\mathbf{H}_{i-1,j}^{k}\right)+\\
&\frac{(p)^2}{8}(\mathrm{BA}+\mathrm{AB})\left(\mathbf{H}_{l+1,m+1}^{n}-\mathbf{H}_{l+1,m-1}^{n}-\mathbf{H}_{l-1,m+1}^{n}+\mathbf{H}_{l-1,m-1}^{n}\right)+\\
&\mathrm{B}^2\frac{(p)^2}{2}\left(\mathbf{H}_{l,m+1}^{n}-2\mathbf{H}_{l,m}^{n}+\mathbf{H}_{l,m-1}^{n}\right),
\end{aligned} \tag{8}$$

where $p=\frac{\Delta T}{\Delta X}$, from (7) dividing the interval $[0,1]$ into L and M subintervals such that $L\Delta X=1$ and $M\Delta Y=1$, and the interval $[0,T]$ into N subintervals such that $N\Delta T=T_{final}$, where $\Delta X, \Delta Y$ and ΔT are step size of space and time. Approximate $\mathbf{H}(X_i,Y_j,T_k)=\mathbf{H}_{i,j}^{k}$ are value of the component vector at point $X=i\Delta X, Y=j\Delta Y$ with $T=k\Delta T$, where $0\le i\le L, 0\le j\le M$ and $0\le k\le N$. In finally, we will transform X,Y,T coordinate into x,y,t coordinate by linear transformation.

B. *Dispersion model*

The equation used to describe the spread of the pollutant concentration of water according to the governing equation Advection-Diffusion equation with source term

$$\frac{\partial C}{\partial t}+u(x,y,t)\frac{\partial C}{\partial x}+v(x,y,t)\frac{\partial C}{\partial y}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+S\,, \tag{9}$$

for all (x,y) in domain Ω , $C=C(x,y,t)$ is the pollutant concentration averaged in depth at the displacement x,y with time t (kg/m^3), $u(x,y,t)$ and $v(x,y,t)$ are the velocities in x and y directions (m/s) , respectively, D is diffusion coefficient of C (m^2/s) and S is the source term is constant function, with initial condition $C(x,y,0)=C_0$, while the boundary condition at horizontal sides $\frac{\partial C}{\partial y}=0$ and boundary condition at vertical sides $\frac{\partial C}{\partial x}=0$.

a) *Numerical method*

For solving the advection diffusion equation (9), we using the backward difference in space and forward difference in time for approximate advection term or first order derivative term:

$$\frac{\partial C}{\partial t}=\frac{C_{i,j}^{k+1}-C_{i,j}^{k}}{\Delta t},\ \frac{\partial C}{\partial x}=\frac{C_{i,j}^{k}-C_{i-1,j}^{k}}{\Delta x}\ \text{and}\ \frac{\partial C}{\partial y}=\frac{C_{i,j}^{k}-C_{i,j-1}^{k}}{\Delta y},$$

diffusion term or second order derivative term:

$$\frac{\partial^2 C}{\partial x^2}=\frac{C_{i+1,j}^{k}-2C_{i,j}^{k}+C_{i-1,j}^{k}}{(\Delta x)^2},\ \frac{\partial^2 C}{\partial y^2}=\frac{C_{i,j+1}^{k}-2C_{i,j}^{k}+C_{i,j-1}^{k}}{(\Delta y)^2}$$

with $u(x,y,t)=u_{i,j}^{k}$ and $v(x,y,t)=v_{i,j}^{k}$, we have the finite difference method for dispersion model as follow by

$$\begin{aligned}&C_{i,j}^{k+1}=D\frac{\Delta t}{\Delta x^2}C_{i+1,j}^{k}+D\frac{\Delta t}{\Delta y^2}C_{i,j+1}^{k}+\\&\left(\frac{\Delta t}{\Delta x}u_{i,j}^{k}+D\frac{\Delta t}{\Delta x^2}\right)C_{i-1,j}^{k}+\left(\frac{\Delta t}{\Delta y}v_{i,j}^{k}+D\frac{\Delta t}{\Delta y^2}\right)C_{i,j-1}^{k}+\\&\left(1-\frac{\Delta t}{\Delta x}u_{i,j}^{k}-\frac{\Delta t}{\Delta y}v_{i,j}^{k}-2D\frac{\Delta t}{\Delta x^2}-2D\frac{\Delta t}{\Delta y^2}\right)C_{i,j}^{k}+\Delta t(S),\end{aligned}\tag{10}$$

where $C(x,y,t)=C_{i,j}^{k}$ are approximation of pollutant concentration at point $x=i\Delta x, y=j\Delta y$ and $t=k\Delta t$, for $0\le i\le L, 0\le j\le M$ and $0\le k\le N$.

III. Wastewater Treatment Model

In this section, we propose a mathematical model describing the spread of pollution when released effective microorganism from the wastewater treatment station for treat water in natural water sources.

Part I: The spread of microorganism is released into water from the wastewater treatment station, governing equation is the Advection-Diffusion equation as:

$$\frac{\partial K}{\partial t}+u(x,y,t)\frac{\partial K}{\partial x}+v(x,y,t)\frac{\partial K}{\partial y}=D_K\left(\frac{\partial^2 K}{\partial x^2}+\frac{\partial^2 K}{\partial y^2}\right),\tag{11}$$

where $K=K(x,y,t)$ is the effective microorganisms concentration at the displacement x,y with time t (kg/m^3) and $u(x,y,t), v(x,y,t)$ are velocities from hydrodynamic model and D is diffusion coefficient of K (m^2/s). Solve by using finite difference method for solving (11),

$$\begin{aligned}&K_{i,j}^{k+1}=D_k\frac{\Delta t}{\Delta x^2}K_{i+1,j}^{k}+D_k\frac{\Delta t}{\Delta y^2}K_{i,j+1}^{k}+\\&\left(\frac{\Delta t}{\Delta x}u_{i,j}^{k}+D_k\frac{\Delta t}{\Delta x^2}\right)K_{i-1,j}^{k}+\left(\frac{\Delta t}{\Delta y}v_{i,j}^{k}+D_k\frac{\Delta t}{\Delta y^2}\right)K_{i,j-1}^{k}+\\&\left(1-\frac{\Delta t}{\Delta x}u_{i,j}^{k}-\frac{\Delta t}{\Delta y}v_{i,j}^{k}-2D_k\frac{\Delta t}{\Delta x^2}-2D_k\frac{\Delta t}{\Delta y^2}\right)K_{i,j}^{k},\end{aligned}\tag{12}$$

where $K(x,y,t)=K_{i,j}^{k}$ are approximation the effective microorganisms concentration at point $x=i\Delta x,\ y=j\Delta y$ and $t=k\Delta t$, for $0\le i\le L, 0\le j\le M$ and $0\le k\le N$, with initial condition $K(x,y,0)=K_0$, while the boundary condition at horizontal sides $\frac{\partial K}{\partial y}=0$ and boundary condition at vertical sides $\frac{\partial K}{\partial x}=0$.

Part II: The effective microorganism can reduce pollutant concentration in domain by equation

$$\begin{aligned}&\frac{\partial \tilde{C}}{\partial t}+u(x,y,t)\frac{\partial \tilde{C}}{\partial x}+v(x,y,t)\frac{\partial \tilde{C}}{\partial y}=D\left(\frac{\partial^2 \tilde{C}}{\partial x^2}+\frac{\partial^2 \tilde{C}}{\partial y^2}\right)+\\&S-K(x,y,t),\end{aligned}\tag{13}$$

where $\tilde{C}=\tilde{C}(x,y,t)$, the effective microorganisms concentration are reduced by K, $u(x,y,t)$ and $v(x,y,t)$ are velocities from hydrodynamic model and D is diffusion coefficient of $\tilde{C}$ (m^2/s). Using finite difference method for solving (13) by

$$\begin{aligned}&\tilde{C}_{i,j}^{k+1}=D\frac{\Delta t}{\Delta x^2}\tilde{C}_{i+1,j}^{k}+D\frac{\Delta t}{\Delta y^2}\tilde{C}_{i,j+1}^{k}+\\&\left(\frac{\Delta t}{\Delta x}u_{i,j}^{k}+D\frac{\Delta t}{\Delta x^2}\right)\tilde{C}_{i-1,j}^{k}+\left(\frac{\Delta t}{\Delta y}v_{i,j}^{k}+D\frac{\Delta t}{\Delta y^2}\right)\tilde{C}_{i,j-1}^{k}+\\&\left(1-\frac{\Delta t}{\Delta x}u_{i,j}^{k}-\frac{\Delta t}{\Delta y}v_{i,j}^{k}-2D\frac{\Delta t}{\Delta x^2}-2D\frac{\Delta t}{\Delta y^2}\right)\tilde{C}_{i,j}^{k}+\Delta t(S)-\Delta t(K),\end{aligned}\tag{14}$$

where $\tilde{C}(x,y,t)=\tilde{C}_{i,j}^{k}$ are approximation the pollutant concentration is reduced at point $x=i\Delta x,\ y=j\Delta y$ and $t=k\Delta t$, for $0\le i\le L, 0\le j\le M$ and $0\le k\le N$, with initial condition $\tilde{C}(x,y,t)=C_0$, while the boundary condition at horizontal sides $\frac{\partial \tilde{C}}{\partial y}=0$ and boundary condition at vertical sides $\frac{\partial \tilde{C}}{\partial x}=0$.

IV. Numerical Results

In this section, we showed many surfaces of water elevation cases with velocities field in hydrodynamic model, surface and contour plot of pollutant concentration in dispersion model. Secondly, we showed the comparison of graph and table of the pollutant concentration at observation points for case of no source term and constant source term. Finally, we showed the comparison of graphs and tables of the pollutant concentration at observation points for case of before treatment and after treatment of pollutant concentration. The simple domain of Nok Phrao Island with size 500×300 m^2 as shown in Fig. 4, consisted of the five rivers and a small island for three rivers flow and two rivers flow out.

There is the water elevation from three rivers flow in area of Nok Phrao Island by trigonometry function $\xi_1=f_1(x,t)$, $\xi_2=f_2(x,t)$ and $\xi_3=f_3(y,t)$, together with pollutant concentration from these rivers flow in by function $C_1=g_1(x,t), C_2=g_2(x,t)$ and $C_3=g_3(y,t)$. In the same time, water flow out by the rate of change of velocities with respect to y-coordinate was $\frac{\partial v}{\partial y}=a_1, \frac{\partial v}{\partial y}=a_2$ along with the rate of change of pollutant concentration with respect to y-coordinate was $\frac{\partial C}{\partial y}=c_1, \frac{\partial C}{\partial y}=c_2$, when initial time in the area of Nok Phrao has velocities, elevation water equal zero and pollutant concentration equal 0.02 (mg/l) as shown in Fig. 4.

A. *Numerical results of hydrodynamic model*

Hydrodynamic model provides velocities field and water elevation, using Lax-Wendroff method for solving two-dimension shallow water equations in non-dimensional form (7), when l is 500 m. Define step size of X and Y in non-dimensional form were 0.025 and step size of time is 0.01, function of water elevation $f_1=f_2=0.1\sin\left(\pi(X+10T)\right)$, $f_3=0.2\sin\left(\pi(Y+10T)\right)$ an finally, the result should be converted to dimension by linear transformation.

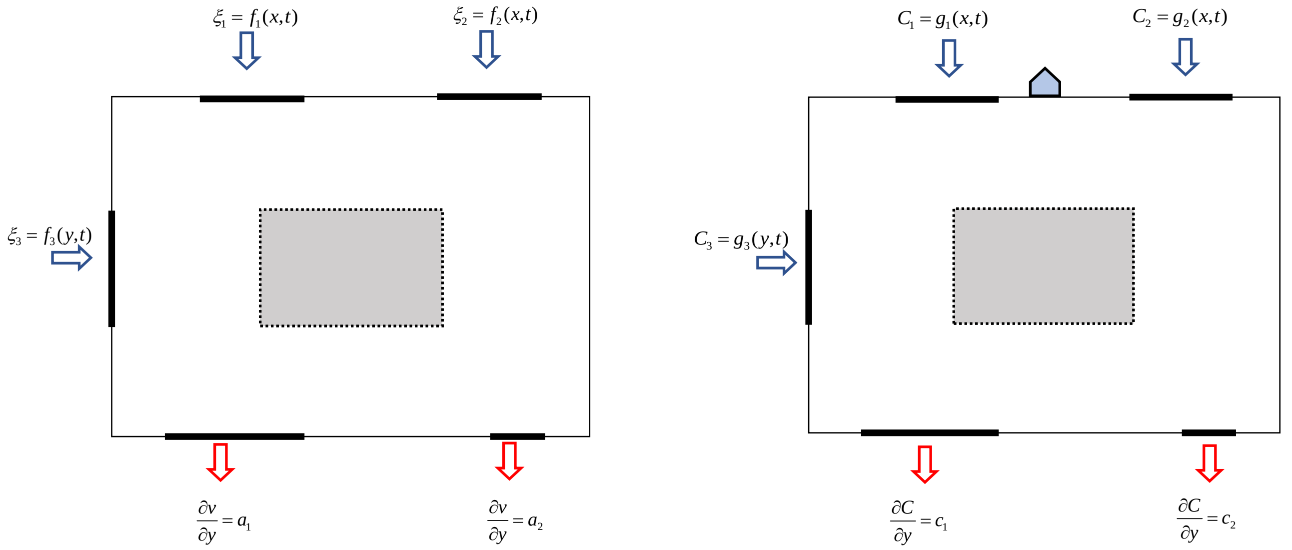

Fig. 4. Initial and boundary condition of Simple model of Nok Phrao Island.

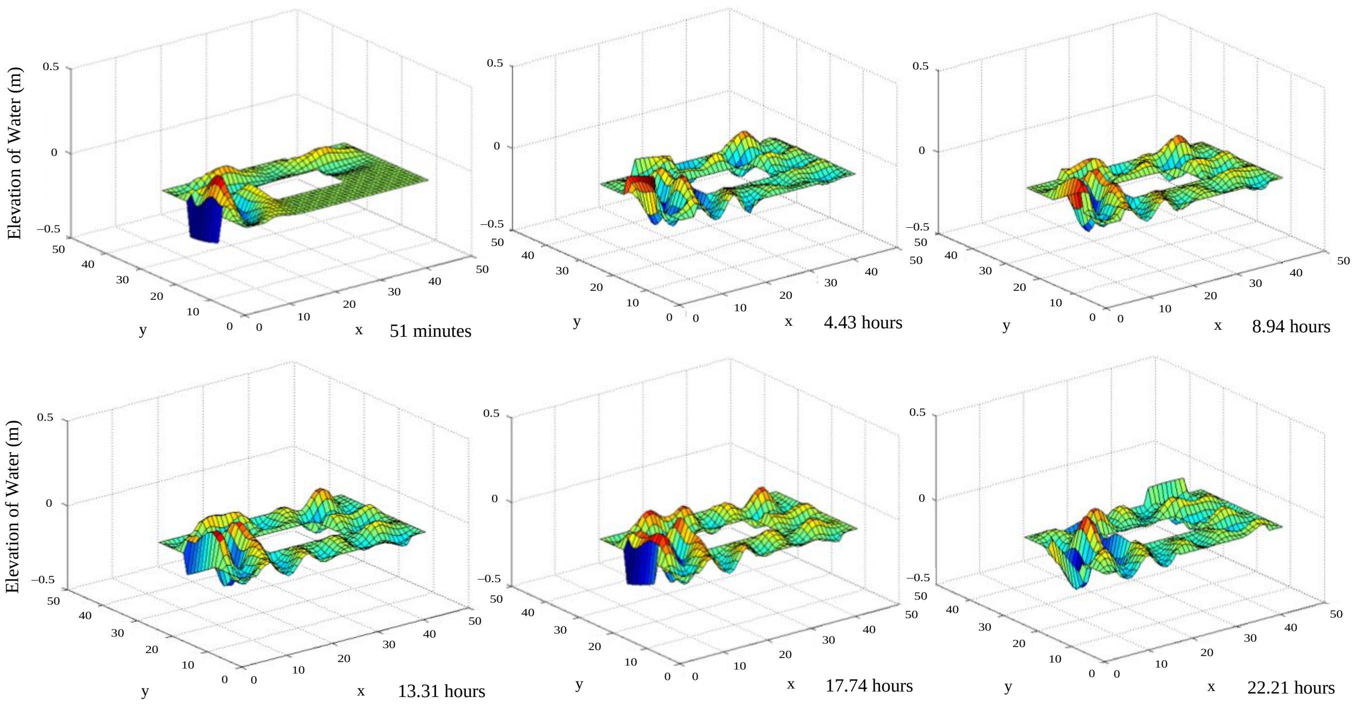

Fig. 5. Surface of water elevation measured successively at certain time spots 51 minutes to 22.21 hours.

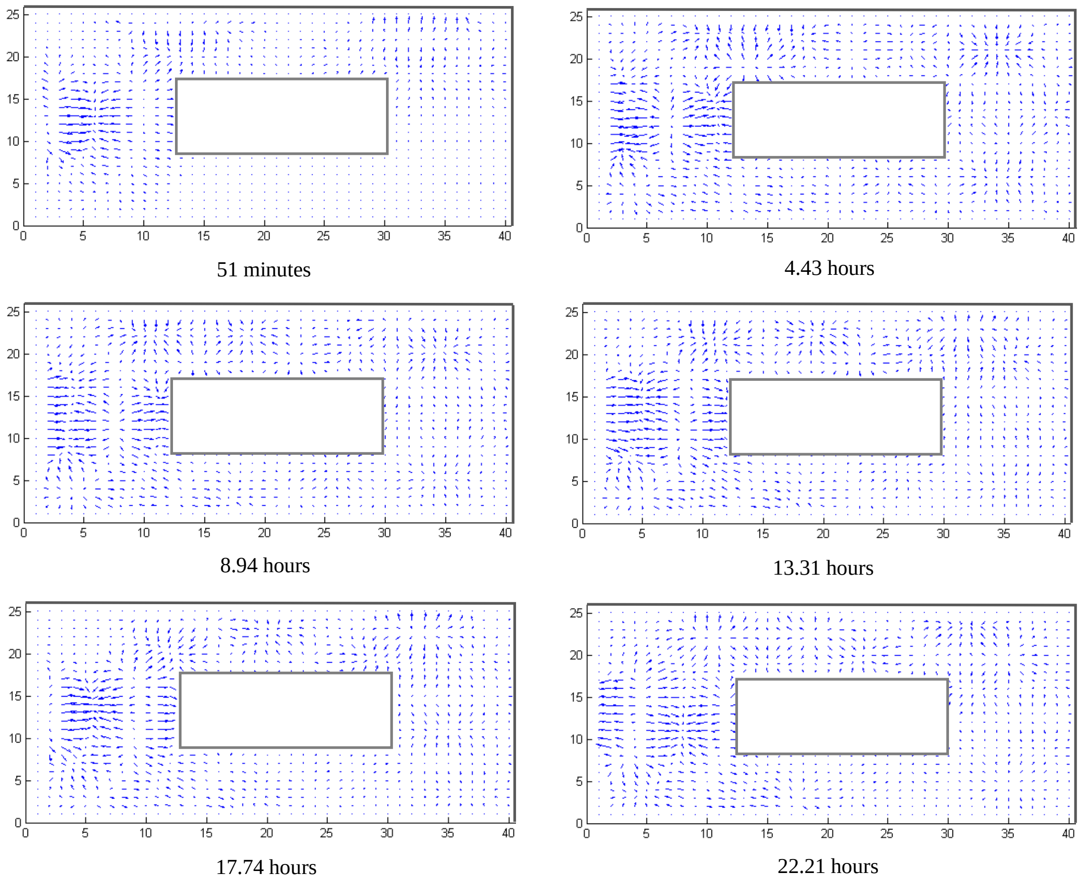

Fig. 6. Vector field of velocities measured successively at certain time spots 51 minutes to 22.21 hours.

B. *Numerical results of dispersion model*

The pollutant concentration level typically caused by wastewater flowing from the river or some kind of algae in the water (source term function), the spread of pollution depends on the velocity of water flow, which data from hydrodynamic model. Dispersion model provided pollutant concentration level, using finite difference method for solving advection-diffusion equations. Define diffusion coefficient 15 m^2/s, function of pollutant concentration flow in $g_1 = e^{-0.01t}$, $g_2 = 0.5$, $g_3 = 0.5$ (mg/l), the rate of change of pollutant concentration flow out, $c_1 = -0.0002$, $c_2 = -0.0002$. The surface and contour of pollutant concentration in case of source term and no source term as shown in Fig. 7-10, assume source term 0.0001 (mg/l) with time difference at 22.17 hours, 32.26 hours and 44.34 hours as shown in Fig. 9-10.

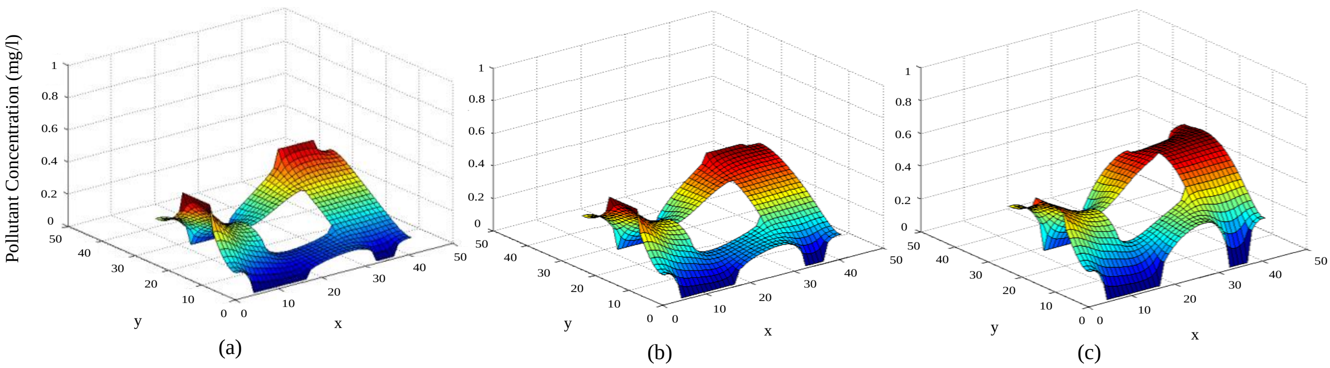

Fig. 7. Pollutant concentration surface (a) no source term (b) source term S=0.0001 (c) source term S=0.0002 with time 13.31 hours.

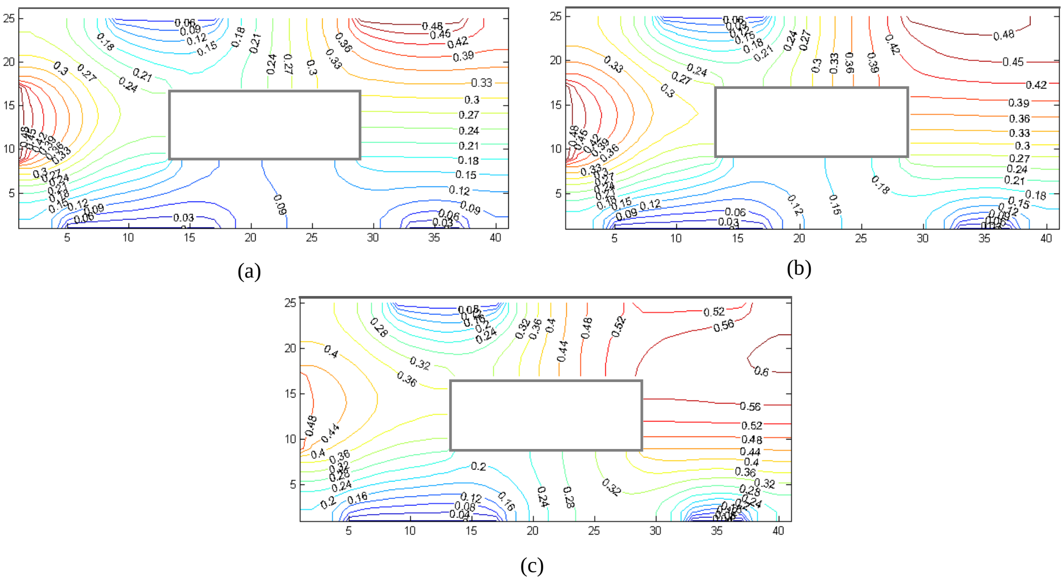

Fig. 8. Pollutant concentration contour (a) no source term (b) source term S=0.0001 (c) source term S=0.0002 with time 13.31 hours.

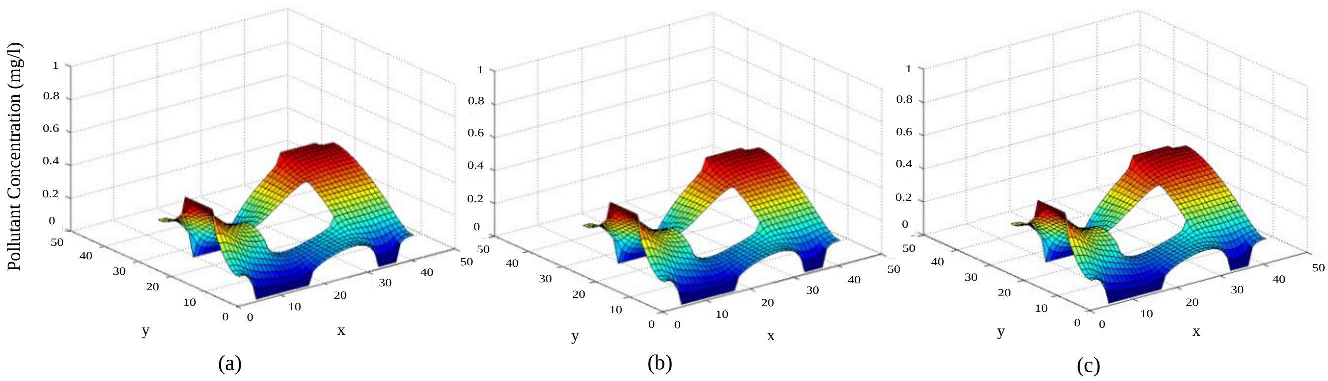

Fig. 9. Pollutant concentration surface with source term S=0.0001 (a) at time 22.17 hours (b) at time 32.26 hours (c) at time 44.34 hours.

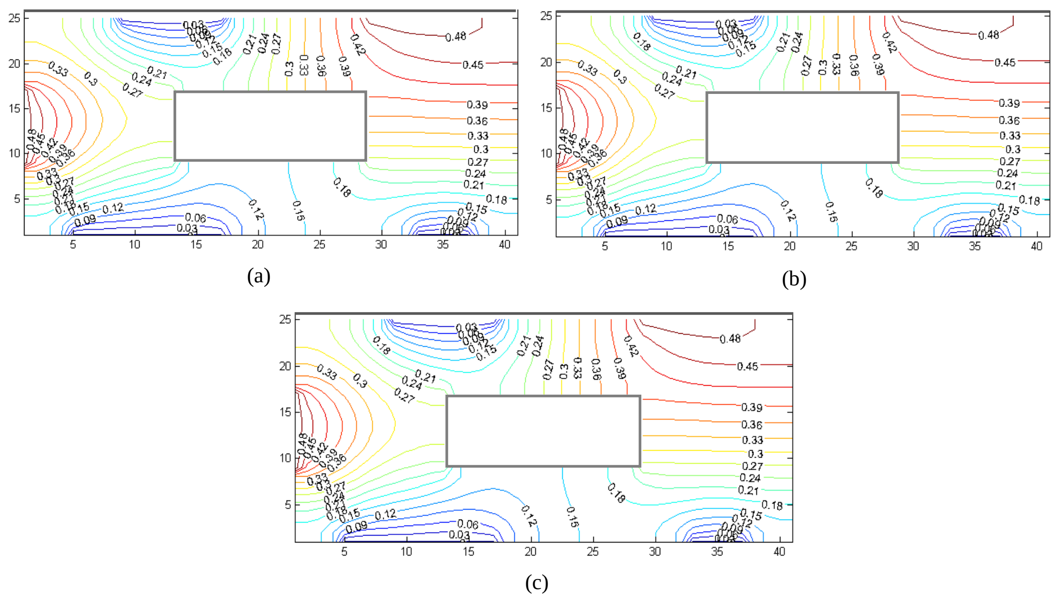

Fig. 10. Pollutant concentration with source term S=0.0001 (a) at time 22.17 hours (b) at time 32.26 hours (c) at time 44.34 hours.

C. *Observation points*

Observation points were the changes of the pollutant concentration in the Nok Phrao Island area at different points of time, a line of points A, H, G and C, D, E make up the vertical sides, whilst a line of points G, F, E and A, B, C make up the horizontal sides as shown in Fig. 11. We showed the comparison of line graphs in the many cases of pollutant concentration in the Nok Phrao Island area by the observation points A to H. This was illustrated by comparing the graph data of the pollutant concentration in the cases of no source term, and source term in the vertical and horizontal nodes as shown in Fig. 12-14. While Fig. 13, showed the comparison of the pollutant concentration before and after treatment of the water by microorganisms released from the wastewater treatment station. It was found that the line of points E, D, C and the line of points G, F, E indicated a higher decrease in pollutant concentration, than in the line of points G, H, A and the line of points E, D, C, because of influence from the water flowing from the river north of the island. Overall, it could be seen that the pollutant concentration of every observation point shows a decrease. Fig. 14 shows the comparison of the pollutant concentration at the observation points (vertical and horizontal lines) in the case of wastewater introduced from the north of the island only. It could be also seen that over time (22.17 hours, 33.26 hours and 44.34 hours), the pollutant concentrations were unchanged (Steady State).

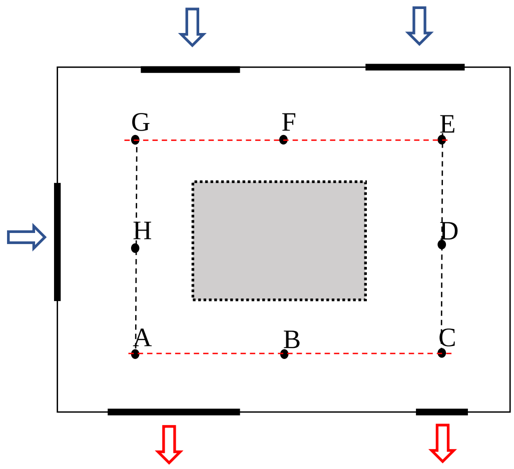

Fig. 11. Observation points A to H in area of Nok Phrao Island.

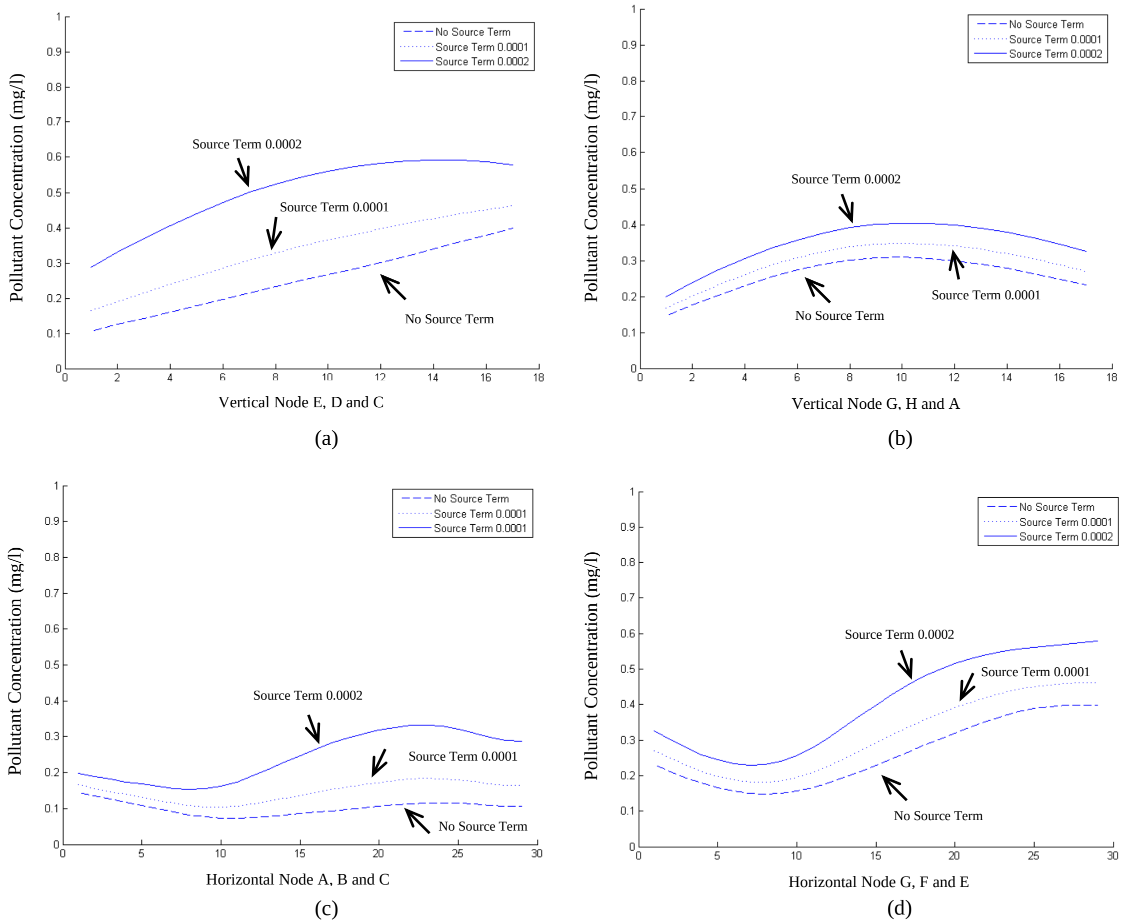

Fig. 12. Comparing graph of many observation points in case of no source term and source term at time 43.44 hours (a) vertical node E to C (b) vertical node G to A (c) horizontal node A to B (d) horizontal node G to E.

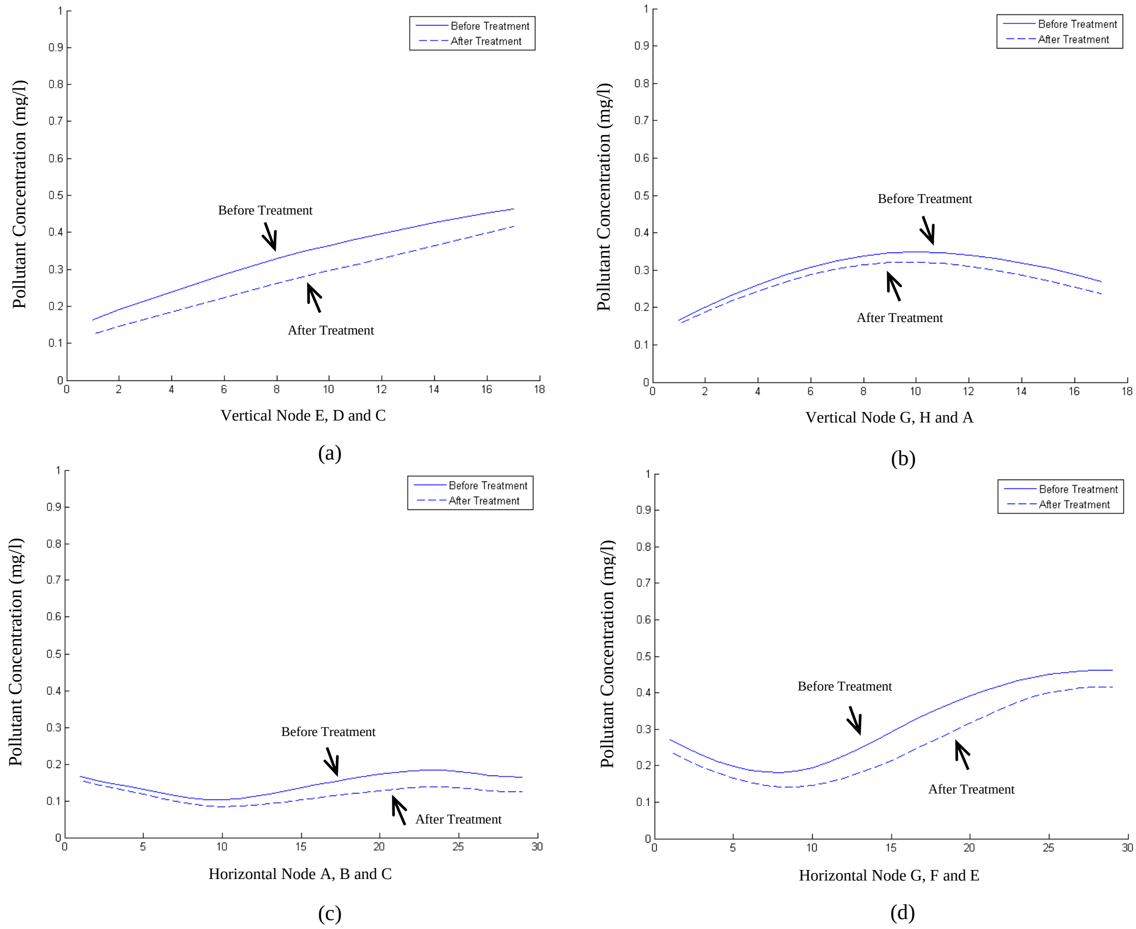

Fig. 13. Comparing graph of many observation points in case of before treatment and after treatment at time 43.44 hours (a) vertical node E to C (b) vertical node G to A (c) horizontal node A to B (d) horizontal node G to E.

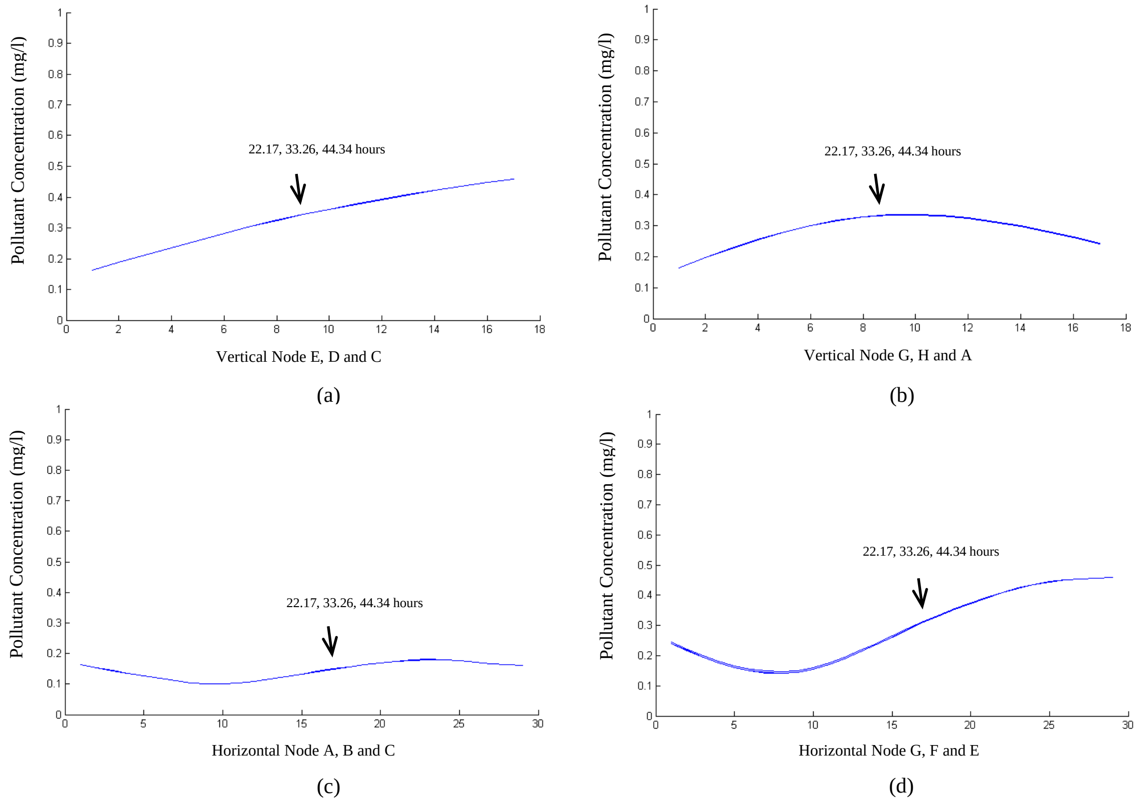

Fig. 14. Comparing graph of many observation points in case of difference time 22.17 hours, 33.26 hours and 44.34 hours (a) vertical node E to C (b) vertical node G to A (c) horizontal node A to B (d) horizontal node G to E.

TABLE I showed that the pollutant concentration at observation points A to H in case of source term S=0.0001 (mg/l), S=0.0002 (mg/l) and no source term. The highest concentration of pollutant was observed at point E whereas the lowest was observed at point A.

TABLE I
POLLUTANT CONCENTRATION AT OBSERVATION POINTS A TO H.

Observation points	S=0.0002	S=0.0001	No source term (S=0)
	Pollutant concentration (mg/l)		
A	0.1986	0.1672	0.1454
B	0.2478	0.1360	0.0854
C	0.2865	0.1640	0.1067
D	0.5440	0.3477	0.2502
E	0.5787	0.4622	0.3986
F	0.3994	0.2920	0.2294
G	0.3252	0.2703	0.2314
H	0.4003	0.3464	0.3079

TABLE II showed that the pollutant concentration at observation points A to H in case of before and after treatment of water by microorganisms from wastewater treatment station at 0.0005 (mg/l) and assuming source term 0.00002 (mg/l). These results show that the pollutant concentration at observation point B and C has been decreased by over 50% while, observation points A, E, G and H has been decreased below 30% compared to other observation points.

TABLE I I
POLLUTANT CONCENTRATION AT OBSERVATION POINTS A TO H

Observation points	Before treatment of water	After treatment of water	Percent of reduction
	Pollutant concentration (mg/l)		(%)
A	0.1986	0.1554	21.75
B	0.2478	0.1030	58.43
C	0.2865	0.1246	56.51
D	0.5440	0.2792	48.68
E	0.5787	0.4165	28.03
F	0.3994	0.2146	46.27
G	0.3252	0.2369	27.15
H	0.4003	0.3210	19.81

V. CONCLUSION AND DISCUSSION

The assessment of water quality could be analyzed by using samples obtained from field measurements, or by using a software program with many parameters to calculate the pollutant concentration of the water, which these methods obtained an accurate values, but required specialized personnel and knowledge of various parameters when carrying out the calculation by software program. In this research, a simple mathematical model explained the spread of pollution around Nok Phrao Island in the situation of wastewater treatment by microorganism. This model consisted of two models, a hydrodynamic model which described the elevation and velocity of water flow which affected the advection of the pollutant concentration, and a dispersion model that describes the quality of water. This research emphasized a non-dimensional form of two-dimensional shallow water equation which could not only overcome the complexity of hydrodynamic model but also consists with the use of Lax-Wendroff method. The first case, was where the wastewater caused by the northern communities of the Nok Phrao Island, whereas the second case was when the wastewater caused by the reaction of water plants that affected the pollutant concentration in Nok Phrao Island, and the third case, was using a biological purifier for the treatment wastewater in Nok Phrao Island by releasing microorganisms from the wastewater treatment station in the north of the island to observe the effects of the changing pollutant concentration, by comparing before and after the treatment of the wastewater. The results obtained from the mathematical models showed a decrease in pollutant concentration at the observation points after releasing microorganisms from the wastewater treatment station, there were therefore, three factors that caused a reduction of the pollutant concentration: location of station, direction of water flow and area. It could be seen that, this mathematical model could be used for the initial assessment in the quality of water around Nok Phrao Island or in similar geographical areas, even with limitations in the number of parameters.

ACKNOWLEDGMENT

The authors would like to thank the reviewers for their careful reading and constructive comments. The authors would like also to thank the "Centre of Excellence in Mathematics Program of the Commission on Higher Education" for financial support this research.

REFERENCES

[1] A. Garzon and L. D'Alpaos, "A Modified Method of the Characteristic Technique Combined with Gelerkin Finite Element Method to Solve Shallow Water Mass Transport Problems," *Proceedings 23rd International Conference in Coastal Engineering,* vol. 3, pp3068-3080, 1992.

[2] N. Pochai, "A Numerical Computation of the Non-Dimensional Form of a Non-Linear Hydrodynamic Model in a Uniform Reservoir," *Journal of Nonlinear Analysis: Hybrid Systems*, vol. 3, no. 4, pp463-466, 2009.

[3] N. Pochai, "A Numerical Computation of a Non-Dimensional Form of Stream Water Quality Model with Hydrodynamic Advection–Dispersion–Reaction Equations," *Journal of Nonlinear Analysis: Hybrid Systems*, vol. 3, no. 4, pp666-673, 2009.

[4] N. Pochai, S. Tangmanee, L.J. Crane and J.J.H. Miller, "A Water Quality Computation in the Uniform Reservoir," *Journal of Interdisciplinary Mathematics*, vol. 11, no. 6, pp803-814, 2008.

[5] N. Pochai, S. Tangmanee, L.J. Crane and J.J.H. Miller, "A Mathematical Model of Water Pollution Control Using the Finite Element Method," *Proceedings in Applied Mathematics and Mechanics*, vol. 6, no. 1, pp755-756, 2006.

[6] N. Pochai, "A Numerical Treatment of Non-Dimensional Form of Water Quality Model in a Non-Uniform Flow Stream Using Saulyev Scheme," *Mathematical Problems in Engineering*, vol. 2011, pp491-317, 2011.

[7] N. Pochai, "A Computation of a Non-Linear Hydrodynamic Model in a Uniform Reservoir," *5th Asian Mathematical Conference Proceedings*, vol. 2, pp596-600, 2009.

[8] N. Pochai, "Numerical Treatment of a Modified MacCormack Scheme in a Non-Dimensional Form of the Water Quality Models in a Non-Uniform Flow Stream," *Journal of Applied Mathematics*, vol. 46, no. 2, pp274-263, 2014.

[9] P. Tabuenca and J. Cardona, "Numerical Model for the Study of Hydrodynamics on Bays and Estuaries," *Applied Mathematical Modeling*, vol. 16, no. 2, pp78–85, 1992.

[10] N. Pochai, S. Tangmanee, L.J. Crane and J.J.H. Miller, "A Finite Element Simulation of Water Quality Measurement in the Open Reservoir," *Thai Journal of Mathematics*, vol. 7, no. 2, pp77–93, 2009.

[11] P. Tabuenca, J. Vila, J. Cardona and A. Samartin, "Finite Element Simulation of Dispersion in the Bay of Santander," *Advanced in Engineering Software*, vol. 28, no. 5, pp313-332, 1997.

[12] C. Wang, Y. Wang and P. Wang, “Water Quality Modeling Pollution Control for the Eastern Route of South to North Water Transfer Project in China,” *Journal of Hydrodynamics*, vol. 18, no. 3, pp253-261, 2006.

[13] M. Galina, “A Study on the Mathematical Modeling of Water Quality in “River-Type” Aquatic Systems,” *WSEAS TRANSACTION on FLUID MECHANICS*, vol. 8, no. 2, pp80-89, 2013.

[14] W. Kraychang and N. Pochai, “A Numerical Treatment of a Non-Dimensional Form of a Water Quality Model in the Rama-nine Reservoir,” *Journal of Interdisplinary Mathematics*, vol. 18, no. 4, pp 375-394, 2015.

[15] W. Kraychang and N. Pochai, “Implicit Finite Difference Simulation of Water Pollution Control in a Connected Reservoir System,” *IAENG International Journal of Applied Mathematics*, vol. 46, no. 1, pp47-57, 2016.

[16] C. Moler, *Experiments with MATLAB.* MathWorks, 2015.

[17] C.R. Robinson, *Shallow Water Equations.* Syracuse University, 2011.

[18] W.Klaychang and N. Pochai, "Numerical Treatment to a Water-Quality Measurement Model in an Opened-Closed Reservoir," *Thai Journal of Mathematics*, vol. 13, no. 3, pp775-788, 2015.

[19] T.H. Tran, D.T. Pham, V.L. Hoang and H.P. Nguyen, “Water Pollution Estimation Based on the 2D Transport – Diffusion Model and the Singular Evaluative Interpolated Kalman Filter,” *Comptes rendus – Mecanique*, vol. 342, no. 2, pp106-124, 2014.

[20] L.J. Alvarez-Vázquez, A. Martínez, C. Rodríguez and M.E. Vázquez-Méndez, “Mathematical Model for Optimal Control in Wastewater Discharge: The Global Performance,” *CR Biol*, vol. 328, no. 4, pp327-336, 2005.

[21] K. Suebyat and N. Pochai, “A Numerical Simulation of a Three-Dimensional Air Quality Model in an Area Under a Bangkok Sky Train Platform Using an Explicit Finite Difference Scheme,” *IAENG International Journal of Applied Mathematics*, vol. 47, no. 4, pp471-476, 2017.

[22] S. A. Konglok and N. Pochai, “Numerical Computations of Three-Dimensional Air Quality Model with Variations on Atmospheric Stability Classes and Wind Velocities Using Fractional Step Method,” *IAENG International Journal of Applied Mathematics*, vol. 46, no. 1, pp112-120, 2016.

[23] G. Zhong, M. Yi and J. Huang, “Numerical Method for Solving Fractional Convection Diffusion Equations with Time-space Variable Coefficients,” *IAENG International Journal of Applied Mathematics*, vol. 48, no. 1, pp62-66, 2018.

[24] S. Zhou, F. Meng, Q. Feng and L. Dong, “A Spatial Sixth Order Finite Difference Scheme for Time Fractional Sub-diffusion Equation with Variable Coefficient,” *IAENG International Journal of Applied Mathematics*, vol. 47, no. 2, pp175-181, 2018.

[25] M. Zhao, H. Guan and P. Yin, “A Stable Mixed Finite Element Scheme for the Second Order Elliptic Problems,” *IAENG International Journal of Applied Mathematics*, vol. 46, no. 4, pp545-549, 2016.